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LIKELIHOOD ANALYSIS OF PARITY-VIOLATING ASYMMETRIES MEASURED FOR COMPOUND NUCLEAR RESONANCES

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Statistical methods are reviewed that have been used to extract the root-mean-squared matrix element, M, of the nuclear parity-violating interaction between compound-nuclear states from parity-violating asymmetries measured in experiments with polarized neutrons [1,2,3,8,9]. The likelihood function is derived for M for the situation where the spins of the p-wave compound-nuclear resonances are unknown. The likelihood functions used by different authors are compared. It is shown that the likelihood function in the approach of Ref. [2,3] is inconsistent with Bayesian statistics.

Анализируются приложения вероятностно-статистических методов для получения (из экспериментальных данных для нейтронных резонансов) матричного элемента M, нарушающего пространственную четность взаимодействия между компаунд-состояниями в ядре. Выводится выражение для функции правдоподобия от M, справедливое для общего случая выборки из совокупности p-волновых резонансов с неизвестными спинами, возбуждаемых при взаимодействии нейтронов с четно-четными ядрами. Это выражение и соответствующие значения для M сравниваются с результатами в подходах других авторов [1,2,3,8,9], и делается вывод о том, что функция правдоподобия в подходе [2,3] несовместима с байесовской статистикой.

1. INTRODUCTION

The TRIPLE Collaboration has published a number of articles reporting the measurement of parity-violating (PV) asymmetries in compound-nuclear (CN) resonances using polarized neutrons at the Los Alamos Neutron Scattering Centre, Ref. [1]. Many non-zero asymmetries were measured in individual nuclei. These data formed the basis of a statistical analysis. Values of the root-mean-squared matrix element, M, of the PV interaction between s-1/2 and p-1/2 CN resonances were extracted from the measured asymmetries using likelihood analysis. Bunakov has criticized the likelihood function introduced by the TRIPLE Collaboration and used by several authors [1,8,9] to extract values of M in his articles «Fundamental Symmetry Breaking in Nuclear Reactions», Ref.[2], and «Statistical Analysis of Imperfect Measurements of Stochastic Variables», Ref. [3].

Bunakov and the TRIPLE Collaboration start from the same assumptions, but reach inconsistent conclusions. Bunakov bases his arguments on Bayesian statistics. The likelihood analysis employed by the TRIPLE Collaboration was in fact a Bayesian analysis. The assumptions made concerning the statistical model of the CN were the same. In order to clarify the nature of the disagreement, we first give a careful derivation of the likelihood function introduced by the TRIPLE Collaboration starting from the fundamental relation of Bayesian statistics. We then show that the likelihood function proposed in Refs.[2,3] is inconsistent with Bayesian statistics. We discuss the results obtained for M by different authors [1,2,3,8,9].

2. ASSUMPTIONS

Bowman et al. [1] showed that, under the assumptions of the statistical model of the CN, the reduced asymmetry, x (defined below), for a p-1/2 ($J^{\pi} = 1/2^{-}$) CN resonance in a spin-zero target nucleus has a Gaussian PDF. Bowman et al. start from the perturbation-theory expression for the PV asymmetry, p_{μ}, of the CN resonance μ [4]

$$p_{\mu} = 2 \sum_{\nu} \frac{V_{\nu\mu}}{E_{\nu} - E_{\mu}} \frac{g_{\nu}}{g_{\mu}} . \tag{1}$$

