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FUNDAMENTAL SYMMETRY BREAKING IN NUCLEAR REACTIONS

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A theoretical review is given of P- and (or) T-invariance violating effects in nuclear reactions. It is demonstrated that all of them are characterized by 2 major enhancement factors — dynamical and resonance ones. The net enhancement effect reaches 5—6 orders of magnitude. Both enhancements are caused by quantum chaoticity (complexity) of compound-nucleus resonances. This complexity, however, demands statistical methods of analysis of observed data in order to extract information on symmetry-breaking interaction strength constants. These methods are also presented and discussed in the review.

Дан теоретический обзор эффектов P- и (или) T-несохранения в ядерных реакциях. Показано, что все они характеризуются двумя основными механизмами усиления — динамическим и резонансным. Полное усиление при этом достигает 5—6 порядков. Оба механизма усиления являются следствием квантовой хаотичности (сложности) структуры резонансов компаунд-ядра. Эта хаотичность, однако, приводит к необходимости использования статистических методов анализа экспериментальных данных для извлечения из них информации о силовых константах, нарушающих симметрию взаимодействий. Эти статистические методы также обсуждаются в обзоре.

It is frequently pointed that discovery of America by Columbus nicely illustrated transformation of scientific hypothesis into discovery. Colubmbus cherished the idea that Earth is round and hoped to reach East India by sailing to the West. Notice that:

a. His idea was by no means original, but he received new information.

b. He faced enormous difficulties both in search for subsidies and in carrying on with his experiment.

c. He did not find a new way to India, but discovered a new continent instead.

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d. In spite of all the arguments to the contrary he continued believing that he discovered a new way to the East.

e. He got neither special respect nor substantial reward in his lifetime.

f. Since then it was proved without doubt that he was not the first European to reach America.

(«Physicists Joking». Mir, Moscow, 1966, p.66)

I. INTRODUCTION

The phase «fundamental symmetry» in the title is the frequently used shorthand for P- and T-invariance. It just reminds us of old times, when both those symmetries were considered to be most unbreakable. Since then P-invariance lost a good deal of its glamour. Its fundamentality was shaken by the discovery of P-violation first in β-decay and then in nucleon-nucleon interactions. For a certain time it seemed possible that weak interaction which caused P-violation in those two cases might be different for leptonic and baryonic processes. This could have added interest to nuclear P-violation studies. However the electroweak interacrtion theory of Weinberg and Salam closed this possibility. The T-invariance fortress remains much more invincible. The only experimental fact known by now is CP-violation in K-mesons decays discovered 30 years ago. On the basis of the CPT-theorem this implies T-violation, which can be explained theoretically in an infinite number of ways. All this makes further search of T-violation a much more exciting problem of general importance in physics. In other words, T-invariance is much more fundamental nowadays than the P-one.

The present wave of interest to P-violation in nuclear reactions was boosted by the theoretical predictions [1,2] of possible huge (6 orders of magnitude) enhancement of these effects in the vicinity of compound-nucleus resonances which were almost immediately confirmed experimentally [3]. A year later the same huge enhancements were predicted [4] for the effects violating both P- and T-invariance. Similar enhancements of P-conserving T-violating effects were predicted a few years later [5,6]. It seemed to us at that time (and still seems to me now), that those huge enhancements should be primarily used in experimental search of T-violating effects, since even establishing new upper bounds on CP-interaction constants might help a lot in selecting the most promising models of CP-violation.

However this natural way of reasoning does not seem to be well understood and shared. The major part of experimental and theoretical efforts up to now was concentrated on P-violation effects. Even there a good deal of energy was wasted on sensation-hunting and theoretical re-discovering of facts known for years.

One of the reasons for this is, according to my experience, a poor understanding of basic and quite general physics, which governs the above enhancements even by those, who work in the field. Lots of them naively believe that one can professionally discuss and analyze symmetry breaking in nuclear reactions without any knowledge of nuclear reaction theory or, at best, with rudimentary knowledge of Breit-Wigner formula. This strange belief is partially explained by bad traditions in nuclear physics, where nuclear structure studies were always considered to be of major importance in spite of the fact that most information for those studies was obtained from nuclear reactions. On deeper level it comes from the fact that quantum mechanics of bound states is in all respects much simpler than for continuum. Therefore it is quite tempting (and often quite misleading) to oversimplify the description of the process by using well-known bound-state analogies.

This unprofessional approach and naive clinging to the bound-state analogies create a fragmentary and wrong impression about the enhancement mechanisms for different observable quantities. For instance, very popular interpretation of neutron transmission enhancement in terms of «structural» enhancement factor $(kR)^{-1}$ does not allow one to understand the significance of P-odd correlation asymmetry measurements in inelastic channels, where the effects might be even larger [7,8] but could not be understood in terms of bound-state parallels. Exactly the same applies to T-violation tests of detailed balance in isolated resonance regime, where the net enhancements might be orders of magnitude larger [6,9] than in transmission experiments and are practically bound only by the experimental energy-resolution. All this leads to prejudiced distortion of the «priority scale» for different observables and often makes the experimentalists to choose rather difficult experiments, which in reality promise no enhancements whatsoever.

Only when the results start to deviate from naive expectations, people start reading papers on nuclear reaction theory. And the lack of professionalism again shows itself — people start mixing one mechanism with the other, reinvent models, which were longly discarded with by professionals or invent «home-brew» models whose validity was never checked in description of nonexotic reaction processes. This in turn often leads to aggressively incompetent statements of the type — «Live me alone with all your fancy reaction models, I had already wasted several days reading some of them and I'm sure, that you are overcomplicating quite simple things».

In view of all this, my present review is primarily addressed to those unprejudiced readers, who realize that nuclear reaction theory is a special branch of nuclear physics developed by generations of professionals. It has only few features in common with the bound-state spectroscopic theories, and I shall try to emphasize them.

Therefore I shall start with brief reminding of P-violation theory in cases of bound states (Sec.II) with special emphasis on its specific enhancements.

Then I shall switch over to my main topic of nuclear reactions for isolated resonances when the average resonance spacing d is much larger than average resonance width Γ (Sec.III).

I shall start this Section by short reminder of some results of nuclear reaction theory for isolated resonances (III, 1), which will be essential for all the further analysis, namely the structure of the wave functions for a system of incident (outgoing) particle and a target (residual) nucleus. In doing this I shall use the best and most physical version of nuclear reaction theory, namely the shell-model with continuum, developed by Mahaux and Weidenmüller [10] as the natural realization of Feshbach's unified theory of nuclear reactions. The advantage of this approach over the more popular R-matrix one lies in much more physical treatment on continuum wave-functions, which allows one to describe both the direct and compound processes in a unified way.

A fairly large Subsection III.2 discusses all the aspects of P-violation in nuclear reactions. Paragraph 2.1 contains the analysis of possible P-violating observables. In paragraph 2.2. a short historical background is given with special emphasis on how erroneous the bound-state parallels might be. The rather lengthy paragraph 2.3 contains the analysis of all the possible mechanisms of P-violation in nuclear reactions (all the 32 terms contributing to P-violating scattering amplitude). Mark that the absolute magnitudes of the corresponding effects are defined by nucleon-nucleon weak-interaction constants, on which up to now we have only educated guesses and whose extraction from experimental observables should be the ultimate aim of our P-violating investigations. Therefore only estimates of relative contributions coming from different mechanisms and of their energy behaviour are meaningful in the analysis of different competing mechanisms. For this reason I expand in paragraph 2.3 on this kind of analysis and emphasize the generality of various enhancement effects specific for each mechanism. It turns out that only 2 major enhancement factors govern the P-violating amplitudes — dynamical enhancement $\tilde{v}/d \sim \sqrt{N}$ ($\tilde{v}$ is the variance of strong interaction matrix element between compound states and $N \sim 10^6$ is the number of basic components, which define the complexity of the compound state wave function) and resonance enhancement d/Γ. While the former enhancement is well-known in the bound-state P-violating theory, the resonance enhancement is a specific feature of continuum spectra, which has no bound-state analogous. After analyzing the energy behaviour of P-violating amplitudes we are forced to come back to observables (paragraph 2.4) in order to investigate their rather complicated energy dependence, caused by the combined influence of P-violating amplitudes in their numerators and P-invariant ones in their denominators. This

allows us to compare the specific enhancements of all the *P*-violating observables in different energy regions and to understand in quite general terms the «priority hierarchy» of observables, which is confirmed by experiments. We also show that the «structural» enhancement factor $(kR)^{-1}$ is an artifact of presenting the auxiliary quantities instead of the really observed ones.

Subsection III.3 deals with *T*-violation. In paragraph 3.1 we discuss specific hidden dangers, which make true *T*-invariance investigations much more subtle than the *P*-invariance ones, and present the list of «true» *T*-violating observables. The theory of *T*-violation in nuclear reactions dates back to late 50-ies and is rather dramatic. However only few people know about it. Therefore I present a short survey of its development in 3.2. Paragraph 3.3 deals with the most important theoretically *P*-odd *T*-odd «triple correlation» (TC). The analysis of possible TC enhancements is given in it together with analysis of specific difficulties in its experimental observation. In 3.4 we analyze the *P*-even *T*-odd correlation (FC) in neutron transmission, demonstrating both its advantages and drawbacks. In 3.5 we analyze the possibilities of *T*-violation detailed balance tests (TVDB) for 2 close-lying resonances, when average spacing d in much larger than average Γ. In paragraph 3.6, we briefly summarize our results on *T*-violation effects showing that all of them are governed by the same dynamical and resonance enhancement effects as the *P*-violating ones. In complete analogy to *P*-violation we present the «priority hierarchy» of *T*-violating observables and conclude that most promising results in the near future might be expected from TVDB tests of paragraph 3.5.

In Section IV the statistical approach to compound-resonance measurements is discussed. In Subsection IV.1 it is demonstrated that both dynamical and resonance enhancements are quite general consequences of quantum chaos characteristic of compound nucleus, which was faced and physically understood in «strong» symmetry-breaking from the dawn of nuclear physics. Therefore the meaningful analysis of weak symmetry-breaking (WSB) matrix elements extracted from experiments should be done with exactly the same mathematical methods which were successfully applied in studies of «strong» symmetry-breaking, namely with the use of randon-matrix theory and Gaussian ensembles of Wigner and Dyson. Since historically such methods were first applied to calculation of energy-averaged WSB quantities, I discuss in Subsection IV.2 the practical disadvantages of «unbiased» energy-averaging and come to the idea of «biased» on-resonance ensemble averaging, which is fully described in Subsection IV.3. In IV.4, I discuss how should one apply the on-resonance theory of IV.3 to the realistic case of necessarily imperfect experimental measurements (small number of independent on-resonance observations with poor experimental accuracy), concluding that the only appropriate way in this case is given by Bayesian statistics (BS) based on the use of standard conditional prob-

ability theory. I also discuss the parallels and differences between BS results and empirical maximum-likelihood method (MLM), which is applied in experimental analysis of *P*-violating effects during recent 5 years. Most obvious disadvantages and nuisances of MLM are shown, especially in the typical case when the spins of observed resonances are unknown. In view of this, I only briefly analyze in Subsection IV.5 the sensational «sign-correlation» effect observed in *P*-violating experiments on ^{232}Th target, and conclude that its statistical significance was greatly exaggerated and is highly questionable.

In Section V, I present a short summary of the most important and general conclusions and recommendations for future.

II. *P*-VIOLATION IN THE CASE OF NUCLEAR BOUND STATES

We shall start with brief reminding of the «classical» *P*-violation experiments in low-energy physics when only the mixing of bound states was considered. Pretty early (see, e.g., [11]) it was realized that the experimental observation of the interference-type phenomena (i.e., observation of *P*-odd correlations in the amplitudes of different processes) has an advantage over the «brute force» violation of probabilities (e.g., *P*-forbidden α-decay) because the latter are quadratic in weak interaction strength constant *F*. There are several possible *P*-odd correlations (see the list in [11]), among them the correlation $(\boldsymbol{\sigma}_\gamma \cdot \mathbf{p}_\gamma) = h_\gamma$ between the spin and momentum of γ-quanta emitted by the excited unpolarized nuclei. The value h is called helicity and leads to circular polarization of the emitted γ-quanta which can be observed experimentally (see, e.g., [12]). Let us consider this experiment in more detail in order to demonstrate various enhancement mechanisms, which might manifest themselves in it. The wave function Ψ_i of the decaying excited state might be presented as a sum

$$\Psi_i = \psi_1 + \alpha \psi_2. \tag{1}$$

Here ψ_1 and ψ_2 are the states of opposite parity, while the coefficient α describes the admixture of the state ψ_2 caused by *P*-violating weak interaction V_W. Standard first-order perturbation theory gives

$$\alpha = \frac{\langle \psi_1 | V_W | \psi_2 \rangle}{E_1 - E_2}. \tag{2}$$

Circular polarization appears as an interference of the electric E_λ and magnetic M_λ transitions of the same multipolarity λ. Therefore

$$h = 2\alpha \langle \psi_f | \hat{O}_\lambda | \psi_1 \rangle \langle \psi_f | \hat{O}'_\lambda | \psi_2 \rangle, \tag{3}$$

where $\hat{O}_\lambda$ is the «regular» γ-transition operator, which connects the main component ψ_1 of Ψ_i with the final state Ψ_f; $\hat{O}'$ is the «irregular» transition operator which, due to P-selection rules, connects only ψ_2 component with Ψ_f. Then the degree of circular pluralization observed will be defined by

$$\delta = \frac{h}{|\langle \psi_f|\hat{O}_\lambda|\psi_1\rangle|^2 + \alpha^2|\langle \psi_f|\hat{O}'_\lambda|\psi_2\rangle|^2} \simeq \alpha \frac{\langle \psi_f|\hat{O}'_\lambda|\psi_2\rangle}{\langle \psi_f|\hat{O}_\lambda|\psi_1\rangle}. \tag{4}$$

It is important to note that δ like *any other observable*, which measures the degree of any symmetry breaking, is always normalized by the *total transition probability*. Therefore, in general, the denominator of (4) contains a sum over all the allowed transitions $\sum_\lambda |\langle \psi_f|\hat{O}_\lambda|\psi_1\rangle|^2$:

$$\delta = \frac{h}{\sum_\lambda |\langle \psi_f|\hat{O}_\lambda|\psi_1\rangle|^2 + \alpha^2|\langle \psi_f|\hat{O}'_\lambda|\psi_2\rangle|^2}. \tag{4a}$$

Only this kind of normalization for observables has real physical meaning — the maximal value of δ is unity, meaning 100% parity violation. This almost trivial rule is taken for granted in all fields of physics, from optics to elementary particles, and all the meaningful enhancements appear only within this normalization. Of course some odd personalities might introduce the normalization of their own by, say, retaining only the weakest term in the normalization sum. If this term is really small, this would immediately enhance the newly introduced quantity. But such a fictitious «enhancement» would have nothing to do with the physics of the process. I have to mention this triviality only because, as we shall see below, even this standard rule is unprofessionally violated all the time in the majority of experimental (and, alas, even theoretical) publications on P-noninvariance in nuclear reactions.

Coming back to ex.(4), we can analyze its structure in order to see the role of different enhancement mechanisms. First of all, we observe that α increases with decreasing level spacing $D = |E_1 - E_2|$ — a natural result of perturbation theory. Therefore naively one should expect the average effect to increase linearly with increasing state density $\rho = 1/d$ of the system (d here is the average level spacing). However the increase of ρ is closely connected with increasing complexity of the mixing states ψ_1 and ψ_2. In terms of basic (so-called «simple configurations») components, which build up the compound state wave function ψ, this means increasing number N of this components and simultaneously the random signs for their admixture coefficients in ψ. Therefore the average value

of $\langle \psi_2 | V_W | \psi_1 \rangle \equiv v_p$ matrix element will be zero in proper statistical treatment (see Sec.IV below), and we can speak only in terms of its variance

$$\tilde{v}_p = \sqrt{(\overline{v}_p^2)}. \tag{5}$$

Using the simple scaling procedure we can express v_p in terms of the strong interaction matrix element v:

$$v_p = F \cdot v, \tag{6}$$

where F is the characteristic ratio of the strengths for weak and strong interaction, which is given by the phenomenological models as $F \approx 10^{-7} \div 10^{-8}$.

The average value of $\tilde{v}$ can be estimated from the usual expression for the spreading width Γ_{spr} of the single-particle resonance (which roughly equals the imaginary part W of optical model potential and in the limit of black nucleus approaches the single-particle level spacing d_0):

$$\Gamma_{spr} = \frac{2\pi \tilde{v}^2}{d} \approx W \sim d_0. \tag{7}$$

This gives us

$$\tilde{v}_p = F_p = \sqrt{\frac{\Gamma_{spr} d}{2\pi}} \approx F_p \sqrt{d_0 d}. \tag{8}$$

Therefore the variance of α in (2) can be estimated as:

$$\tilde{\alpha} = \frac{\tilde{v}_p}{D} = F_p \frac{\tilde{v}}{D} = F_p \sqrt{\frac{\Gamma_{spr}}{2\pi D}} \approx F_p \sqrt{\frac{d_0}{d}} = F_p \sqrt{N}. \tag{9}$$

Thus we see, that the many-body aspect of the compound nuclear system manifests itself in the systematic enhancement of P-violating effect by roughly a factor of $\sqrt{N}$, where N is the number of basic components forming the compound states. The namber N increases with increasing excitation energy E^* and nuclear mass number and reaches $\sim 10^6$ for $E^* \approx B_n$ (B_n is the neutron binding energy) in medium and heavy nuclei. This sort of enhancement was considered several times [13,14,15,16] and received the name [15] of *«dynamical enhancement»*.

One can also see from (4) that increasing the value of «irregular» amplitude and decreasing the value of «regular» one leads to the additional enhancement of the effect. This usually happens, when for purely structural reasons the «regular» component is strongly forbidden, while the «irregular» one is favoured. Therefore this kind of enhancement was called (see [15]) *«structural enhancement»*.

These enhancements made possible one of the earliest P-violating observation in γ-channel [17] on the level of $6 \cdot 10^{-6}$. However even earlier [18], similar experiments were done in (n,γ) reaction with thermal neutrons showing the effects up to $\sim 10^{-4}$. As we shall see below, these more impressive results can be easily understood in the framework of nuclear reaction theory.

As we see, the general trick of any enhancement mechanism is to make the numerator of the observable (4) as large as possible and the denominator small. The same trick will be used in case of nuclear reactions. Since however both the numerators and denominators of observables in that case are rapidly varying functions of energy, this allows a much larger variety of situations and leads to quite specific enhancements, with we consider in the next sections.

III. NUCLEAR REACTIONS (ISOLATED RESONANCES)

1. Elements of Nuclear Reaction Theory for Isolated Resonances. We shall introduce the basic results of nuclear reaction theory which will be extensively used in all our further applications. In doing this, we shall follow the approach of Mahaux and Weidenmüller [10], which is a projection of Feshbach's unified theory of nuclear reactions on the realistic shell-model basis.

The most essential for our purposes result of this approach is that in the region of isolated resonances, where $\Gamma << d$, the wave function of the system of incident (outgoing) particle plus target (residual) nucleus is given by:

$$\Psi^{(\pm)}_{i,f}(E) = \sum_k a^{(\pm)}_{k(i,f)}(E)\varphi_k + \sum_c \int b^{(\pm)}_{c(i,f)}(E,E')\chi^{(\pm)}_c(E')dE'. \qquad (10)$$

Here φ_k is the wave function of the so-called «bound state embedded in continuum» (BSEC) or, roughly speaking, the wave function of the k-th compound state, where all the nucleons of the system occupy only the bound states in average nuclear potential, but are not allowed to collect all the excitation energy via pair-wise collisions on a single particle. The $\chi_c(E)$ is the continuum c-th channel wave function which, describes the (infinite) motion of a particle in the average field of the target (residual) nucleus. In the particular case of a neutron incident on the ground state target (elastic channel $c = i$), χ_i is the antisymmetrized product of the target nucleus wave function in its ground state times the wave function of a neutron moving in the average field of the target. The φ_k and χ_c correspond to Feshbach's projections on closed (Q) and open (P) channels, respectively. Mind that unless you switch on the pair-wise residual interactions V between the channels (PHQ = QHP in Feshbach's notations), you do not allow the incident neutron to share its energy with the

target nucleons. This does not allow this neutron to form a compound resonance. Therefore in the absence of pair-wise interaction V the BSEC's would never decay, while χ's would describe only potential scattering in the mean field. This unphysical situation changes as soon as we switch on the residual interaction V. Then each resonance k receives partial decay widths $\Gamma_k^{i,f}$ with amplitudes:

$$(\Gamma_k^{i,f})^{1/2} = \gamma_k^{i,f} = (2\pi)^{1/2} \langle \chi_{i,f}(E)|V|\varphi_k \rangle. \tag{11}$$

In other words, the pair-wise residual interaction allows the nucleons of BSEC to collect their total excitation energy on one of the particles and emit it into the open channel i or f.

For the expansion coefficients a and b in this case the theory gives rather transparent expressions:

$$a_{k(i,f)}^{(\pm)}(E) = \frac{\exp(\pm i\delta_{i,f})}{\sqrt{2\pi}} \frac{\gamma_k^{i,f}}{E - E_k + i\,\Gamma_k/2}. \tag{12}$$

Here E_k is the energy of a given compound resonance, $\Gamma_k = \sum \Gamma_k^c$ is its total width.

The open channel wave functions (second part of (10)) are governed by the coefficients:

$$b_c^{(\pm)}(E,E') = \delta_{c(i,f)}\delta(E - E')\exp(\pm i\delta_c) + \\ + \frac{1}{E^{\pm} - E'} \sum_k a_k^{\pm}(E) \langle \chi_c(E')|V|\varphi_k \rangle. \tag{13}$$

Here $E^{\pm} = E \pm i\varepsilon$ is the usual notation for pole shifts in the complex energy plane.

Let us now simplify the picture, neglecting all the BSEC's in the sums of (10) and (13) besides the one, whose energy E_k is the closet to the energy E of the incident neutron (this might always be done when Γ_k is much smaller than the distance between resonances of the same spin and parity). Let us also consider the case when only the neutron elastic scattering channel is open ($c = i = f$). In this case the continuum term in (10) can be written as:

$$e^{i\delta_i}\chi_i(E) + a_{k(i)}^{+}(E) \int \frac{dE'}{E + i\varepsilon - E'} \langle \chi_i(E')|V|\varphi_k \rangle = e^{i\delta_i}\chi_i(E) + \\ + a_{k(i)}^{+}(E)\left[i\pi \langle \chi_i(E)|V|\varphi_k \rangle \chi_i(E) + \mathcal{P}\int \frac{dE'\chi_i(E)}{E - E'} \langle \chi_i(E')|V|\varphi_k \rangle \right], \tag{14}$$

where $\mathcal{P}$ stands for the integral principal value.

The first term in (14) describes the potential elastic scattering of a neutron. The resonance behaviour of $a_k(E)$ (see (12)) shows that the two terms in square brackets of (14) are the «imprints» of compound resonance at $E = E_k$ on the elastic continuun. Using eq.(11) one can express the first of these terms as

$$i\sqrt{\frac{\pi}{2}}\,\gamma_k^i\,\chi_i(E). \tag{15}$$

Recollecting now that $\chi_i(E)$ describes the single neutron (*valence* particle) in the mean field of a ground-state target, we see that ex.(15) is exactly what was called the single-particle (or *valence*) component u of the compound resonance wave function in the simplified R-matrix theory. The wave function $\chi_i(E)$ belongs to the continuum, as it should. However, if it has a potential resonance at $E = E_0$ with (single-particle) width Γ_0, one can use the approximate expression (see [19]) valid inside the nuclear potential radius R (for simplicity we suppress the coordinates of the target nucleons):

$$\chi(E) \simeq \sqrt{\frac{\Gamma_0}{2\pi}}\,\frac{u_0(r)}{E - E_0 + i\,\Gamma_0/2}.$$

Here $u_0(r)$ is the solution of the Schrödinger equation describing particle motion in average field, which is normalized to unity inside the nuclear volume $r \le R$. Substituting this expression into (15), we get in the Γ_0 vicinity of $E \approx E_0$

$$i\sqrt{\frac{\pi}{2}}\,\gamma_k^i\chi_i(E) \simeq \left(\frac{\Gamma_k^i}{\Gamma_0}\right)^{1/2} u_0(r), \tag{16}$$

restoring thus the approximate result of R-matrix approach. The quantity

$$S_k^n = \frac{\Gamma_k^n}{\Gamma_0} \approx \frac{1}{N}$$

is usually called the spectroscopic factor. It defines the probability of finding a single-particle (valence) component in the compound-resonance state, and is equal in the black-nucleus approximation to $1/N \sim (d/\bar{v})^2$, that is, to the inverse square of the dynamical enhancement factor of (9).

