«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
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MANY-BODY THEORY FOR SYSTEMS OF COMPOSITE HADRONS

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Many-body systems of composite hadrons are characterized by processes that involve the simultaneous presence of hadrons and their constituents. We briefly review several methods that have been devised to study such systems and present a novel method that is based on the ideas of mapping between physical and ideal Fock spaces. The method, known as the Fock–Tani representation, was invented years ago in the context of atomic physics problems and was recently extended to hadronic physics. Starting with the Fock-space representation of single-hadron states, a change of representation is implemented by a unitary transformation such that composites are redescribed by elementary Bose and Fermi field operators in an extended Fock space. When the unitary transformation is applied to the microscopic quark Hamiltonian, effective, hermitian Hamiltonians with a clear physical interpretation are obtained. The use of the method in connection with the linked-cluster formalism to describe short-range correlations and quark deconfinement effects in nuclear matter is discussed. As an application of the method, an effective nucleon–nucleon interaction is derived from a constituent quark model, and used to obtain the equation of state of nuclear matter in the Hartree–Fock approximation.

Многочастичные системы составных адронов характеризуются процессами, которые включают в себя одновременно и адроны, и их составляющие. Кратко рассмотрены несколько методов, специально разработанных для изучения таких систем, и представлен новый метод, который основан на идее соответствия между физическим и идеальным пространствами Фока. Этот метод, известный как представление Фока–Тани, был разработан уже давно для решения задач атомной физики, а недавно был расширен и для применения в адронной физике. Начиная с одночастичных адронных состояний в пространстве Фока с помощью унитарного преобразования представление меняется так, что составляющие переписываются в терминах элементарных полевых операторов Бозе и Ферми в расширенном пространстве Фока. Когда это унитарное преобразование применяется к микроскопическому кварковому гамильтониану, получаются эффективные эрмитовы гамильтонианы с ясной физической интерпретацией. Обсуждается использование этого метода в связи с формализмом связанных кластеров для описания корреляций на малых расстояниях и эффектов кваркового деконфайнмента в ядерной материи. В качестве применения этого метода рассмотрены: вывод эффективного нуклон–нуклонного взаимодействия из конституентной кварковой модели и получение уравнения состояния ядерной материи в приближении Хартри–Фока.

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1. INTRODUCTION

One of the most central problems of contemporary particle and nuclear physics is the description of the interactions among hadrons and the properties of high temperature and/or density hadronic matter in terms of quark and gluon degrees of freedom. The mathematical description of such systems is complicated due to the simultaneous presence of composites (hadrons) and constituents (quarks and gluons). The early studies of the hadron–hadron interaction using quark models employ cluster techniques such as adiabatic methods [1] and resonating group or generator coordinate methods [2]. More recently Barnes and Swanson [3] introduced a formalism based on the concepts of constituent interchange and quark line diagrams, known as the «quark Born diagram» formalism. This method is similar to the «constituent exchange» mechanisms proposed by Gunion, Brodsky and Blankenbecler [4] many years ago for high-energy hadron scattering. Also recently, Blaschke and Röpke [5] considered a thermodynamic Green's function approach for meson–meson scattering in a constituent quark model. Thermodynamic Green's functions have been used to study many-body problems in many areas of physics and seem appropriate for the study of quark-nuclear physics problems.

A different approach to the problem is the use of mapping representations, in which composites are redescribed by elementary particles. There exists an extensive literature on the subject; in nuclear physics mapping representations are mainly used in the study of collective oscillations of the nucleus. Examples of such mappings include the Holstein–Primakoff representation [6], the boson expansion of Belyaev and Zelevinskii [7] and the Marumori mapping [8]. A good review on these can be found in Ref. 9. Although such techniques are available for a long time, only recently there have been made attempts to extend them to treat hadron–hadron interactions in the context of quark models. References 10–15 are examples of extensions of such techniques to quark models. Here we discuss in some detail the Fock–Tani representation, which was originally developed for atomic physics applications and recently was extended to hadronic physics problems [16–18]. It was invented independently by Girardeau [19] and Vorob'ev and Khomkin [20]. It has been continuously improved during the last two decades, and has been used with success by Girardeau and collaborators in several areas of atomic physics [21–23]. Like many other mapping formalisms, the method is based on a change of representation by introducing fictitious elementary hadrons in close correspondence to the real hadrons. The change of representation is implemented by means of a unitary transformation such that the composite hadrons are redescribed by elementary-particle field operators. The unitary transformation is a generalization of a transformation employed by S. Tani [24] in 1960 to study single-particle scattering by a potential with a bound state. In the new representation the microscopic interquark forces change, they become weaker, in the sense they cannot bind the quarks into hadrons, they describe only truly scattering processes. In the new representation, in addition to the modified interquark forces, one obtains effective interactions describing all possible processes between hadrons and their constituents. In the new representation all field operators representing quarks, antiquarks, gluons and *hadrons* satisfy canonical commutation relations and therefore the traditional methods of quantum field theory can be readily applied.