Here the $V_{\nu\mu}$ are the matrix elements of the PV interaction between the p-1/2 CN state μ and s-1/2 ($J^{\pi} = 1/2^{+}$) state ν, E_{μ}, E_{ν}, and g_{μ}, g_{ν} are the energies and neutron decay amplitudes of the resonances. The sum extends over all s-1/2 resonances. According to the statistical model of the CN, the matrix elements $V_{\nu\mu}$ and the decay amplitudes g_{μ} and g_{ν} behave as statistically independent Gaussian random variables with mean zero*. Furthermore, the matrix elements and decay amplitudes for different resonances are statistically independent. We assume that the relevant CN amplitudes satisfy the ergodic hypothesis. We assume that the energies and the neutron decay widths Γ_μ and Γ_ν ($\Gamma_\mu = |g_\mu|^2$ and $\Gamma_\nu = |g_\nu|^2$) are known from experiment. Bunakov makes the same assumptions. It then follows, using the assumption of statistical independence, that the asymmetry, p_μ, is the sum of independent Gaussian random variables, and that p_μ itself has a Gaussian PDF with mean zero. The variance of p_μ is given by:

$$E(p_\mu^2) = E(|V_{\nu\mu}|^2) \sum_\nu \frac{4}{(E_\nu - E_\mu)^2} \frac{\Gamma_\nu}{\Gamma_\mu} \equiv E(|V_{\nu\mu}|^2) A_\mu^2, \tag{2}$$

where $E(u)$ denotes the expectation value of the quantity u over an ensemble of CN states. The reduced asymmetry,

$$x_\mu = \frac{p_\mu}{A_\mu}, \tag{3}$$

has the same variance, M^2, as the $V_{\nu\mu}$. M^2, defined in this way, is the mean-squared matrix element of the PV interaction between s-1/2 and p-1/2 states,

$$M^2 \equiv E(|V_{\nu\mu}|^2) = \frac{1}{ST} \sum_{E_\nu \in \Delta E,\, E_\mu \in \Delta E} |V_{\nu\mu}|^2, \tag{4}$$

where S and T are the numbers of s-1/2 and p-1/2 states contained in the energy interval ΔE. We assume that ΔE is small compared to the width of any structure in the strength functions of the p-1/2 and s-1/2 resonances. The PDF of the reduced asymmetry for p-1/2 resonances is given by:

$$P(x|M, 1/2) = \frac{1}{\sqrt{2\pi M^2}} \exp\left(-\frac{1}{2}\frac{x^2}{M^2}\right). \tag{5}$$

*Although statistical independence was assumed, this condition is in general not fulfilled. The statistical model of the CN assumes that the wave functions of CN resonances are superpositions of a large number, N, of basis states and that the amplitude of no individual basis state dominates all others. The correlations between the matrix elements of few-body operators that are unrelated to the Hamiltonian of the system and to each other are then of order $1/N$. Experiments in ^{232}Th have found non-statistical correlations between PV asymmetries. Large amplitudes of individual doorway configurations in the wave functions of p- and s-wave resonances have been invoked to explain these correlations [5].

The TRIPLE Collaboration used a likelihood analysis based on Eq.5 to extract values of M from measured reduced PV asymmetries. The spin of p-wave resonances can be 1/2 or 3/2, but only p-1/2 resonances can mix with s-1/2 resonances to exhibit parity violation. For most nuclei, the spins of p-wave resonances are not known, and in order to write down the likelihood function of M it is necessary to take into account the fact that the spins are unknown. In the present article we do this accounting within the framework of Bayesian statistics.

3. BAYESIAN STATISTICS AND LIKELIHOOD ANALYSIS

We will next give a brief discussion of the relationship between likelihood analysis and Bayesian statistics. We follow the discussion given by Eadie et al. [6]. We begin with the fundamental equation of Bayesian statistics:

$$P_p(t \mid z) = \frac{P(z \mid t) P_A(t)}{P_z(z)}, \tag{6}$$

where the normalization factor $P_z(z)$ is the unconditional probability of obtaining an experimental outcome z;

$$P_z(z) = \int P(z \mid t) P_A(t) dt. \tag{7}$$

$P(z \mid t)$ is the conditional PDF of observing an experimental result, z, given a value of the parameter, t. The *a priori* PDF of t, $P_A(t)$, expresses the knowledge of t before the experiment to measure z was done. The *a posteriori* PDF, $P_P(t \mid z)$, is the PDF of t after the result from the experiment is included. Equation 6 is identical to Bunakov's Equation 154[2]. We identify the likelihood function of t, $L(t \mid z)$, with the *a posteriori* PDF, where the value of z, w, actually obtained in an experiment is substituted into $P_P(t \mid z)$:

$$L(t \mid w) \equiv P_P(t \mid w). \tag{8}$$

Equations 6 and 8 **define** the *a posteriori* PDF, or the likelihood function. The definition given in Eq.6 is general and applies for any sets of parameters and experimentally measured quantities $\{t_i\}$ and $\{w_i\}$. The likelihood function can be used to estimate values of the parameters from experimental data. The maximum likelihood estimate (MLE) of a parameter t is the value of t that maximizes $L(t \mid w)$. Several methods of establishing confidence intervals for the MLE are discussed in Eadie et al. [6]. Bowman and Sharapov [7] have discussed the reliability of the confidence intervals determined by the TRIPLE Collaboration using Monte Carlo methods (see section 4 below).

4. DERIVATION OF THE LIKELIHOOD FUNCTIONS

First we derive likelihood function of M when the spins of the levels are known. In this case, only the levels with spin 1/2 need be considered, since the levels with spin 3/2 carry no information about M (see section 5 below). The measured reduced PV asymmetry, y (as opposed to the reduced asymmetry, x, defined in Eq.3, which does not include the experimental error), of a p-1/2 level will have a variance larger than M^2, the variance of the reduced asymmetry, due to the (Gaussian) experimental error, σ, of y. The PDF of the measured asymmetry is

$$P(y \mid M, 1/2) = \frac{1}{\sqrt{2\pi(\sigma^2 + M^2)}} \times \exp\left(-\frac{1}{2}\frac{y^2}{(\sigma^2 + M^2)}\right). \tag{9}$$

Substituting Eq.9 into Eq.6 yields the following for the likelihood function for a single p-1/2 level:

$$L(M \mid y, 1/2) \equiv P_p(y \mid M, 1/2) =$$

$$= \frac{1}{\sqrt{2\pi(\sigma^2 + M^2)}} \times \exp\left(-\frac{1}{2}\frac{y^2}{(\sigma^2 + M^2)}\right) P_A(M) \tag{10}$$

(In Eq.10 and in most subsequent expressions for likelihood functions the normalization factors are omitted for brevity). The reduced asymmetries of different levels are statistically independent, and the same value of M applies to all the levels in the energy interval ΔE. It follows that for a set of measurements of reduced asymmetries of several p-1/2 resonances, $\{y_i\}$, $1 \le i \le n$, the joint PDF of the y_i is the product of the individual PDFs

$$P(\{y_i\} \mid M, 1/2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi(\sigma_i^2 + M^2)}} \exp\left(-\frac{1}{2}\frac{y_i^2}{(\sigma_i^2 + M^2)}\right). \tag{11}$$

The likelihood function is obtained by substituting Eq.11 into Eq.6:

$$L(M \mid \{y_i\}, 1/2) = P_A(M) \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi(\sigma_i^2 + M^2)}} \exp\left(-\frac{1}{2}\frac{y_i^2}{(\sigma_i^2 + M^2)}\right). \tag{12}$$

Next we derive the likelihood function for M when the spins of the levels are unknown. All s-wave resonances have spin 1/2. Since the PV interaction commutes with the angular momentum operator, the matrix elements of the PV interaction between p-3/2 ($J^{\pi} = 3/2^{-}$) states and s-1/2 states are zero. There is no PV asymmetry for p-3/2 states (we neglect the very small PV asymmetry due to mixing of p-3/2 states with very weak d-3/2 states) whatever the value of M. The PDF of the measured asymmetry is Gaussian with variance σ^2, given by the experimental error in the reduced asymmetry:

$$P(y \mid M, 3/2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma^2}\right). \tag{13}$$

