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QUARK-HADRON PHASE TRANSITION IN NEUTRON STARS

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In part one of this paper we introduce a suitable formulation of the hydrodynamical description of compact stars in a curved space-time manifold using the tetrad formalism. In the second part of the paper we review new developments concerning the possibility of quark deconfinement in neutron stars. Contrary to earlier claims, it is shown that "neutron" stars may very well contain quark matter in their cores, which may register itself in an observable signal – an anomaly in the braking index of pulsars.

В первой части обзора при использовании формализма тетрад предложена удобная формулировка гидродинамического описания компактных звезд в искривленном пространственно-временном множестве. Во второй части представлен обзор современного развития представлений о возможности кваркового деконфайнмента в нейтронных звездах. Вопреки ранним утверждениям показано, что "нейтронная" звезда может с большой вероятностью содержать в своей сердцевине кварковую материю, присутствие которой проявляет себя в доступном для наблюдения явлении — в аномалии индекса торможения пульсара.

1. INTRODUCTION

Shortly after the discovery of the neutron in 1932, the hypothesis that compact superdense stars made up of neutron matter might exist as remnants of star collapse within a supernova explosion scenario has been given for the first time by Baade and Zwicky [1]. However, the verification of the exciting neutron star hypothesis by observations seemed to be hopeless until 1967, when the first pulsar had been detected and the interpretation as a rotating neutron star has been invoked. The body of evidence for the scenario of superdense star formation was complete when in the center of the Crab nebula which is a remnant of the 1034 supernova explosion a pulsar was found.

Tolman [2], Oppenheimer and Volkoff [3] were the first to solve the problem of hydrostatic equilibrium for neutron stars within the framework of general relativity. Meanwhile it has become mandatory to investigate the evolution of supermassive as well as superdense stellar bodies in the framework of general relativistic hydrodynamics (hydrostatics) [4,5] and an introduction to this theoretical basis will therefore constitute the first part of the present review.

Supernova matter is known to be made up of relativistic neutrons, protons, electrons, and degenerate neutrinos [6]. Relativistic neutrons, protons, electrons, and a certain fraction of muons form the fundamental constituents of the supernova remnants – that is, neutron stars – too. The occurrence of other particles, like hyperons, and a K^--meson condensate, appears to be likely [7]. Moreover, some of the latest investigations on the composition of superdense neutron star matter make a strong case in favor of the presence of deconfined quark matter, too [8–10]. This topic will be reviewed in the second part of this paper.

Neutron stars are described by the (general relativistic) equations of hydrostatic equilibrium, which are relatively easy to deal with. This is not so for the evolution of such stars, starting from the initial stages of stellar collapse to the final state that results in either a stable neutron star or, alternatively, in a black hole. In the early investigations on this issue, the attempt was made to solve the dynamical problem of the dust matter in the comoving frame. In this case, the solution to the problem can be found analytically. This is however hampered by the fact that the comoving frame of the observer is very inappropriate for numerical calculations, because the gravitational field is time dependent in this frame. Today, we have a long list of investigations dealing with numerical studies of stellar evolution, as well as structure and stability calculations [11–15], most of which being devoted to different numerical solution of Einstein's equations [16, 17].

All algorithms use the gravitational field equations, besides the equation of state (EOS) of matter, as the basic dynamic equations, assuming that hydrodynamics is a consequence of general relativity theory. The formulation of the dynamics of stellar evolution, independently of the gravitational field equations, provides the opportunity to compare the formulation and the result of the problem in different alternative theories of gravitation [18].

Of course the description of gravity as an effect of curvature of a space-time manifold remains the most useful generalization of the gravitational field in the case of general relativity. This description implies that the gravitational field has to be described in terms of a metric space-time tensor. The next approximation, which is also useful in case of quasi-stationary slow processes, consists in assuming that the flux of matter is adiabatic and has negligibly small viscosity. We try to represent the dynamics of the self-consistent flux of the matter in the form of wave-propagation equations. This method is a generalization of the well-known method of the characteristics. This approach solves two problems:

1. It includes the possible shock wave propagation (the so-called weak solutions), which cannot be investigated within the method of finite differences or methods using spline or series representations.

2. It automatically controls the physical and mathematical instabilities.

The Oppenheimer-Volkoff equations have been generalized in case of time evolution.

The problem of the dynamics of rotating configurations has been discussed, e.g., in Refs.19,20. In the present work we consider the dynamics of the pulsars as quasi-stationary processes with variable rotation frequency.

2. BASIC EQUATIONS OF HYDRODYNAMICS

The algorithm is essentially a chain of calculations starting with a given initial matter density and flux velocity distributions at the initial time, and evolves the physical quantities stepwise towards the time direction in 4-dimensional space.

In our case, during the determination of the stellar configuration, the spherical symmetry does not change and the dynamics of the star can be considered in two dimensions (r and t).

Formulating the equations in the invariant form, makes the calculations applicable for all observation frames. The hydrodynamic equations have the following form

$$\nabla \cdot \mathbf{T} = 0, \tag{1}$$

where we neglect all the external and internal sources of energy, in particular the viscosity of the matter is ignored. So, the energy-momentum tensor can be written in the form

$$\mathbf{T} = (\epsilon + P)\ \mathbf{u} \otimes \mathbf{u} - P\ \mathbf{g}, \tag{2}$$

where $\mathbf{u}$ is the 4-velocity, $\mathbf{g}$ is the metric tensor of space-time, P is the pressure; and ϵ, the density of the matter in the comoving frame. The quantity ∇ is the covariant 4-gradient, and we are using units for which $c = G = 1$.

The system (1) can be rewritten as

$$\begin{aligned} \mathbf{u}[\epsilon] + (\epsilon + P)\nabla \cdot \mathbf{u} &= 0\,, \\ \mathbf{n}[P] - (\epsilon + P) < \mathbf{n}, \nabla_{\mathbf{u}}\mathbf{u} > &= 0\,. \end{aligned} \tag{3}$$

Here we use the notations $\mathbf{n}[P] = \nabla_{\mathbf{n}} P$, which is the gradient of a scalar towards a vector $\mathbf{n}$, and $<>$ is a projection operator. The vector $\mathbf{n}$ is orthogonal to $\mathbf{u}$. The 4-velocity has the normalization $\mathbf{n} \cdot \mathbf{n} = -1$ (space-like), while the normalization of $\mathbf{u} \cdot \mathbf{u} = 1$.

In the coordinate frame which is connected with objects at infinity (t, r, θ, ϕ), one has

$$\mathbf{u} = u^t(\frac{\partial}{\partial t} + v\frac{\partial}{\partial r}), \quad (4)$$

$$\mathbf{n} = u^t(v\frac{\partial}{\partial t} + \frac{\partial}{\partial r}), \quad (5)$$

where $v = \frac{dr}{dt}$ is coordinate velocity, and u^t is the Lorentz factor.