The use of the Fock–Tani representation for studying hadronic interactions at the quark–gluon level shares some similarities with the program outlined by Weinberg in the last section of his 1979 paper on effective Lagrangians [25]. Weinberg makes the suggestion of using the «quasiparticle» approach [26] for making perturbative calculations in QCD at low energies. The quasiparticle approach is a formalism developed by Weinberg in the 60's to deal with potentials that are too strong to allow the use of perturbation theory. In the quasiparticle approach the bound states of the theory are redescribed by fictitious elementary particles and, in order not to change the physics of the problem, the original potential is modified in such a way that the new potential does not produce the elementary particles as bound states of the theory. With such a modification the potential becomes sufficiently weak that scattering amplitudes can be calculated perturbatively. Weinberg imagines the possibility of implementing a quasiparticle approach to QCD. The program would start by weakening the forces of QCD with the introduction of an infrared cut-off. In order to preserve the physical content of the theory, the bound states (hadrons) are introduced as fictitious elementary particles which should be described by an effective chirally invariant Lagrangian. The parameters of the effective Lagrangian would have to be functions of the cut-off, defined by differential equations which guarantee the cut-off independence of the S-matrix, with the boundary condition that for higher enough energies one recovers pure QCD, where there is no cut-off. The program would work in practice if the solutions of the equations could be continued at low energies to cut-off values sufficiently small that perturbation theory could be employed.

A major complication for the applications of many-body techniques to systems containing composite particles is the absence of a good understanding of the low-energy regime of quantum chromodynamics (QCD). The phenomena of confinement of quarks and gluons and the formation of the hadron bound states in QCD requires the use of effective models, which in many cases are over-simplifications of reality. The techniques mentioned above invariably employ nonrelativistic quark models, or semi-relativistic ones. Obviously for the study of high temperature and/or density hadronic matter, relativity seems to be essential. On the other hand, for the study of low-energy hadron–hadron scattering, or even to low-temperature and/or density nuclear matter, the use of nonrelativistic or semirelativistic models might still be of interest for obtaining insight into the problem.

In the present paper we discuss the application of the Fock–Tani representation to study the properties of nuclear matter in terms of composite nucleons. We start in Section 2 with a brief discussion on quark models and possible connections to some aspects to QCD. In particular, for low-energy applications, the connection of the constituent quark model to the dynamical chiral symmetry breaking in QCD is discussed. In order to make contact with other composite-particle formalisms commonly used in this context, we briefly review in Section 3 the resonating group method (RGM), the quark-Born diagram (QBD) method and thermodynamic Green's function formalisms. In Section 4 we present in some detail the basic ideas and methods of the Fock–Tani representation. Section 5 presents the derivation of an effective nucleon–nucleon interaction from a microscopic quark model. In Section 6 we discuss the use of a linked-cluster formalism in the Fock–Tani representation for the problem of nuclear matter and the onset of quark deconfinement. We also present one application of the effective nucleon–nucleon interaction derived in Section 5 to the calculation of the equation of state of cold nuclear matter. Conclusions and future perspectives are presented in Section 7.

2. QCD, CHIRAL SYMMETRY AND THE QUARK MODEL

There is a widespread belief that there exists an intermediate energy region in which it makes sense to describe the strong interactions in terms of an effective field theory of *constituent quarks* subject to weak color forces that become strong only at large separations and keep the quarks confined. The u and d constituent quarks have a mass of $m \sim 300$ MeV, which are believed to be the result of the spontaneous breakdown of the $SU(2) \otimes SU(2)$ chiral symmetry. If this is so, the Goldstone bosons of the spontaneous symmetry breakdown (pions in the case of u and d quarks only) must be included among the degrees of freedom of the effective theory. The lowest order terms of the Lagrangian of such an effective field theory were written down by Manohar and Georgi [27]. Many of the successes of the simple nonrelativistic quark model can be understood in this framework with a chiral symmetry breaking scale $\Lambda_{\chi\mathrm{SB}} \sim 1$ GeV, which is significantly larger than the confinement scale Λ_{conf}. This scenario of weakly interacting constituent quarks has recently been shown [28] to provide a nice interpretation of lattice calculations. Also it has been shown recently that the Manohar and Georgi model can be derived from QCD models of the Nambu–Jona-Lasinio type and QCD effective action calculations [29].

The use of the Fock–Tani representation in connection with an effective quark–gluon Lagrangian involves a two-step process, as in Weinberg's program outlined in the Introduction. In the first step the QCD forces are weakened by the introduction of an infrared cut-off Λ, which we choose to be $\Lambda_{\mathrm{conf}} <$

$\Lambda < \Lambda_{\chi SB}$, and the QCD Lagrangian is replaced by an effective Lagrangian, as for example the one of Manohar and Georgi. In the next step, fictitious elementary particles with the quantum numbers of hadrons are introduced and their effective interactions are derived from the microscopic effective quark–gluon Lagrangian through the Fock–Tani unitary transformation. The parameters of the resulting effective hadronic interactions are functions of those of the quark–gluon Lagrangian. The program will be completed, in the sense of Weinberg's program, when the cut-off independence of the S-matrix elements is enforced. Of course, this is the most difficult part of the entire program and not much progress can be made without a better understanding of the underlying mechanisms which govern the confinement and dynamical chiral symmetry breaking phenomena of QCD. While such an understanding is not reached, progress in the study of the hadronic interactions at the quark–gluon level can be made by fixing the parameters of the effective quark–gluon theory experimentally.