Although we include the symbol M in the expression for the conditional PDF of y for purely formal reasons, the PDF does not depend on M. As before, we assume that a set of reduced asymmetries $\{y_i\}$, $1 \le i \le n$ has been measured. We assume that the spins of the p-wave resonances, $J_i = 1/2$ or 3/2, are not known. We take p and q as the probabilities that the spin of a resonance is 1/2 or 3/2. The conditional PDF for each of the y_i given M is the sum of two terms, the first associated with spin 1/2 and the second with spin 3/2:

$$P(y_i \mid M) = \left[\frac{p}{\sqrt{2\pi\,(\sigma_i^2 + M^2)}} \exp\left(-\frac{1}{2}\frac{y_i^2}{(\sigma_i^2 + M^2)}\right) + \right.$$
$$\left. + \frac{q}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{1}{2}\frac{y_i^2}{\sigma_i^2}\right)\right]. \tag{14}$$

Since the y_i are independent random variables, the joint PDF of the set of measurements is the product of the individual PDFs:

$$P(\{y_i\} \mid M) = \prod_{i=1}^{n}\left[\frac{p}{\sqrt{2\pi(\sigma_i^2 + M^2)}} \exp\left(-\frac{1}{2}\frac{y_i^2}{(\sigma_i^2 + M^2)}\right) + \right.$$
$$\left. + \frac{q}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{1}{2}\frac{y_i^2}{\sigma_i^2}\right)\right]. \tag{15}$$

Combining Eq.15 and Eq.6 gives:

$$L(M\,|\,\{y_i\}) = \prod_{i=1}^{n}\left[\frac{p}{\sqrt{2\pi(\sigma_i^2+M^2)}}\exp\left(-\frac{1}{2}\frac{y_i^2}{(\sigma_i^2+M^2)}\right)+\right.$$
$$\left.+\frac{q}{\sqrt{2\pi\sigma_i^2}}\exp\left(-\frac{1}{2}\frac{y_i^2}{\sigma_i^2}\right)\right]P_A(M). \qquad (16)$$

This is the likelihood function used by the TRIPLE Collaboration.

5. COMPARISON WITH BUNAKOV'S APPROACH

In section IV, «Statistical Approach to Compound-Resonance Measurements», of his article, [2] Bunakov challenges the applicability of the above form of the likelihood function stating, in his words: «We see that this expression (Eq.16 in the present article) is built in violation of both Bayes statistics (since the 3/2 term with the q coefficient should be delta-shaped — see (159)) and the rule (159) of conditional probability theory (conditional probability of n independent measurements differs from a product of n conditional probabilities)».

We have given a derivation of the likelihood function, Eq.16, from the Bayesian point of view. We disagree with assertions that Eq.16 is incorrect and that the q-term is δ shaped. In the derivation we have shown that the q-term is a Gaussian with variance σ^2 and that the product form of the likelihood function follows from the product form of $P(\{y_i\}\,|\,M)$ (Eq.15). Why our likelihood function (Eq.16) is different from the expression of Refs. [2,3] (Eq.160 [3]) for the *a posteriori* PDF, $P_P(M\,|\,x)$? In deriving his expression Bunakov assumes that the *a priori* PDF for M is different depending on whether one is considering a p-1/2 level or a p-3/2 level. This assumption contradicts the fundamental equation of Bayesian statistics. We now duscuss this in some detail.

Bunakov asserts (just before Eq. 159 [2]) that for a p-3/2 level,

$$P_A(M) = \delta(M), \qquad (17)$$

and for a p-1/2 level

$$P_A(M) = \theta(M). \qquad (18)$$

Equations 17 and 18 are diametrically opposed. Equation 17 implies complete knowledge of M, namely $M = 0$, and Eq.18. implies no knowledge of M. The *a priori* PDF of M cannot depend on whether one is considering a p-1/2 or a p-3/2 level. The quantity M is a property of all p-1/2 levels and s-1/2 levels in the averaging interval ΔE. The *a priori* PDF of M is the same for all levels in the energy range ΔE. The fact that a p-3/2 level must have zero asymmetry does not imply that $M=0$ or that $P_A(M)=\delta(M)$ if one is considering a p-3/2 level. To the contrary, the reduced asymmetry for a p-3/2 level is zero whatever the value of M.