Adding up this term (16) to the BSEC's wave function φ just gives us the R-matrix compound resonance wave function:

$$\Phi_k = \varphi_k + \sqrt{S_k^i}\,u_0. \tag{17}$$

Therefore the wave function (10) of the system in the vicinity of isolated compound resonance E_k can be expressed as:

$$\Psi_i^{(\pm)}(E) = a_{k,i}^{\pm}(E)\left[\varphi_k + \sqrt{S_k^i}\, u_0\right] + e^{i\delta_i}\chi_i(E) +$$

$$+ a_k^{\pm}(E)\, \mathcal{P}\int \frac{dE'\chi_i(E')}{E-E'} \langle \chi_i(E')|V|\varphi_k\rangle = a_k^{\pm}(E)\Phi_k + e^{i\delta_i}\chi_i(E) +$$

$$+ a_{k,i}^{\pm}(E)\, \mathcal{P}\int \frac{dE'\chi_i(E')}{E-E'} \langle \chi_i(E')|V|\varphi_k\rangle. \qquad (18)$$

2. *P*-Violation Let us consider now the case of *P*-violation in neutron-induced reactions, which would demonstrate all the specific features of any symmetry-breaking in them.

We shall first discuss the *P*-violating quantities, which might be observed experimentally in these reactions.

2.1. P-Violating Observables in Neutron-Induced Reactions. Using the polarized neutron beam, one can observe the P-violating correlation $(\boldsymbol{\sigma}_n \cdot \mathbf{k}_n)$ between the spin $\boldsymbol{\sigma}_n$ and momentum $\mathbf{k}_n$ of a neutron. There are several possibilities of doing it. One can consider the transmission of neutrons with opposite helicities through a target sample and measure the difference of the corresponding total cross-sections:

$$\Delta_{\text{tot}}^{P} = \sigma_{\text{tot}}^{+} - \sigma_{\text{tot}}^{-} = \frac{4\pi}{k}\,\text{Im}(f_+ - f_-). \qquad (19)$$

Here $f_{\pm}$ defines the forward scattering amplitudes for neutrons with opposite helicities. To obtain the second equality we used the optical theorem.

The corresponding dimensionless measure of this transmission asymmetry effect is

$$P = \frac{\Delta_{\text{tot}}^{P}}{\sigma_{\text{tot}}^{+} + \sigma_{\text{tot}}^{-}} \simeq \frac{\Delta_{\text{tot}}^{P}}{2\sigma_{\text{tot}}}. \qquad (20)$$

One should point (see, e.g., [20]) that in reality the experimentalists do measure the numbers $N_{\pm}$ of neutrons with opposite helicities transmitted through the target sample with thickness x and calculate the ratio:

$$P_{\text{exp}} = \frac{N_+ - N_-}{N_+ - N_-}. \qquad (21)$$

Now for counter efficiency $\varepsilon = 1$

$$N(x) = N_0\, e^{-x\sigma_{\text{tot}}\rho}, \qquad (22)$$

where N_0 is the intensity of the incident beam and ρ is the density of nuclei in a target sample. Expressing $\sigma^{\pm}_{tot}$ as

$$\sigma^{\pm}_{tot} = \sigma^{0}_{tot} \pm \frac{\Delta^{P}_{tot}}{2} \tag{23}$$

one can write

$$N_{+} - N_{-} = N_0 \, e^{-x\sigma^{0}_{tot}\rho}(e^{+x\Delta^{P}_{tot}\rho/2} - e^{-x\sigma^{P}_{tot}\rho/2}) \simeq N_0 \, e^{-x\sigma^{0}_{tot}\rho} \cdot \Delta^{P}_{tot} \cdot x \cdot \rho. \tag{24}$$

Therefore

$$P_{exp} \simeq \frac{\Delta^{P}_{tot}}{2} x\rho. \tag{25}$$

It seems from (25) that since the experimentally observed effect increases with x, one should use very thick targets. However (see (22)) the counting rates $N(x)$ go down exponentially with increasing x. The relative counting statistical error equals $1/\sqrt{N}$ and increases exponentially with x:

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{N_0}} e^{\sigma_{tot}\rho x/2}. \tag{26}$$

In order to maximize (25) retaining the minimal possible error (26) one has to choose $x\rho \approx (1/\sigma_{tot})$. Thus *really measured* quantity (25) coincides with the expression (25):

$$P_{exp} = \frac{\Delta^{P}_{tot}}{2\sigma_{tot}} \equiv P. \tag{27}$$

One can easily see [21] that the same quantity could be obtained with unpolarized neutron beam. Then P is just a measure of the longitudinal polarization of the initially unpolarized beam arising after passing a distance in the sample equal to mean free path (hence the symbol P, denoting this quantity). Sometimes one measures the difference in radiative capture cross sections $\sigma^{(\pm)}_{n\gamma}$ and gives the quantity:

$$A = \frac{\sigma^{+}_{n\gamma} - \sigma^{-}_{n\gamma}}{\sigma^{+}_{n\gamma} + \sigma^{-}_{n\gamma}}. \tag{28}$$

The same $(\boldsymbol{\sigma}_n \cdot \mathbf{k}_n)$ correlation in the elastic scattering amplitude also causes the rotation of the neutron polarization around $\mathbf{k}_n$. The angle of this rotation per unit length of the target sample is defined [22] as follows:

$$\frac{d\Phi}{dz} = \frac{2\pi\rho}{k} \mathrm{Re}\,(f_+ - f_-), \tag{29}$$

where ρ is the density of nuclei in the sample. For the same reasons of better statistics the experimentally defined angle Φ is measured for neutrons, which travelled the distance z equal to mean free path $1/\rho\sigma_{tot}$ in the target:

$$\Phi = \frac{1}{\rho\sigma_{tot}} \frac{d\Phi}{dz} = \frac{\mathrm{Re}\,(f_+ - f_-)}{\mathrm{Im}\,(f_+ + f_-)}. \tag{30}$$

One can also look for inelastic reaction (n,f) and measure the P-odd correlation $(\boldsymbol{\sigma}_n \cdot \mathbf{k}_f)$ between the initial neutron polarization and the momentum $\mathbf{k}_f$ of the outgoing particle in channel f. This is done by measuring the asymmetry of the final products with respect to $\boldsymbol{\sigma}_n$:

$$\Delta_{nf} = \frac{d\sigma_{nf}}{d\Omega}\uparrow\uparrow - \frac{d\sigma_{nf}}{d\Omega}\uparrow\downarrow. \tag{31}$$

The corresponding dimensionless degree of asymmetry is:

$$\alpha_{nf} = \frac{\Delta_{nf}}{\frac{d\sigma_{nf}}{d\Omega}\uparrow\uparrow + \frac{d\sigma_{nf}}{d\Omega}\uparrow\downarrow}. \tag{32}$$

2.2. Historical Background. The possibility of using low energy neutron-nucleus interactions and all sorts of neutron coherent scattering processes (neutron optics) in studies of P-violation was considered long ago (see [21,22,23,24]). But these theoretical investigations were concerned only with potential scattering models completely disregarding the presence of compound-resonances. Some of these approaches ([23,24]) made a point of possible enhancement of the effects in the vicinity of potential (single-particle) p-wave resonance. The first theoretical paper [25] mentioning the possible enhancement of γ-ray circular polarization in the vicinity of compound resonance remained unnoticed. The first simplified approach to compound resonance analysis which really encouraged the experimental investigations was done only in 1980 ([1], see also [26]). In this approach the p-wave compound resonance was treated in complete analogy with the bound-state case above (see Sec.II). Indeed, Sushkov and Flambaum took the case of two closely-lying bound states (imitating p- and s-resonances) with corresponding wave functions ψ_1 and ψ_2. Then in complete analogy to (1), the p-resonance wave function, which takes into account the possible parity admixture, looks like:

$$\Psi(E_p) = \Psi_p + \alpha\,\Psi_s. \tag{33}$$

Now one might just say that in case of elastic scattering both states decay by neutron emission and substitute the γ-ray transition probabilities in (3) and (4) by the corresponding partial neutron widths Γ_s^n and Γ_p^n. Then one immediately obtains for, say, *P*-value in analogy to (4):

$$P \approx \alpha \frac{\sqrt{\Gamma_s^n}}{\sqrt{\Gamma_p^n}}. \tag{34}$$

In slow neutron case $\Gamma_p^n \approx (kR)^2 \Gamma_s^n$, where $(kR)^2$ comes from the centrifugal barrier penetration factor. Thus

$$P \approx \frac{\alpha}{kR}. \tag{35}$$

The typical value of (kR) for eV energy region in medium and heavy nuclei is ~ 10^{-3}. Thus in addition to the dynamical enhancement contained in α, they got a particular case of «structural enhancement» factor ~ 10^3.

This way of arguing *sometimes* gives, as we shall see later, the correct order of magnitude estimate of the effect, but is quite misleading. To start with, the initial equation (33) for the continuum wave function is meaningless, since each continuum wave function with fixed momentum $\mathbf{k}_n$ *is always* a linear superposition of states with opposite parities (i.e., superposition of partial waves). Therefore the compound nucleus wave function $\Psi(E)$ even in the simplest case of slow neutron elastic scattering without any *P*-violating forces is a sum of *p*- and *s*-compound resonance wave functions Ψ_p and Ψ_s with corresponding «mixture» coefficients (see eq.(10) or any sound reaction theory):

$$\frac{e^{i\delta_p}(\Gamma_p^n)^{1/2}}{E - E_p + i\Gamma_p/2} \quad \text{and} \quad \frac{e^{i\delta_s}(\Gamma_s^n)^{1/2}}{E - E_s + i\Gamma_s/2}.$$

Thus even in the maximum of *p*-resonance ($E = E_p$) we have:

$$\Psi(E_p) \sim \Psi_p + \alpha' \Psi_s,$$

where

$$\alpha' = i \frac{\Gamma_p}{2D} \left(\frac{\Gamma_s^n}{\Gamma_p^n} \right)^{1/2}.$$

Proceeding now with the bound-state arguments which lead us from (33) to (34), we obtain

$$|P| = |\alpha'| \left(\frac{\Gamma_s^n}{\Gamma_p^n} \right)^{1/2} = \left(\frac{\Gamma_s^n}{\Gamma_p^n} \right) \frac{\Gamma_p}{2D} \approx \frac{1}{(kR)^2} \frac{\Gamma_p}{2D} \sim 10^6 \frac{\Gamma}{D}.$$

For the famous *La* case this would give us $P \approx 10^3$ *without any weak interaction!*

There are also other striking absurdities in (34), (35): a) Consider its energy behaviour. Since all the E dependence enters (34), (35) essentially through the energy dependence of partial widths, we see that the effect (35) blasts to infinity for very small E (small k). b) We know that neutron partial widths vary in a rather wide range obeying Porter-Thomas law. Eq.(34) clearly indicates that the largest P effects would be observed for the smallest Γ_p^n possible — the less observable p-resonance is in total cross-section the more it would stick out in P-violation. Even more tempting is to repeat the whole above reasoning for mixing of resonances in higher partial waves (say, $l = 3$ ones) with s-wave resonance. (This is perfectly legitimate if one considers the target with spin $I \geq 2$). Then the «regular» neutron partial widths would be even smaller ($\Gamma_l^n \approx (kR)^{2l} \Gamma_s^n$) and for $l = 3$ we obtain «structural enhancement» factor $1/(kR)^3 \sim 10^9$ in eq. (39). This allows the quantity P, which by definition (20) cannot exceed unity, to reach the value 10^5. Obviously Sushkov and Flambaum were too good theorists to be caught into such traps, but I have seen an experimental proposal with clearly stated intentions to hunt for the weakest p-resonances in order to obtain maximal P effects. I also know experimental group, which made special efforts to perform transmission experiments with thermal neutrons and was quite disappointed when the effect turned out to be about 10^{-6} instead of huge increase predicted by eq. (35). To mix up things even more, the above «structural enhancement» of Sushkov — Flambaum nowadays is called in a lot of experimental papers «the kinematic enhancement» (originally this name was given by Shapiro [15] to the typical ratio of electric to magnetic transition amplitudes, which might really cause additional enhancement of P-violating observables in γ-transitions).

In spite of all these inconsistencies, these theoretical results, as I had already pointed out, greatly encouraged the preparation of on-resonance experimental measurements of the Dubna group [3].

The first proper theoretical treatment of the problem in the framework of nuclear reaction theory was given by us in the beginning of 1981 [2]. We had derived the expressions for Δ_{tot}^P and $\text{Re}(f_- - f_+)$ essential for the description of P-violation in neutron transmission. Since the Dubna on-resonance measurements were still in preparation at that time, we had to check our theory [2] by comparing the theoretical relations between P and Φ values at thermal energies

with the existing experimental measurements in *Sn* performed by the Grenoble group [27]. A few months later Dubna group has performed the first on-resonance observations in *Sn* [3] and checked our expression for $P(E)$ by comparing their on-resonance results with thermal-energy ones, obtained by Lobashov's group in Gatchina [28]. This was the first experimental confirmation of the resonance enhancement mechanism. Ironically enough that, although our expressions for $\Delta^{P}_{\text{tot}}(E)$ derived in [2] clearly manifested the resonance enhancement parameter D/Γ, we fully understood its physical meaning and generality only a year later, while finishing a big paper on general theory of *P*- and *T*-violating effects [7]. Some of our expressions obtained in that paper were re-derived later on in the framework of *R*-matrix theory [29,30]. Quite apart stands the theoretical investigation [31] of the $\alpha_{n,f}$-type correlation in the particular case of $(\mathbf{p}, \alpha)$ reactions. The authors obtained fairly large estimates, but did not realize that they hitted the new far-leading enhancement mechanism for $\alpha_{n,f}$ correlation. This fact, together with principal possibility of observing *P*-violating effects of the order of unity, was pointed out in [8].

2.3. P-Mixing Mechanisms and Specific Enhancements in Neutron-Induced Reactions. Any proper treatment of *P*-violation in nuclear reactions consists of two steps: 1. Expressing the observed quantity in terms of the *P*-violating part of *T*- (or *S*) matrix T_W. 2. Calculating T_W in the first-order Born approximation with respect to weak interaction V_W.

The first part of the task involves the standard theory of reaction kinematics with polarized beams (see, e.g., [32,21,22]). As it is usual in any of angular correlations, this gives rather awkward combinations of vector-coupling coefficients, which are all of the order of unity and which do not contribute to the understanding of essential physics. The general expressions for them can be found in [7,29,30] (two latter references even give numerical values for some target spins). We shall further on omit them in the majority of expressions. The effects also depend linearly on the incoming beam polarization p, which will be set to unity in all the further expressions. In case of slow neutrons one can also restrict the neutron angular momenta by $l_n = 0$ and $l_n = 1$.

With these remarks one obtains the following expressions:

$$\Delta^{P}_{\text{tot}} = \frac{\pi}{k^2} \operatorname{Im} [\langle p, j = 1/2 | T_W | s \rangle + \langle s | T_W | p, j = 1/2 \rangle], \tag{36}$$

$$\frac{d\Phi}{dz} = \frac{2\pi\rho}{k^2} \operatorname{Re} [\langle p, j = 1/2 | T_W | s \rangle + \langle s | T_W | p, j = 1/2 \rangle], \tag{37}$$

$$\Delta_{nf} = \frac{2\pi}{k^2} \sum_{l_f, l_n} \operatorname{Im} \{\langle l_f, f | T | l_n \rangle \langle (l_f + 1), f | T_W | l_n \rangle\}. \tag{38}$$

Here $\langle l_f+1, f|T_W|l_n\rangle$ means the parity violating element of T-matrix describing the transition from initial state with $l_n = 0,1$ (s-, p-correspondingly) to the final state with angular momentum (l_f+1); f means all the additional quantum numbers defining the channel f (j in case of elastic p-wave); $\langle l_f, f|T|l_n\rangle$ defines the corresponding P-allowed transition.

Now one can use the Born approximation to calculate the P-forbidden transition

$$T_W = \langle \Psi_f^{(-)}|V_W|\Psi_i^{(+)}\rangle. \tag{39}$$

To simplify the problem even more, we shall retain only one s- and p-wave resonance in expressions (18) for the initial and final states. In this case the first-order Born amplitude (39) contains 9 terms:

$$\begin{aligned}
T_W = \langle \Psi_s^{(-)}|V_W|\Psi_p^{(+)}\rangle &= a_p^+(E)\langle \Psi_s|V_W|\Psi_p\rangle a_s^+(E) + \\
&+ e^{i\delta_s} a_p^+(E)\langle \Psi_p|V_W|\chi_s(E)\rangle + \\
&+ a_p^+(E)a_s^+(E)\,\mathcal{P}\int \frac{dE'}{E-E'}\langle \Phi_p|V_W|\chi_s(E')\rangle\langle \chi_s(E')|V|\varphi_s\rangle + \\
&+ e^{i\delta_p}\langle \chi_p(E)|V_W|\Phi_s\rangle a_s^+(E) + e^{i(\delta_s+\delta_p)}\langle \chi_p(E)|V_W|\chi_s(E)\rangle + \\
&+ e^{i\delta_p} a_s^+(E)\,\mathcal{P}\int \frac{dE'}{E-E'}\langle \chi_p|V_W|\chi_s(E')\rangle\langle \chi_s(E')|V|\varphi_s\rangle + \\
&+ a_p^+(E)a_s^+(E)\,\mathcal{P}\int \frac{dE'}{E-E'}\langle \varphi_p|V|\chi_p(E')\rangle\langle \chi_p(E')|V_W|\varphi_s\rangle + \\
&+ a_p^+(E)\,e^{i\delta_s}\,\mathcal{P}\int \frac{dE'}{E-E'}\langle \varphi_p|V|\chi_p(E')\rangle\langle \chi_p(E')|V_W|\chi_s\rangle + \\
&+ a_p^+(E)a_s^+(E)\,\mathcal{P}\int \frac{dE'dE''}{(E-E')(E-E'')}\langle \varphi_p|V|\chi_p(E')\rangle \times \\
&\times \langle \chi_p(E')|V_W|\chi_s(E'')|V|\varphi_s\rangle.
\end{aligned} \tag{40}$$

In order to understand the physical meaning of each term it is useful to introduce the graphical technique with the following correspondence rules: the wavy line with indices (p- or s-) means an $l=1$ or $l=0$ neutron in the mean field of the target; thin solid line means the ground state target; empty circles correspond to strong interaction amplitudes $\gamma_s^n \exp i\delta_s/\sqrt{2\pi}$ and $\gamma_p^n \exp i\delta_p/\sqrt{2\pi}$; crossed circles correspond to weak interaction matrix

elements; double solid lines with indices correspond to resonance propagators $1/[E-E_s)+i\,\Gamma_s/2]$ or $1/[E-E_p)+i\,\Gamma_p/2]$ for s- or p- BSEC's φ_s or φ_p; the boldface solid lines mean the same propagators, but for the «full» compound-resonance wave functions Φ_s or Φ_p (see eq.(17)); the closed loop of neutron and target lines implies the principal value integration over neutron energy.

The first term in (40)

$$T_1=\frac{e^{i\delta_p}\gamma_p^n}{\sqrt{2\pi}}\,\frac{1}{(E-E_p)+i\,\Gamma_p/2}\,\langle\,\Phi_p|V_W|\Phi_s\,\rangle\,\frac{1}{(E-E_s)+i\,\Gamma_s/2}\,\frac{e^{i\delta_s}\gamma_s^n}{\sqrt{2\pi}} \tag{41}$$

describes (see diagram 1 of Fig.1) the p-neutron strong absorption into compound resonance Φ_p, p-resonance propagation, its weak-interaction mixing with compound resonance Φ_s, propagation of s-resonance and its strong decay.

The second term

$$T_2=\frac{e^{i\delta_p}\gamma_p^n}{\sqrt{2\pi}}\,\frac{1}{(E-E_p)+i\,\Gamma_p/2}\,\langle\,\Phi_p|V_W|\chi_s(E)\,\rangle \tag{42}$$

describes (diagram 2 of Fig.1) the p-neutron strong absorption into compound resonance Φ_p, p-resonance propagation and its subsequent «weak» decay into the s-wave continuum state.

The third term

$$T_3=\frac{e^{i\delta_p}\gamma_p^n}{\sqrt{2\pi}}\,\frac{1}{(E-E_p)+i\,\Gamma_p/2}\,\mathcal{P}\int\frac{dE'}{E-E'}\,\langle\,\Phi_p|V_W|\chi_s(E')\,\rangle\times$$

$$\times\,\langle\,\chi_s(E')|V|\Phi_s\,\rangle\,\frac{1}{(E-E_s)+i\,\Gamma_s/2}\,\frac{e^{i\delta_s}\gamma_s^n}{\sqrt{2\pi}} \tag{43}$$

describes (diagram 3 of Fig.1) p-neutron strong absorption, p-resonance propagation of Φ_p, its «weak» decay into s-neutron continuum with immediate strong reabsorption of s-neutron into the s-resonance state Φ_s. Then follows s-resonance propagation of Φ_s, and its strong neutron decay. We see that this is another way of mixing p- and s-resonance (compare with T_1) by virtual emission and reabsorption of s-wave neutron. The processes of this type were first encountered in isospin symmetry breaking, where they played an important role. Historically they were first analyzed in terms of R-matrix theory and called «external mixing» processes, contrary to the «internal mixing» of T_1.

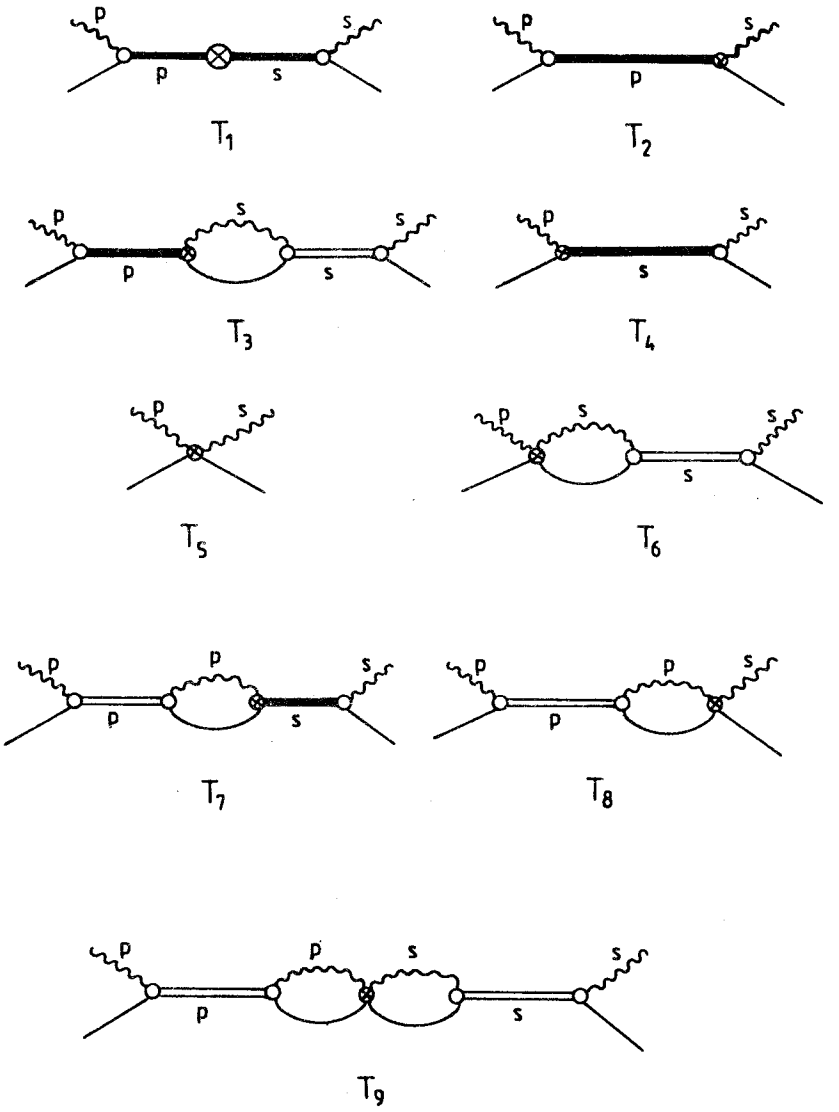

Fig. 1. Diagrams of possible processes contributing to p-violation in neutron-nucleus elastic scattering

The fourth term T_4 describes p-neutron «weak» absorption forming s-resonance compound state Φ_s which then decays in a normal «strong» way. It is obvious (see Fig.1) that this amplitude closely resembles the T_2 amplitude.