As discussed previously, since the effects of dynamical chiral symmetry breaking are included in the constituent quark mass the interquark forces become weaker in the effective theory. This allows one to identify the low-lying hadrons with nonrelativistic bound states of the constituent quarks. The quarks are presumably bound by the confining QCD interactions, along with effects of multiquark and multigluon operators that appear in high orders of $1/\Lambda_{\rm conf}$ in the effective Lagrangian. Calculations of matrix elements of strong and electroweak couplings of quarks are performed using perturbation theory or large N_c expansion techniques, where N_c is the number of colors. For the calculation of matrix elements involving hadrons, such as the calculation of baryon magnetic moments and the G_A/G_V ratio in β-decay, the usual nonrelativistic quark-model wave functions are used for the hadron bound states. The nonrelativistic wave functions are obtained by solving the Schrödinger equation for three quarks (baryons) or a quark–antiquark pair (mesons) using a phenomenological confining interaction.

For future purposes, let us consider just the pion–quark interaction piece of the Manohar and Georgi model. At tree level, it is the standard pseudovector coupling

$$\mathcal{H}_{\pi q} = \frac{1}{f_\pi}\bar{\psi}\gamma_5 t^a \boldsymbol{\alpha}\cdot\boldsymbol{\nabla}\pi^a\psi, \tag{1}$$

where $t^a = 1/2\,\tau^a$. This leads to an effective nonrelativistic quark–quark interaction of the form

$$V_{\pi q} = -\left(\frac{1}{f_\pi}\right)^2 t^{a(1)}t^{a(2)}\,\frac{\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(2)}\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\,. \tag{2}$$

This effective quark–quark interaction will be used to study the short-range part of the nucleon–nucleon interaction obtained via mapping to the Fock–Tani representation. This will be discussed in Section 5.

Before proceeding to the next section, let us introduce some notations. A meson state composed of one quark and one antiquark can be written in terms of constituent quark and antiquark creation operators $q^\dagger$ and $\bar{q}^\dagger$ as

$$|\alpha\rangle = M^\dagger_\alpha |0\rangle, \tag{3}$$

where $|0\rangle$ is the vacuum state for the constituent quarks, defined by

$$q_\mu |0\rangle = \bar{q}_\nu |0\rangle = 0, \tag{4}$$

$M^\dagger_\alpha$ is the meson creation operator

$$M^\dagger_\alpha = \Phi^{\mu\nu}_\alpha q^\dagger_\mu \bar{q}^\dagger_\nu, \tag{5}$$

and $\Phi^{\mu\nu}_\alpha$ is the Fock-space meson amplitude. The index α identifies the quantum numbers of the meson, $\alpha = \{\mathrm{spatial}, \mathrm{spin}, \mathrm{isospin}\}$. The indices μ and ν denote the spatial, spin, flavor, and color quantum numbers of the quarks. A summation over a repeated index is implied. It is convenient to work with Φ orthonormalized

$$\langle\alpha|\beta\rangle = \Phi^{*\mu\nu}_\alpha \Phi^{\mu\nu}_\beta = \delta_{\alpha\beta}. \tag{6}$$

The quark and antiquark operators satisfy canonical anticommutation relations,

$$\begin{aligned} &\left\{q_\mu, q^\dagger_\nu\right\} = \left\{\bar{q}_\mu, \bar{q}^\dagger_\nu\right\} = \delta_{\mu\nu}, \\ &\{q_\mu, q_\nu\} = \{\bar{q}_\mu, \bar{q}_\nu\} = \{q_\mu, \bar{q}_\nu\} = \left\{q_\mu, \bar{q}^\dagger_\nu\right\} = 0. \end{aligned} \tag{7}$$

Using these quark anticommutation relations and the normalization condition of Eq. (6), one can easily show that the meson operators satisfy the following commutation relations

$$\begin{aligned} &[M_\alpha, M^\dagger_\beta] = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \\ &[M_\alpha, M_\beta] = 0, \end{aligned} \tag{8}$$

where

$$\Delta_{\alpha\beta} = \Phi^{*\mu\nu}_\alpha \Phi^{\mu\sigma}_\beta \bar{q}^\dagger_\sigma \bar{q}_\nu + \Phi^{*\mu\nu}_\alpha \Phi^{\rho\nu}_\beta q^\dagger_\rho q_\mu. \tag{9}$$

In addition, one has

$$\begin{aligned} &[q_\mu, M^\dagger_\alpha] = \delta_{\mu\mu'} \Phi^{\mu'\nu}_\alpha \bar{q}^\dagger_\nu, \qquad [\bar{q}_\nu, M^\dagger_\alpha] = -\delta_{\nu\nu'} \Phi^{\mu\nu'}_\alpha q^\dagger_\mu, \\ &[q_\mu, M_\alpha] = [\bar{q}_\nu, M_\alpha] = 0. \end{aligned} \tag{10}$$

The single-composite baryon creation operator, $B^\dagger_\alpha$, is written in terms of three constituent-quark creation operators as

$$B^\dagger_\alpha = \frac{1}{\sqrt{3!}} \Psi^{\mu_1\mu_2\mu_3}_\alpha q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\mu_3}. \tag{11}$$