The gravitational metric tensor (invariant interval) in the most common type for the spherically symmetry can be written as

$$ds^2 = e^{2\Phi}dt^2 - e^{2\Psi}(dr + \beta dt)^2 - r^2e^{2\chi}(d\theta^2 + \sin^2\theta d\phi^2)\,. \quad (6)$$

We use the tetrad basis

$$\begin{aligned}
\omega^{\mathbf{0}} &= e^{\Phi}dt\,, && (7)\\
\omega^{\mathbf{1}} &= e^{\Psi}(dr + \beta dt)\,, &&\\
\omega^{\mathbf{2}} &= re^{\gamma}d\theta\,, &&\\
\omega^{\mathbf{3}} &= r\sin\theta e^{\chi}d\phi\,. && (8)
\end{aligned}$$

In the conjugate basis (e_μ), the 4-vector $\mathbf{u}$ has a simple form

$$\begin{aligned}
\mathbf{u} &= \gamma(\mathbf{e}_0 + w\mathbf{e}_1)\,, && (9)\\
\mathbf{n} &= \gamma(w\mathbf{e}_0 + \mathbf{e}_1)\,, &&\\
\gamma &= 1/\sqrt{1-w^2}\,, &&\\
w &= (v-\beta)e^{\Phi-\Psi}\,. &&
\end{aligned}$$

The quantity w is the actual velocity of the matter on the sphere for an observer at infinity, while β is the velocity of the local inertial frames (frame dragging).

Using the Cartan equations for the structure, it is easy to find the set of the 1-forms of connection,

$$\omega = \Gamma^{\alpha}_{\alpha\beta}\,\omega^{\gamma}\;E^{\beta}_{\alpha}\,,$$

where E^{β}_{α} is the basis of the 4×4 matrix algebra;

$$\begin{aligned}
\omega &= \alpha\,(E^0_1{+}E^1_0)\\
&+ a\,\{\omega^{\mathbf{2}}\,(E^0_2{+}E^2_0) + \omega^{\mathbf{3}}\,(E^0_3{+}E^3_0)\}\\
&+ b\,\{\omega^{\mathbf{2}}\,(E^1_2{-}E^2_1) + \omega^{\mathbf{3}}\,(E^1_3{-}E^3_1)\}{+}\,c\,\omega^{\mathbf{3}}\,(E^2_3{-}E^3_2)\,, \quad (10)
\end{aligned}$$

with $\alpha = f\,\omega^{\mathbf{2}}{+}h\,\omega^{\mathbf{1}}$ a l-form determining the influence of gravity (gravitational forces corresponding to Newtonian limit), $f = \mathbf{e}_1[\Phi]$ and $h = \mathbf{e_0}[\Psi] - e^{\Psi-\Phi}\mathbf{e}_1[\beta]$. $a = \mathbf{e}_0[\chi]$, $b = \mathbf{e}_1[\chi + \ln(r)]$ and $c = \mathbf{e_2}[\ln(\sin\theta)]$ are scalar functions.

Using Eq. (10) and the notation of the angular velocity $\zeta = \operatorname{atanh}(w)$, we obtain

$$\nabla \cdot \mathbf{u} = \mathbf{n}[\zeta] + \alpha(\mathbf{n}) + 2\mathbf{u}[\chi + \ln(r)]\,, \tag{11}$$

$$< \mathbf{n}, \nabla_u \mathbf{u} >= -\mathbf{u}[\zeta] - \ \alpha(\mathbf{u})\,,$$

where $\alpha(\mathbf{u})$ and $\alpha(\mathbf{n})$ are the values of the 1-form α on the vector fields $\mathbf{u}$ and $\mathbf{n}$. One can now rewrite Eq. (3) in the following form

$$\begin{aligned} \mathbf{u}[\sigma] + c\,\mathbf{n}[\zeta] + \alpha(c\,\mathbf{n}) + 2c\,\mathbf{u}[\chi + \ln(r)] &= 0\,, \\ c\,\mathbf{n}[\sigma] + \mathbf{u}[\zeta] + \alpha(\mathbf{u}) &= 0\,, \end{aligned} \tag{12}$$

or in the more compact form

$$\begin{aligned} \mathbf{D}_\pm[J_\pm] + \alpha(\mathbf{D}_\pm) \pm 2c\,\mathbf{u}[\chi + \ln(r)] &= 0\,, \qquad (13) \\ J_\pm &= \zeta \pm \sigma\,, \\ \mathbf{D}_\pm &= \mathbf{u}\ \pm c\,\mathbf{n}\,. \qquad (14) \end{aligned}$$

Here we have used the notations for the speed of sound, c, in the medium and the wave enthalpy, σ,

$$\sigma = \int \frac{dP}{c(\epsilon + P)}\,, \tag{15}$$

$$c = \left(\frac{\partial P}{\partial \epsilon}\right)_S\,.$$

From the representation of $\mathbf{D}_\pm$ in the coordinate basis one can find the expression for the sound propagation velocity,

$$V_s^\pm = c\frac{1 - w^2}{1 \pm wc} e^{\Phi - \Psi}\,. \tag{16}$$

The $+$ sign in all equations shows the direction outwards from the center of the star; while the $-$ sign, towards the center. The propagation of the waves can be determined by Eq. (13) which is the generalization of the Riemann equations in case of relativity including the self-consistent account of the gravitational effects.

Eq. (16) shows that besides the Doppler effect (which results in an anisotropy of the wave propagation), the effect of change of the speed in the gravitation field is valid for the sound like the red shift effect for the light.

3. EQUATION OF THE GRAVITATIONAL FIELD

Up to now we did not fix neither the gravitational field equations nor the observer's coordinate frame in order to describe the hydrodynamics independently of the gravitational field theory.

It is necessary to have conditions that determine the gravitational force at each moment, since we have the distribution of the matter and its flux. We have to write equations for the functions f , h and $\Phi - \Psi$. Let us choose the general theory of relativity in the framework of which most of the stellar models are generally considered. The invariant form of the Einstein equations for the metric (6) and energy-momentum tensor $\mathbf{T}$, in Eq. (2), is given by

$$\mathbf{G} = 8\pi\mathbf{T}\,, \tag{17}$$

where $\mathbf{G}$ denotes the Einstein tensor:

$$\begin{aligned}\mathbf{G} \;=\; & (E+2D)\omega^0\otimes\omega^0 + (2B-E)\omega^1\otimes\omega^1 + 2C(\omega^0\otimes\omega^1+\omega^1\otimes\omega^0) \\ & +(A+B-D)(\omega^2\otimes\omega^2+\omega^3\otimes\omega^3)\,. \end{aligned} \tag{18}$$

The coefficients A, B, C and D can be obtained from the set of the following equations

$$\begin{aligned} A &= \mathbf{e}_1[f] - \mathbf{e}_2[h] + f^2 - h^2, \\ B\omega^0 + C\omega^1 &= -\mathbf{d}a + b\alpha - a^2\omega^0 - ab\omega^1, \\ C\omega^0 + D\omega^1 &= -\mathbf{d}b + a\alpha - b^2\omega^1 - ab\omega^0, \\ E &= e^{2\chi}/r^2 + a^2 - b^2. \end{aligned} \tag{19}$$

To simplify the problem we choose the Schwarzschild coordinates, where $\chi = 0$ and $\beta = 0$, that enables us to find analytic time-independent external solution for the gravitational field equations (Birkoff's theorem). Finally from Eq. (17) we have

$$\begin{aligned} E + 2D &= 8\pi\varepsilon\,, \\ 2B - E &= 8\pi\Pi\,, \\ C &= -4\pi(\varepsilon + P)\gamma^2 w\,, \end{aligned} \tag{20}$$

where ε and Π are the energy density and pressure of the matter in the frame connected with the center of the star (the rest frame of the observer). One has

$$\begin{aligned} \varepsilon &= \gamma^2(\epsilon + P\,w^2)\,, \\ \Pi &= \gamma^2(\epsilon\,w^2 + P)\,. \end{aligned} \tag{21}$$