The fifth term T_5 describes just the potential scattering of p-wave neutron in the weak mean field of the target. This type of process was historically discussed in P-violation first of all (see, e.g., [22]) and is important, since all the specific enhancements below should be defined with respect to this simplest amplitude.

The term T_6 describes re-scattering of p-wave neutron in the weak mean field of the target with subsequent «strong» absorption of the created s-neutron forming the s-resonance BSEC φ_s. This resonance propagates and then decays in a normal «strong» way governed by γ_s^n amplitude. This process is topologically close to the process T_8 (see below).

The term T_7 describes strong p-neutron absorption into p-compound BSEC φ_p, resonance propagation and strong decay, which is followed by «weak» re-absorption of neutron into s-compound resonance state Φ_s. This state then decays in a normal «strong» way. Obviously T_7 is very similar to the above T_3 one.

We have already mentioned that T_8 is very similar to T_6. It describes the strong formation of the p-resonance BSEC and its strong decay followed by weak re-scattering in the target field. Exactly this amplitude was considered recently by Weidenmüller and Lewenkopf [33,34].

Finally T_9 describes «strong» p-compound formation of Φ_p and its «strong» decay, which is followed by «weak» re-scattering in the target mean field. The s-neutron created in this re-scattering is then «strongly» re-absorbed into s-compound BSEC, which finally decays in a normal «strong» way.

Now, as we know all the essential mechanisms contributing to P-violation in slow neutron-nucleus elastic scattering, it is high time to estimate which of them gives the most important contribution and why. In doing this estimates we shall drop all the phase shift exponentials because in the energy region of interest to us $\delta_s \sim (kR) \sim 10^{-3}$, $\delta_p \sim (kR)^2 \sim 10^{-6}$. Since the total resonance widths in this region are defined essentially by γ-emission we shall assume that $\Gamma_s \approx \Gamma_p = \Gamma$. For simplicity we shall denote $|E_s - E_p|$ as D and assume that $D \sim d$.

We shall tell the reader in advance the result of our analysis carried out back in 1982 (see [7]) — the dominant contribution comes from the mechanism

of T_1, which is usually called compound-compound (c-c) mixing. Other mechanisms' contributions are smaller by, at any rate, the above factor of dynamical enhancement $v/D \sim \sqrt{N} \sim 10^3$ (see eq.(9)). Therefore we shall estimate the ratios of T_1 to all other amplitudes in (40).

We shall start with standard weak potential scattering T_5, which defines the process in the simplest systems of n—p type. In order to estimate T_5 we shall first take the weak interaction scaling factor F out of $\langle \chi_p(E)|V_W|\chi_s(E) \rangle$. Then we shall proceed by removing the extra barrier penetration factor out of the amplitude, thus converting T_5 into the strong interaction amplitude $\langle \chi_s(E)|V|\chi_s(E) \rangle$, which roughly equals the s-wave phase shift $\delta_s \sim (kR)$. Thus:

$$\langle \chi_p(E)|V_W|\chi_s(E) \rangle \approx F \cdot (kR)^2. \tag{44}$$

The exact calculation of T_5 done in [34] shows that ex.(44) is correct to within a constant factor of ~ 7. Now we can estimate the ratio

$$\frac{T_1}{T_5} \approx \frac{\gamma_p^n \gamma_s^n \langle \Phi_p|V_W|\Phi_s \rangle}{(E - E_p + i\,\Gamma_p/2)(E - E_s + i\,\Gamma_s/2)} \frac{1}{F(kR)^2} \approx$$

$$\approx \frac{S_n \Theta_0^2 v}{(E - E_p + i\,\Gamma_p/2)(E - E_s + i\,\Gamma_s/2)}. \tag{45}$$

Here we have done the scaling (see eq.(6)) of weak interaction matrix element $v_p = Fv$ and used the standard estimates of neutron partial widths, factoring out the barrier penetration $(kR)^{2l+1}$, spectroscopic factor S_n and the «single-particle reduced width» $\Theta_0^2 = 2\hbar^2/mR^2$. The resonance denominators of T_1 give the smallest ratio (45) exactly between the E_s and E_p. In this energy point (we consider the case of $D >> \Gamma$):

$$\frac{T_1}{T_s} \approx \frac{S_n \Theta_0^2 v}{D^2} \approx \frac{\Theta_0^2 v}{d_0 D} \approx \frac{v}{D} \sim \sqrt{N}. \tag{46}$$

In estimating (46) we assumed that $S_n \sim d/d_0$, where the single-particle level spacing d_0 was taken to be roughly equal to Θ_0^2 (see, e.g., [12]). Thus we see that under the worst «off-resonance» conditions c-c mixing mechanism T_1 gives us the dynamical enhancement factor. We also see that this enhancement disappears for the simplest systems with $N \sim 1$. This, however, is not the whole story. We also see from (41), (45) that in the vicinity of each

resonance pole ($E \approx E_s$ or E_p) T_1 presents us with extra resonance enhancement factor D/Γ, providing thus for the overall enhancement of the ratio

$$\left(\frac{T_1}{T_5}\right)_{\text{res}} \approx \frac{v}{D}\frac{D}{\Gamma} = \frac{v}{\Gamma}. \tag{47}$$

The above *resonance enhancement* is a specific feature of nuclear reactions which has no analogues in case of bound states. Its meaning is, however, quite transparent — the magnitude of P-violating effects is proportional to the time τ spent by incident neutron in the weak-interaction field of the target. The role of the complicated compound resonance is to capture the neutron and keep it inside the compound system for a long time $\tau = h/\Gamma$. This kind of effect was first mentioned by Mahaux and Weidenmüller [35].

Consider now the ratio of T_1 to T_2:

$$\frac{T_1}{T_2} \approx \frac{\gamma_s^n \langle \Phi_p | V_W | \Phi_s \rangle}{(E - E_s + i\,\Gamma_p/2)\langle \Phi_p | V_W | \chi_s \rangle} \approx \frac{v}{(E - E_s + i\,\Gamma_p/2)}. \tag{48}$$

In performing the estimate we introduced the scaling $\langle \Phi_p | V_W | \chi_s \rangle \sim$ $\sim F \langle \Phi_s | V | \chi_s \rangle \approx F \cdot \gamma_s^n$. We see again that even in the worst case of $E = E_p$ the T_1 dominates by the dynamical enhancement factor v/D, while at $E = E_s$ the resonance enhancement D/Γ is added.

Since T_4 amplitude is topologically close to T_2 the ratio T_1/T_4 demonstrates exactly the same enhancements with exchange of E_s by E_p.

All the remaining diagrams in Fig.1 contain closed loops of principal value integrals. In estimating those loops we shall follow the arguments of Weidenmüller and Lewenkopf [34,35] carried for the case of T_8. Crudely their argument was that the main E' dependence in the integral

$$\mathcal{P} \int \frac{dE'}{E - E'} \langle \varphi_p | V | \chi_p(E') \rangle \langle \chi_p(E') | V_W | \chi_s(E) \rangle \tag{49}$$

comes from the barrier penetration factor $(kR)^{2l+1}$ of $|\chi_p(E')|^2$. This allows one to drop the principal-value symbol and carry the integrands at $E' = E$ out of the integral. Thus (49) becomes:

$$\langle \varphi_p | V | \chi_p(E) \rangle \langle \chi_p(E) | V_W | \chi_s(E) \rangle \int \frac{dE'}{E'} \frac{\chi_p^2(k'R)}{\chi_p^2(kR)}. \tag{50}$$

In case of square well potential $\chi_p(kR) \sim j_1(kR)$, and one gets the analytical result

$$\langle \varphi_p | V | \chi_p(E) \rangle \langle \chi_p(E) | V_W | \chi_s(E) \rangle \cdot \frac{3\pi}{(kR)^3} \approx \gamma_p^n \frac{F}{(kR)} . \tag{51}$$

In order to obtain the final result we used the above estimate (44) for weak-interaction amplitude. More exact numerical calculations of [34] for Woods-Saxon potential show that instead of 3π one gets the value $C = 3.1$. This obviously does not affect our order-of-magnitude estimates.

Now we can estimate the ratio of T_1 to

$$T_8 \simeq \frac{\Gamma_p^n F}{(E - E_p + i\,\Gamma_p/2)(kR)} . \tag{52}$$

We see that this ratio is

$$\frac{T_1}{T_8} \approx \frac{\gamma_s^n}{\gamma_p^n} \frac{(kR)v}{(E - E_s + i\,\Gamma_s/2)} \approx \frac{v}{(E - E_s + i\,\Gamma_s/2)} . \tag{53}$$

Again we observe that even at $E = E_p$ the T_s is smaller by the dynamical enhancement factor $v/D \sim \sqrt{N}$.

Since T_6 is topologically close to T_8, the same dynamical enhancement is lacking in T_6 even at $E = E_s$, while at $E = E_p$ the resonance enhancement of T_1 is added to the ratio T_1/T_6 .

Now we can use the above procedure for estimation of the principal value integral in T_3 (see (43)):

$$\mathcal{P} \int \frac{dE'}{E - E'} \langle \Phi_p | V_W | \chi_s(E') \rangle \langle \chi_s(E') | V | \varphi_s \rangle \approx \langle \Phi_p | V_W | \chi_s(E) \rangle \times$$

$$\times \langle \chi_s(E') | V | \varphi_s \rangle \int \frac{dE'}{E'} \frac{\chi_s^2(k'R)}{\chi_s^2(kR)} \approx \frac{F \cdot \gamma_s^n \gamma_s^n]}{kR} . \tag{54}$$

Therefore

$$T_3 \approx \frac{\gamma_p^n F (\gamma_s^n)^3}{(E - E_p + i\,\Gamma_p/2)(E - E_s + i\,\Gamma_s/2)(kR)} \tag{55}$$

and

$$\frac{T_1}{T_3} \approx \frac{(kR)v}{\Gamma_s^n} \approx \frac{v}{S_n \Theta_0^2} \approx \frac{d_0}{\Theta_0^2} \frac{v}{d} \approx \frac{v}{d} . \tag{56}$$

In doing this estimate we used the same factorization as in (45), (46) for neutron width Γ_s^n.

In the same way we get the same estimate (55) for T_7 and (56) for the ratio T_1/T_7.

The only remaining term now is T_9. Each integral in it can be estimated in the way we have already used several times, giving:

$$\mathcal{P}\int \frac{dE'dE''}{(E-E')(E-E'')} \langle \varphi_p|V|\chi_p(E')\rangle\langle \chi_p(E')|V_W|\chi_s(E'')\rangle\langle \chi_s(E'')|V|\varphi_s\rangle \approx$$

$$\approx \gamma_p^n \langle \chi_p(E)|V_W|\chi_s(E)\rangle \gamma_s^n \frac{1}{(kR)^4} \approx \gamma_p^n\gamma_s^n \frac{F}{(kR)^2}. \tag{57}$$

Therefore

$$T_9 \approx \frac{\Gamma_p^n F \Gamma_s^n}{(kR)^2(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)} \tag{58}$$

and

$$\frac{T_1}{T_9} \approx \frac{(kR)^2 v}{\gamma_p^n\gamma_s^n} \approx \frac{v}{S_n\Theta_0^2} \approx \frac{d_0}{\Theta_0^2}\frac{v}{d} \approx \frac{v}{d}. \tag{59}$$

To finish with our analysis we shall recollect that some of the above amplitudes (T_1, T_2, T_3, T_4 and T_7) contained the «full» compound nucleus functions $\Phi_k = \varphi_k + (S_k^n)^{1/2}u$. Substitution of φ_k instead of Φ_k in the above amplitudes would not affect, as long as we see, the above estimates of their relative contribution to (40). The additional valence terms $(S^n)^{1/2}u$ would give us seven more amplitudes with the exchange of corresponding Φ by $(S^n)^{1/2}u$. All of them would contain at any rate the additional small factor $\sqrt{S^n} \approx 1/\sqrt{N}$ (inverse dynamical enhancement). Therefore in all the amplitudes but T_1 they should be disregarded as small additions to the *already small* amplitudes. In the case of T_1 there will be 2 «mixed» terms containing the products $(\varphi\cdot\sqrt{S^n}u)$ and one term of the form:

$$T_{10} = \frac{\Gamma_p^n}{(\Gamma_p^0)^{1/2}} \frac{\langle u_p|V_W|u_s\rangle}{(E-E_p+i\Gamma_p/2)(E-E_s+i\Gamma_s/2)} \frac{\Gamma_s^n}{(\Gamma_s^0)^{1/2}}. \tag{60}$$

Since this term contains extra smallness $S^n \sim 1/N$ in comparison with T_1 it seems that it should be dropped first of all. However the compound-compound matrix element v_p (to be exact, its variance $\tilde{v}_p$) of (41) goes down with increasing complexity N of the wave function Φ_k as $d_0/\sqrt{N}$ (see eq.(9)). Therefore the «single-particle» matrix element $\langle u_p|V_W|u_s\rangle$ should be larger than $\tilde{v}_p$ by roughly a factor of $\sqrt{d_0/d} \approx \sqrt{N}$. Thus the overall ratio of T_1/T_{10} is $1/\sqrt{N}$ rather than $1/N$.

An important point is that, contrary to partial amplitudes γ^n of (41), whose signs vary randomly from resonance to resonance, all the partial widths in (60) are positive. Similarly, contrary to randomly varying sign of v_p in (41), the sign of single-particle matrix element in (60) (defined by u_p, u_s) might vary only over the energy range of single-particle levels spacing d_0. The overall sign of (60) at a given resonance, say E_p, seems to be also defined by the sign of $(E_p - E_s)$. If, however, we switch over to multi-resonance case, we should sum T_{10} at E_p over all the s-resonances which might mix with a given p-one:

$$\sum_s T_{10} = \frac{\Gamma_p^n}{(\Gamma_p^0)^{1/2}} \frac{\langle u_p | V_W | u_s \rangle}{(E - E_p + i\,\Gamma_p/2)} \frac{\Gamma_s^0}{(\Gamma_s^0)^{1/2}} \sum_s \frac{S_n(s)}{E - E_s} . \qquad (60a)$$

Since the spectroscopic factor saturates to unity over the energy range Γ_{spr} around the position E_{s0} of a single-particle level:

$$\sum_s T_{10} \approx \frac{\Gamma_p^n \langle u_p | V_W | u_s \rangle}{(E - E_p + i\,\Gamma_p/2)} \left(\frac{\Gamma_s^0}{\Gamma_p^0} \right)^{1/2} \frac{1}{E - E_{s0}} , \qquad (60b)$$

therefore the overall sign of the effect caused by valence neutron remains constant over the range d_0. However the same is true for mechanisms of T_9 and T_8 (see (58), (52)). All these mechanisms, which essentially sprang to life because of the single-particle (valence) contributions $\chi(E)$ to the BSEC wave functions ϕ_k would provide for the constant sign contributions to the effect. But as we pointed already in [7] *all of them lack the factor of dynamical enhancement* $v/D \sim \sqrt{N}$ and should be dropped on this grounds. We shall briefly come back to this problem in discussing the «sign correlation effect» below (see Sec. IV. 5).

Up to now we considered (see (40) and Fig.1) only the case, when the initial p-wave neutron is transformed by weak interaction into the final state s-one — i.e., the second term in (36). Repeating the above analysis for the first term of (36) will add 16 more amplitudes similar to those of Fig.1. One can easily see that diagrams 1,3,5,7 and 9 are symmetric with respect to the exchange of s- and p-neutron states. Therefore such an exchange will just double the contributions of corresponding mechanisms to (36). The same exchange in amplitudes 2, 8 of Fig.1 would shift their resonance poles to E_s. However the poles of T_4 and T_6 after this exchange would be shifted to E_p. Therefore the addition of the first term in (36) would just completely restore the symmetry of the whole expression (36) with respect to the exchange of initial and final states,

which is expected for *any* elastic scattering T-invariant amplitude. The relative dominance of c–c mixture amplitude T_1 remains unaltered.

To sum up, we have seen that proper nuclear reaction theory allows us to find all the contributions to the weak interaction elastic scattering amplitude of neutrons on a nuclear target. The leading contribution to this amplitude comes from compound-compound mixing mechanism T_1 of eq.(41). This mechanism shows two kinds of enhancement factors: a) the dynamical enhancement factor $v/D \sim \sqrt{N}$; b) the resonance enhancement factor D/Γ. The physical reason of both enhancements is the complexity (or quantum chaoticity) of the compound nucleus resonances. The lack of symmetries characteristic of quantum chaotic system (see, e.g., [36]) removes all the degeneracies of the independent particles' shell-model and thus exponentially decreases the resonance spacing. At the same time it complicates the structure of the compound-resonance wave function, hindering in this way all the decay processes and reducing the total resonance width Γ.

One should also mention that in our analysis we met no traces of the mystical «structural enhancement factor» $1/(kR)$. As is mentioned above, theoretically this factor is a false result of inconsistent application of bound-state theory to the continuum nuclear reaction case.

2.4. Back to Observables. In the previous paragraph we performed an analysis of all possible P-violating amplitudes in the simplest case of elastic neutron-nucleus scattering, understood the physics of their enhancements and have chosen our «favourite» — c–c mixing amplitude T_1, which exceeds the others by at any rate dynamical enhancement factor of $\sqrt{N} \sim 10^3$.

Inserting T_1 into eqs. (36), (37) we obtain

$$\Delta^{P}_{\text{tot}} = \frac{2\pi}{k^2} \frac{\gamma_p^n \cdot v_p \cdot \gamma_s^n \cdot e^{i(\delta_s + \delta_p)}}{[(E - E_p)^2 + \Gamma_p^2/4]\,[(E - E_s)^2 + \Gamma_s^2/4]} \times$$

$$\times [(E - E_s)\,\Gamma_p + (E - E_p)\,\Gamma_s], \tag{61}$$

$$\frac{d\Phi}{dz} = \frac{4\pi\rho}{k^2} \frac{\gamma_p^n \cdot v_p \cdot \gamma_s^n \cdot e^{i(\delta_s + \delta_p)}}{[(E - E_p)^2 + \Gamma_p^2/4]\,[(E - E_s)^2 + \Gamma_s^2/4]} \times$$

$$\times \left[(E - E_s)\,(E - E_p) - \frac{\Gamma_s\,\Gamma_p}{4} \right]. \tag{62}$$

Here v_p stands for the weak interaction matrix element $i\langle \Phi_p | V_W | \Phi_s \rangle$. The presence of Breit — Wigner denominators shows that both effects de-

monstrate symmetric resonance enhancements in the vicinities of both s- and p-resonances. Δ^{P}_{tot} reaches its maxima at E_s and E_p:

$$\Delta^{P}_{\text{tot}}(E_{\text{res}}) \approx \frac{8\pi}{k^2} \frac{\gamma_p^n \gamma_s^n}{\Gamma} \frac{\nu}{D} . \tag{63}$$

The quantity (62), however, changes sign at points $E \approx E_{s,p} + \Gamma_s \Gamma_p / 4D \approx E_{s,p}$ and reaches its maxima

$$\left(\frac{d\Phi}{dz} \right)_{\text{res}} \approx \pm \frac{8\pi p}{k^2} \frac{\gamma_p^n \gamma_s^n}{\Gamma} \frac{\nu}{D} \tag{64}$$

at points $E \approx E_{s,p} \pm \Gamma_{s,p}/2$. For the characteristic curves of the energy behaviour of (61), (62) see our paper [2].

Provided that the asymmetry in total cross-sections Δ^{P}_{tot} is dominated by resonance-resonance mixture of T_1, we can write (see, e.g., [2]) for the asymmetry $\Delta^{P}_{n,\gamma}$ in the denominator of ex. (28):

$$\Delta^{P}_{n,\gamma} = \sigma^{+}_{n,\gamma} - \sigma^{-}_{n,\gamma} \approx \frac{\Gamma_\gamma}{\Gamma} \Delta^{P}_{\text{tot}} \approx \Delta^{P}_{\text{tot}} . \tag{61a}$$

Mind that for other mixture mechanisms this might be not true. Even in this case the last equality holds only for low-energy neutrons incident on non-fissioning target.

We have not yet performed the analysis of the P-violating amplitudes in the inelastic channels, which are essential for calculation of the quantity $\Delta_{n,f}$ of eq. (38). This can be easily done on the same lines as the previous paragraph. Those, who are interested in more details, might look through our papers [7] and [8]. The net result of such an analysis is again the conclusion that c–c mixture amplitudes are dominant. For incident $l_n = 0$ and $l_n = 1$ those amplitudes are (see Fig.2):

$$T^{(1)}_{nf} = \langle\, l_f + 1, f | T_W | 0 \,\rangle = i \frac{\gamma_s^{n} \cdot \nu_p \cdot \gamma_p^{f} \cdot e^{\,i(\delta_s^n + \delta^f_{l_f+1})}}{(E - E_s + i\,\Gamma_s/2)\,(E - E_p + i\,\Gamma_p/2)} ,$$

$$T^{(2)}_{nf} = \langle\, l_f, f | T_w | 1 \,\rangle = i \frac{\gamma_p^{n} \cdot \nu_p \cdot \gamma_s^{f} \cdot e^{\,i(\delta_p^n + \delta^f_{l_f})}}{(E - E_s + i\,\Gamma_s/2)\,(E - E_p + i\,\Gamma_p/2)} . \tag{65}$$

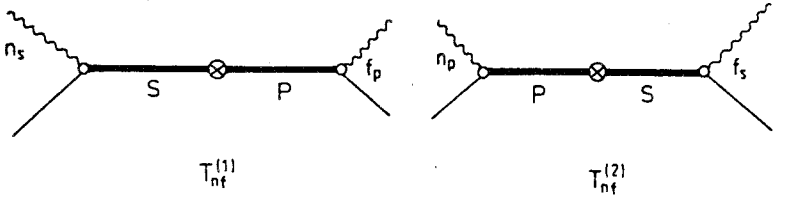

Fig. 2. Two P-violating compound-compound mixing processes in (n, f) inelastic channel

Both amplitudes demonstrate resonance enhancement of D/Γ at E_s and E_p plus the dynamical enhancement of v/D. However their ratio is

$$\frac{T^{(2)}}{T^{(1)}} = \frac{\gamma_p^n \gamma_s^f}{\gamma_s^n \gamma_p^f} \sim (kR) \frac{\gamma_s^f}{\gamma_p^f}.$$

Therefore for low-energy ($E \leq 1$ Mev) neutrons the second diagram contains «*the initial channel hindrance*» factor (kR) (see [7]), which again has no bound-state analogues, but can be easily understood in terms of nuclear reaction theory. Contrary to P-violating elastic amplitudes of the above paragraph the inelastic amplitudes are not symmetric with respect to the exchange of s- and p-neutron waves, and it is highly preferable to excite the s-wave compound resonance in the initial neutron channel rather than p-wave one. This initial channel hindrance leads to even more general and important consequences (see below), making the «inelastic» observables more preferable in general than the «transmission» ones. The corresponding «allowed» reaction amplitudes are:

$$\langle l_f, f | T | 0 \rangle = \frac{\gamma_s^n \cdot \gamma_s^f \cdot e^{i(\delta_s^n + \delta_{l_f}^f)}}{(E - E_s + i\Gamma_s/2)}, \tag{66}$$

$$\langle l_f, f | T | 1 \rangle = \frac{\gamma_p^n \cdot \gamma_p^f \cdot e^{i(\delta_p^n + \delta_{l_f}^f)}}{(E - E_p + i\Gamma_p/2)}. \tag{67}$$

Inserting (65)—(67) into (38) and retaining only the largest terms, we obtain:

$$\Delta_{nf} = \frac{2\pi}{k^2} \sum_{l_f} \frac{\gamma_s^f \cdot v_p \cdot \gamma_p^f}{[(E - E_s)^2 + \Gamma_s^2/4]\,[(E - E_p)^2 + \Gamma_p^2/4]} \times$$

$$\times \operatorname{Re}\left[(E - E_p)\, \Gamma_s^n \cdot e^{i(\delta_{l_f}^f - \delta_{l_{f+1}}^f)} \right]. \tag{68}$$

In this expression we already neglected the neutron potential phase shifts δ_s, δ_p. When the final channel is a γ-emission one, we get:

$$\Delta_{n\gamma_0} = \frac{2\pi}{k^2} \frac{\gamma_s^n \cdot v_p \cdot \gamma_p^f}{[(E-E_s)^2 + \Gamma_s^2/4]\,[(E-E_p)^2 + \Gamma_p^2/4]} (E-E_p)\,\Gamma_s^n. \qquad (69)$$

Mind that γ_0 here denotes the *particular* γ-transition (say, to the ground state of initial nucleus). Mark also, that the effect changes its sign in close vicinity to $E = E_p$ in analogy to $d\Phi/dz$ of eq. (62).