$\Psi_\alpha^{\mu_1\mu_2\mu_3}$ is the Fock-space baryon amplitude, where the index α identifies the quantum numbers of the baryon, and μ those of the quarks. As for the mesons, it is convenient to take the Fock-space amplitude orthonormalized

$$\langle\alpha|\beta\rangle = \Psi_\alpha^{*\mu_1\mu_2\mu_3}\Psi_\beta^{\mu_1\mu_2\mu_3} = \delta_{\alpha\beta}\,. \tag{12}$$

Using the quark anticommutation relations, Eq. (7), and the normalization condition above, it can easily be shown that the baryon operators satisfy the following noncanonical anticommutation relations

$$\{B_\alpha, B_\beta^\dagger\} = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \qquad \{B_\alpha, B_\beta\} \;=\; 0, \tag{13}$$

where

$$\Delta_{\alpha\beta} = 3\Psi_\alpha^{*\mu_1\mu_2\mu_3}\Psi_\beta^{\mu_1\mu_2\nu_3}q_{\nu_3}^\dagger q_{\mu_3} - \frac{3}{2}\Psi_\alpha^{*\mu_1\mu_2\mu_3}\Psi_\beta^{\mu_1\nu_2\nu_3}q_{\nu_3}^\dagger q_{\nu_2}^\dagger q_{\mu_2}q_{\mu_3}. \tag{14}$$

In addition,

$$\{q_\mu, B_\alpha^\dagger\} = \sqrt{\frac{3}{2}}\Psi_\alpha^{\mu\mu_2\mu_3}q_{\mu_2}^\dagger q_{\mu_3}^\dagger, \qquad \{q_\mu, B_\alpha\} = 0. \tag{15}$$

The bound state amplitudes $\Phi_\alpha^{\mu\nu}$ and $\Psi_\alpha^{*\mu_1\mu_2\mu_3}$ are obtained from a microscopic quark Hamiltonian. The commonly-used quark-model Hamiltonians can be written generically as

$$\begin{aligned} H \;&=\; T(\mu)\,q_\mu^\dagger q_\mu + T(\nu)\,\bar{q}_\nu^\dagger \bar{q}_\nu + V_{q\bar{q}}(\mu\nu;\sigma\rho)q_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho q_\sigma + \\ &+\; \frac{1}{2}V_{qq}(\mu\nu;\sigma\rho)q_\mu^\dagger q_\nu^\dagger q_\rho q_\sigma + \frac{1}{2}V_{\bar{q}\bar{q}}(\mu\nu;\sigma\rho)\bar{q}_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho \bar{q}_\sigma, \end{aligned} \tag{16}$$

where the convention of a summation over repeated indices is again assumed. Strong decays and baryon–meson couplings are described by terms involving annihilation terms such as $\bar{q}^\dagger q^\dagger q^\dagger q$, which we do not write explicitly.

From Eq. (16), the equation of motion for the single meson state, in free space, is given by

$$\begin{aligned} &H\,(\mu\nu;\mu'\nu')\,\Phi_\alpha^{\mu'\nu'} = \\ &= \left\{\delta_{\mu[\mu']}\delta_{\nu[\nu']}\left[T(\mu) + T(\nu)\right] + V_{q\bar{q}}\,(\mu\nu;\mu'\nu')\right\}\Phi_\alpha^{\mu'\nu'} = E_{[\alpha]}\Phi_{[\alpha]}^{\mu\nu}\,, \end{aligned} \tag{17}$$

where E_α is the total energy of the meson. Here we are using the convention that there is no sum over repeated indices inside square brackets. A similar equation follows for the baryon amplitude.

The composite nature of the mesons and baryons is manifest in the terms $\Delta_{\alpha\beta}$ in Eqs. (8) and (13). Because of these terms, the usual field theoretic techniques, such as the Green's function method, Wick's theorem, etc., cannot be directly applied to such operators. In the same way, the fact that the commutators $[q_\mu, M^\dagger_\alpha]$ and $[\bar{q}_\nu, M^\dagger_\alpha]$, and the anticommutator $\{q_\mu, B^\dagger_\alpha\}$ are not equal to zero, is a manifestation of the lack of kinematic independence of the hadron operators from the quark and antiquark operators. The point is that the hadron operators M_α, $M^\dagger_\alpha$, B_α and $B^\dagger_\alpha$ are not convenient dynamical variables to be used.

Of course, as will be shown in the next section for the case of the Blaschke and Röpke approach, the traditional methods can be directly applied to the microscopic degrees of freedom. But then, the hadron degrees of freedom will appear as poles of Green's functions and their role as independent degrees of freedom is difficult to assess. The aim of changing representation is precisely to isolate the hadronic degree of freedom from the microscopic ones, and transfer the complicated interactions among the hadrons themselves and with their constituents to effective interactions.

We next briefly review the traditional methods RGM, QBD and Green's functions. We make contact between these and the Fock–Tani representation in Section 4.