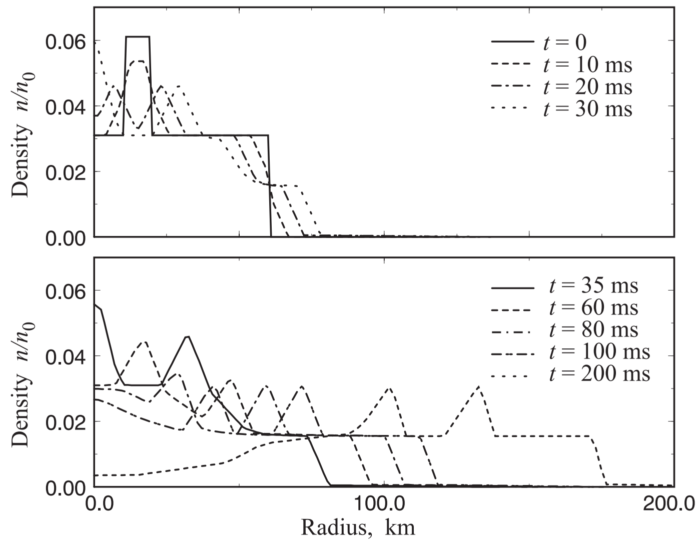

Fig. 1. Time evolution of the process of the neutron matter explosion

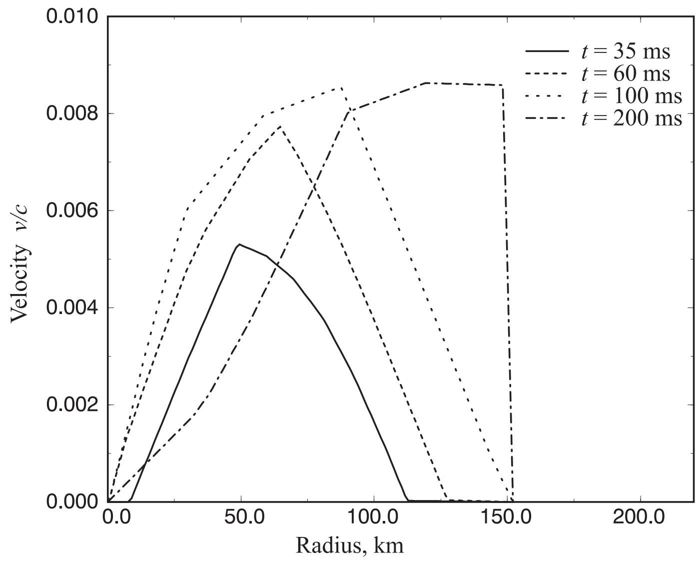

Fig. 2. Time dependence of the velocity distribution of a homogeneous compressed neutron gas at $T = 0$

Since Eq. (1) is a consequence of the field equations (Bianchi identity) and since in case of spherical symmetry there is no gravity-wave radiation, we can consider only that part of the system, i.e., Eq. (17), which does not contain time derivatives. It is sufficient to have a complete system of equations adding the wave equation (13) and the equation of state. The formal solution of Eq. (21) is given by

$$\Psi(r,t) = -\frac{1}{2}\ln\big(1-\frac{2m(r,t)}{r}\big)\,, \tag{22}$$

$$\Phi(r,t) = \Psi(r,t)+\int_0^r \frac{2m(r',t)+4\pi(P(r',t)-\epsilon(r',t))(r')^3}{r'(r'-2m(r',t))}dr' + \Phi_0(t)\,,$$

where $\Phi_0(t)$ is the gravitational potential in the center and $m(r,t)$ is the accumulated mass at a distance r from the center,

$$m(r,t) = 4\pi\int_0^r \varepsilon(r',t)(r')^2 dr'\,. \tag{23}$$

These equations are the boundary conditions for the dynamical equations of waves.

A model calculation of the time evolution of a configuration of the ideal gas of neutrons at zero temperature is shown in Figs. 1 and 2. Figure 1 illustrates the process of the explosion of the matter with arbitrarily chosen distribution in case of weak self-gravity. In Fig. 2 one can see the changes of the matter velocity for the hydrodynamically not stable homogeneous neutron matter configuration.

4. SHOCK WAVE CONDITIONS

During the evolution of the matter of the star configuration it is possible to have a situation when the continuity of the matter distribution will be broken. The system of Eq. (13) is not valid on the point of the discontinuity of the functions $J_\pm$. Such jumps are of various physical nature, they could be shock waves (SW), the edges of the different phases of the matter, the configuration surface (matter–vacuum boundary) or others. To include the propagation of such points we need to fulfill the conditions of energy, momentum and baryon number conservations (let us call them SW conditions) on the surface of the discontinuity.

The presence of the gravitation field does not play a role, because the SW conditions are local properties of the matter flux through the SW front and one can formulate them in the frame of the local observer. Moreover, due to the continuity of the gravitational field we don't need the equations (1) of hydrodynamics written in curved space. One can simply use the following equations

$$\partial_k T_i^k = 0\ , \tag{24}$$

$$\partial_i(su^i) = 0\ , \tag{25}$$

$$\partial_i(nu^i) = 0\ , \tag{26}$$

where T_i^k is the energy-momentum tensor

$$T_{\mu\nu} = w u_\mu u_\nu - P \; g_{\mu\nu} \quad , \tag{27}$$

$w = \epsilon + P$ is the enthalpy density, s is the entropy density and n is the baryon number density. In the case of spherical symmetry the SW conditions have the form

$$[n u^r] = 0 \tag{28}$$

for the conservations of the particle number,

$$c[T^{tr}] = 0 \tag{29}$$

for the continuity of the energy flux, and

$$[T^{rr}] = 0 \tag{30}$$

for the continuity of the flow of momentum in front and behind the shock wave.

Let us choose the coordinate frame connected with the shock wave and suppose that the motion of the matter flux is towards the r coordinate, then the equations (28) - (30) can be written as

$$n u_b^r = n u_f^r, \tag{31}$$

$$w(u_b^r)^2 + p_b = w u_f^{r2} + p_f, \tag{32}$$

$$w u_b^r u_f^t = w u_b^t u_f^r. \tag{33}$$

From the last two equations one can find the velocities v_b and v_f of the matter on both sides of the SW

$$\frac{v_b}{c} = \sqrt{\frac{(p_f - p_b)(\varepsilon_f + p_b)}{(\varepsilon_f - \varepsilon_b)(\varepsilon_b + p_f)}} \quad , \tag{34}$$

$$\frac{v_f}{c} = \sqrt{\frac{(p_f - p_b)(\varepsilon_b + p_f)}{(\varepsilon_f - \varepsilon_b)(\varepsilon_f + p_b)}} \quad , \tag{35}$$

and the equation of the Taub adiabate has the form

$$(\frac{w_b}{n_b})^2 - (\frac{w_f}{n_f})^2 + (p_b - p_f)\left[\frac{w_b}{n_b^2} - \frac{w_f}{n_f^2} + (p_b - p_f)\right] = 0. \tag{36}$$

Using the equations (32) and (33) with the Gibbs fundamental equation in the form

$$d\frac{w}{n} = T d\frac{s}{n} + \frac{1}{n} dp \quad , \tag{37}$$

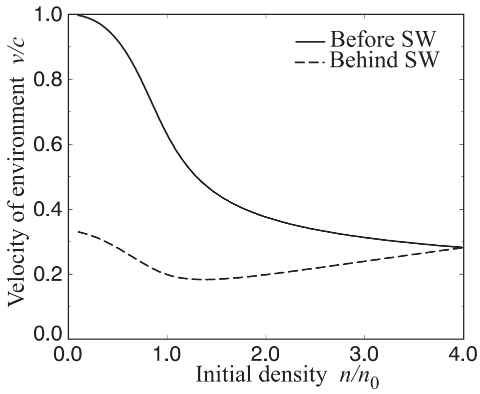

Fig. 3. Matter velocity before and behind the shock front during the explosion of an ideal neutron gas configuration as a function of the compression

one can define the condition of the isentropic flow of the matter

$$u^k \frac{\partial}{\partial x^k} \frac{s}{n} = \frac{d}{d\tau} \frac{s}{n} \,. \tag{38}$$

This condition is valid if the flow of the matter is adiabatic during the motion along the ”world line”. It is important also for the static configurations if we consider hot stars [10], when the thermal energy dissipation is negligible.