Generally speaking, the most nontrivial part of P-violation theory ends with expressions (61), (62), (61a), (68) and (69) for Δ_{tot}^p, $d\Phi/dz$, $\Delta_{n,\gamma}^p$ and $\Delta_{n,f}$. In order to find the dimensionless ratios P, Φ, A and $\alpha_{n,f}$ observed experimentally, one should just divide those expressions by $2\sigma_{\text{tot}}$, $2\sigma_{n,\gamma}$ or $2d\sigma_{n,f}/d\Omega$, respectively. However it turns out that even this seemingly simple arithmetical operation is full of intricate tricks, because, as we have already mentioned, the denominators also exhibit rapid and sometimes complicated energy dependencies.

Indeed, even in the simplest case of one s- and one p-compound resonances the simplified (i.e., without interference terms) expression for σ_{tot} is:

$$\sigma_{\text{tot}}(E) \approx \sigma_s(E) + \sigma_{\text{pot}}(E) + \sigma_p(E) \approx$$

$$\approx \frac{2\pi}{k^2}\left[\frac{\Gamma_s^n \Gamma_s}{(E-E_s)^2 + \Gamma_s^2/4} + 4(kR)^2 + \frac{\Gamma_p^n \Gamma_p}{(E-E_p)^2 + \Gamma_p^2/4}\right]. \qquad (70)$$

Here σ_s, σ_p and σ_{pot} are the contributions to total cross-section coming from s-, p-compound resonances and potential elastic scattering. As is pointed above, the numerators Δ_{tot}^p, $d\Phi/dz$ display resonance enhancement *both* in s- and p-resonances. However in the energy region of major interest to us (from thermal neutrons to few eV) $\sigma_{\text{tot}}(E)$ and $\sigma_{n,f}(E)$ are dominated by s-resonance contribution: $\sigma_s(E)/\sigma_{\text{tot}}(E) \sim 1$; $\sigma_{n,f}^s(E)/\sigma_{n,f}(E) \sim 1$. Therefore in this region the resonance enhancement at $E \approx E_s$ is completely cancelled from the «observable» ratios:

$$P(E) \approx \frac{\gamma_p^n}{\gamma_s^n} \frac{v_p}{(E-E_p)^2 + \Gamma_p^2/4} \frac{[(E-E_s)\,\Gamma_p + (E-E_p)\,\Gamma_s]}{2\Gamma_s}, \qquad (71)$$

$$\Phi(E) \approx \frac{\gamma_p^n}{\gamma_s^n} \frac{v_p}{(E-E_p)^2 + \Gamma_p^2/4} \frac{(E-E_s)(E-E_p) - \Gamma_s\Gamma_p/4}{\Gamma_s}, \qquad (72)$$

$$\alpha_{n\gamma_0} \approx \frac{\gamma_p^{\gamma_0}}{\gamma_s^{\gamma_0}} \frac{v_p}{(E-E_p)^2+\Gamma_p^2/4} \frac{(E-E_p)}{2}. \tag{73}$$

Since the p-resonance contribution to cross-section is usually *only a small bump* on the large smooth tail of s-resonance (and σ_{pot} background of σ_{tot}) all the observables demonstrate characteristic resonance enhancement in the vicinity of E_p, although the latter two change their signs at $E \approx E_p$ and therefore display more complicated patterns typical of optical dispersion rather than simple Breit — Wigner ones. The largest among them is the «inelastic channel» observable (73) which reaches at $E \approx E_p \pm \Gamma_p/2$ the value:

$$\alpha_{n,\gamma_0} \sim \frac{\gamma_p^{\gamma_0}}{\gamma_s^{\gamma_0}} \frac{v_p}{\Gamma}. \tag{73a}$$

And here come a few more specific features of nuclear reactions. Consider now the ratio of observables at maximum $E \approx E_p \pm \Gamma_p/2$:

$$\left(\frac{P(E)}{\alpha_{n\gamma_0}(E)}\right)_{\max} \approx \frac{\gamma_p^n}{\gamma_s^n} \frac{\gamma_s^{\gamma_0}}{\gamma_p^{\gamma_0}} \frac{D}{\Gamma} \sim (kR) \frac{\gamma_s^{\gamma_0}}{\gamma_p^{\gamma_0}} \frac{D}{\Gamma},$$

$$\left(\frac{\Phi(E)}{\alpha_{n\gamma_0}(E)}\right)_{\max} \approx \frac{\gamma_p^n}{\gamma_s^n} \frac{\gamma_s^{\gamma_0}}{\gamma_p^{\gamma_0}} \frac{D}{\Gamma} \sim (kR) \frac{\gamma_s^{\gamma_0}}{\gamma_p^{\gamma_0}} \frac{D}{\Gamma}. \tag{73b}$$

We see that both observables P and Φ connected with the elastic channel correlation $(\boldsymbol{\sigma}_n \cdot \mathbf{k}_n)$ contain the already familiar entrance channel hindrance factor (kR). This demonstrates a very *general law* — if any symmetry-breaking correlation contains a certain power of $\mathbf{k}_n$, all the corresponding observables will contain hindrance factors (k_nR) of at least the same power. (We shall return to this point in our discussion of T-violation below). This fact puts *all the «transmission» observables of symmetry-breaking into unfavourable position with respect to inelastic channel ones* from the very beginning.

Thus we at last encountered the factor (kR) in the observables (71), (72). But, contrary to naive expectations of bound-state parallels, it is a *hindrance* factor rather than enhancement one.

On the other hand, we have a factor γ_p^f/γ_s^f in inelastic channel observables, which in general might play both ways, but for some special cases might serve a role of the only true «structural enhancement» factor (see,e.g.,[8]) increasing the *P-violation effects in inelastic channels practically to 100% level.*

We also see that the interference patterns of resonance enhancement are rather complicated, which often results in extra resonance enhancements D/Γ (see, e.g., Φ and P observables). These extra enhancement factors in most favourable on-resonance situations (see below) might almost compensate the general smallness (kR) pertinent to transmission experiments.

Since historically P-nonconserving effects were first observed at thermal energies, it is instructive to classify the magnitudes of these effects at $E = E_{th}$:

$$P(E_{th}) \approx -\frac{\gamma_p^n}{\gamma_s^n}\frac{v_p}{E_p^2}\frac{E_s\Gamma_p + E_p\Gamma_s}{2\Gamma_s} \approx -\frac{\gamma_p^n}{\gamma_s^n}\frac{v_p}{E_p}\frac{E_s + E_p}{E_p}, \tag{74}$$

$$\Phi(E_{th}) \approx \frac{\gamma_p^n}{\gamma_s^n}\frac{v_p}{E_p^2}\frac{E_s E_p - \Gamma_s\Gamma_p/4}{2\Gamma_s} \approx \frac{\gamma_p^n}{\gamma_s^n}\frac{v_p}{E_p}\frac{E_s}{\Gamma_s}, \tag{75}$$

$$\alpha_{n\gamma_0}(E_{th}) \approx -\frac{\gamma_p^{\gamma_0}}{\gamma_s^{\gamma_0}}\frac{v_p}{E_p^2}\frac{1}{2}. \tag{76}$$

We see that both P and Φ contain the above strong hindrance factor $\gamma_p^n/\gamma_s^n \sim (kR) \sim 10^{-4}$, which is not present in inelastic case of α_{n,γ_0}. Therefore the «inelastic» value α_{nf} is the largest ($10^{-3} + 10^{-4}$) at thermal energies and was experimentally observed for γ's and fission fragments in almost «prehistoric» times (see [18,37]). It was exactly those unbelievably large (compared to $10^{-7} \div 10^{-8}$ effects in n–p scattering) effects observed in neutron-induced fission that initiated the theoretical studies of Sushkov — Flambaum and Bunakov — Gudkov, which led both groups to the prediction of p-resonance enhancements. Next in magnitude (typically 10^{-5}) comes the value of Φ, which contains additional large factor E_s/Γ_s. The P value containing instead the factor $(E_s + E_p)/E_p$ is usually smaller (typically 10^{-6}). Consequently the first experimental observations for them were done later (see [27,28]). Comparison with experiment nicely confirms the above «hierarchy» of observables (see, e.g., [2,7]).

Let us come back to the behaviour of P in the vicinity of p-resonance:

$$P(E) \approx \frac{2\pi}{k^2}\frac{v_p}{D}\frac{\Gamma_p}{(E - E_p)^2 + \Gamma_p^2/4}\frac{\gamma_p^n \cdot \gamma_s^n}{\sigma_{tot}(E)}. \tag{77}$$

Mind that this expression is valid *only* for $D > \Gamma_s/2$.

Since *even at its maximum p-resonance contributes only a small fraction to* $\sigma_{tot}(E_p)$, the overall behaviour of $\sigma_{tot}(E)$ in the vicinity of *p*-resonance *is quite smooth*. Therefore the resonance enhancement of $P(E)$, represented by Breit — Wigner denominator of (77) fully displays itself in experiment. Seemingly everything is clear — we have both this resonance enhancement mechanism plus a familiar factor v_p/D of dynamical enhancement in (77).

Nevertheless, here starts the «mythology» of experimentalists, which prefer to stick to naive bound-state analogies, whose physical inconsistency we have already analyzed at length. *Instead* of presenting the observable P, given by (77), they prefer to introduce the *auxiliary* quantity $\mathcal{P}$ by relating observed Δ^P_{tot} to a *small fraction* of the observed σ_{tot}, namely to the *p*-resonance contribution $\sigma_p(E)$ (see third term in ex. (70)):

$$\mathcal{P} = \frac{\Delta^P_{tot}}{2\sigma_p(E)} = P\frac{\sigma_{tot}}{\sigma_p}. \tag{78}$$

The *purely technical* reason for such a renormalization is explained as follows. We have already mentioned that the really measured quantity (25) is:

$$P_{exp}(E) = \Delta^P_{tot}(E) \cdot C,$$

where the constant C depends linearly on the target sample thickness x. In order to optimize the statistical significance of measurements this x is chosen in such a way that $C \approx 1/2\sigma_{tot}$. While measuring $P_{exp}(E)$ in the Γ_p vicinity of E_p, the experimentalists do not re-adjust x for each energy point (again because the *smallness* of σ_p/σ_{tot} ratio allows this). Therefore the measured value $P_{exp}(E)$ performs a «full-scale» resonance behaviour of Δ^P_{tot} in the vicinity of *p*-resonance:

$$\Delta^P_{tot}(E) \approx \frac{2\pi}{k^2}\frac{v_p}{D}\frac{\gamma_s^n \gamma_p^n \Gamma_p}{(E-E_p)^2 + \Gamma_p^2/4}. \tag{79}$$

In order to avoid quoting a whole set of numbers $P_{exp}(E)$ at all the energy point E measured on the resonance curve (79), the experimentalist prefers to *cancel* the resonance behaviour of the effect by normalizing it to $\sigma_p(E)$. This allows one to present only one value of $\mathcal{P}$ instead of the whole resonance curve.

Of course this makes some sense, although one might rather use the knowledge of C to quote directly the measured matrix elements v_p — that will be again one number and exactly the only one we are looking for in performing our experiments. One should, however, realize that this *artificial normalization* by one of the weakest components of the total cross-section gives you *only the*

auxiliary quantity without much physical meaning (see eq. (4a) of Sec.II and the discussion which follows it). The non-physical normalization of this quantity produces fictitious enhancement which has nothing to do with reality — one might as well normalize Δ^{p}_{tot} by the neutrino cross-section and surprise the world with huge unobservable effects. To mix up things even more, nowadays all the experimental papers use for this auxiliary quantity of eq. (78) the same notation as for the physical observable P, which was defined already for 20 years by eq. (20). Dubna experimental group in the past at any rate bothered to introduce different notations for those two quantities, although they never advertised the difference between them and always presented $\mathcal{P}$ as the *observed* result. The main reason for such an «absent-minded» mixing of two physically different quantities becomes quite obvious when one presents (78) in a slightly different form:

$$\mathcal{P} = \frac{\nu_p}{D}\,\frac{\gamma_s^n}{\gamma_p^n} \sim \frac{\nu_p}{D}\,\frac{1}{(kR)} \,. \tag{80}$$

This is exactly the result which was so easily obtained (see (34), (35)) in a simple but inconsistent attempt to apply bound-state perturbation theory to the reaction continuum case. The physically meaningful resonance enhancement mechanism is «swallowed» in it by the renormalization of $\mathcal{P}$, while instead of it out of nowhere appears the misleading factor of «structural enhancement» $(kR)^{-1}$. If one recollects that the majority of experimental papers and reviews practically start with quoting the simple bound-state expressions (33), (34), one realizes how tempting it is to make a small step, substituting the observed P by the auxiliary $\mathcal{P}$: no need to study reaction theory with its strange terminology of continuum spectra, all the theory you need to understand the results boils down to the above 2 simple expressions (33) and (34). This is exactly the case to apply the Russian proverb: «Simplicity worse than robbery».

To summarize, we have shown that the «structural (or kinematical) enhancement factor» $(kR)^{-1}$ is an artifact produced by renormalization (78). It immediately disappears when you come back to the observable

$$P(E) \equiv \mathcal{P}\,\frac{\sigma_p(E)}{\sigma_{\text{tot}}(E)} \approx P_{\text{exp}} \tag{81}$$

which always contains a small factor $\sigma_p(E)/\sigma_{\text{tot}}(E) \sim (\Gamma_p^n/\Gamma_s^n) \sim (kR)^2$, overcompensating the above «structural enhancement».

As to the resonance enhancement mechanism sitting in resonance denominator of (77), one often hears naive statements: «Why, it is quite trivial, everybody knows that compound-resonance effects are of Breit — Wigner shape, and we do not need your fancy theories to prove it». This is again a

wrong nonprofessional statement which might lead to erroneous conclusions. To begin with, the energy behaviour of Δ^{n}_{tot} in eq. (61) is more complicated than simple Breit — Wigner formula combined with bound-state perturbation theory, as it might seem from, say, eq. (79). When one divides it by the energy-dependent $\sigma_{\text{tot}}(E)$ the resulting expression becomes even more complicated (see, e.g., [7,8]). For instance, the observable P at the p-resonance maximum is given by:

$$P(E_p) \approx 8 \frac{\gamma_p^n}{\gamma_s^n} \frac{\nu_p}{\Gamma_p} \frac{(E_p - E_s)}{\Gamma_s} \left[1 + \frac{\sigma_p}{\sigma_s} + \frac{\sigma_{\text{pot}}}{\sigma_s} \right]^{-1}. \tag{82}$$

Suppose now that we face a situation when s- and p-resonances almost overlap $E_s \approx E_p$ (the normal Wigner repulsion does not apply to resonances of opposite parity, so this can easily happen). Then the maximal observed effect *goes down* linearly with decreasing spacing $D = |E_s - E_p|$, contrary to naive expectations of bound-state perturbation theory and even to our eq. (77) which was valid only for $D > \Gamma/2$. All these intricacies become quite essential in the attempts to calculate the energy-averaged effects (see below). Another illuminating example is provided by the «capture transmission» observable A of eq. (28). In view of eq. (61a) we can write this observable at $E = E_p$ as

$$A \approx \frac{\sigma^s_{n,\gamma}(E_p)\, \sigma^p_{n,\gamma}(E_p)}{\sigma^s_{n,\gamma}(E_p) + \sigma^p_{n,\gamma}(E_p)} B.$$

One can see that, in analogy to all the interference type quantities, A would be maximal when the s-resonance contribution σ^s to the (n, γ) cross-section at $E = E_p$ exactly equals the p-resonance one σ^p. Then and only then A would reach its maximal possible value

$$A^{\max} \approx 2 \frac{\nu_p}{\Gamma}.$$

The observable $P(E)$ differs from A by a factor of $\sigma_{n,\gamma}/\sigma_{\text{tot}}$. Therefore $P^{\max}$ would *never* reach $2\nu_p/\Gamma$. However (see [7,8,38]) the most optimal situation for $P(E_p)$ happens again when $\sigma_s(E_p) = \sigma_p(E_p)$. Then

$$P^{\max}(E_p) \approx 2 \frac{\nu_p}{\Gamma} \frac{\sigma_{n,\gamma}}{\sigma_{\text{tot}}}.$$

The famous *La* resonance with $\mathcal{P} \approx 10\%$ satisfies this condition: $\sigma_p(E_p) \approx \sigma_s(E_p) \approx \sigma_{\text{pot}}(E_p)$, thus providing for the observed $P \approx 3\%$.

Eqs. (78), (81) and above considerations show that for *strong* observable p-resonances (present beam intensities force us to select just those for measurements) the enhancement of P would be maximal. Since in those cases $\sigma_p(E_p)/\sigma_{tot}(E_p)$ is about $0.1 \div 0.3$, the difference between quoted $\mathcal{P}$ and observed P would not be too large, although $\mathcal{P}$ values already sound more impressive than the really measured P. However with increasing intensities experimentalists will start observing effects on weaker p-resonances, and this difference might rise to orders of magnitude. The impressive 10% effects, recently observed in ^{232}Th, when expressed in terms of physical observables P, turn out to be more modest 1% effects. So it is high time to stop mixing the 2 quantities and fooling each other.

To finish this paragraph we should mention that we paid special attention to transmission measurements of quantity P (E) since this type of experiments is most popular nowadays. This feeds a constant stream of publications, where the same physical errors in interpretation are repeated again and again.

A special case is P-violation in neutron-induced fission. I am not going to expand on it for several reasons. First of all, this interesting subject is worth a separate review. I shall only mention that there are 2 theoretical approaches to it. Sushkov and Flambaum (see, e.g., [26]), likewise in case of P-quantites, used intuitive bound-state analogies to construct $\alpha_{n,\,fis}$. Gudkov and I (see [7] and especially [39]) tried to apply the general expression (68) to this case. However the most striking fact connected with experimental observation of $\alpha_{n,\,fis}$ dates back to the midst of 50-ies, when it was discovered (see [40,41]) that, in spite of the fact that all the experimental observables in fission are sums over the enormous amount of outgoing reaction channels seemingly with random signs of γ_f, this summation does not destroy the interference effects in (n, *fission*) cross-section. This difficulty was bypassed at that time by the fission transition-state hypothesis of A.Bohr (see [42,43]). Now the *same* story repeated in the measurements of $\alpha_{n,\,fis}$. This quantity is also an interference type phenomenon (see eq.(68)) and again the summation over all the outgoing channels γ_f does not destroy the P-violation effects caused by c–c mixing mechanism in each channel. In view of this common origin of the difficulty we tried to resolve it by generalizing Bohr's hypothesis of transition states. Sushkov and Flambaum used instead of it a purely classical model of fission-fragments motion plus a hypothesis of pear-like shape of the fissioning nuclei *at the saddle-point*. Both approaches have their weaknesses. Applying the classical trajectory notion to the analysis of quantum interference effects seems quite hazardous to us. Moreover, while most people agree that for asymmetric fission the fissioning nucleus has a pear-like shape *near the scission-point,* the same assumption *on*

the top of fission barrier seems quite dubious and contradicts some experimental evidence (see [7,39]). We are, however, not very happy with our's (or, rather, with A.Bohr's) transition-state hypothesis because it seems up to now a rather artificial construction in the framework of quantum reaction theory. I am sure that P-violation in fission is just an additional guide-light in search for yet nonexistant quantum theory of fission.

3. T-Violation

3.1. Specific Intricacies of T-Invariance and Observables. One of the greatest dangers in the analysis of T-violation is to follow too closely the parallels with P-violation — these parallels might be quite wrong. To demonstrate this point we shall consider the cases of P-odd and T-odd correlations. Everybody knows the mnemonic rule — if the transition operator $\hat{T}$ changes sign under the space reflection operation $\mathcal{P}$ (is P-odd), then it has nonzero matrix elements between states of opposite parity. This is perfectly true. Therefore if you observe nonzero amplitude of P-odd correlation (say, $\sigma \cdot \mathbf{k}$) this means P-violation. However, if you observe nonzero amplitude of T-odd correlation, this fact in itself in the majority of cases has nothing to do with T-violation. In order to understand this let us see, how P-violation is mathematically connected with P-odd operators (or correlations) T. Acting by unitary transformation operator $\mathcal{P}$ on a state $|A\rangle$ we get a number π_A (parity quantum number) equal to + 1 or − 1:

$$\mathcal{P}|A\rangle = \pi_A |A\rangle. \tag{83}$$

For operators the space reflection looks like:

$$\mathcal{P}\hat{T}\mathcal{P}^{-1} = \pi_T \hat{T}. \tag{84}$$

Consider now the transition amplitude:

$$\langle B|\hat{T}|A\rangle = \langle B|\mathcal{P}^{-1}\mathcal{P}\hat{T}\mathcal{P}^{-1}\mathcal{P}A\rangle = \pi_B \pi_T \pi_A \langle B|\hat{T}|A\rangle. \tag{85}$$

The first line in this equation uses the fact that $\mathcal{P}\mathcal{P}^{-1} = 1$. The second line makes use of (83), (84) and the unitarity of $\mathcal{P}$. The whole equation gives us a selection rule:

$$\pi_A \pi_B \pi_T = 1. \tag{86}$$

Thus the P-odd operator ($\pi_T = -1$) leads to nonzero amplitudes only in case when $|A\rangle$ and $|B\rangle$ are of opposite parity.

However the action of time-reversal operator $\mathcal{T}$ changes the signs of momenta and spins and exchanges the initial and final states. Therefore its action on any state $|A\rangle$ cannot be expressed in terms of eigenvalues, like (83). If one also adds that $\mathcal{T}$ is not unitary, one sees that it is impossible to construct (85) for $\mathcal{T}$ and get selection rule (86).

Therefore $\mathcal{T}$-invariance leads only to 2 immediate consequences (see, e.g., [11,12]). The first is the detailed-balance principle. For binary process $A + a \to B + b$ it looks like:

$$\frac{(2s_a + 1)\,(2s_A + 1)\,k_a^2\, d\sigma_{ab}/\, d\Omega}{(2s_b + 2)\,(2s_B + 1)\,k_b^2\, d\sigma_{ba}/\, d\Omega} = 1. \tag{87}$$

Here s_i are the corresponding particles' spins.