3. THE RESONATING GROUP, GREEN'S FUNCTIONS AND QUARK-BORN-DIAGRAMS

Let us consider, for simplicity, the scattering of two composite mesons. The baryon–baryon and baryon–meson cases follow similar path. In a RGM calculation, the two-cluster state is introduced by writing

$$|\Lambda\rangle = \frac{1}{\sqrt{2}}\psi^{\alpha\beta}_\Lambda M^\dagger_\alpha M^\dagger_\beta|0\rangle, \tag{18}$$

where $\psi^{\alpha\beta}_\Lambda$ is the ansatz wave function for the meson pair; it describes the c.m. and relative motions of the two-meson clusters. The $M^\dagger$'s are the meson creation operators as defined in Eq. (5). Λ identifies the set of quantum numbers of the two-cluster state. Using the commutation relation of the meson operators, Eq. (8), the normalization condition for the $\psi^{\alpha\beta}_\Lambda$ is obtained to be

$$\langle\Lambda|\Lambda'\rangle = \psi^{*\alpha\beta}_\Lambda N(\alpha\beta;\alpha'\beta')\psi^{\alpha'\beta'}_{\Lambda'} = \delta_{\Lambda'\Lambda}, \tag{19}$$

where $N(\alpha\beta;\alpha'\beta')$ is the «normalization kernel»,

$$N(\alpha\beta;\alpha'\beta') = \delta_{\alpha\alpha'}\delta_{\beta\beta'} - N_E(\alpha\beta;\alpha'\beta') = \delta_{\alpha\alpha'}\delta_{\beta\beta'} - \Phi^{*\mu\nu}_\alpha\Phi^{*\rho\sigma}_\beta\Phi^{\mu\sigma}_{\beta'}\Phi^{\rho\nu}_{\alpha'}. \tag{20}$$

The exchange kernel, $N_E(\alpha\beta;\alpha'\beta')$, comes from the noncanonical part of the meson commutation relation of Eq. (8), and it reflects the Pauli principle among the quarks and antiquarks in the clusters α and β. The equation of motion for $\psi_\Lambda^{\alpha\beta}$ is determined by means of the variational principle

$$\delta\langle\Lambda|(H-E_\Lambda)|\Lambda\rangle=0, \tag{21}$$

where H is the quark–antiquark Hamiltonian given by Eq. (16). Eq. (21) leads to the RGM equation,

$$[H_{RGM}(\alpha\beta;\gamma\delta)-E_\Lambda N(\alpha\beta;\gamma\delta)]\,\psi_\Lambda^{\gamma\delta}=0, \tag{22}$$

with

$$H_{RGM}(\alpha\beta;\gamma\delta)=T_{RGM}(\alpha\beta;\gamma\delta)+V_{mm}(\alpha\beta;\gamma\delta), \tag{23}$$

where the kinetic term $T_{RGM}(\alpha\beta;\gamma\delta)$ is given by

$$T_{RGM}(\alpha\beta;\gamma\delta)=\delta_{\beta\delta}\Phi_\alpha^{*\mu\nu}H(\mu\nu;\mu'\nu')\Phi_\gamma^{\mu'\nu'}+\delta_{\alpha\gamma}\Phi_\beta^{*\mu\nu}H(\mu\nu;\mu'\nu')\Phi_\delta^{\mu'\nu'}, \tag{24}$$

and the potential term $V_{mm}(\alpha\beta;\gamma\delta)$ can be written as a sum of three contributions

$$V_{mm}(\alpha\beta;\gamma\delta)=V_{mm}^{dir}(\alpha\beta;\gamma\delta)+V_{mm}^{exc}(\alpha\beta;\gamma\delta)+V_{mm}^{int}(\alpha\beta;\gamma\delta), \tag{25}$$

where each of these is given by

$$\begin{aligned}V_{mm}^{dir}(\alpha\beta;\gamma\delta)&=2\Phi_\alpha^{*\mu\sigma}\Phi_\beta^{*\rho\nu}V_{q\bar q}(\mu\nu;\mu'\nu')\Phi_\delta^{\rho\nu'}\Phi_\gamma^{\mu'\sigma}+\\&+\Phi_\alpha^{*\mu\sigma}\Phi_\beta^{*\rho\nu}V_{qq}(\mu\rho;\mu'\rho')\Phi_\delta^{\rho'\nu}\Phi_\gamma^{\mu'\sigma}+\Phi_\alpha^{*\mu\sigma}\Phi_\beta^{*\rho\nu}V_{\bar q\bar q}(\sigma\nu;\sigma'\nu')\Phi_\delta^{\rho\nu'}\Phi_\gamma^{\mu\sigma'},\end{aligned} \tag{26}$$

$$\begin{aligned}V_{mm}^{exc}(\alpha\beta;\gamma\delta)&=-\frac{1}{2}\Big[\Phi_\alpha^{*\mu\nu}\Phi_\beta^{*\rho\sigma}V_{q\bar q}(\mu\nu;\mu'\nu')\Phi_\delta^{\mu'\sigma}\Phi_\gamma^{\rho\nu'}+\\&+\Phi_\alpha^{*\rho\sigma}\Phi_\beta^{*\mu\nu}V_{q\bar q}(\mu\nu;\mu'\nu')\Phi_\delta^{\rho\nu'}\Phi_\gamma^{\mu'\sigma}+\Phi_\alpha^{*\mu\sigma}\Phi_\beta^{*\rho\nu}V_{q\bar q}(\mu\nu;\mu'\nu')\Phi_\delta^{\mu'\nu'}\Phi_\gamma^{\rho\sigma}+\\&+\Phi_\alpha^{*\rho\nu}\Phi_\beta^{*\mu\sigma}V_{q\bar q}(\mu\nu;\mu'\nu')\Phi_\delta^{\rho\sigma}\Phi_\gamma^{\mu'\nu'}+2\Phi_\alpha^{*\mu\sigma}\Phi_\beta^{*\rho\nu}V_{qq}(\mu\rho;\mu'\rho')\Phi_\delta^{\mu'\nu}\Phi_\gamma^{\rho'\sigma}+\\&+2\Phi_\alpha^{*\mu\sigma}\Phi_\beta^{*\rho\nu}V_{\bar q\bar q}(\sigma\nu;\sigma'\nu')\Phi_\delta^{\mu\nu'}\Phi_\gamma^{\rho\sigma'}\Big],\end{aligned} \tag{27}$$