From the assumption that $P_A(M)=\delta(M)$, it follows that the Bayesian *a posteriori* PDF for M given y for a single p-3/2 resonance is a delta function (the Eq.159 [2])

$$P_p(M\,|\,y, 3/2)=\delta(M). \tag{19}$$

If the *a priori* PDF of M is a δ function, then there is perfect knowledge of M before any experiment has been done. Additional experimental data cannot improve the knowledge of M. It follows that the *a posteriori* PDF of M is a δ function whatever the conditional probability of observing an experimental outcome given a value of M. Equation 19 follows from the assumption of complete *a priori* knowledge of M, not from any information on M that is contributed by experimental data.

It is illuminating to derive the expression for $P_p(M\,|\,y, 3/2)$ without the assumption of complete knowledge of M. We make no assumption for the form of $P_A(M)$ other than that it is normalized. We start with the definition of the *a posteriori* PDF (Eq.6) and the conditional PDF for the reduced asymmetry given a value of M for a p-3/2 resonance, (Eq.13). Recall that $P(y\,|\,M, 3/2)$ does not depend on M. We combine Eq.13 and Eq.6 to obtain the *a posteriori* PDF for a p-3/2 resonance. (The normalization factor has been included).

$$P_p(M\,|\,y, 3/2)=\frac{P_A(M)\dfrac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\dfrac{1}{2}\dfrac{y^2}{\sigma^2}\right)}{\int\limits_0^\infty P_A(M)\dfrac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\dfrac{1}{2}\dfrac{y^2}{\sigma^2}\right)dM}=P_A(M). \tag{20}$$

Equation 20 demonstrates that a measurement of the PV asymmetry for a p-3/2 resonance gives no information on M. The *a posteriori* PDF is identical to the *a priori* PDF. Equation 20 follows from the general result that if the conditional PDF of an experimental outcome is independent of a parameter, then the *a posteriori* PDF is the same as the *a priori* PDF and the experimental result does not affect our knowledge of the parameter. Equation 20 is intuitively appealing because the quantity M has nothing to do with p-3/2 resonances. Equation 19 is consistent with Eq.20, but Eq.20 is more general.

Articles [2,3] contain the following expression for the likelihood function for a single p-wave level whose spin is unknown (Eq.160 [2] expressed in our notation):

$$P(M\,|\,y) = \frac{p}{\sqrt{2\pi(\sigma^2 + M^2)}} \exp\left(-\frac{1}{2}\frac{y^2}{(\sigma^2 + M^2)}\right)\theta(M) + q\,\delta(M). \qquad (21)$$

The first term is associated with spin 1/2; and the second, with spin 3/2. The fundamental equation of Bayesian statistics (Eq.6), involves a single *a priori* PDF of the parameters. The conditional PDF of the experimental outcome given values of the parameters is a sum over the PDFs for different spins. However, the *a priori* PDF cannot be different for terms associated with the different spins. The assumption that the *a priori* PDF depends on the spin is therefore inconsistent with the fundamental equation of Bayesian statistics, Eq.6.

6. DETERMINATION OF ROOT-MEAN-SQUARED MATRIX ELEMENTS OF THE PARITY-VIOLATING INTERACTION

Bunakov, ([2] after Eq.162), argues that if the spins of the p-wave levels are not known, then only upper limits can be determined for M. The TRIPLE Collaboration using Eq.16 has determined non-zero for M when the level spins are unknown [1]. It is not surprising that the approach of Ref. [2] leads only to upper limits for M. The assertion that $P_p(M\,|\,y, 3/2) = \delta(M)$ is equivalent to assuming that the consideration of the PV asymmetry of any p-3/2 resonance determines that $M = 0$. As the number of resonances with unknown spin is increased, the probability that any of them has spin 3/2 rapidly approaches unity. Then the *a posteriori* probability that $M = 0$ also approaches unity. However, as discussed above, the same *a priori* PDF of M must apply to p-1/2 levels as well as p-3/2 levels. If it is assumed that $P_A(M) = \delta(M)$, then all p-1/2 levels must have reduced asymmetry zero (to within experimental uncertainty). The results of experiments show many resonances with highly significant non-zero reduced asymmertries [1]. Therefore the assumption that $P_A(M) = \delta(M)$ and the requirement that there be a single *a priori* PDF of M are inconsistent with experiment.