The other consequence is the so-called P–A theorem which connects the polarization P and asymmetry A. For elastic scattering of spin 1/2 particles it states:

$$P = A. \tag{88}$$

However lots of papers seriously discussed T-odd correlations, hoping to measure T-violation. Was it completely meaningless? The answer is no, but the arguments are quite subtle and tricky (see, e.g., [44,45,46]). We start with unitarity of S-matrix:

$$SS^{\dagger} = 1.$$

Inserting this into the expression

$$S = 1 + i\hat{T}$$

for the transition matrix, we get:

$$\hat{T} - \hat{T}^{\dagger} = i\,\hat{T}\hat{T}^{\dagger}. \tag{89}$$

In case of transition from the initial state $|i\rangle$ to final state $|f\rangle$ this looks like:

$$\langle f|\hat{T}|i\rangle - \langle f|\hat{T}^{\dagger}|i\rangle = i\sum_n \langle f|\hat{T}|n\rangle\,\langle n|\hat{T}^{\dagger}|i\rangle, \tag{90}$$

where $|n\rangle$ forms a complete set of all possible intermediate states. Up to this point everything was quite exact. And now starts the approximation. Suppose that the interaction which defines our transition T contains a small parameter F. Then the l.-h. side of (90) is linear in F, while the r.-h. side is quadratic. Therefore in the first-order approximation

$$\langle f|\hat{T}|i\rangle \approx \langle i|\hat{T}^{*}|f\rangle. \tag{91}$$

This means that matrix $\hat{T}$ is Hermitean. Let us now combine (91) with the condition of T-invariance (see definition of $\mathcal{T}$-operation above):

$$\langle f|\hat{T}|i\rangle = \langle -i|\hat{T}|-f\rangle. \tag{92}$$

Here minus signs mean changes of signs for momenta and spins.

Eqs. (91) and (92) give us:

$$\langle f|\hat{T}|i\rangle = \langle -f|\hat{T}|-i\rangle^{*}, \tag{93}$$

or

$$|\langle f|\hat{T}|i\rangle|^2 = |\langle -f|\hat{T}|-i\rangle|^2. \tag{94}$$

The last equation means that in case of $\mathcal{T}$-invariance (92) the transition probability should be an even function under the sign exchange of all spins and momenta. Now (93) shows that sign-inversion operation for initial and final states means just complex conjugation. Therefore the overall sign of transition probability under this sign inversion is completely defined by the sign of transition operator $\hat{T}$. If this operator is T-odd, (94) demands that transition probability should be zero.

Thus we have seen that nonzero T-odd correlations *are* connected to T-violation only when the transition matrix $\hat{T}$ is approximately Hermitean and *within the accuracy of this approximation*. The last point is very delicate. Consider it in more detail, taking as an example the T-odd correlation $\sigma_n \cdot [\mathbf{k}_e \times \mathbf{k}_\nu]$, which is measured in neutron β-decay. Seemingly this is a weak interaction process, which is governed by weak interaction constant F and therefore deviations from Hermiticity are

$$T - T^{\dagger} \sim F^2.$$

However we should not forget about final-state electromagnetic interaction (Coulomb scattering of electron on proton). This means that the non-Hermitean r.-h. side of (90) should contain terms of the type:

$$i\langle pe\tilde{\nu}|\hat{T}|p'e'\tilde{\nu}\rangle\langle p'e'\tilde{\nu}|\hat{T}^{\dagger}|n\rangle. \tag{95}$$

While the second amplitude in (95) is really of the order of F, the first one is proportional to the fine-structure constant α. Therefore the deviation of $\hat{T}$ from Hermiticity is of the order of αF, and this would imitate T-violation even when it does not exist (see also [44]). In principle one can calculate this final-state interaction correction and subtract it from the experimentally observed value of T-odd correlation. However all the existing experiments of this type were giving only the experimental upper bounds on the effect. While experimental accuracy is more or less easily defined, the accuracy of theoretical estimates of final-state interactions is usually much less reliable. This makes the estimates of the upper bounds on «real» T-violation correspondingly unreliable. Therefore experiments of this type are gradually dying out.

If strong interaction is present in the process, the final-state interaction corrections become of the same order as the Hermitean part of the amplitude, and situation becomes completely hopeless.

The only exclusion, when T-odd correlation is a *direct* evidence of T-violation, is the transmission-type experiment. Then the σ_{tot} quantity, which

defines the transmission, is expressed via the optical theorem (see, e.g., eq. (19)) as the imaginary part of zero-angle elastic scattering amplitude $f(0)$. In this case the initial and final states coincide $|i\rangle = |f\rangle$ and the T-invariance conidition (92) by itself (*without approximation of (91)*) immediately gives us (93) and (94).

There exist 2 types of those «true» T-odd correlations in elastic forward scattering amplitude, which can manifest themselves in neutron transmission experiments with nonzero target spins I.

One of them is the correlation $\sigma_n \cdot (\mathbf{k}_n \times \mathbf{I})\,(\mathbf{k}_n \cdot \mathbf{I})$. One can measure this correlation in polarized neutron transmission through the oriented-nuclei samples. Observing that this correlation depends on the angle θ between $\mathbf{k}_n$ and the target alignment axis as $\sin 2\theta$, one immediately sees that the best observation conditions would be for $\theta = 45°$ and neutron spins σ_n directed parallel or antiparallel to $(\mathbf{k}_n \times \mathbf{I})$ axis. The presence of T-violating interaction would cause the difference in σ_{tot} for those two choices of neutron polarization

$$\Delta^T = \sigma_{\rightarrow} - \sigma_{\leftarrow} \,. \tag{96}$$

Using the optical theorem, one can express this quantity in terms of T-violating part of the scattering amplitude f_T :

$$\Delta^T = \frac{4\pi}{k}\ \mathrm{Im}\, f_T \tag{97}$$

in complete analogy with eqs. (19), (36). Since the experimentalists would always prefer to measure relative quantities rather than absolute ones (see (20)—(27)), the experimentally observed T-violation effect will be:

$$\beta = \frac{\Delta^T}{\sigma_{\rightarrow} + \sigma_{\leftarrow}} \simeq \frac{\Delta^T}{2\sigma_{tot}} \,. \tag{98}$$

Observe that the above correlation is T-violating but P-conserving. The conventional name for it is «five-fold correlation» (FC).

There also exists another correlation, namely $\sigma_n\,[\mathbf{k}_n \times \mathbf{I}]$, which is both P- and T-violating. This «triple correlation» (TC) should be measured with polarized neutron beam and polarized target nuclei. Performing transmission experiments with beam polarization parallel or antiparallel to $[\mathbf{k}_n \times \mathbf{I}]$ axis, one might observe the cross-section difference:

$$\Delta_{PT} = \sigma_{\uparrow} - \sigma_{\downarrow} = \frac{4\pi}{k}\ \mathrm{Im}\,(f_{\uparrow} - f_{\downarrow}) \tag{99}$$

and the corresponding P- and T-violation effect:

$$\eta = \frac{\Delta_{PT}}{\sigma_{\uparrow} + \sigma_{\downarrow}} \simeq \frac{\Delta_{PT}}{2\sigma_{\text{tot}}} . \tag{100}$$

In complete analogy with P-violating effects (see (29)), this correlation also causes the precession of neutron spin around the $[\mathbf{k}_n \times \mathbf{I}]$ axis. The corresponding value of the rotation angle χ per unit length in a target sample is:

$$\frac{d\chi}{dz} = \frac{4\pi\rho}{k} \text{Re}\,(f_{\uparrow} - f_{\downarrow}). \tag{101}$$

In the optimal experimental conditions $z = 1/N\sigma_{\text{tot}}$ the corresponding angle of rotation is:

$$\chi = \frac{\text{Re}\,(f_{\uparrow} - f_{\downarrow})}{\text{Im}\,(f_{\uparrow} + f_{\downarrow})} . \tag{102}$$

3.2. Historical Background. Although the general remark that nuclear reactions of strong dynamical complexity are most likely to be sensitive to T-violation was done by Henley and Jackobsohn [47] long ago, this remark seemingly passed unnoticed till the experimental discovery [48] of CP-violation in K-meson decay. In the framework of CPT theorem that meant T-violation. This discovery brought a new wave of experimental and theoretical studies of T-violation in nuclear reactions. First experimental tests of T-violation in detailed balance (TVDB, see (87)) were carried out [49] in 1966. Simultaneously appeared first publications on nuclear-reaction theory in the presence of T-violation [50,35]. Mahaux and Weidenmüller [35] obtained the theoretical expression for T-violating amplitudes in case of two near-lying compound resonances and were the first to understand the above mechanism of resonance enhancement. However both experimental and theoretical efforts at that time were concentrated on the energy domain of overlapping resonances $\Gamma >> d$ (Ericson regime). Therefore Ericson [50] claimed the enhancement parameter to be $\sqrt{(W/\Gamma)} \sim 10$, where W was supposed to be of the order of spreading width of eq. (7). Mahaux and Weidenmüller [35] pointed out that W should be much smaller, reducing the enhancement factor $\sqrt{W/\Gamma}$ to unity. Thus the possibilities of isolated resonance region with really large resonance enhancements $d/\Gamma >> 1$ remained unnoticed. Much later Pearson and Richter [51] considered TVDB for *one* isolated resonance. This case (for its analogues in P-violation see diagrams T_2 and T_4 of Fig.1) in principle contains resonance enhancement but lacks the dynamical enhancement factor $\sqrt{N}$ typical for 2-resonance interaction. Moreover, in case of TVDB experimental observable (see below) this resonance enhancement is completely cancelled by the resonance enhancement of the T-invariant cross-section in the denominator. Therefore this mechanism remained unnoticed and main theoretical investigations of TVDB

[52,53,54,16] were centered on energy-averaged quantities for strongly-overlapping resonances.

The full significance of both dynamical and resonance enhancements in T-violation was first realized by Gudkov and the author [4,7] in 1982, when we started the theoretical analysis of the newly suggested [55,56] P- and T-violating triple correlation in neutron transmission and predicted 5 + 6 orders of possible enhancement for this effect on p-resonances interacting with near-lying s-ones. Later on we [57,38] studied the resonance enhancement of P-conserving five-fold correlations (96)—(98). The same investigations were done independently by Barabanov [58]. Ironically enough, in both investigations the resonance-resonance T-violating term in the amplitude (analogue of T_1 term in P-violation) was unnoticed and only analogues of T_2 and T_4 were considered. This mistake was finally corrected in 1988 by the author [8], who realized the possibilities of both dynamical and resonance enhancements for this type of correlation. Since however the FC contains an extra $|\mathbf{k}|$ factor in comparison to TC, this results in the extra hindrance factor (see eqs. (74)—(76) and the discussion following them) of (kR) order which reduces the overall enhancement of FC to more modest 2 + 4 orders of magnitude.

Detailed balance tests for a close-lying pair of resonances in isolated resonance regime was first considered by Weidenmüller and the author [6]. Both dynamical and resonance enhancement effects were found in that case together with possible «true» structural enhancement. However, the measured quantities in TVDB show even more complicated interference energy behaviour and the conditions for observing the maximal possible effect are even more involved. The net enhancement in realistic conditions was found to essentially depend on experimental energy resolution and was estimated by us as $10^3 + 10^4$. However, recently Mitchel and co-workers [9] generalized our analysis by including the angular dependence of observable quantities. This led to even more complicated two-dimensional picture for the effect as a function of energy and angle. However, this more complicated analysis brought even more optimistic estimates. Analyzing their own high-resolution experimental data on (p, p) and (p, α) reactions, obtained at Duke, the authors proved that it is possible to obtain enhancements up to $10^4 + 10^5$. This fact together with various difficulties characteristic of other types of T-violating experiments (see below) makes the TVDB tests for interfering resonances perhaps the best possible way of T-noninvariance observation in the nearest future.

3.3. T- and P-Violation (Triple Correlation). The triple correlation quantities Δ_{PT} and $d\chi/dz$ of (99) and (100) can be analyzed in complete analogy to P-odd quantities Δ_P and $d\Phi/dz$ (see [3,7]) by substituting the T- and P-violating interaction iV_{PT} instead of P-violating weak interaction V_W into the Born

amplitude (39). Analysis of the resulting analogue of eq. (40) shows again that c–c mixing amplitude

$$T_1^{PT} \approx \frac{\gamma_p^n \cdot v_{PT} \cdot \gamma_s^n}{(E - E_p + i\,\Gamma_p/2)\,(E - E_s + i\,\Gamma_s/2)} \tag{103}$$

dominates, since it contains both factors of dynamical v/D and resonance D/Γ enhancement.

Inserting this amplitude into (99) gives [3,7]:

$$\Delta_{PT} = \frac{2\pi}{k^2}\, G_J\, \frac{\gamma_p^n \cdot v_{PT} \cdot \gamma_s^n}{[(E - E_p)^2 + \Gamma_p^2/4]\,[(E - E_s)^2 + \Gamma_s^2/4]} \times$$

$$\times\, [(E - E_s)\,\Gamma_p + (E - E_p)\,\Gamma_s] \tag{104}$$

with

$$G_J = \frac{1}{2}\sqrt{\frac{3}{2(2I+1)}}\left[\sqrt{\frac{2I+1}{2I+3}}\,\delta_{J,I+1/2}\,\delta_{c,I-1/2} + \right.$$

$$\left. + \sqrt{\frac{I}{I+1}}\,\delta_{J,I-1/2}\,\delta_{c,I+1/2}\right]. \tag{105}$$

Here J is the total compound resonance spin and c is the channel spin.

In p- and s-resonance points we have maxima:

$$(\Delta_{PT})_{\text{res}} \approx \frac{8\pi}{k^2}\, G_J\, \frac{v_{PT}}{D}\, \frac{\gamma_p^n \cdot \gamma_s^n}{\Gamma}\,. \tag{106}$$

Exactly in the same way we obtain:

$$\frac{d\chi}{dz} = -\frac{4\pi}{k^2}\, G_J\, \frac{\gamma_p^n \cdot v_{PT} \cdot \gamma_s^n}{[(E - E_p)^2 + \Gamma_p^2/4]\,[(E - E_s)^2 + \Gamma_s^2/4]} \times$$

$$\times \left[(E - E_p)\,(E - E_s) - \frac{\Gamma_s\,\Gamma_p}{4}\right]. \tag{107}$$

Observe that (107) changes sign at $E \approx E_p$ or E_s and shows maxima at $E_{p,s} \pm \Gamma_{p,s}/2$:

$$\left(\frac{d\chi}{dz}\right)_{\text{res}} \approx \frac{8\pi}{k^2}\, G_J\, \frac{v_{PT}}{D}\, \frac{\gamma_p^n \cdot \gamma_s^n}{\Gamma}\,. \tag{108}$$

Introducing the scaling factor λ between the P-, T-violating interaction $\tilde{v}_{PT}$ and the weak one $\tilde{v}_p$, we see that in average the P-, T-violating observed effects will be enhanced in the same p-resonances as P-violating ones:

$$\eta \approx \lambda P, \tag{109}$$

$$\chi \approx \lambda \Phi. \tag{110}$$

It is worth mentioning that all the present gauge-invariant theoretical models consider only the simultaneous P- and T-violation giving a wide range of λ between 10^{-4} and 10^{-15} (see, e.g., [46, 59]). Since the existing experimental constraints on the theory are given only by the case of K-meson decay and by the upper bound on neutron electric dipole moment (EDM), there is a special branch of CP-violation theory called «model-building» — anybody can construct his own branch of CP-violation theory provided that it does not contradict the above 2 experimental constraints. Moreover, λ enters those constraints in a model-dependent way. Thus *any* additional constraint on λ obtained from TC measurements could help a lot in narrowing the class of acceptable CP-violation models. It also seems that after 3 decades of constant improving the methodics of EDM measurements, the experimentalists had already exhausted their possibilities there. Mind also, that EDM does not contain the above 6 orders of enhancement, which somewhat resemble the good old Wolfenstein enhancement of CP-violation in K-mesons. Therefore FC measurements might rank as highest-priority ones from the point of view of its «fundamentality».

There are however grave difficulties with the idealized transmission experiments we were analyzing above in search for FC, and we ourselves were the first to realize their presence [60]. Indeed, the simplest way seems to choose $\mathbf{I}$ and $\mathbf{k}$ directions along, say, x and z axes, while directing $\boldsymbol{\sigma}$ parallel or anti-parallel to y axis. However, in order to polarize the target we need external (and rather strong) magnetic field $\mathbf{H}$. This field would cause Larmour precession of $\boldsymbol{\sigma}$ around $\mathbf{H}$ as soon as the neutron enters the target. This precession in its turn produces non-zero helicities (of different signs for initial cases of $\boldsymbol{\sigma} \uparrow\uparrow \mathbf{y}$ and $\boldsymbol{\sigma} \uparrow\downarrow \mathbf{y}$). Those helicities would cause the P-odd difference in transmission coming from «normal» weak interaction and considered in the previous subsection. Moreover, this weak interaction would start rotating $\boldsymbol{\sigma}$ around the $\mathbf{k}$ direction as well (see ex. (62)). Therefore the neutron spin starts wobbling in 3 dimensions in almost unpredictable way. Since all the CP-violating theories agree that $\lambda << 1$, those effects would completely camouflage the T-odd correlation we are looking for. To make the situation worse, even if we manage to keep the target polarized without the strong external magnetic field, we still will face the so-called «nuclear pseudo-magnetism» (see [61]). This is the phenomenon caused by $\boldsymbol{\sigma} \cdot \mathbf{I}$ dependent part of nuclear strong interaction, which imitates the external magnetic field causing the $\boldsymbol{\sigma}$ precession around $\mathbf{I}$. Since this effect arises from nuclear interaction, it is quite strong (usually equivalent to several KG of magnetic field).

The only crude remedy we could suggest in 1984 (see [60]) was to compensate the nuclear pseudomagnetic field by fine-tuning the external field **H** (this is in principle possible since the direction of pseudomagnetic precession is not correlated with neutron magnetic moment). We fully realized that such a solution is rather awkward in practice, since one needs to control this compensation by measuring neutron spin rotation angle with high precision $-\delta\phi \sim 10^{-(3+5)}$. In principle much higher precision of 10^{-6} was reached [27] in measurements of P-violating value Φ of eq. (75), but in our case one should do these high-precision measurements simultaneously with the measurements of FC itself.

On publishing this kind of «experimental» proposal [60] we expected to hear the reaction of professional experimentalists. Our expectations lasted for about a decade. Only in 1993 we received first response. One suggestion was presented by KEK group of Masuda [62] and actually contains a refined version of our magnetic field fine-tuning. The other suggestion came from PNPI group of Serebrov [63]. It involves the simultaneous measurements of polarization and asymmetry in transmission of initial longitudinally-polarized neutrons, and seem to be free of the above camouflaging effects. It remains, however, to check the energy dependence of the much more complicated observable suggested in this experiment in order to see whether the above enhancement effects survive in it and to estimate the accuracy of this experiment in realistic conditions.

3.4. P-Conserving T-Violating Transmission (Five-Fold) Correlation. We shall start with the T-violating part of the scattering amplitude f_T in eq. (97). It can be expressed (see [57,58]) for low-energy neutrons ($l = 0, 1, 2$):

$$f_T = \frac{A}{k}\Bigg[3\sqrt{2}\begin{Bmatrix} I & 2 & I \\ I-1/2 & J & I+1/2 \end{Bmatrix}[\langle I+1/2,\ 1|T^{TJ}|I-1/2,\ 1\rangle - \langle I-1/2,\ 1|T^{TJ}|I+1/2,\ 1\rangle] +$$

$$+ (-1)^{2I}\sqrt{\frac{3}{2I}}\,[\langle I+1/2,\ 2|T^{TJ}|I-1/2,\ 0\rangle - \langle I-1/2,\ 0|T^{TJ}|I+1/2,\ 2\rangle]\,\delta_{J,I-1/2} +$$

$$+ (-1)^{2I}\sqrt{\frac{3}{2(I+1)}}\,[\langle I+1/2,\ 2|T^{TJ}|I-1/1,\ 0\rangle - \langle I-1/2,\ 0|T^{TJ}|I+1/2,\ 2\rangle]\,\delta_{J,I+1/2}, \qquad (111)$$

$$A = (2J+1)\sqrt{\frac{5I(2I-1)}{(2I+1)(2I+3)}}\,(-1)^{I+3/2-J}.$$

Here $T = 1 - S$, S being the scattering matrix. Notation $\langle c', l'|T|c, l \rangle$ is used for transition matrix elements, where l and l' are orbital momenta of the initial and final channels, c and c' are the corresponding channel spins and J is the compound system spin. Upper T indices denote T-violating part of the T-matrix.

One might introduce, following Mahaux and Weidenmüller [35], the quantity $\delta S_{\lambda\mu} = S_{\lambda\mu} - S_{\mu\lambda}$ arising in the presence of T-noninvariant part V_T in the Hamiltonian, and consider the matrix $\delta S_{\lambda\mu} = -T^T_{\lambda\mu}$ in case of two interacting resonances, using the wave function expressions (18) for the initial and final channels μ and λ. This would give us the whole set of amplitudes similar to those of Fig.1 — the role of parity quantum number is «mimicked» in our case by channel spins. The analysis of different contributions to $T^T_{\lambda\mu}$ gives us essentially the same results as in P-violation. This analysis was schematically mentioned in connection with detailed-balance tests already in [35] and later in [5,6]. As usual, it boils down [5] to the dominant contribution of c–c mixing term, which demonstrates both dynamical and resonance enhancements:

$$T^T_{\lambda\mu} = 4\sqrt{2\pi}\cdot i \; \frac{\gamma_{1\mu}\,\gamma_{2\lambda}\,(V_T)_{12} + \gamma_{2\mu}\,\gamma_{1\lambda}\,(V_T)_{21}}{(E - E_1 + i\,\Gamma_1/2)\,(E - E_2 + i\,\Gamma_2/2)} \,. \tag{112}$$

Here E_i, Γ_i are the energies and total widths of the mixing compound resonances. The T-violating matrix elements between the compound states Φ_1 and Φ_2 $(V_T)_{12} = \langle \Phi_1|V_T|\Phi_2 \rangle = -(V_T)_{21}$ are purely imaginary

$$(V_T)_{12} = iv_T \,. \tag{113}$$

Therefore:

$$\mathrm{Im}\; T^T_{\lambda\mu} = \frac{(\gamma_{1\mu}\,\gamma_{2\lambda} - \gamma_{2\mu}\,\gamma_{1\lambda})\,v_T}{[(E - E_1)^2 + \Gamma_1^2/4]\,[(E - E_2)^2 + \Gamma_2^2/4]} \times$$

$$\times\,[(E - E_1)\,\Gamma_2 + (E - E_2)\,\Gamma_1] \,. \tag{114}$$

Substituting (114) into (111) and (97), we get the terms of p–p resonance mixing (we omit the geometrical factors):

$$\Delta_T(p_1, p_2) \approx \frac{4\pi}{k^2}\,\frac{(\gamma^n_{1(-)}\,\gamma^n_{2(+)} - \gamma^n_{2(-)}\,\gamma^n_{1(+)})\cdot v_T}{[(E - E_1)^2 + \Gamma_1^2/4]\,[(E - E_2)^2 + \Gamma_2^2/4]} \times$$

$$\times\,[(E - E_1)\,\Gamma_2 + (E - E_2)\,\Gamma_1] \tag{115}$$

and s–d resonance mixing terms:

$$\Delta_T(s, d) \approx \frac{4\pi}{k^2} \frac{(\gamma^n_{s(-)}\,\gamma^n_{d(+)} - \gamma^n_{s(+)}\,\gamma^n_{d(-)}) \cdot v_T}{[(E - E_s)^2 + \Gamma_s^2/4]\,[(E - E_d)^2 + \Gamma_d^2/4]} \times$$

$$\times [(E - E_s)\,\Gamma_d + (E - E_d)\,\Gamma_s]. \qquad (116)$$

The indices (+) and (−) stand for the channel spins. The order-of-magnitude estimates for the brackets with γ-amplitudes in (115), (116) are $(\gamma^n_p)^2$ and $\gamma^n_S\,\gamma^n_d$, respectively. Comparing this with the value Δ^p_{tot} of (61), we see that in both cases Δ_T is smaller by roughly (kR) factor arising from extra $|\mathbf{k}|$ value in FC. Otherwise both (115) and (116) show dynamical and resonance enhancements. However the optimal conditions for maximal observable $\beta(E)$ of eq. (98) differ from those of $P(E)$. Detailed analysis (see, e.g., [5]) demonstrates that observations in strong isolated p-resonance (when $\sigma_p(E_p) \approx \approx \sigma_{\text{tot}}(E_p)$) are most favourable. In this case the overall $(kR)^2$ smallness is compensated by resonance enhancement $(D/\Gamma)^2$ to give:

$$\beta_{\max}(E_p) \approx \frac{v_T}{D_{12}} \frac{\sigma_p(E_p)}{\sigma_{\text{tot}}(E_p)}. \qquad (117)$$

In the vicinity of s-resonance one gets from (116):

$$\beta(E_s) \approx \frac{\gamma^n_d}{\gamma^n_s} \frac{v_T}{D_{sp}} \sim (kR)^2 \frac{v_T}{D_{sp}}. \qquad (118)$$

The additional small (kR) factor makes those observations impractical. The main trouble with observations of (116) in d-resonance lies in the fact that exceedingly small Γ^n_d values make those resonances unobservable in σ_{tot}. If only we knew E_d in advance, then:

$$\beta(E_d) \approx \frac{\gamma^n_d}{\gamma^n_s} \frac{v_T}{\Gamma_d} \frac{D_{sp}}{\Gamma_s} \frac{\sigma_s(E_d)}{\sigma_{\text{tot}}(E_d)} \sim (kR)^2 \frac{v_T}{D_{sd}} \left(\frac{D_{sd}}{\Gamma}\right)^2. \qquad (119)$$

Observe that for very small D_{sd} (119) decreases drastically and transforms into (118) for $D_{sd} \approx \Gamma$. If the d-resonance lies sufficiently far away from s-resonance (say, $D_{sd} \sim 10$ eV) then the resonance enhancement factor $(D_{sd}/\Gamma)^2$ in (119) might almost compensate the smallness of $(kR)^2 \sim$ $\sim 10^{-5} \div 10^{-6}$. Since, however, the E_d are not known in advance, the d-resonance enhancement seems to be of purely academic interest, as it was

pointed in my ref. [5]. This point was misunderstood by the authors of ref. [30] who «re-discovered» the *s–d* mixing two years later (see also [64]). Unfortunately the only known target of ^{165}Ho suitable for FC measurements does not show any *p*-wave resonances [64]. Therefore the above possibilities of resonance enhancement (117) in FC were not used by the experimentalists up to now.