$$\begin{aligned}V_{mm}^{int}(\alpha\beta;\gamma\delta)&=-\frac{1}{2}\Big[\Phi_\alpha^{*\mu\nu}\Phi_\beta^{*\rho\sigma}H(\mu\nu;\mu'\nu')\Phi_\delta^{\mu'\sigma}\Phi_\gamma^{\rho\nu'}+\\&+\Phi_\alpha^{*\rho\sigma}\Phi_\beta^{*\mu\nu}H(\mu\nu;\mu'\nu')\Phi_\delta^{\rho\nu'}\Phi_\gamma^{\mu'\sigma}+\Phi_\alpha^{*\mu\sigma}\Phi_\beta^{*\rho\nu}H(\mu\nu;\mu'\nu')\Phi_\delta^{\mu'\nu'}\Phi_\gamma^{\rho\sigma}+\\&+\Phi_\alpha^{*\rho\nu}\Phi_\beta^{*\mu\sigma}H(\mu\nu;\mu'\nu')\Phi_\delta^{\rho\sigma}\Phi_\gamma^{\mu'\nu'}\Big].\end{aligned} \tag{28}$$

The two-meson wave function is not normalized in the usual quantum mechanical way, because of the presence of normalization kernel in Eq. (19). It is common practice [2] to introduce a «renormalized» wave function defined as

$$\bar\psi_\Lambda^{\alpha\beta}\equiv N^{\frac{1}{2}}(\alpha\beta;\alpha'\beta')\psi_\Lambda^{\alpha'\beta'}, \tag{29}$$

where $N^{1/2}$ is the square root of the RGM normalization kernel. This clearly leads to

$$\bar{\psi}^{*\alpha\beta}_{\Lambda'}\bar{\psi}^{\alpha\beta}_{\Lambda} = \delta_{\Lambda'\Lambda}. \tag{30}$$

In terms of the renormalized wave function, the RGM equation can be rewritten as

$$\left[\bar{H}_{RGM}(\alpha\beta;\gamma\delta) - E_\Lambda \delta_{\alpha\gamma}\delta_{\beta\delta}\right]\bar{\psi}^{\gamma\delta}_{\Lambda} = 0, \tag{31}$$

where the «renormalized» RGM Hamiltonian is

$$\bar{H}_{RGM}(\alpha\beta;\gamma\delta) \equiv N^{-\frac{1}{2}}(\alpha\beta;\alpha'\beta')H_{RGM}(\alpha'\beta';\gamma'\delta')N^{-\frac{1}{2}}(\gamma'\delta';\gamma\delta). \tag{32}$$

Now, let us expand $N^{-\frac{1}{2}}$ in Eq. (32) as

$$N^{-\frac{1}{2}} = (1 - N_E)^{-\frac{1}{2}} = 1 + \frac{1}{2}N_E + \frac{3}{8}N_E^2 + \cdots, \tag{33}$$

where N_E is the exchange kernel defined in Eq. (20). Now, if only the first order term is retained, then the lowest order correction to the RGM Hamiltonian is

$$\begin{aligned}
\bar{H}_{RGM}(\alpha\beta;\gamma\delta) &= T_{RGM}(\alpha\beta;\gamma\delta) + V^{dir}_{mm}(\alpha\beta;\gamma\delta) + V^{exc}_{mm}(\alpha\beta;\gamma\delta) - \\
&-\frac{1}{2}\left\{\Phi^{*\mu\nu}_{\alpha}\Phi^{*\rho\sigma}_{\beta}\left[H(\mu\nu;\mu'\nu') - H(\mu\nu;\lambda\tau)\Delta(\lambda\tau;\mu'\nu')\right]\Phi^{\mu'\sigma}_{\delta}\Phi^{\rho\nu'}_{\gamma} + \right.\\
&\left. + (\alpha\leftrightarrow\beta;\gamma\leftrightarrow\delta)\right\} - \\
&-\frac{1}{2}\left\{\Phi^{*\mu\sigma}_{\alpha}\Phi^{*\rho\nu}_{\beta}\left[H(\mu\nu;\mu'\nu') - \Delta(\mu\nu;\lambda\tau)H(\lambda\tau;\mu'\nu')\right]\Phi^{\mu'\nu'}_{\delta}\Phi^{\rho\sigma}_{\gamma} + \right.\\
&\left. + (\alpha\leftrightarrow\beta;\gamma\leftrightarrow\delta)\right\} .
\end{aligned} \tag{34}$$

If the Φ's are chosen to be the eigenstates of the microscopic quark Hamiltonian, Eq. (17), the intra-exchange term V^{int}_{mm} is obviously canceled. This cancellation is the main effect of the renormalization of the wave function, higher order terms in the expansion give small corrections. This can be explicitly demonstrated in a simple example.