SW could be considered as a process of the hadron-quark phase transition during the explosion of the matter, when the SW propagates through the hadronic crust of the star. Passing the SW front, the neutron gas can be compressed up to the critical densities of the phase transition. In Fig. 3 we show the dependence of the velocity of the compressed matter before the SW as a function of the density of the hadronic crust (we took the critical density of the phase transition $n = 4n_0$, where n_0 is the nuclear density).

5. QUARK–HADRON PHASE TRANSITION IN NEUTRON STARS

At present one does not know from experiment at what density the expected phase transition to quark matter occurs, and one has no conclusive guide yet from lattice QCD simulations. From simple geometrical considerations it follows that nuclei begin to touch each other at densities of $\sim (4\pi r_N^3/3)^{-1}$, which, for a characteristic nucleon radius of $r_N \sim 1$ fm, indicates this to happen at just a few times nuclear matter density [8]. Above this density, it appears plausible that the nuclear boundaries of particles like p, n, Σ^-, Λ, K^- will dissolve and the formerly confined quarks populate free states outside the baryons. The high-pressure environment that exists in the cores of neutron stars, with densities up to an order of magnitude higher than that of nuclear matter, constitutes ideal sites where hadrons may transform into quark matter, forming a *permanent* component of such matter inside neutron stars.

The possibility of quark deconfinement in the cores of neutron stars has already been suggested in the 1970's by several authors (see, for instance, [21]). However until recently no observational signal has ever been proposed. This is so because whether or not the quark–hadron phase transition occurs in such stars, makes only little difference to their static properties, such as the range of possible masses, radii, or even their limiting rotational periods. This however turns out to be strikingly different for the timing structure of pulsars that develop quark matter cores in the course of spin-down (spin-up), as we shall see in the following sections.

6. CONSERVED CHARGES

The reason why the likely occurrence of the quark-hadron phase transition in neutron stars has been overlooked in all earlier works prior to Glendenning's [8, 22] is that a degree of freedom was frozen out which yielded a description of the quark-hadron phase transition as a *constant* pressure one. Since pressure in a neutron star is monotonically decreasing – an inevitable consequence that follows from hydrostatic equilibrium – this had the explicit consequence of excluding

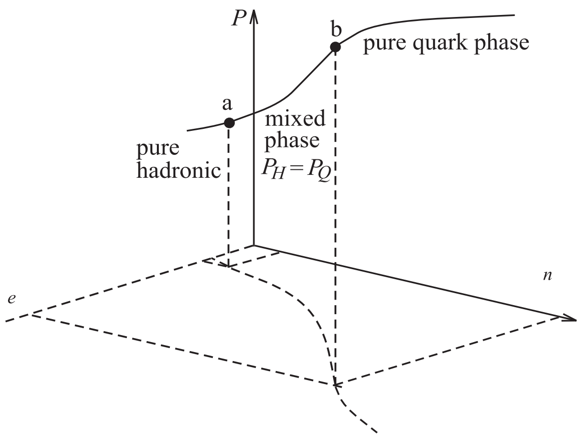

Fig. 4. Phase transition in a body (i.e., neutron star) with *two* conserved entities, that is, electric charge and baryon charge. In contrast to the phase transition in a body with only one conserved charge, here pressure in the mixed phase varies with density. Therefore the mixed phase is not excluded from the star, as would be the case if the pressure were incorrectly taken to be constant

the coexistence phase of hadrons and quarks from neutron stars. The degree of freedom that was frozen out is the possibility of reaching the lowest possible energy state by rearranging electric charge between the regions of hadronic matter and quark matter in phase equilibrium. Because of this freedom the pressure in the mixed phase *varies* as the proportions of the phases, and therefore the coexistence phase is not excluded from the star, as pointed out first by Glendenning [8, 22] (see also Hermann [23]).

The physical reason behind this is the conservation of baryon charge and electric charge in neutron star matter. Correspondingly, there are *two* chemical potentials – one associated with baryon charge and the other associated with electric charge – and therefore the transition of baryon matter to quark matter is to be determined in three-space spanned by pressure and the chemical potentials of the electrons and neutrons (rather than two-space). A schematic illustration is given in Fig. 4. Again, this circumstance has not been realized in the numerous investigations published on this topic prior to Glendenning's paper [22].

7. CHEMICAL THERMODYNAMICS

Let us explore now how the qualitative arguments given just above can be put into a proper mathematical description of the quark-hadron phase transition in neutron stars (or, more generally, complex bodies that are characterized by more than one conserved charge). In the trivial case of a star made from pure neutron matter, electric charge neutrality is automatically satisfied and so its matter is characterized by a *single* chemical potential, that for the neutrons, μ^n. According to chemical thermodynamics, the Gibbs condition for phase equilibrium between neutron matter and quark matter at a given temperature would then read

$$P_{\rm H}(\mu^n, \{\phi\}, T) = P_{\rm Q}(\mu^n, T)\,, \tag{39}$$

where the subscripts 'H' and 'Q' denote the confined hadronic and quark phase, respectively, and $\{\phi\}$ denotes the field variables and Fermi momenta that characterize a solution to the equations of confined hadronic matter exclusive of the chemical potential [22]. The Lagrangian of pure neutron matter is obtained from

$$\begin{aligned}\mathcal{L}(x) \quad = \quad & \sum_{B=p,n,\Sigma^{\pm,0},\Lambda,\Xi^{0,-},\Delta^{++,+,0,-}} \mathcal{L}^0_B(x) \\ + \quad & \sum_{M=\sigma,\omega,\pi,\rho,\eta,\delta,\phi} \Big\{ \mathcal{L}^0_M(x) \; + \sum_{B=p,n,\ldots,\Delta^{++,+,0,-}} \mathcal{L}_{BM}(x) \Big\} \\ + \quad & \mathcal{L}^{(\sigma^4)}(x) \; + \sum_{L=e^-,\mu^-} \mathcal{L}_L(x)\end{aligned} \tag{40}$$

Table 1. Masses, electric charges ($q_{\rm el}^B$) and quantum numbers (spin, J_B; isospin, I_B; strangeness, S_B; hypercharge, Y_B; third component of isospin, I_{3B}) of those baryons whose thresholds may be reached in the cores of neutron stars

Baryon (B)	m_B (MeV)	J_B	I_B	S_B	Y_B	I_{3B}	q_B
n	939.6	$1/2$	$1/2$	0	1	$-1/2$	0
p	938.3	$1/2$	$1/2$	0	1	$1/2$	1
Σ^+	1189	$1/2$	1	-1	0	1	1
Σ^0	1193	$1/2$	1	-1	0	0	0
Σ^-	1197	$1/2$	1	-1	0	-1	-1
Λ	1116	$1/2$	0	-1	0	0	0
Ξ^0	1315	$1/2$	$1/2$	-2	-1	$1/2$	0
Ξ^-	1321	$1/2$	$1/2$	-2	-1	$-1/2$	-1
Δ^{++}	1232	$3/2$	$3/2$	0	1	$3/2$	2
Δ^+	1232	$3/2$	$3/2$	0	1	$1/2$	1
Δ^0	1232	$3/2$	$3/2$	0	1	$-1/2$	0
Δ^-	1232	$3/2$	$3/2$	0	1	$-3/2$	-1

by setting $B = n$. We stress that, in general, all baryon states B whose thresholds will be reached in the dense interiors of neutron stars (see [7] for details) are to be summed self-consistently. The interaction between the baryons is described by the exchange of mesons with masses up to about 1 GeV, depending on the many-body approximation. At the level of the simplest approximation – the relativistic mean-field (or Hartree) approximation – these are the σ, ω and ρ mesons only.