3.5. Detailed Balance Tests (TVDB). The simplest quantity, which describes the *T*-violation in detailed balance (TVDB) is (see eq. (87)):

$$\Delta_{DB}(E, \theta) = 2 \frac{\sigma_{ab}(E, \theta) - \sigma_{ba}(E, \theta)}{\sigma_{ab}(E, \theta) + \sigma_{ba}(E, \theta)} . \tag{120}$$

Since, as usual, the Δ_{DB} value is a ratio of experimentally measured quantities, we included the kinematic factors $k_a^2 (2s_a + 1) (2s_A + 1)$ into the value $\sigma_{ab}(E, \theta) = k_a^2 (2s_a + 1) (2S_A + 1) \, d\sigma_{ab}/ d\Omega (E, \theta)$. To simplify our analysis we shall also restrict ourselves with only E dependence of the cross section, omitting the θ dependence for the time being. Thus (120) would read:

$$\Delta_{DB}(E) = 2 \frac{\sigma_{ab}(E) - \sigma_{ba}(E)}{\sigma_{ab}(E) + \sigma_{ba}(E)} . \tag{121}$$

Any statistically meaningful deviation of this quantity from 0 would mean *T*-violation. However the experimental accuracy for absolute cross-section measurements is much less than for relative ones. Therefore it is preferable to do measurements at any rate in 2 different energy points E_I and E_{II} and construct a quantity:

$$\tilde{\Delta}_{DB}(E_I, E_{II}) = \frac{\sigma_{ab}(E_I) \, \sigma_{ba}(E_{II})}{\sigma_{ab}(E_{II}) \, \sigma_{ba}(E_I)} - 1. \tag{122}$$

Here one of the points, say E_I, is chosen for the normalization of the ratio. This allows one to cancel out most of systematic errors. This can be seen, since to first order in $\Delta(E)$ we have:

$$\tilde{\Delta}_{DB}(E_I, E_{II}) \simeq \Delta_{DB}(E_I) - \Delta_{DB}(E_{II}). \tag{123}$$

Therefore in most cases we shall proceed working with the simplest form (121). In doing so we shall use the general expression obtained in [35] for the difference $\delta S_{ab} = S_{ab} - S_{ba}$ between *S*-matrix elements, connecting channels *a* and *b* and caused by the presence of *T*-violating part V_T in the Hamiltonian. As usual, we shall consider the situation for only a pair of close-lying compound resonances 1 and 2. As we have already mentioned in the previous paragraph, the use of the wave functions (18) would give to first order in V_T

amplitudes similar to those considered in P-violation (Figs.1,2). These amplitudes are characterized by the same dynamical v/D and resonance D/Γ enhancement factors. Therefore the dominant contribution to δS_{ab} would come from c–c mixing amplitude of type T_1:

$$\delta S_{ab} = 4\sqrt{2\pi}\ \frac{(\gamma_{2a}\,\gamma_{1b} - \gamma_{2b}\,\gamma_{1a})\cdot v_T}{(E - E_1 + i\,\Gamma_1/2)\,(E - E_2 + i\,\Gamma_2/2)}\,. \tag{124}$$

Here $v_T = -\,i\,\langle\,\Phi_1|V_T|\Phi_2\,\rangle = i\,\langle\,\Phi_2|V_T|\Phi_1\,\rangle$ is the matrix element of V_T interaction between the compound resonances' wave functions Φ_1 and Φ_2 (see (17)).

Now, for the numerator of (121) we have:

$$\sigma_{ab} - \sigma_{ba} = 2\,[\mathrm{Re}\,(\delta S_{ab})\ \mathrm{Re}\,(S^0_{ab}) + \mathrm{Im}\,(\delta S_{ab})\ \mathrm{Im}\,(S^0_{ab})]. \tag{125}$$

While the corresponding expression for the denominator is:

$$\frac{1}{2}\,(\sigma_{ab} + \sigma_{ba}) = |S^0_{ab}|^2 = \left|\frac{\gamma_{1a}\,\gamma_{1b}}{E - E_1 + i\,\Gamma_1/2} + \frac{\gamma_{2a}\,\gamma_{2b}}{E - E_2 + i\,\Gamma_2/2}\right|^2. \tag{126}$$

Then for the case of two weakly overlapping resonances ($\Gamma < |E_1 - E_2| \equiv D$) we get (see [6]):

$$\Delta_{DB}(E) = \frac{v_T\cdot(|\gamma_{1b}\,\gamma_{2a}| - |\gamma_{2b}\,\gamma_{1a}|)\,(\Gamma_1\,|\gamma_{2a}\,\gamma_{2b}| + \Gamma_2\,|\gamma_{1a}\,\gamma_{1b}|)}{[(E - E_2)\,|\gamma_{1a}\,\gamma_{1b}| + (E - E_1)\,|\gamma_{2a}\,\gamma_{2b}|]^2 + 1/4\,(\Gamma_2|\gamma_{1a}\,\gamma_{1b}| + \Gamma_1|\gamma_{2a}\,\gamma_{2b}|)^2}\,. \tag{127}$$

The analysis of this expression shows that it reaches its maximal value in the interference minimum of the cross-sections $|S^0_{ab}|^2$, i.e., when

$$E = E_0 \equiv \frac{|\gamma_{1a}\,\gamma_{1b}|\,E_2 + |\gamma_{2a}\,\gamma_{2b}|\,E_1}{|\gamma_{1a}\,\gamma_{1b}| + |\gamma_{2a}\,\gamma_{2b}|}\,. \tag{128}$$

In case of $\Gamma_1 \approx \Gamma_2 = \Gamma$ this yields for (127):

$$\Delta_{DB}(E_0) \simeq 4\,\frac{v_T}{\Gamma}\,\frac{|\gamma_{2b}\,\gamma_{1a}| - |\gamma_{1b}\,\gamma_{2a}|}{|\gamma_{1a}\,\gamma_{1b}| - |\gamma_{2a}\,\gamma_{2b}|}\,. \tag{129}$$

Supposing for simplicity $\Gamma_{1a}\,\Gamma_{1b} \approx \Gamma_{2a}\,\Gamma_{2b}$ (i.e., equally strong resonances) we obtain:

$$\Delta_{DB}(E_0) = 2\,\frac{v_T}{\Gamma}\left[\left|\frac{\gamma_{2b}}{\gamma_{1b}}\right| - \left|\frac{\gamma_{2a}}{\gamma_{1a}}\right|\right] \equiv \frac{v_T}{\Gamma}\,f, \tag{130}$$

where $f = 2\,[|\gamma_{2b}/\gamma_{1b}| - |\gamma_{2a}/\gamma_{1a}|]$, and the position of the interference dip of σ_{ab} is given by:

$$E_0 \approx \frac{1}{2}\,(E_1 + E_2).$$

Thus we observe how the already familiar factors of dynamical and resonance enhancement give in (130) the overall enhancement factor v/Γ. We also see that (130) can be sometimes enhanced even more by the «real» structural enhancement factor f (compare with eq. (73a) for the «inelastic channel» observable α_{nf} in P-violation). To be realistic however, one should take into account the finite experimental energy resolution ΔE, which usually exceeds Γ and smears the whole interference picture, bringing the observed effect down to $(v_T/\Delta E)\,f$. Making now a conservative assumption $f \approx 1$, we get the overall enhancement factor in $\Delta_{DB}(E_0)$ to be

$$\frac{\tilde{v}}{\Delta E}\,. \tag{131}$$

Remember that $\tilde{v}$ is the variance of strong-interaction matrix element (see (9)) which is of the order of 1 MeV. Therefore the net enhancement of TVDB is about 10^3.

This was essentially the result of our analysis with Weidenmüller [6]. As I already mentioned, recently the TUNL-Duke group [9], which is the world authority in fine-resolution experiments, generalized our approach to include the θ-dependence of eq. (120) in it. After performing a tedious analysis of their own experimental data on (p, p) and (p, α) reactions, they concluded that there are *real experimental* situations when the enhancement in TVDB reaches $10^4 \div 10^5$. In view of specific difficulties which mark the TC and FC experiments, this seems to be the most realistic experimental way of T-invariance measurements in the nearest future.

3.6. Brief Summary of Possible Enhancements. Thus we have seen that the dominant contribution to all the above effects of T-violation comes from c–c mixing amplitude of the type T_1 in Fig.1. This amplitude contains two basic enhancement mechanisms — dynamical enhancement $\tilde{v}/D$ and resonance enhancement D/Γ. Since however the experimental observables are different, those mechanisms manifest themselves in different manner. Therefore in the optimal conditions we might have the following enhancements:

For TC in the vicinity of E_p, provided that $\sigma_s(E_p) \approx \sigma_p(E_p) \geq \sigma_{\text{pot}}$ (see (82a)):

$$\eta_{\max} \sim \frac{\tilde{v}}{\Gamma}\,, \qquad \chi_{\max} \sim \frac{\tilde{v}}{\Gamma}\,. \tag{132}$$

For FC in the vicinity of strong isolated p-resonance ($\sigma_p(E_p) \geq \geq \sigma_s(E_p) + \sigma_{pot}(E_p)$):

$$\beta_{max} \sim \frac{\tilde{v}}{D}. \tag{133}$$

In this case the extra (kR) factor with respect to FC cancels D/Γ.

For TVDB in interference minimum of two close-lying resonances:

$$\Delta_{DB}^{max} \sim \frac{\tilde{v}}{\Gamma} f_1(\theta), \tag{134}$$

where $f_1(\theta)$ might serve as an additional enhancement factor. In realistic conditions, when experimental energy resolution $\Delta E > \Gamma$, we have:

$$\Delta_{DB}^{max} \sim \frac{\tilde{v}}{\Delta E} f_1(\theta). \tag{135}$$

Of greatest interest to the «high-brow» gauge theories of CP-violation is TC, since it is both T- and P-violating. However, I remind that estimates (152) were obtained by us [4,7] for idealized experimental case without Larmour and pseudomagnetic effects.

Among the remaining, purely T-violating effects, TVDB has obvious advantages compared to transmission FC. Its enhancements (135) for already known experimental cases reach $10^4 + 10^5$, while the only available target for FC shows no p-resonances and therefore lacks even the enhancement of (133).

IV. STATISTICAL APPROACH TO COMPOUND-RESONANCE MEASUREMENTS

1. Nuclear Chaos and Necessity of Statistical Approach. The analysis of the previous section shows that in the isolated resonance regime $\Gamma << d$ (d is the *average* spacing between compound resonances of the same spin) the symmetry breaking interaction of a pair of close-lying resonances with energy separation D leads to two major enhancements — dynamical enhancement v/D and resonance enhancement D/Γ, which very often combine with other specific factors of nuclear reaction theory to produce the net enhancement v/Γ, reaching $5 \div 6$ orders of magnitude. Both enhancements result from a complexity of nuclear compound resonances and practically disappear in simple nucleon-nucleon scattering (see eq. (47)). To be more specific, they manifest quantum chaos, whose idea is still rejected by the majority of professional «chaotists», but accepted (at any rate on intuitive level) by all the nuclear

physicists. Establishing the connection between the fully recognized chaos of classical mechanics and quantum chaos of nuclear physics is an interesting and perspective problem (see, e.g., [36]), whose solution shows that a generic feature of any chaoticity (both in quantum and classical mechanics) is the lack of symmetries in the Hamiltonian of the system. In compound nucleus this lack of symmetries is caused in the first place by the strong pair-wise «residual» interactions, which remove practically all the degeneracies connected with the mean field symmetries (let us call it «strong» chaos) and thus lead to the exponential increase of level density $1/d$. This chaoticity also increases the complexity N of the compound resonance wave function and randomizes the signs of its basic («simple») components' amplitudes. In plain words this means that the incident nucleon quickly distributes its energy among the target nucleons and gets entrapped in a compound system. This, together with small barrier penetration factors, strongly reduces the contribution of particle-emission channels to the resonance total width Γ leaving only the γ-emission. The same complexity of the resonance wave functions considerably reduces the gamma widths. All this results in large compound-resonance lifetimes $\tau \sim 1/\Gamma$ which enter the above enhancement factors. This seems to be a rare occasion, when complexity helps us, leading directly to 6 orders of magnitude enhancement of the experimentally observed effects.

However, one has to pay for everything. The above complexity of compound resonance wave functions Φ makes the head-on calculations of the «weak chaos» symmetry-breaking (WSB) matrix elements $\langle \Phi_1 | V_{SB} | \Phi_2 \rangle$ completely hopeless. Therefore even if we observe the WSB effect and manage to extract the corresponding v_{WSB} out of it (which might be a problem in itself), we seem to learn nothing about the strength constant of the WSB interaction. Coming back to the origin of strong chaos, we see that the above problem with weak interaction matrix elements v_{WSB} differs from the same problem with strong interaction ones only by the strength constants F (see (6)—(9)) which we are hunting for. The problem of strong quantum chaos and strong symmetry breaking matrix elements was faced and physically understood at the dawn of nuclear physics and led to Niels Bohr's hypothesis of compound nucleus, which does not «remember» its formation, and to Weisskopf's idea of black absorbing nucleus. A more refined mathematical technique of nuclear Hamiltonian random matrices was developed by Wigner, Dyson, Mehta and other outstanding physicists in 50-ies. In this approach the expansion coefficients c_i of Φ as well as the matrix elements $\langle \Phi_i | V | \Phi_k \rangle$ are considered to be random numbers varying from resonance to resonance while the value v for the ensemble of individual resonances obeys the normal distribution law with zero mean and variance $\tilde{v} = \sqrt{\langle v^2 \rangle}$. This led to various statistical predictions concerning the properties of

resonances which were all brilliantly confirmed experimentally: the Wigner distribution law for level spacing, the Porter-Thomas law for neutron width distribution and corresponding laws for γ-widths distributions. Therefore the problem of WSB presented practically nothing new to us. This fact was intuitively recognized in the analysis of WSB for a pair of isolated bound states or resonances from the very beginning (see, e.g., the conception of dynamical enhancement factor $\sim \sqrt{N}$ in [15]). Therefore all the order-of-magnitude theoretical estimates of enhancements from the very beginning (see, e.g., [26,4,7]) were done actually for ensemble-averaged variance. We also predicted the sign randomness of the observed effects in c–c mixing mechanism and possible constancy of sign (connected however with loss of dynamical enhancement factor) for valence mechanism [7]. However the intricacies of specific nuclear reaction enhancements for various observables discussed above were so exciting, while the statistics of experimental observations was so meager, that we were postponing the problem of meaningful analysis for experimentally observed values.

2. Energy Averaging. In the meanwhile a highly professional and sophisticated statistical theory of symmetry breaking in nuclear reactions was developing. For purely historical reasons it was essentially concentrated on TVDB effects. After the observation was made [47] that in the two-channel case detailed balance follows from unitary alone (without T-invariance), theoretical [50,52,53,54] and experimental [49,65] interest shifted to the domain of many open channels and strongly overlapping resonances ($\Gamma >> d$). Even the explicit appearance of Γ in the denominator of TVDB energy-averaged expression [50] remained unnoticed. Mahaux and Weidenmüller derived the two-resonance expression (112), (124) which allowed 2 decades later me [5], Weidenmüller and me [6] to see the large enhancements in FC and TVDB, discussed in the previous section. But they also applied it to $\Gamma >> d$ regime of Ericson fluctuations to show that the only enhancement in this regime is the structural factor f (see eq. (130)). Later on Moldauer [52] obtained the same results in R-matrix formalism, again sticking to $\Gamma >> d$ region. He also considered TVDB in direct reactions (see also [66]) proving that direct reaction mechanism contribution to T-violation is 3 orders of magnitude smaller than that of compound resonance mechanism for $\Gamma >> d$ (unfortunately, on obtaining this result he formulated it in a somewhat misleading way — the direct reaction sensitivity to T-violation is 3 orders of magnitude smaller than that of compound resonance reactions).

A new wave of statistical approach to energy-averaged TVDB which involved a full scale mathematics of random-matrix theory developed in [67,68] started with the papers [53,54]. Only in 1989 the technique developed in those papers was first applied by Davis [69] to numerical calculations of energy-averaged FC and TVDB — numerical since the method involved the computation of rather complicated multi-dimensional integrals. «An informed

guess» allowed Davis to approximate in semi-analytical form the results of exact numerical integration for the energy-averaged values of $\langle \Delta_T^2 \rangle$ (see eq. (115) for FC) and $\langle (\sigma_{ab} - \sigma_{ba})^2 \rangle$ (see eqs. (124)—(125) for TVDB) in the limit of isolated resonances $\Gamma < 0.1d$. The corresponding expression for FC was given (omitting the trivial $4\pi/k^2$ factor) as

$$\langle \Delta_T^2 \rangle \approx 4T_{n(-)}T_{n(+)} \frac{2\pi^2 \langle v_T^2 \rangle}{d^2} \frac{(0.35 - 0.17 \ln t)}{t}, \qquad (136)$$

where the neutron transmission coefficients for channel spins (±) are given by

$$T_{(\pm)} = \frac{2\pi \Gamma_{n(\pm)}}{d}; \qquad t = \frac{2\pi \Gamma}{d}.$$

Since by this time the resonance enhancement effect for 2 interacting isolated resonances was already recognized, ref. [69] was the first recognition of the fact that resonance enhancement survives also in *energy-averaged* effects. Moreover, the necessity to pay attention to the isolated resonance regime ($\Gamma << d$) was also accepted. However a statement was made in favour of using *energy-averages* of observables for this regime as opposed to the statistics of individual *on-resonance* observations, which I (and the experimentalists) kept in mind all the time. It took me some time and a bit of reasoning to persuade the author of [69] that his point of view is rather academic than practical. My arguments were as follows: Consider the FC expression (115) in the Γ-vicinity of each resonance energy assuming the typical case $(E_1 - E_2) \approx d$, $\Gamma_1 \approx \Gamma_2 \approx \Gamma$:

$$\bar{\Delta}_T^{\text{res}} \approx 4 \frac{v_T}{d} \frac{a_{12}}{\Gamma}. \qquad (137)$$

Here $a_{12} = \gamma_{1(-)}^n \gamma_{2(+)}^n - \gamma_{2(-)}^n \gamma_{1(+)}^n$ and the $4\pi/k^2$ factor is omitted for the sake of comparison with (136). The same expression for (113) in a typical situation between the resonances $|E - E_1| \approx |E - E_2| \sim d$ would be:

$$\bar{\Delta}_T = 2 \frac{v_T}{d} \frac{a_{12}}{d} \frac{\Gamma}{d}. \qquad (138)$$

This expression is a factor of $(d/\Gamma)^2$ smaller than Δ_T^{res}. Let us estimate in a simple-minded way the energy-averaged effect (136) starting from (137) and (138) and averaging their squares over an interval d. Then the $(\Delta^{\text{res}})^2$ contribution should be weighed by roughly a factor of Γ/d, while the $(\bar{\Delta})^2$ should have a weighing factor $(d - \Gamma)/d \approx 1$:

$$\langle \Delta_T^2 \rangle \approx (\Delta_T^{res})^2 \frac{\Gamma}{d} + \bar{\Delta}_T^2 \approx 16 \frac{v_T^2}{d^2} \frac{a_{12}^2}{\Gamma^2} \frac{\Gamma}{d} + 4 \frac{v_T^2}{d^2} \frac{\Gamma^2}{d^2}. \qquad (139)$$

Thus we calculated the contribution to (136) from the interval d around one particular resonance. It remains now to average the v_T^2 and a_{12}^2 parameters over all the possible resonances. Since γ's in a_{12} are uncorrelated random variables $\langle a_{12}^2 \rangle = 2\Gamma_{(-)}^n \Gamma_{(+)}^n$. Using the Γ/d smallness, we shall retain in (138) *only the on-resonance contribution.* Thus

$$\langle \Delta_T^2 \rangle \approx 32 \frac{\Gamma_{n(-)}}{d} \frac{\Gamma_{n(+)}}{d} \frac{\langle v_T^2 \rangle}{d^2} \frac{d}{\Gamma} = 4T_{n(-)}T_{n(+)} \frac{2\pi^2 \langle v_T^2 \rangle}{d^2} \frac{2}{\pi^2 t}. \qquad (140)$$

We see that this crude but simple picture almost exactly reproduces the results of (136). The only marked difference is substitution of $2/\pi^2 \approx 0.2$ factor for the $(0.35 - 0.17 \ln t)$. This difference comes essentially because we took from the start a fixed value d for the inter-resonance distance $(E_1 - E_2)$. The actual distribution of this distance obeys a more complicated two level correlation law (see the correlation function R_2 of [70]), which, apart from Wigner repulsion at very small distances allows for all the possible values between 0 and d. If one allows $(E_1 - E_2)$ to vary in this way, the distances smaller than d will contribute more to the resonance effect (137) thus increasing the value $\langle \Delta_T^2 \rangle$. Indeed, energy integration of (115) with the correlation function (see [71]) gives the analytical result exactly equal to (136).