We consider meson–meson scattering, where the quark and the antiquark have equal masses, m_q, and use an harmonic potential for the microscopic interaction. The Fock-space amplitude is then a gaussian whose width b is related to the r.m.s. radius of the meson by $< r^2 > \; = \; \sqrt{3/2}\, b$. The total energy of a single meson is [18]

$$E(\boldsymbol{P}) = \frac{\boldsymbol{P}^2}{4m_q} + 2m_q + \frac{3}{m_q b^2} + \frac{4C}{3}, \tag{35}$$

where C is the spring constant.

The evaluation of normalization kernel and its square root can be done analytically. The results are [18],

$$N(\alpha\beta;\gamma\delta) = \delta^{(3)}(\boldsymbol{P}_\alpha - \boldsymbol{P}_\gamma)\delta^{(3)}(\boldsymbol{P}_\beta - \boldsymbol{P}_\delta) - \frac{1}{6}\mathcal{N}_E(\boldsymbol{P}_\alpha\boldsymbol{P}_\beta;\boldsymbol{P}_\gamma\boldsymbol{P}_\delta), \quad (36)$$

$$N^{-\frac{1}{2}}(\alpha\beta;\gamma\delta) = \delta_{\alpha\gamma}\delta_{\beta\delta}\delta^{(3)}(\boldsymbol{P}_\alpha - \boldsymbol{P}_\gamma)\delta^{(3)}(\boldsymbol{P}_\beta - \boldsymbol{P}_\delta) + C_N\mathcal{N}_E(\boldsymbol{P}_\alpha\boldsymbol{P}_\beta;\boldsymbol{P}_\gamma\boldsymbol{P}_\delta), \quad (37)$$

where

$$\mathcal{N}_E(\boldsymbol{P}_\alpha\boldsymbol{P}_\beta;\boldsymbol{P}_\gamma\boldsymbol{P}_\delta) = \delta^{(3)}(\boldsymbol{P}_\alpha + \boldsymbol{P}_\beta - \boldsymbol{P}_\gamma - \boldsymbol{P}_\delta)\left(\frac{b^2}{2\pi}\right)^{3/2} \times$$
$$\times \exp\left\{-\frac{b^2}{4}\left[\boldsymbol{P}_\alpha^2 + \frac{\boldsymbol{P}_\gamma^2}{2} + \frac{\boldsymbol{P}_\delta^2}{2} - \boldsymbol{P}_\alpha\cdot(\boldsymbol{P}_\gamma + \boldsymbol{P}_\delta)\right]\right\} \quad (38)$$

with

$$C_N = \frac{\omega}{2}\lim_{k\to\infty}\sum_{m=1}^{k}\left(\frac{\omega}{2}\right)^{m-1}\left(\prod_{n=1}^{m}\frac{2n-1}{n}\right), \quad (39)$$

with $\omega = 1/6$. The partial sums $C(k)$,

$$C(k) = \sum_{m=1}^{k}\left(\frac{\omega}{2}\right)^{m-1}\left(\prod_{n=1}^{m}\frac{2n-1}{n}\right), \quad (40)$$

are plotted in Fig. 1 below. It is seen that the series is rapidly convergent, for $k \geq \geq 2$, the $C(k)$'s have almost reached their asymptotic value, $C(\infty) \sim 1.145$. The meaning of this is that retention of only the first term in Eq. (33) is a very good approximation to the exchange kernel.

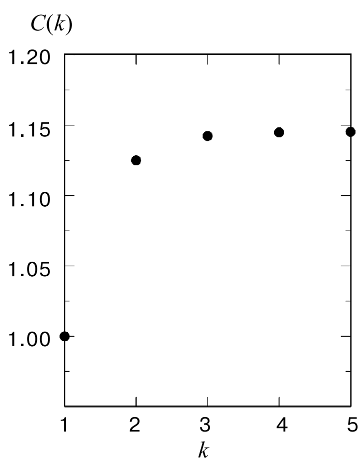

Fig. 1. The partial sums $C(k)$ of Eq. (39)

One can also show [18], by solving the full RGM equation exactly, that the effect of the higher order terms in Eq. (33) is less than 5% on the effective meson–meson potentials. Obviously, these results are for a microscopic harmonic interaction. For other types of interactions, Fock-space amplitudes Φ will not be a gaussian, and a check on the rate of convergence of the expansion in Eq. (33) is advisable.

We next consider the thermodynamic Green's function method of Blaschke and

Röpke [5]. In order to simplify the discussion these authors considered a static quark–antiquark potential $V_{q\bar{q}}$ which operates within color neutral pairs only. Therefore, only ladder diagrams contribute. The starting point is the single-quark Green's function (we follow Ref. 5, which uses the notations of Ref. 30)

$$G(\mu, z) = \frac{1}{z - E_\mu}, \tag{41}$$

where, as above, μ represents collectively the spatial, spin, flavor, and color quantum numbers of the quarks and E_μ is the energy of the quark. The two-quark Green's function, which contains information on the meson bound-states, is the solution of the equation represented in Fig. 2.