Now, such a treatment, which incorrectly describes neutron star matter as being composed of only neutrons, is to be contrasted with the phase transition between confined hadronic matter and quark matter both in full β-equilibrium, in which case all the baryons listed in Table 1 are summed self-consistently. In this case one has *two* conserved laws, one which conserves baryon charge and the other which conserves electric charge. The Gibbs condition for phase equilibrium then is that the two chemical potentials μ^n and μ^e, corresponding to baryon and electric charge conservation, and the pressure in the two phases be equal,

$$P_{\rm H}(\mu^n, \mu^e, \{\phi\}, T) = P_{\rm Q}(\mu^n, \mu^e, T)\,. \tag{41}$$

The pressure $P_{\rm Q}$ of quark matter is obtainable from the bag model,

$$P + B = \sum_{i=u,d,c,s;e^-,\mu^-} P_i\,, \tag{42}$$

$$\epsilon = \sum_{i=u,d,c,s;e^-,\mu^-} \epsilon_i + B\,, \tag{43}$$

where P, ϵ and B refer to the external pressure, total energy density, and the bag constant, respectively, and P_i and ϵ_i are the contributions of individual quarks and leptons contained in the bag [9]. The quark chemical potentials are related to the baryon and charge chemical potentials as

$$\mu^u = \mu^c = \frac{1}{3}\,\mu^n - \frac{2}{3}\,\mu^e\,, \qquad \mu^d = \mu^s = \frac{1}{3}\,\mu^n + \frac{1}{3}\,\mu^e\,. \tag{44}$$

Equation (41) is to be supplemented with the two relations for conservation of baryon charge and electric charge. In general the conservation laws in chemical thermodynamics are *global*, instead of being local, which constitutes a weaker condition on the system than the assumption of the local conservation does. Global conservation of baryon charge within an unknown volume V containing A baryons is expressed as

$$\rho \equiv \frac{A}{V} = (1-\chi)\,\rho_{\rm H}(\mu^n, \mu^e, T) + \chi\,\rho_{\rm Q}(\mu^n, \mu^e, T)\,, \tag{45}$$

where $\chi \equiv V_{\rm Q}/V$ denotes the volume proportion of quark matter, $V_{\rm Q}$, in the unknown volume V. The global neutrality of electric charge within the volume V means that the integral over the charge density, $Q \equiv 4\pi \int_V dr\, r^2\, q(r)$, must vanish rather than $q(r)$ itself *. One thus has

$$0 = \frac{Q}{V} = (1-\chi)\,q_{\rm H}(\mu^n, \mu^e, T) + \chi\,q_{\rm Q}(\mu^n, \mu^e, T) + q_{\rm L}\,, \tag{46}$$

where $q_{\rm L}$ denotes the electric charge density of leptons.

For a given temperature, Eqs. (41) through (46) serve to determine the two independent chemical potentials and the volume V for a specified volume fraction χ of the quark phase in equilibrium with the hadronic phase. This being done $V_{\rm Q}$ is obtained as $V_{\rm Q} = \chi V$. In these equations the chemical potentials obviously depend on the proportion χ of the phases in equilibrium, and hence so also all properties that depend on them: the energy densities, baryon and charge densities of each phase, and the common pressure. For the mixed phase, the volume proportion of quark matter varies from $0 \le \chi \le 1$, and the energy density is the linear combination of contributions coming from the two phases,

$$\epsilon = (1-\chi)\,\epsilon_{\rm H}(\mu^n, \mu^e, \{\phi\}, T) + \chi\,\epsilon_{\rm Q}(\mu^n, \mu^e, T)\,. \tag{47}$$

Solving the models of confined and deconfined phases, in both pure phases and in the mixed phase, we can compute the baryon, lepton and quark populations

---

*Since the substances are uniform in any small locally inertial region V of the star, the integral over the charge densities that expresses global neutrality takes the simple form

$$Q \equiv 4\pi \int_V dr\, r^2\, q(r) = (V - V_{\rm Q})\,q_{\rm H}(\mu^b, \mu^q) + V_{\rm Q}\,q_{\rm Q}(\mu^b, \mu^q)\,,$$

where $q_{\rm H}$ and $q_{\rm Q}$ denote the net electric charges carried by hadronic and quark matter [8].

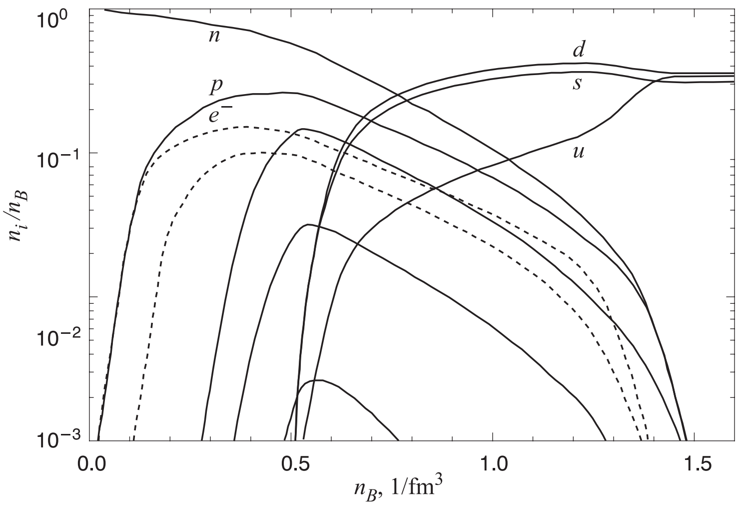

Fig. 5. Composition of chemically equilibrated, stellar quark–hadron (hybrid star) matter as a function of baryon density. Hadronic matter is described by a relativistic Hartree Lagrangian (HV of [7, 27], the bag constant is $B = 250\ \mathrm{MeV/fm^3}$)

in charge-neutral β-stable neutron star matter from Eqs. (41) through (46). The outcome is shown in figure 5 for different, representative bag constants as well as different many-body approximations employed to model confined hadronic matter. (For more details, see [7].)

Three features emerge immediately from this population. Firstly, one sees that the transition from pure hadronic matter to the mixed phase occurs at rather low density of about $3\,\rho_0$ or less [23]. Depending on parameters like the bag constant as well as the underlying nuclear many-body approximation, threshold values even as small as about $2\,\rho_0$ were found, as pointed out first by Glendenning [22, 24] and confirmed later, on the basis of different many-body approximations, by Hermann [23]. Secondly, we emphasize the saturation of the leptons as soon as quark matter appears. At this stage, charge neutrality is achieved more economically among the baryon-charge carrying particles themselves. Thirdly, the presence of quark matter enables the hadronic regions of the mixed phase to arrange to be more isospin symmetric (i.e., closer equality in proton and neutron number) than in the pure phase by transferring charge to the quark phase in equilibrium with it. Symmetry energy will be lowered thereby at only a small cost in rearranging the quark Fermi surfaces. Electrons play only a minor role when neutrality can be realized among the baryon-charge carrying particles. Thus the mixed phase region of the star will have positively charged regions of nuclear matter and negatively charged regions of quark matter.