The main point of the above simple arithmetic is that *only the small* Γ*-vicinity of each resonance contributes* to $\langle \Delta^2 \rangle$. It turns out therefore that in order to compare my theoretical value (136) with experiment and to extract $\langle v_T^2 \rangle$, I ask the experimentalist to make accurate measurements not only on the resonance curve, but also in the whole «empty» interval $d >> \Gamma$ between the resonances. At that I know for sure that all these tedious off-resonance measurements would give null results. I also know that in this way I would decrease the sensitivity of $\langle v_T^2 \rangle$ definition by a factor of d/Γ. In order to compensate this loss the experimentalist has to increase the accuracy in measuring each null effect by, say, increasing the beam flux by a huge factor $(d/\Gamma)^2$. One should also recollect that transmission experiments around the strong s-resonances are impossible, or, at best suffer poor statistics. All this almost devoids energy-averaged calculation *for isolated resonances* of any practical meaning. I want to stress this point because of repeated attempts to compare the energy-averaged quantities in this regime with experimentally observed ones. Even in our joint

paper [71], after reading the above critical comments on energy-averaging, one meets a vague statement that it is still appropriate for ^{165}Ho case where there are no observable p-wave resonances below 100 eV. This statement is again a purely academic one* — if you observe no p-resonances in σ_{tot} this just means that either the p-resonance spacing d in this energy-region is anomalously large or that Γ_p^n are anomalously small. Both facts should be somehow taken into account in the «unbiased» estimates (136) as additional biasing and this would lower its value. Even then it would be necessary to drop out of thus biased estimate (136) the unobservable regions around strong s-resonances. One might still hope to get something from averaging the $(s - d)$ mixture term (116). Seemingly one should get for $\langle \Delta_T^2 (s - d) \rangle$ practically the same value as (136). However here comes another very serious danger of energy-averaging. We had already noticed that the $(-\ln t)$ term in brackets of (136) which dominates in our case of small t appears because unbiased energy-averaging procedure favours the situations when the mixing resonances lie anomalously close to each other ($D_{sp} << d$). But exactly in those situations the denominators of observable β (see (98)) would exhibit strong s-resonance maxima which reduces the resonance enhancement of the numerators practically to zero. Therefore the averaged *observed* quantity β would strongly deviate from the calculated $(\langle \Delta^2 \rangle)^{1/2} / \langle \sigma_{tot} \rangle$ because of the strong correlation of the numerator and denominator. In terms of experimentally measured quantities of the type (21)—(25) this means that experimental measurements would be impossible or give extremely poor statistics in exactly the same energy intervals (near strong s-resonances) which mostly contribute to the calculated values of $\langle \Delta^2 (s - d) \rangle$. Together with the above loss of sensitivity by a factor of Γ / d and the abundance of strong s-wave resonances in experimentally observed spectrum of Ho this just means that Ho measurements might be a waste of time. Conclusions to the same effect concerning the energy-averaged estimates of P-violation observables done by Koonin et al. [72] with the aid of optical model functions were reached by Weidenmüller and Lewenkopf [34] — thus averaged quantity lacks resonance enhancement and *bears no relation to the experimental observable P of eq. (20).*

To finish the matter of energy averaging, I mention the recent publication [73] on FC experiments with 2 MeV polarized neutrons in ^{165}Ho, where the theoretical analysis is done in the spirit of a very simple model of T-violation in direct reactions considered almost 30 years ago by Moldauer [66]. I already

*It is unusual to disagree with one's own publication. The main conception and the analytical part of [71] were finished before the end of 1989. However the final text was written only 4 months later, when I was out of reach in Russia and 2 other co-authors in Arizona and Heidelberg, respectively.

mentioned that actually Moldauer had shown in [52,66] that the contribution to T-violating amplitudes from compound resonance mechanisms is about 3 orders of magnitude larger than from direct interaction ones — the fact, which is quite obvious in terms of resonance enhancement physics (time spent by the particles inside the T-violating nuclear field is much larger for compound processes than for direct ones). Therefore I cannot understand why (apart from its extreme simplicity) the authors of [73] applied the direct reaction analysis to their data. This is especially strange, since this experimental energy range might be the most appropriate place to apply the «big guns» of complicated numerical integration worked out [69,74] by one of the authors of ref. [73].

3. «On-Resonance» Ensemble Averaging. Having thus discussed the drawbacks of «unbiased» energy-averaging in isolated resonance regime $\Gamma << d$, we are coming back to the natural idea discussed in Subsection IV.1, namely, to follow the lines of well-developed statistical approach to «strong» chaos of neutron resonances and consider the ensemble of weak-interaction v_{pi} values, measured in different p-resonances as an ensemble of random numbers obeying the normal distribution law with zero mean and variance $\tilde{v}_p \equiv M = \sqrt{\langle v_p^2 \rangle}$:

$$P(v_p) = \frac{1}{\sqrt{2\pi M^2}} \exp\left(- \frac{v_p^2}{2M^2} \right). \tag{141}$$

Thus any particular value of v_p obtained from one on-resonance measurement is of minor importance and the main interest is shifted to the variances M. If we are able to extract the value of M from a set of on-resonance observations (and we shall see below how intricate this extraction might be), then we can use the scaling trick (see eqs. (6)—(8)) and compare this value with its equivalent $\tilde{v}$ for strong interactions, obtained from a well-established quantity of the spreading width Γ_{spr} of eq. (7). This comparison would give us the scaling constant

$$F = \frac{M}{\tilde{v}}, \tag{142}$$

or, at any rate, an upper bound on it.

Everything seems fine in such a simplified scheme. However, the realistic situation is much more complicated. To begin with, in deriving all the expressions for weak symmetry breaking quantities of the previous Section we retained for simplicity only one resonance in the initial and one in the final channel. In principle all the observables of Sec. III should contain a double sum over all the mixing resonances. Since we consider the on-p-resonance measurements in the isolated-resonance regime, the contributions of all the other p-re-

sonances to the observed effect would be smaller by at any rate a factor of $(d/\Gamma)^2$ and can be discarded. However the other sum still remains, and the correct expression for, say, $\Delta^P_{\rm tot}$ at E_p would be:

$$\Delta^P_{\rm tot}(E_p) = \frac{8\pi}{k^2}\frac{\gamma^n_{pi}}{\Gamma_{pi}}\sum_s \frac{\langle s|V_W|p_i\rangle}{E_p - E_s} \equiv \sum_s A_{is}(v_p)_i ,$$

$$A_{is} = \frac{8\pi}{k^2}\frac{\gamma^n_{pi}\cdot\gamma^n_s}{E_p - E_s}, \qquad (v_p)_i = \langle s|V_W|p_i\rangle. \tag{143}$$

Assuming that all the parameters γ^n_p, γ^n_s and E_s, are known, $\Delta^P_{\rm tot}(E_{pi})$ is a sum if Gaussian distributed random variables $(v_p)_i$ with fixed coefficients. Since a sum of Gaussian random variables is also a Gaussian random variable $\Delta^P_{\rm tot}(E_p)$ itself is a Gaussian. However it is not ergodic in the sense of ref. [75]. Ergodicity here means that the statistical ensemble average of an observable (whose behaviour we know theoretically) is equal to the running average of the same observable taken over a set of experimentally observed resonances. For $\Delta^P_{\rm tot}(E_p)$ to be ergodic, it is necessary that it should be independent of the parameters of the actually investigated resonances. In plain words this means that we should get rid of all the trivial constants known for each particular resonance p_i, and consider the new value whose variance is given by $\langle v^2_p\rangle$. Since all the $(v_p)_i$ in (143) have the same variance, such an ergodic variable in the present case would be

$$\Delta^P_i = \frac{\Delta^P_{\rm tot}(E_{pi})}{|\sum_s A^2_{is}|^{1/2}}. \tag{144}$$

In the one s-resonance approximation $\Delta^P_i = (v_p)_i$.

Let us now slightly complicate the situation. Before doing this, we shall step back to the original expression (61) for $\Delta^P_{\rm tot}$. In order to simplify the derivation we considered the case of zero-spin target $I = 0$. In this case the total compound-resonance spin J is completely defined by $\mathbf{j} = \mathbf{l} + \mathbf{s}$ of the neutron, and the partial width of the p-resonance admixed to s-one is simply $\Gamma^n_p = \Gamma^n_{p\,1/2}$, where 1/2 is neutron j-value. If, however, we remove the restriction $I = 0$, then the p-resonance partial width would contain two components $\Gamma^n_p = \Gamma^n_{p\,1/2} + \Gamma^n_{p\,3/2}$. In the channel spin representation (see, e.g.,

(111)—(116)) this corresponds to 2 different values (±) of the channel spin c. Since T-invariant interaction conserves c, only the $\Gamma^n_{p\,1/2}$ widths (and corresponding amplitudes $\gamma^n_{p\,1/2}$) would enter the Δ^P_{tot} expression. However in the majority of cases we know only the total Γ^n_p, while the values $\gamma^n_{p\,1/2}$ and $\gamma^n_{p\,3/2}$ are unknown. Therefore for $I \neq 0$ even the simplest form of on-resonance Δ^P_{tot} would be

$$\Delta^P_{\text{tot}}(E_{pi}) = \frac{8\pi}{k^2} \frac{\gamma_s^n}{\Gamma_{pi}(E_s - E_{pi})} \gamma^n_{p\,1/2} \cdot \nu_p \equiv B_{pi}\, \gamma^n_{pi}\, \nu_{pi}\,, \tag{145}$$

i.e., the product of a known constant B_p and the unknown $\hat{\Delta}_p = \gamma^n_{p\,1/2}\, \nu_p$. Now for different choices of p-resonances $\gamma^n_{p\,1/2}$ behaves as a Gaussian random variable with zero mean and variance $\langle \Gamma^n_{p\,1/2} \rangle$, which can be easily related to, say, neutron strength function. The p-wave amplitude $\gamma^n_{p\,1/2}$ and the matrix element ν_p are independent random variables. Thus we can introduce on-resonance ensemble of ergodic values:

$$\hat{\Delta}_{pi} = \frac{\Delta^P_{\text{tot}}(E_{pi})}{B_i}\,. \tag{146}$$

This random values are, however, distributed according to the law which governs the distribution of a product of 2 independent Gaussian random variables:

$$P(\hat{\Delta}_p) = \frac{1}{\pi\omega} K_0(\hat{\Delta}_p/\omega), \tag{147}$$

where K_0 is MacDonald's function and $\omega^2 = \langle \Gamma^n_{p\,1/2} \rangle \langle \nu_p^2 \rangle$ defines the variance of $\hat{\Delta}_p$.

In the many-level generalization we have (see (143)):

$$\Delta^P_{\text{tot}}(E_p) = \gamma^n_{p\,1/2} \sum_s B_{ps}(\nu_p)_s \tag{148}$$

and can introduce the ergodic variable:

$$\hat{\Delta}_i = \frac{\Delta^P_{\text{tot}}(E_{pi})}{|\sum_s B_{is}^2|^{1/2}}\,, \tag{149}$$

which obeys the same distribution law (147).

Observe how the lack of knowledge of only one additional parameter $\gamma^n_{p\,1/2}$ of «standard» nuclear spectroscopy complicates the statistical analysis of WSB and data interpretation — instead of the well known analytic Gaussian shape of (142) for Δ^p_i, we get the much more complicated low (148) for $\hat{\Delta}_i$. One can also be sure that statistical confidence levels of variances, extracted with the help of (148) would be much lower than those obtained using (142).

The case of T-violating FC (eq. (116)) generalized for many levels gives the on-resonance expression:

$$\Delta_T(p) \approx \frac{16\pi}{k^2}\frac{1}{\Gamma_p}\sum_k \left(\gamma^n_{p\,1/2}\,\gamma^n_{pk\,3/2} - \gamma^n_{p\,3/2}\,\gamma^n_{pk\,1/2}\right) \times$$

$$\times \frac{(v_T)_k\,(E_p - E_{pk})\,\Gamma}{(E_p - E_{pk})^2 + \Gamma^2/4}\,. \qquad (150)$$

In a rather optimistic case we may know in this expression the parameters of the p-resonance, where we do the measurements, namely E_p, $\Gamma_p \approx \Gamma_{pk} = \Gamma$, $\gamma^n_{p\,1/2}$ and $\gamma^n_{p\,3/2}$. Since our information about p-resonances in general is very poor, the rest of parameters in (150) are very likely to be unknown. The analysis of this situation in [71] demanded numerical Monte-Carlo simulation for the distribution of distant E_{pk} and allowed one to built two approximate analytical expressions for the distribution of ergodic variable δ_T corresponding to (150). Both of them contain two-fold integrals (see [71]) for details).

This strikingly increasing complexity of distributions with the increase of unknown spectroscopic parameters serves a good lesson for experimentalists. If they want to extract useful information on WSB interaction constants rather than surprise the world with large P-violation effects, then they should try to do a good deal of dull job in «standard» spectroscopy in order to define as many spectroscopic parameters as possible. We shall have to strengthen this statement in the analysis below.

4. Analysis of Realistic Imperfect Experimental On-Resonance Measurements. The distribution analysis of the previous subsection concerned only the statistics for the «theoretical» values of observables arising from the chaotic nature of compound-resonances. However each on-resonance measurement of, say, $\hat{\Delta}_i$ value can be done with finite experimental error σ_i, and the experimental results x_i of the measurement in majority of cases would obey the normal distribution law:

$$P\left(x_i|\hat{\Delta}_i\right) = \frac{1}{\sqrt{2\pi\,\sigma_i^2}}\exp\left\{-\frac{(x_i - \hat{\Delta}_i)^2}{2\sigma_i^2}\right\}. \qquad (151)$$

Here we introduced the notation of conditional probability $P(a|b)$ of a given b. In our case the measured value $\hat{\Delta}_i$ itself is randomly distributed in accordance with the Gaussian law (141), which we shall denote as $P(\hat{\Delta}_i|M)$. Therefore the connection between the measured value x_i and M will be given by:

$$P(x_i|M) = \int d(\hat{\Delta}_i)\, P(x_i|\hat{\Delta}_i)\, P(\hat{\Delta}_i|M) =$$

$$= \frac{1}{\sqrt{2\pi\,(\sigma_i^2 + M^2)}} \exp\left\{-\frac{x_i^2}{2(\sigma_i^2 + M^2)}\right\}. \qquad (152)$$

For an infinite set of experimental measurements, x_i, with small errors, σ_i, one could plot the curve (152) and extract the M value by applying, say, the least square method. Even in this idealized case the problem of finding the confidence intervals ΔM for M is not clearly defined. However the realistic situation of imperfect measurements is much worse. In practice we might hope to get only few experimental points x_i with accuracy not exceeding a few σ_i. To dramatize the problem even more, consider a case when after years of hard experimental work we shall finally get an upper bound $x_1 \leq \sigma_1$ in the triple-correlation measurements on *La* resonance. What shall we do then in order to connect this upper bound with the corresponding upper bound on M?

Up to now denoting (151), (152) as conditional probabilities might seem to be an unnecessary terminological complication of simple things. But when we start considering the above case of imperfect measurements, only the conditional probability theory allows to solve our problems. Indeed, the exact formulation of the problem is: We have a theoretical expression (152) defining probability of experimental result x_i (with σ_i) for a given M value. We need to «inverse» (152) and find a probability $P(M|x_i)$ of M given an experimental result $x_i(\sigma_i)$. This problem is in principle easily solved by using the well-known Bayes theorem of standard conditional probability (CPr) theory:

$$P(M|x)\cdot P(x) = P(x|M)\cdot P(M) \qquad (153)$$

and putting it into the form:

$$P(M|x) = \frac{P(x|M)\cdot P(M)}{P(x)}. \qquad (154)$$

According to the same standard CPr theory the «unconditioned» probability

$$P(x) = \int P(x|M)\, P(M)\, dM \equiv N(x). \qquad (155)$$

Expressions (153)—(155) are given in any textbook on CPr and accepted by all the mathematicians. However the interpretation of (154) given by Bayes himself makes a special branch of *Bayesian statistics* (BS), which is criticized by the representatives of the more orthodox «frequency» school in mathematical statistics for its «subjectivity» (see an excellent and very brief review of this topic in [76]). Unfortunately BS is practically unknown to physicists. Unfortunately, since every physicist who ever worked with experimental data or with any kind of the above «inverse» problem intuitively felt the necessity of BS or even tried to apply it without realizing that this is BS. Bayes supposed that *before* we do any kind of measurements x of the physical quantity M, we often have some «a priori» knowledge concerning $P(M)$ — e.g., *before* measuring the mass we know that it is positive. Bayes theorem (154) formulates in a mathematically precise way how to combine this «a priori» knowledge with the results x of our measurement in order to obtain the «a posteriori» probability $P(M|x)$. One might associate this «a priori» $P(M)$ with considerations of common sense, which prompts to a physicist that application of orthodox statistical prescriptions to, say, negative experimental x_i obtained in the mass-measurement experiment leads to nonsense — he is sure that mass is positive, and that only poor accuracy of his measurement produced the negative x_i. But without BS the physicist does not know how to get out of this trap. The best solution might be to throw this result away and take a more precise measuring device. But what to do if this is the best at your disposal (and often the only one in the world)? In BS approach you just suppose the «a priori» $P(M) = \theta(M)$ and go ahead through (154), (155), obtaining a sensible upper limit on mass as an «a posteriori» result of your imperfect measurement (see [77], where Philip Anderson describes BS as «the correct way to do inductive reasoning from necessarily imperfect data»). In case when nothing is known «a priori» about M the standard assumption is (see [76]) that $P(M)$ is uniform and constant.

There is a close, but sometimes misleading connection between the Bayesian post probability (BPP) given by (154), and the maximal likelihood method (MLM) described in numerous manuals on statistics for the experimentalists (e.g., [78]). One might characterize MLM as an attempt to use Bayesian statistics without recognizing it. Indeed, the $P(x|M)$ function of (154) is often called «likelihood function» $L(M)$ and MLM says that the best estimate of M is the value $M_{\max}$, which maximizes $P(x|M)$ considered as a function of M. One easily sees from (154) that in case of complete «a priori» ignorance (i.e., $P(M) = \text{const}$) BPP coincides with the likelihood function to within the normalization (155). Therefore the BPP in this case has exactly the same maximum. As to the definition of the confidence, the MLM usually prescribes to assign the errors of M by finding the values of M for which $L(M)$ is reduced

from its maximal value by a factor of exp (– 1/ 2). This prescription is obviously based on the assumption that $L(M)$ is a Gaussian centered at M_{max}. For this (and *only for this*) assumption the above prescription indeed gives the conventional «one σ» confidence level of 68%. What is even more important, the ratio of confidence intervals ΔM for 99% and 68% confidence levels in this case is only a factor of 2.6, and this is known to everybody. We shall see, however, that for a small number n of independent experimental measurements $L(M)$ is highly non-Gaussian — in terms of BPP this means that ΔM for 99% confidence might be larger than ΔM for 68% by a factor of $10^2 \div 10^5$ (see, e.g., [78])! Therefore for small n the MLM confidence prescription becomes senseless. For this reason the most accurate manuals on MLM warn against the use of this method for small n cases and vaguely state that the actual accuracy of M_{max} definition in MLM should not exceed the characteristic width (whatever it is) of the $L(M)$ function maximum (see, e.g., [80]).

Thus we see that standard MLM coincides with BS only when $P(M) = \text{const}$ and the ensemble of experimental measurements n is large. In all the other cases MLM strongly deviates from BS and should not be applied to data analysis at all.

Since most physicists do not know the ordinary CPr theory (not to mention the BS) I shall briefly mention most dangerous points, where CPr and BS disagree with our «intuitive» expectations based on rudimentary knowledge of the ordinary (non-conditioned) probability in its «frequency» modification.

First of all, the BPP of eq. (154) (as well as *any* conditional probability) for 2 independent measurements *is not* equal to the product of BPP's for each measurement:

$$P(M|x_1x_2) = \frac{P(x_1x_2|M)\,P(m)}{N(x_1, x_2)} \neq P(M|x_1)\,P(M|x_2). \qquad (156)$$

Here $N(x_1, x_2) = \int P(x_1, x_2|M)\,P(M)\,dM$.

Thus for n independent on-resonance measurements $x_1, x_2, \ldots x_n \equiv \{x_i\}^n$ we obtain:

$$P(M|\{x_i\}^n) =$$

$$= \frac{\theta(M)}{N(\{x_i\}^n)} \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi(\sigma_i^2 + M^2)}} \exp\left\{-\frac{x_i^2}{2(\sigma_i^2 + M^2)}\right\} \qquad (157)$$

with normalization

$$N(\{x_i\}^n) = \int_0^\infty dM \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi(\sigma_i^2 + M^2)}} \exp\left\{-\frac{x_i^2}{2(\sigma_i^2 + M^2)}\right\}. \qquad (158)$$

Expressions of this type were used by us [79] for the analysis of TVDB experiments [49,65], which were unfortunately done in the regime of strongly overlapping resonances long before the discovery of resonance enhancement. Our analysis revealed the already mentioned highly non-Gaussian shape of $P(M|x_i^n)$ curves for small n. Therefore in case of [49] with its $n = 2$ we got at confidence level 85% the upper bound on T-violation constant $\xi \leq 4 \cdot 10^{-3}$, which is comparable to $3 \cdot 10^{-3}$ quoted in [49]. However at confidence level 99% our result was $\xi \leq 8.8 \cdot 10^{-2} \approx 0.1$. Mark 2 orders of magnitude difference between confidence levels 85% (not even 68%!) and 99%. Only after combining the $n = 6$ independent observations of [49,65] the BPP curve started approaching Gaussian and allowed us to obtain $\xi \leq 3.5 \cdot 10^{-3}$ at 99% confidence level.

We also observe that for one measurement the $N(x)$ integral of (155) diverges logarithmically at the upper limit M_u *even for infinite accuracy* $\sigma = 0$. This means that a single experimental upper bound (however accurate) would *never* allow you to extract the upper bound of random variable variance M. This is obvious, since observed $x_i \approx 0$ might emerge both in case of $M \approx 0$ and for large M — as an unlucky fluctuation of random variable.

All this was true for $p_{1/2}$ resonances. For $p_{3/2}$ ones, which can't mix with s-resonances, we know «a priori» that $M = 0$. Here the BS results differ drastically from our naive expectations. Since now the «a priori» probability $P(M) = \delta(M)$, we get from (152)—(155):

$$P(M|x, 3/2) = \frac{P(x|0)}{P(x|0)} \delta(M) = \delta(M). \tag{159}$$

The meaning of this purely Bayesian result is also quite simple — if we know *for sure* the exact value of M, before the experiment, this knowledge would not be changed by any further measurements.

The situation becomes much more complicated if we do not distinguish between $p_{1/2}$ and $p_{3/2}$ resonances. Then we can only use their statistical weights and claim that while probing p-resonances at random, one gets spin 1/2 with probabiliy $p = 1/3$ and spin 3/2 with probability $q = 2/3$. If only one measurement is done, we can use the ordinary CPr expression:

$$P(M|x) = \sum_{\beta} P(M|x, \beta) P(\beta) \tag{160}$$

in order to combine the BPP's of (157)—(158) (for $n = 1$) for spin 1/2 and (159) for spin 3/2 with the aid of corresponding probabilities $P(\beta)$ (equal to p or q). However, the trivial $\delta(M)$ arising in (160) from the $J = 3/2$ term of

(159) is not of interest for us. Therefore we can easily subtract it from (160) and consider only the $P\ (M|x\ 1/2)$ term given by (157).

The case of 2 measurements x_1, x_2 is more subtle. Then we should consider different possibilities. With probability A both resonances might be $J = 1/2$; with probability B only one of them is $J = 1/2$, and it is equally possible that this is either x_1 or x_2. Therefore:

$$P\ (M|x_1\ x_2) = A{\cdot}P\ (M|x_1\ x_2,\ 1/2) + B\ [P\ (M|x_1,\ 1/2) + P\ (M|x_2,\ 1/2)]. \quad (161)$$

The coefficients A and B are well known in statistics of Bernoulli trials and are closely related to the binomial distribution coefficients. The general analysis of [81] gives for a case of n measurements:

$$P\ (M|\{x_i\}^n) = \frac{1}{1-q^n}\sum_{r=1}^{n} p^r q^{(n-r)} \sum_{K_r} P\ (M|\{x_i\}_r^{K_r},\ 1/2). \quad (162)$$

Here r denotes the number of $1/2$ resonances which we might hit in our n trials, and coefficients in the first sum give the probabilities of this to happen. However for each given r we need to find all the possible combinations K_r of r particular x_i values $\{x_i\}_r^{K_r}$. For each particular combination $\{x_i\}_r^{K_r}$ the BPP $P\ (M|\{x_i\}_r^{K_r}\ 1/2)$ is given by (157)—(158). The number of K_r increases with increasing $r \leq n$ in a factorial way. Therefore trying all the options in the sums of (163) for, say, $n = 30$ takes weeks of fast-speed computer time, while the calculation of each particular $P\ (M|\{x_i\}_r^{K_r}\ 1/2)$ defining the M-distribution for a chosen set of $1/2$ resonances takes only seconds. This is another example of how the lack of elementary spectroscopic information on resonance spins enormously complicates the P-violation analysis. Unfortunately this is not the whole truth. Much worse is the fact that this lack of information on spins makes the results of all P-violation measurements practically meaningless. We shall show below that *without the spin assignment* all the measurements performed on ^{238}U and ^{232}Th in recent 6 years allow only to find *the statistically significant upper bound* $F \leq 10^{-6}$ on the strength of P-violating weak interaction in those nuclei. It is just a lucky chance that the purely phenomenological expression used for likelihood function in the analysis of these data produced for ^{238}U the results close to those obtained with the spin assignment.