To allow $M \neq 0$ as required by experiment it is necessary to choose an *a priori* PDF other than $\delta(M)$. We expect that the value of M that is extracted from data will not be strongly dependent on the form of the *a priori* PDF that is assumed. Two groups have analysed the data on ^{238}U assuming the spins to be unknown. Bowman et al. [1] took $P_A(M)$ to be constant between 0 and $M_{max} = 10$ meV and 0 elsewhere and obtained $M = 0.57^{+0.39}_{-0.21}$ meV. The confidence interval was determined by finding the values of M at which the likelihood function decreases by a factor of exp (−1/2) from its maximum value. Bowman and Sharapov [7] studied the reliability of this confidence interval using Monte Carlo techniques. Bowman and Sharapov generated one thousand sets of pseudo-random data that had statistical properties similar to the experimental data. They determined the maximum likelihood estimate and the confidence interval for each of these pseudo-random data sets. The average value of the MLE was 0.96 of the value of M used to generate the pseudo-random data. For 76% of the data sets the assumed value of M fell within the confidence interval. If the likelihood function had a Gaussian shape the confidence interval would correspond to ± one standard deviation and for 68% of the data sets the assumed value of M would fall within the confidence interval. The study of Ref. [7] shows that the assigned confidence interval is somewhat smaller than a 68% confidence interval. Davis [8] took

$$P_A(M) = \frac{1}{M_\alpha} \exp\left(-\frac{M}{M_\alpha}\right), \tag{22}$$

where M_α is a scale parameter determined from a self-consistency condition, and obtained $M = 0.49$ meV. The analysis of data using these two choices gave similar (non-zero) results. After the TRIPLE results had been published, Corvi et al. [9] measured the spins of p-wave resonances in ^{238}U and carried out a likelihood analysis for the p-1/2 levels using Eq.12 to obtain $M = 0.56^{+0.32}_{-0.20}$ meV. The value of M obtained by the TRIPLE Collaboration [1] without knowledge of the spins is non-zero and consistent with the value determined by Corvi et al. [9]. The knowledge of the spins leads to a modest improvement of the confidence interval. The reason for this behavior is that in the situation when the spins are not known and a given resonance shows a statistically significant PV asymmetry, then that resonance is with high probability a p-1/2 resonance. Resonances that show null results may be either p-1/2 or p-3/2. As the experimental error decreases, the probability that a p-1/2 resonance shows a null result decreases. Most of the p-1/2 resonances identify themselves by showing PV. Use of the likelihood method correctly estimates and includes in the analysis the small probability that some p-1/2 resonances have null asymmetries by chance.

We have derived the form of the likelihood functions used by the TRIPLE Collaboration to analyse parity-violating asymmetries measured for compound-nuclear resonances starting explicitly from Bayesian statistics. The forms of the likelihood functions for the root-mean-squared matrix element of the parity-violating interaction between s-1/2 and p-1/2 levels are given when the spins of the p-wave levels are known to be 1/2 and when the spins are unknown. We compared the likelihood functions used by different authors. We show that the approach of Refs. [2,3] to likelihood analysis for the situation where the spins of the p-wave levels are unknown is not correct. The stem of an error is an assumption of different *a priori* probability density functions for the root-mean-squared matrix element for terms in the likelihood function arising from spins 1/2 and 3/2. This assumption is contrary to the fundamental equation of Bayesian statistics and is the root cause of the many disagreements between approaches of Refs. [2,3] on one side and of Refs. [1,7,8,9] on the other side.

7. ACKNOWLEDGEMENTS

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