8. MODELS FOR THE EQUATION OF STATE

To explore the implications of the mixed phase for the structure of neutron stars, we shall employ a collection of different models for the equation of state derived for three different assumptions about the composition of 'neutron' star matter. In the most primitive conception, a neutron star is constituted from neutrons. At a slightly more accurate representation, a beta stable compact star will contain neutrons and a small number of protons whose charge is balanced by leptons. We represent the interactions among baryons in the relativistic mean field theory. Details can be found elsewhere [25–27]. The coupling constants in the theory are chosen so that for symmetric nuclear matter, the five important bulk properties (energy per baryon, incompressibility, effective nucleon mass, asymmetry energy, saturation density) are reproduced [28, 29].

At the densities in the interior of neutron stars, the neutron chemical potential will exceed the mass (modified by interactions) of various members of the baryon octet [30]. So, in addition to neutrons, protons and electrons, neutron stars are expected to have populations of hyperons which together with nucleons and leptons are in a charge neutral equilibrium state. Interactions among the baryons are incorporated, and coupling constants chosen, as above [25–28]. Hyperon coupling constants are chosen: (1) to reproduce the binding of the lambda in the

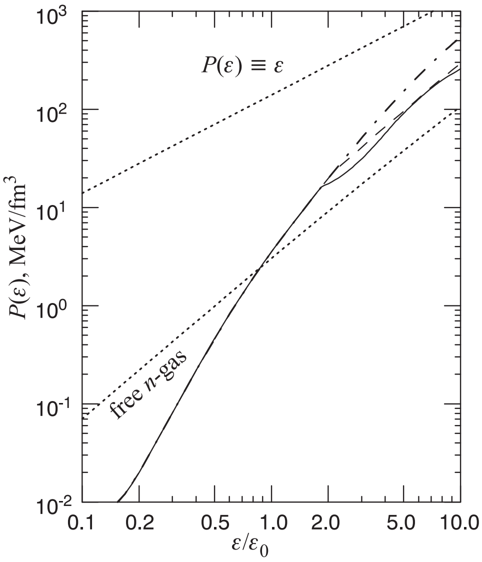

Fig. 6. Three models for the equation of state. Solid curve: equation of state of hybrid star ($G^{\rm K240}_{\rm B180}$), dashed curve: hyperon star ($G^{\rm K240}_{\rm M78}$), dash-dotted curve: neutron star ($G^{\rm K240}_{\rm M78}$), protons and neutrons only. For details about these equations of state, see Ref.29

nuclear matter, (2) to be compatible with hypernuclei, and (3) to support at a minimum a neutron star of at least $1.5\,M_\odot$ [22].

How to handle the phase equilibrium in the dense neutron star matter having two conserved charges, baryon and electric, is described in detail in the Glendenning paper [22]. As we have seen qualitatively in Section 1, the properties of a phase transition in a multi-component system are very different from the familiar ones of a single-component system formerly (and incorrectly) used to describe the deconfinement phase transition in neutron stars [8, 22]. Models for the equation of state of neutron, hyperon, and hybrid star matter are shown in Fig. 6. One sees that the transition of confined baryonic matter to quark matter sets in at about twice the nuclear matter density ϵ_0 $(= 140\ \mathrm{MeV/fm^3})$, which leads to an additional softening of the equation of state. Pure quark matter is obtained for densities $\gtrsim 7\epsilon_0$.

9. SEQUENCES OF ROTATING STARS WITH CONSTANT BARYON NUMBER

Having models for the equation of state at hand which account for the possible quark–hadron phase transition in dense neutron-star matter, we now proceed to the question of how this transition may register itself in an observational stellar signal.

Neutron stars are objects of highly compressed matter so that the geometry of space-time is changed considerably from flat space-time. Thus for the construction of realistic models of rapidly rotating pulsars one has to resort to Einstein's theory of general relativity. In the case of a star rotating at its *absolute* limiting rotational period, that is, with the Kepler (or mass-shedding) frequency, Einstein's field equations,

$$G^{\kappa\lambda} \equiv R^{\kappa\lambda} - \frac{1}{2}\, g^{\kappa\lambda}\, R = 8\,\pi\, T^{\kappa\lambda}(\epsilon, P(\epsilon))\,, \tag{48}$$

are to be solved self-consistently in combination with the general relativistic expression which describes the onset of mass-shedding at the equator [27, 31, 32]:

$$\Omega_{\mathrm{K}} = \beta + \frac{\beta'}{2\psi'} + e^{\Phi-\psi}\sqrt{\frac{\Phi'}{\psi'} + \left(\frac{\beta'}{2\psi'}e^{\psi-\Phi}\right)^2}\,. \tag{49}$$

The metric for a rotating star, suitable for both the interior and exterior, reads [32, 33]

$$ds^2 = e^{2\Psi}dt^2 - e^{2\lambda}dr^2 - e^{2\mu}d\theta^2 - e^{2\psi}(d\phi - \beta\, dt)^2\ . \tag{50}$$

Because of the underlying symmetries, the metric functions Ψ, λ , μ, and ψ

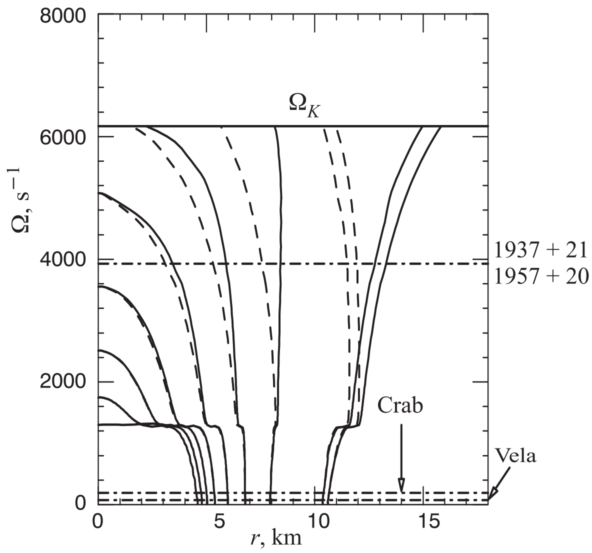

Fig. 7. Frequency dependence of quark structure in rotating hybrid stars. The radial direction is along the star's equator (solid curves) and pole (dashed curves). The nonrotating star mass is $\sim 1.42\, M_\odot$. The rotational frequency ranges from zero to Kepler frequency

are independent of t and ϕ but depend on r, θ, and Ω. The quantity β denotes the angular velocity of the local inertial frames (frame dragging frequency) and depends on the same variables as the metric. The frequency $\bar{\beta} \equiv \Omega - \beta(r, \theta, \Omega)$, is the star's rotational frequency relative to the frequency of the local inertial frames, and is the one the centrifugal force acting on the mass elements of the rotating star's fluid depends on [34]. The quantities $R^{\kappa\lambda}$, $g^{\kappa\lambda}$, and R denote respectively the Ricci tensor, metric tensor, and Ricci scalar (scalar curvature). The dependence of the energy-momentum tensor $T^{\kappa\lambda}$ on pressure and energy density, P and ϵ, respectively, is indicated in Eq. (48). The primes in (49) denote derivatives with respect to Schwarzschild radial coordinate, and all functions on the right are evaluated at the star's equator. All the quantities in the right-hand side of Eq.(49) depend also on $\Omega_{\rm K}$, so that it is not an equation for $\Omega_{\rm K}$, but a transcendental relationship which the solution of the equations of stellar structure, resulting from Eq. (48), must satisfy if the star is rotating at its Kepler frequency. (Details can be found in [27].)