Before showing this, let us summarize the difference between Bayes method and MLM prescriptions of the orthodox statistics in application to our problem. When resonance spins are known the only principal difference for $J = 1/2$ resonances is that BPP of eq. (157)—(158) are normalized to unity,

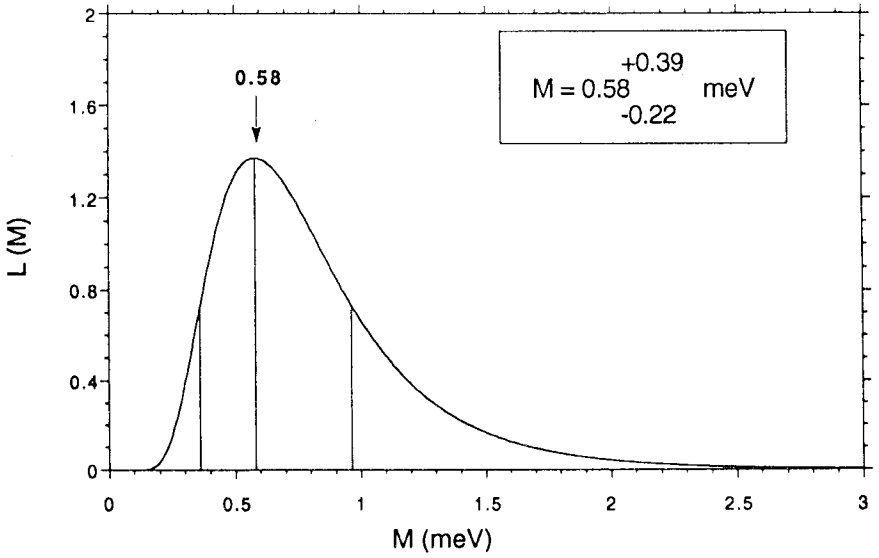

Fig. 3. Likelihood function $L(M)$ for seven $J = 1/2$ p-resonances in ^{238}U based on experimental data of [83] and spin assignment of [84]

while the MLM functions are not. This seemingly insignificant detail leads in case of small ensembles n to quite different results: in case of one measurement the BPP cannot be normalized (and we have seen the deep physical reasons behind it), while standard prescription of the MLM still gives the 68% «confidence level» which is quite meaningless. In case of $n \geq 2$ measurements the BPP curves are normalizable, but the MLM prescription for 68% confidence remains misleading since: a) The exp $(-1/2)$ prescription deviates from actual 68% confidence level; b) The ΔM intervals for confidence levels of 68% and 99% might differ by several orders of magnitude. With increasing n the difference between BS and MLM becomes less marked because $L(M)$ gradually approaches the Gaussian. To illustrate this we show in Fig.3 the $L(M)$ behaviour for seven $p_{(1/2)}$ resonances in ^{238}U based on P-violation measurements of [82,83] and spin assignment of [84] together with «one-σ intervals» of MLM prescription. Mark that even for $n = 7$ the $L(M)$ is still not Gaussian. Therefore the correct definition of 99% confidence would raise the upper limit of M to $M_{up} \approx 1.5$ meV.

In case of $J = 3/2$ the BPP would always give $\delta(M)$ distribution, while MLM will not — see $L(M)$ for nine $3/2$ resonances in ^{238}U of ref. [84] shown in Fig.4. This case shows how misleading the MLM prescriptions might be even

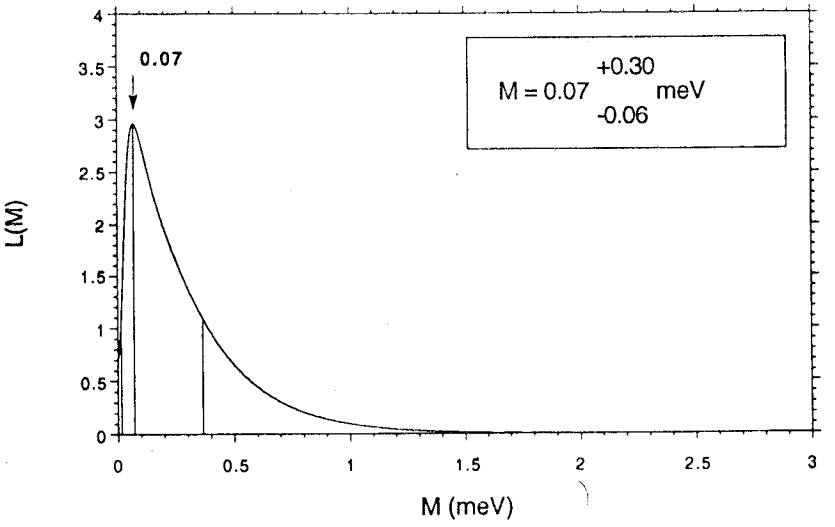

Fig. 4. Likelihood function $L(M)$ for seven $J = 3/2$ p-resonances in ^{238}U based on experimental data of [83] and spin assignment of [84]

for the case of $n = 9$. We all understand that for 3/2 resonances $L(M)$ results should be compatible with $M = 0$ and that all the extra maxima of $L(M)$ curve should be associated only with poor statistical ensemble (small n). The Bayesian results agree quite well with the above natural expectations — on normalizing the $L(M)$ curve of Fig.4 and looking for 99% confidence level we obtain only the upper bound of $M \leq 1.5$ meV. Definitely this upper bound is a rather poor one, but this reflects the basic fact that $n = 9$ ensemble is still a poor one. Bayesian statistics can't produce miracles and heal this drawback, but it warns that *the positions of L (M) maxima are much less important than the correct definition of their confidence levels.*

In case of no spin assignment the *purely empirical* expression was suggested for $L(M)$ in [82]:

$$L(M) = \prod_{i=1}^{n} \left[\frac{p}{\sqrt{2\pi\,(\sigma_i^2 + M^2)}} \exp\left\{ -\frac{x_i^2}{2\,(\sigma_i^2 + M^2)} \right\} + \right.$$
$$\left. + \frac{q}{\sqrt{2\pi\,\sigma_i^2}} \exp\left\{ -\frac{x_i^2}{2\sigma_i^2} \right\} \right]. \qquad (163)$$

We see that this expression is built in violation of both Bayes statistics (since the 3/2 term with q coefficient should be δ-shaped — see (159)) and the rule (159) of conditional probability theory (conditional probability of n inde-

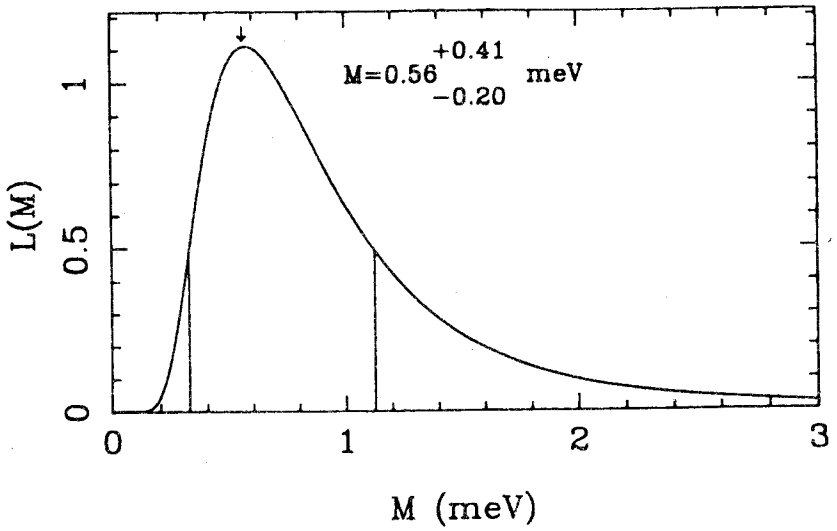

Fig. 5. Likelihood function of ex. (163) for 16 p-resonances in ^{238}U without spin assignment (see [83])

pendent measurements differs from a product of n conditional probabilities). The plot of (163) for 16 resonances measured in ^{238}U as given by [83] is shown in Fig.5. When a similar plot of ref. [82] was first demonstrated at 1989 Alushta School, the response of the experimental audience was: «How do you manage to extract the M values so precisely, when only 6 out of your 17 measured x_i deviate from zero by more than 2σ?» (One should add that now we know that only 3 of those 6 non-zero results are *actually* $J = 1/2$!) This perfectly sound remark aroused my interest to imperfect data statistical analysis and led finally to the above Bayesian results. These results, given by (162), are demonstrated for the case of ^{238}U measurements in Fig.6. Their obvious meaning is (compare with Fig.4) that without spin assignments the extracted value of M is *compatible with zero and gives only the upper bound* $M \leq 3$ *meV at 95% confidence level*, in complete agreement with sound expectations. The oscillations of L (M) curve, likewise in case of Fig.4, result only from the poorness of the statistical ensemble.

The problem now is to understand the striking similarity between the curves of Fig.5 obtained by using the erroneous analysis (163) and Fig.3, whose analysis (besides the above remarks on confidence levels) is correct. The plain answer is — the occurrence of several lucky and unpredictable coincidences.

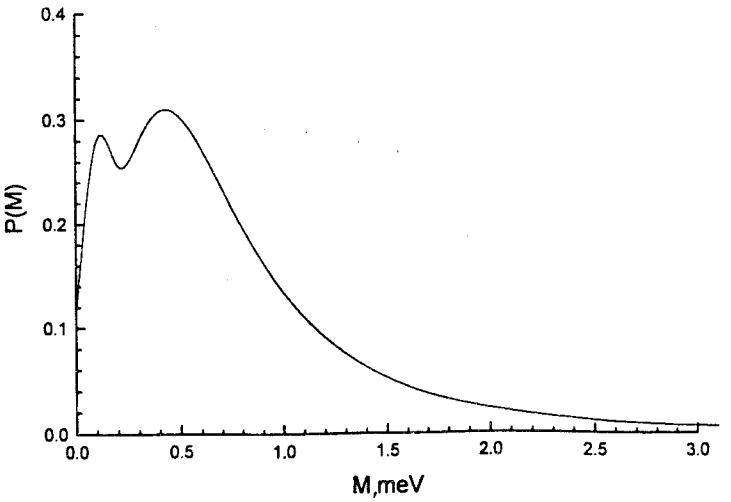

Fig. 6. Bayesian post probability of ex. (162) (normalized to $p = 1/3$) for 16 p-resonances in ^{238}U without spin assignment

Let us compare the phenomenology of (163) with the Bayesian ex. (162). By removing the constant factors $\exp - (x_i^2/ 2\sigma_i^2)$ out of each factor in the product of (163) and presenting (163) as an n-th power of binomial we can simplify this comparison. Now we see that the main difference between (162) and (163) lies in the fact that each term of the binomial expansion in (163) contains an extra weight factor $\exp (x_i^2/ 2\sigma_i^2)$, which exponentially enhances the contribution of each significant deviation $x_i^2/ 2\sigma_i^2$ of the effect from zero. In particular ^{238}U, case a single 63.5 eV resonance with $x_0 \approx 7\sigma_0$ contains an enormous enhancement factor $A_0 = 10^{10}$. Therefore the maximum of this term $M_{\max}^0 = \sqrt{x_0^2 - \sigma_0^2} = 0.65$ meV practically defines the maximum of the whole expression (163). The remaining 4 statistically significant results, whose maxima lie on both sides of M_0, only slightly shift the overall maximum to $0.58^{+0.5}_{-0.2}$ meV of ref. [82]. In order to estimate the influence of those remaining terms, observe that omission of one of the 2σ effects at 57.9 meV in [83] shifted this maximum to $0.56^{+0.4}_{-0.2}$ meV. Naturally, all the abundant zero-effects, mentioned above, produce no influence whatsoever on the L (M) curve of (163).

Thus the phenomenological construction (163), built in violation of conditional probability theory, has a surprising feature of artificial «exponential» selection of the most statistically significant results, or, to put in the other way, exponential suppression of all the null results. Intuitively one might expect that this lucky feature of (163) (quite unexpected by its creators) is a strange but reasonable way to select 1/2 resonances, since we expect that *in average* only those resonances would show *marked* deviations from zero. To a certain extend this is true, but only to a certain extend. Let us come back to our basic expression (152) to understand how sound this selection might be. First of all, we observe that for *any* value of M (that means 1/2 resonances!) the most probable result is $x_i = 0$. ^{238}U case is not an exclusion — 4 out of 7 resonances $J = 1/2$ show zero results. However, the width of the distribution curve (152) depends both on M and σ_i values. Consider first a limiting case of extremely high experimental accuracy $(M/\sigma_i)^2 >> 1$. In this case practically all the non-zero results x_i would come from 1/2 resonances with $M \neq 0$. The «contamination» from 3/2 resonances would be negligible because of small σ values. Therefore in this idealized situation the suppression of null results is perfectly correct and the ex. (163) might be a reasonable approximation of (157).

Consider now the case $(M/\sigma)^2 \simeq 1$. In this case the distributions (152) for $M = 0$ and $M \neq 0$ would come closer to each other and 3/2 resonances would considerably «contaminate» our measurements contributing a lot of non-zero results x_i. However, if we have a very large ensemble of resonance-measurements $n >> 1$ and exponentially select the most significant results, we can still be sure that more $x_i \neq 0$ would come from 1/2 resonances. Mind that for small n ensembles this will not work and a «degree of contamination» coming from 3/2 resonances would randomly vary from ensemble to ensemble.

Finally consider $(M/\sigma_i)^2 << 1$. Then the distributions (152) for $M = 0$ and $M \neq 0$ would practically coincide. Only by taking the unrealistic limit of infinite ensemble $n \to \infty$ plus some kind of selection of largest results one might hope to select the 1/2 contributions.

Coming back to ^{238}U results, we know *only after spin assignment* that $M = 0.56$ meV. Comparing it with σ_i of [82,83] we see that experimental accuracy parameter $(M/\sigma_i)^2$ is more or less evenly distributed between 400 and 0.05. Therefore the chances that non-zero effects come only from 1/2 resonances are roughly «fifty-fifty». Indeed, as we know now, 4 out of 7 non-zero results in U were coming from 3/2 resonances. Therefore the above

coincidence of 2 maxima positions is an unpredictable chance coming essentially from the fact that the main 7σ contribution to (153) *alone* was giving the value M_0^{max} already within σ_0 of the true M-value of [84]. Therefore it would be much simpler and honest to say plainly: «We have reasons to hope that 7σ effect is a $1/2$ resonance one, while with the rest we have no guaranties. So we identify the matrix element of this large effect with the variance M». Obviously, the statistical significance of such a statement if quite unpredictable, but so is the statistical significance of results obtained with the much less transparent ex. (163).

We can also understand now the results of Monte-Carlo simulation [85] of Th case. The authors of [85] considered the ensemble of $n = 7000$ resonances with non-zero effect in case when $(M/\sigma_i)^2$ parameter was distributed in the range from 10^3 to 1. We have already shown above that for such a choice of $(M/\sigma)^2$ the unrealistically large ensemble of 7000 non-zero observations would almost certainly produce the correct results, which was the outcome of [85]. The trouble is that in actual ^{232}Th of [86,87] we have only 7 non-zero effects and do not know in advance the $(M/\sigma_i)^2$ values. Therefore without the spin assignment we cannot even make a clever guess how many non-zero effects arise from «contaminating» $3/2$ resonances and how many null effects come from $1/2$ resonances (in U those were 4 out of 7 resonances with $J = 1/2$).

Therefore in realistic measurements even the correctness of M_{max} derived from (163) would be always unpredictable without spin assignment, not to mention the confidence levels given by (163). This unpredictable character of (163) is mathematically reflected in Bayesian expression (162) and in the results of Fig.6.

5. Sign-Correlation Effect. Returning back to the multilevel ex. (145) for Δ^p_{tot}, we see that the signs of the observed effect should vary randomly from resonance to resonance. This comes from sign-randomness of 3 quantities in it: matrix element $(v_p)_i$, partial amplitudes γ_s^n and γ_p^n, and energy denominators $(E_p - E_s)$.

However the measurements in ^{232}Th (without spin assignment) demonstrated that 7 out of the 7 statistically significant $(x_i \geq 2\sigma_i)$ effects have the same positive sign (see [86,87]). This poses a question — is it a fluctuation or it comes from some systematic effect, which is not included into (143) and which has constant sign. The answer to this question might be best found on the lines of Bayesian statistics (see [77], where Anderson demonstrates how efficient BS is in «null-hypothesis» tests, discarding with notorious «fifth-

force» experiments). However authors of [88] preferred to introduce «ab initio» the constant-sign term in addition to (143). This resulted in constant-sign addition $B\sqrt{1\ \mathrm{eV}/E}$ (%) to the quoted effect $\mathcal{P}$ (see (78), (81)). The magnitude B of this addition was defined in the 2-dimensional MLM analysis, using $L(M)$ of (164) modified by the presence of the B term. The result of this analysis was $B = 8^{+6.2}_{-6.0}$ %, i.e., of the same order of magnitude as the effect $\mathcal{P}$ itself (which ranges between 1% + 10%).

This highly sensational result was taken for its face value as an experimental one by a lot of theorists and experimentalists and produced an avalanche of publications with attempts to explain it theoretically. Since it might take a special review to analyze (and even mention) all of them, I will just classify the main trends in those publications. Some of them concentrated on the analysis of possible sign-correlation between matrix elements $(\nu_p)_i$, completely ignoring the random signs of γ's and $(E_p - E_s)$. Others re-discovered our old statement (see, e.g., [7]) that valence mechanism leads to sign correlation of the P-effects (see ex. (60) for valence part T_{10} of T_1 amplitude), ignoring however that we discarded this mechanism in [7] because it lacks the dynamical enhancement factor $\sqrt{N} \approx 10^3$. Apart stand papers [33,34] which concerned the amplitudes of T_8 type. Since all the diagrams containing the χ wave function (see wavy lines in loops of T_3, T_5–T_9 in Fig.1) describe the motion of valence particle in target mean field, we called them all «the valence mechanism» in [7]. We estimated them with a crude procedure similar to that of eq. (15), (16) and, on discovering that they lack the dynamical enhancement factor $\sqrt{N}$, simply stated this fact in [7]. Weidenmüller in [33] developed a much more elegant technique of principal value integrals evaluation which I borrowed from him in the above estimates of Sec.III.

Practically all the authors concluded that valence mechanism needs extra enhancement of $10^2 + 10^3$ in order to explain the above B-value. Practically nobody mentioned that this extra factor is exactly the same dynamical enhancement which provided for large observed effects in complex nuclei in the first place.

Nobody also questioned the reliability of the above huge B-value, obtained from MLM analysis, whose drawbacks we have just discussed. To those, who do not believe in Bayesian approach, I can suggest to compare the above $B = 8^{+6.2}_{-6.0}$ % value with MLM value $M = 0.07^{+0.30}_{-0.06}$ meV in Fig.4. It is believed by all that the results of Fig.4 should be compatible with $M = 0$. The same must be said about B even by the MLM adepts. So the actual statistical significance of sign correlation effect remains an open question which could be solved, if

really necessary, in terms of Bayesian approach. Since this approach gives only the upper bounds even for M-values without spin assignment, the same will be even more true for B-values. Therefore spin assignment in Th is essential also if one really wants to consider the sign-correlation problem seriously.

V. SUMMARY

Thus we have seen that the enhancements of *all* the symmetry-breaking effects in nuclear reactions on isolated resonances comes essentially from dynamical factor $\tilde{v}/d \sim \sqrt{N}$ and resonance enhancement factor d/Γ, which combine in most optimal situations to the overall factor $\tilde{v}/\Gamma$. For inelastic channels (including TVDB) there might be an additional structural enhancement factor $f \sim (\gamma_1/\gamma_2)$ (see eqs. (73a), (130)). For «elastic channel» observables probed in transmission experiments the situation is more complicated. All of them contain instead of the above f the barrier penetration hindrance factors (kR) or $(kR)^2$ (for FC). However, they also contain an extra resonance enhancement d/Γ (see, e.g., eq. (73b), (73c)) which might almost completely (in case of P-odd correlation) or partially (in case of FC) compensate those hindrance factors.

We have also seen that the «structural (kinematic) enhancement factor» $1/(kR)$, so often used to explain the enhancements for the «elastic channel» P-odd correlations, is merely an artifact of misleading analogy with bound states in theory and of quoting the auxiliary value $\mathcal{P}$ instead of the really observed P in experiment.

Both major enhancements (dynamical and resonance) are quite general results of quantum chaoticity of compound resonances, which increases the complexity N of the compound-resonance wave function and reduces their total widths Γ. This reduction is most efficient in the low-energy region of isolated resonances $\Gamma << d$.

The same chaoticity which produced huge enhancements also necessitates the use of statistical methods for the analysis of observables and their proper connection with strength parameters of the symmetry breaking interactions. This does not lower the reliability of information obtained as compared to simple nucleon-nucleon interaction processes, provided that one obeys certain general rules and uses the correct statistical methods. In theory this boils down to using the methods of random matrix approach, whose reliability was established during half a century in neutron-resonance spectroscopy. In processing the experimental data one should use Bayesian statistics (BS), whose significance is gradually realized by all the physical community.

Both statistical approaches (in theory and experiment) show an important general feature — lack of information on «standard» spectroscopic parameters immediately complicates the analysis and enormously lowers the statistical

significance of the symmetry-breaking observations. BS also strongly vouches for the increase of independent experimental observations rather than their individual accuracy.

In this review I had to concentrate essentially on reaction aspects of symmetry breaking and on proper statistical analysis, paying much less attention to the ultimate aim of on-resonance experiments — analysis of new information on symmetry breaking interaction constants. As mentioned in Introduction, since the creation of Weinberg-Salam electro-weak interaction theory nobody would be surprised by P-violation caused by weak components of nucleon-nucleon interaction. Therefore in P-violation one should try to study the systematic behaviour of F (e.g., its A-dependence). Even from this point of view P-violation is only a particular case of symmetry breaking in nuclei. Studies of «strong» symmetry breaking (see Sec.IV.1) are by no means less important — they are only much more advanced. Even in those studies there are still open problems whose solution might be of major importance for P-violation — see Weidenmüller's comments on isospin-violating spreading widths and their implications in [89]. Therefore even more important is to use P-violation as «test sites» for future T-violation «on-resonance» experiments by developing most reliable experimental and theoretical technique.

I have to point that although on-resonance enhancements were continuously predicted for various T-violating observables since 1982, the experimental situation in this field lacks dynamics. Perhaps too much experimental energy is wasted on P-violation, especially on creating and discussing «quasi-sensations». The same applies to FC correlation measurements in almost hopeless Ho. In view of the facts stated in the end of Sec.III the on-resonance TVDB observations seem much more promising than the FC ones in general. I must also repeat that discussion of all the experimentally realistic modifications of TC measurements should be a highest priority. In planning the experimental strategy of T-violation measurements one should especially follow the rule resulting from BS — more independent on-resonance observations even with smaller accuracy. Mind that in the optimistic case of non-zero effect one measurement would only produce a sensation, telling nothing about the interaction constant. In the more probable case of experimental upper-bound observation one point, however accurately measured, would be quite meaningless.

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