The outcome of two self-consistent calculations, one for a hybrid and the other for a conventional hyperon star, is compared in Figs. 7 and 8. The stars' baryon number is kept constant during spin-down from the Kepler frequency to zero rotation, as it should be. The frequency Ω is assumed to be constant throughout the star's fluid since uniform rotation is the configuration that minimizes the

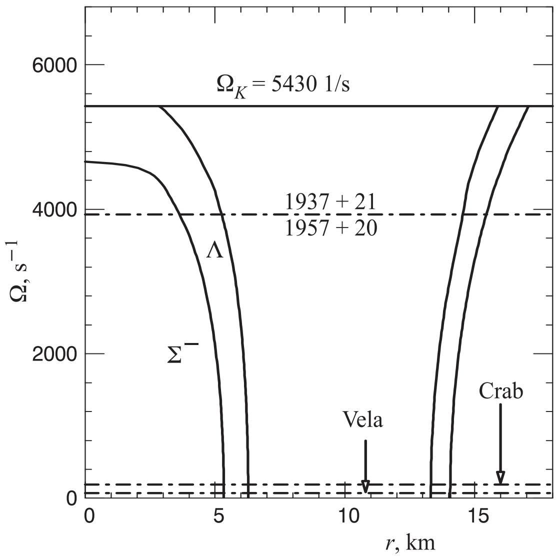

Fig. 8. Same as Fig. 7, but for a conventional hyperon star ($M \sim 1.40\,M_\odot$), that is, transition to quark matter is suppressed. The Σ^- is absent for $\Omega \gtrsim 4700$ s^{-1} because the central density falls below the threshold density of the Σ^- particle

mass-energy at specified baryon number and angular momentum [35].

According to the mass of a hybrid star, it may consist of an inner sphere of purely quark matter (lower-left portion of Fig. 7 for which $\Omega \lesssim 1250$ s^{-1} and $r \lesssim 4.5$ km), surrounded by a few kilometers thick shell of mixed phase of hadronic and quark matter arranged in a lattice structure, and this surrounded by a thin shell of hadronic liquid, itself with a thin crust of heavy ions [8]. The Coulomb lattice structure of varying geometry introduced to the interior of neutron stars [8], which may have dramatic effects on pulsar observables including transport properties and the theory of glitches, is a consequence of the competition of the Coulomb and surface energies of the hadronic and quark matter phase. This competition establishes the shapes, sizes and spacings of the rarer phase in the background of the other (that is, for decreasing density: hadronic drops, hadronic rods, hadronic plates immersed in quark matter followed by quark plates, quark rods and quark drops immersed in hadronic matter) so as to minimize the lattice energy. For an investigation of the structure of the mixed phase of baryons and quarks predicted by Glendenning, we refer to [36].

When the frequency of rotation of the hybrid star diminishes, it becomes less deformed and the central density *rises*. For some pulsars the mass and initial rotational frequency Ω may be such that the central density rises from below the critical density for dissolution of baryons into their quark constituents. This is accompanied by a sudden shrinkage of the hybrid star, which dramatically effects its moment of inertia and hence the braking index of a pulsar, as we shall see in the next sections.

10. MOMENT OF INERTIA

Elsewhere we have obtained an expression for the moment of inertia of a relativistic star that accounts for the centrifugal flattening of the star [31, 37]. It is given by

$$I(\Omega) = 4\pi \int_0^{\pi/2} d\theta \int_0^{R(\theta)} dr \, \frac{e^{\lambda+\mu+\nu+\psi} \, [\epsilon + p]}{e^{2(\nu-\psi)} - \bar{\omega}^2} \, \frac{\Omega - \omega}{\Omega} \, . \tag{51}$$

The radial distribution of the energy density and pressure, $\epsilon(r)$ and $p(r)$, are found from the solution of the equations for rotating relativistic stars. For slow rotation this expression reduces to the well-known, and frequency *independent* result [37].

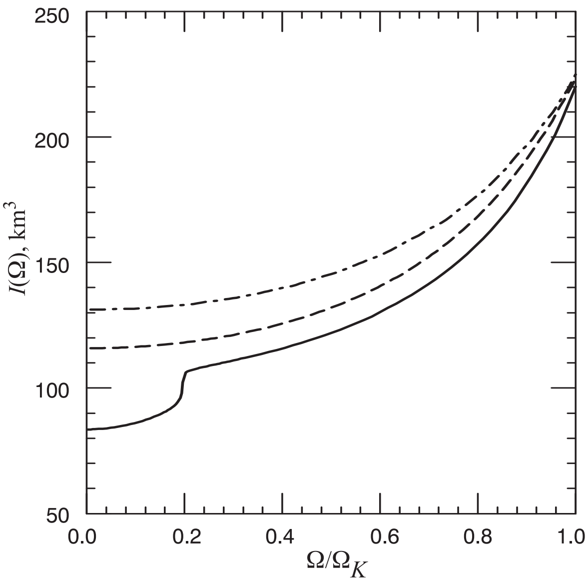

Fig. 9. Moment of inertia as a function of rotational frequency (in units of Kepler) for three neutron stars (solid curve: hybrid star, dashed curve: hyperon star, dash-dotted curve: neutron star made up of only protons and neutrons) with different constitutions as described in Sect. 8. The baryon number is constant along each curve. The development of a quark matter core for decreasing frequency (increasing density) causes a sudden reduction of I.

We show how the moment of inertia changes with frequency in Fig. 9 for stars having the same baryon number but different constitutions, as described in Sect. 8. The two curves without a drop at low frequencies are for conventional neutron (dot-dashed line) and hyperon (dashed) stars of non-rotating mass $M \sim 1.45\, M_\odot$. The solid line is for a hybrid star of roughly the same mass and baryon number. The shrinkage of the hybrid star due to the development of a quark matter core at low frequencies, known from Fig. 7, manifests itself in a sudden reduction of I, which is the more pronounced the bigger the quark matter core (i.e., the smaller Ω) in the centre of the star.

11. SIGNAL OF QUARK-DECONFINEMENT IN THE BRAKING INDICES OF PULSARS

Pulsars are identified by their periodic signal believed to be due to a strong magnetic field fixed in the star and oriented at an angle from the rotation axis. The period of the signal is therefore that of the rotation of the star. The angular velocity of rotation decreases slowly but measurably over time, and usually the first and occasionally the second time derivative can also be measured. Various energy loss mechanisms could be at play such as the dipole radiation, part of which is detected on each revolution, as well as other losses such as ejection of charged particles [38]. The measured frequency and its time derivative have been used to estimate the spin-down time or age of pulsars. The age is very useful for classifying and understanding pulsar phenomena such as glitch activity.

Let us assume, as usual, that the pulsar slow-down is governed by a single mechanism or several mechanisms having the same power law. Let us write the energy balance equation as

$$\frac{dE}{dt} = \frac{d}{dt}\left\{\frac{1}{2}\, I\,\Omega^2\right\} = -\,C\,\Omega^{n+1}\,, \tag{52}$$

where, for magnetic dipole radiation, $C = \frac{2}{3}m^2 \sin^2\alpha$, $n = 3$, m is the magnetic dipole moment and α is the angle of inclination between magnetic moment and rotation axis. If, as is customary, the angular velocity Ω is regarded as the only time-dependent quantity, one obtains the usual formula for the rate of change of pulsar frequency,

$$\dot{\Omega} = -\,K\,\Omega^n\,, \tag{53}$$

with K a constant and n the *braking index.* From the braking law (53) one usually defines from its solution, the spin-down age of the pulsar

$$\tau = -\,\frac{1}{n-1}\,\frac{\Omega}{\dot{\Omega}}\,. \tag{54}$$

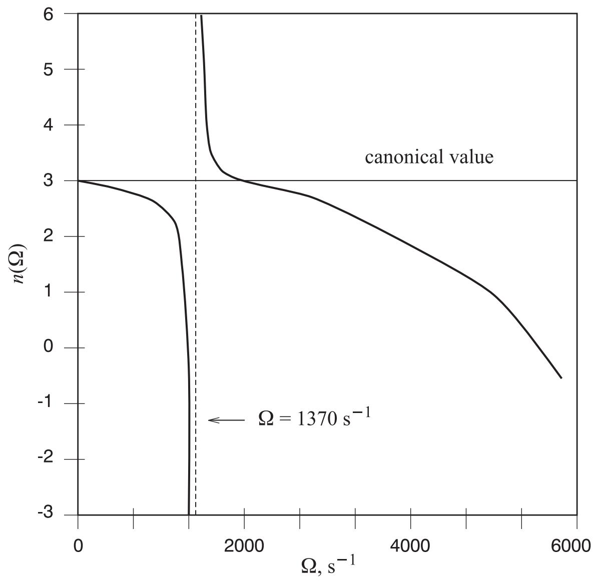

Fig. 10. Braking index as a function of rotational frequency for a hybrid star. The dip at frequencies $\Omega \sim 1370\ \mathrm{s}^{-1}$ originates from *quark deconfinement*. The overall reduction of n below 3 is due to the frequency dependence of I and, thus, is independent of whether or not quark deconfinement takes place in pulsars

However the moment of inertia is *not* constant in time but responds to changes in rotational frequency, as can be seen in Fig. 9, more or less according to the softness or stiffness of the equation of state (that is, the star's internal constitution) and depending on whether the stellar mass is small or large. This response changes the value of the braking index in a frequency-dependent manner, even if the sole energy-loss mechanism were *pure* dipole as in Eq. (52). Thus during any epoch of observation, the braking index will be measured to be different from $n = 3$ by a certain amount. How large will this amount depend, for any given pulsar, on its rotational frequency and for different pulsars of the same frequency, on their mass and on their internal constitution?

When the frequency response of the moment of inertia is taken into account, Eq. (53) is replaced by

$$\dot{\Omega} = -\,2IK\,\frac{\Omega^n}{2I + I'\Omega} = -\,K\,\Omega^n \left\{ 1 - \frac{I'}{2I}\,\Omega + \left(\frac{I'}{2I}\,\Omega\right)^2 - \cdots \right\}, \qquad (55)$$

where $I' \equiv dI/d\Omega$ and $K = C/I$. This explicitly shows that the frequency dependence of $\dot{\Omega}$ corresponding to *any* mechanism that absorbs (or deposits) rotational energy such as Eq. (52) cannot be a power law, as in Eq. (53) with

K a constant. It must depend on the mass and internal constitution of the star through the response of the moment of inertia to rotation as in Eq. (55).

Equation (55) can be represented in the form of Eq. (53) (but now with a frequency dependent prefactor) by evaluating

$$n(\Omega) = \frac{\Omega\,\ddot{\Omega}}{\dot{\Omega}^2} = n - \frac{3\,I'\,\Omega + I''\,\Omega^2}{2\,I + I'\,\Omega}\,. \tag{56}$$

Therefore the effective braking index depends explicitly and implicitly on Ω. The right side reduces to a constant n only if I is independent of frequency. But this cannot be, not even for slow pulsars if they contain a quark matter core. The centrifugal force ensures the response of I to Ω. As an example, we show in Fig. 10 the variation of the braking index with frequency for the rotating hybrid star of Fig. 7. For illustration we assume dipole radiation. As before, the baryon number of the star is kept constant. Because of the structure in the moment of inertia, driven by the phase transition into the deconfined quark matter phase, the braking index deviates dramatically from 3 at small rotation frequencies. Such an anomaly in $n(\Omega)$ is not obtained for conventional neutron or hyperon stars because their moments of inertia increase smoothly with Ω (cf. Fig. 9). The observation of such an anomaly in the timing structure of pulsars may thus be interpreted as a signal for the development of quark-matter cores in the centers of pulsars.

As a very important subject on this issue, we estimate the duration over which the braking index is anomalous. It can be estimated from

$$\Delta T \simeq -\frac{\Delta\Omega}{\dot{\Omega}} = \frac{\Delta P}{\dot{P}}\,, \tag{57}$$

where $\Delta\Omega$ is the frequency interval of the anomaly. The range over which n is smaller than zero and larger than six (Fig. 10) is $\Delta\Omega \approx -100$ s^{-1}, or $\Delta P \approx -2\pi\Delta\Omega/\Omega^2 \approx 3\times10^{-4}$ s at $\Omega = 1370$ s^{-1}. So, for a millisecond pulsar whose period derivative is typically $\dot{P} \simeq 10^{-19}$, we find $\Delta T \simeq 10^8$ years (see Fig. 11). The dipole age of such pulsars is about 10^9 years. So as a rough estimate we may expect about 10% of the millisecond pulsars * to be in the transition epoch and so could be signaling the ongoing process of quark deconfinement in their cores! To avoid confusion, we point out that the spin-up has nothing to do with the minuscule spin-up known as a pulsar glitch. In the latter case the relative change of the moment of inertia is very small, $\Delta I/I \simeq -\Delta\Omega/\Omega \simeq 10^{-6}$ or smaller, and approximates closely a continuous response of the star to changing frequency on any time scale that is large compared to the glitch and recovery interval.

*Presently about 25 solitary millisecond pulsars are known.

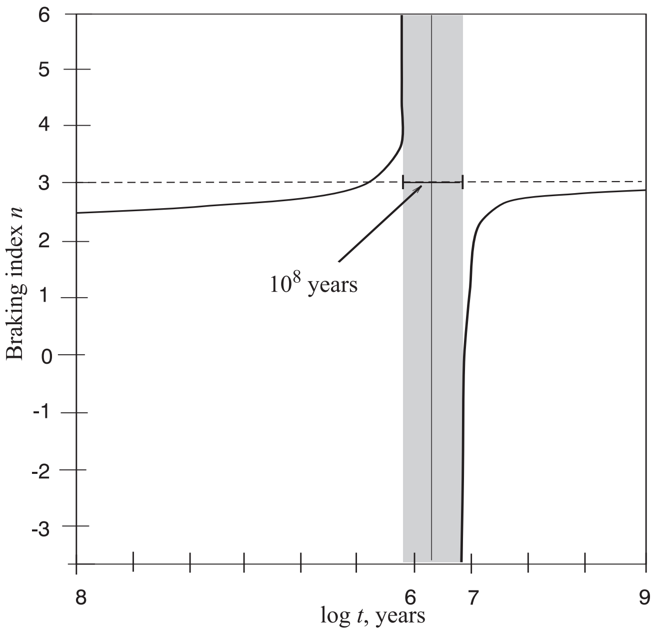

Fig. 11. Braking index as a function of time. The epoch over which n is anomalous (i.e., $-\infty < n < +\infty$) because of quark deconfinement, $\sim 10^8$ years, is indicated by the shaded area (after Glendenning [39]).

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