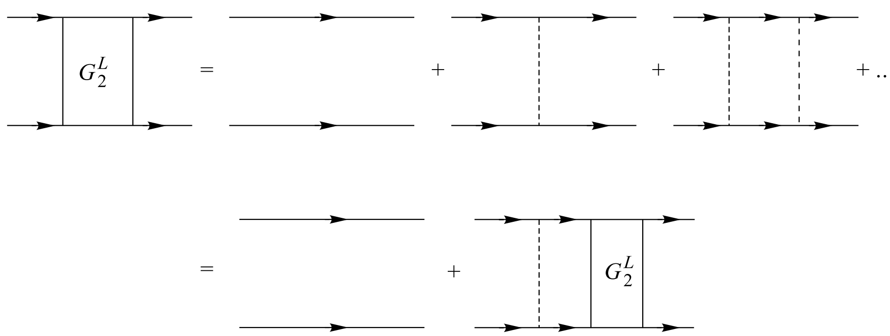

Fig. 2. Equation for the two-particle Green's function in the ladder approximation

The equation for G_2^L is

$$G_2^L(\mu\nu, \mu'\nu', \Omega_2) = G_2^0(\mu, \nu, \Omega_2)\left[\delta_{\mu\mu'}\delta_{\nu\nu'} + V(\mu\nu, \sigma\rho)G_2^L(\sigma\rho, \mu'\nu', \Omega_2)\right], \tag{42}$$

where L indicates ladder approximation. Here the notation of sum over repeated indices is used (wherever clarity demands, a summation will be explicitly indicated), and $G_2^0(\mu, \nu, \Omega_2)$ is the free two-quark Green's function

$$G_2^0(\mu, \nu, \Omega_2) = \sum_n G(\mu, z_n)\, G(\nu, \Omega_2 - z_n) = \frac{1 - f(E_\mu) - f(E_\nu)}{\Omega_2 - E_\mu - E_\nu}. \tag{43}$$

The Fermi–Dirac distributions can be neglected at low densities.

Let, as above, $\Phi_\alpha^{\mu\nu} \equiv < \mu\nu|\alpha >$ denote the solution of the two-quark equation of motion with energy E_α. Obviously,

$$G_2 = \sum_\alpha |\alpha > \mathcal{G}_{\rm meson}(\alpha, \Omega_2) < \alpha|, \tag{44}$$

where the summation is over the discrete and continuum states and $\mathcal{G}_{\rm meson}(\alpha, \Omega_2)$ is the meson propagator,

$$\mathcal{G}_{\rm meson}(\alpha, \Omega_2) = \frac{1}{\Omega_2 - E_\alpha}. \tag{45}$$

Again, in the low density limit, where one expects that the quarks remain confined into the mesons, the summation is only over discrete states (bound meson states).

The effective meson–meson potential can be identified by considering the four-quark T_4-matrix or, equivalently, the four-quark Green's function G_4. The Green's function G_4 is obtained from the T_4-matrix by the usual amputation procedure, as explained in Ref. 30. A typical diagram that contributes to G_4^L (where L again means ladder approximation) is shown in Fig. 3 below.

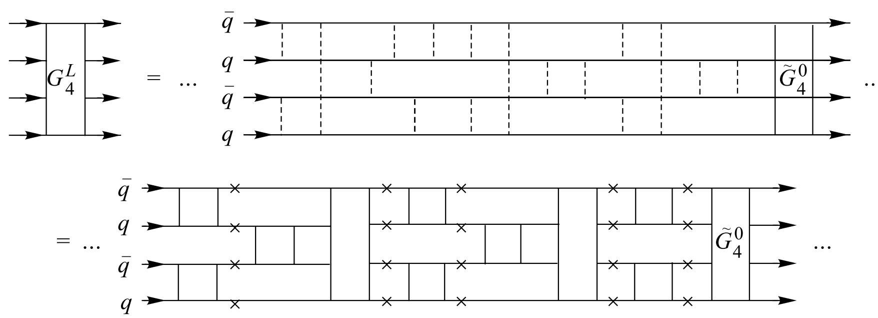

Fig. 3. A typical contribution to G_4^L

This particular diagram is represented by

$$G^{I}_{2+2}\, G_4^{0-1}\, G^{II}_{2+2}\, G_4^{0-1}\, G^{I}_{2+2}\, G_4^{0-1}\, G^{II}_{2+2}\, G_4^{0-1}\, G^{I}_{2+2}\, G_4^{0-1}\, \widetilde{G}^0_4. \tag{46}$$

Here, G^I_{2+2} and G^{II}_{2+2} are the two possible two-meson interactions

$$G^I_{2+2}(\mu\nu\sigma\rho, \mu'\nu'\sigma'\rho', \Omega_4) =$$

$$= \frac{\Phi^{*\mu\nu}_{\alpha_1}\, \Phi^{*\sigma\rho}_{\alpha_2}\, \Phi^{\sigma'\rho'}_{\alpha'_2}\, \Phi^{\mu'\nu'}_{\alpha'_1}}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}} - \frac{\delta_{\mu\mu'}\,\delta_{\nu\nu'}\,\delta_{\sigma\sigma'}\,\delta_{\rho\rho'}}{\Omega_4 - E_\mu - E_\nu - E_\sigma - E_\rho}, \tag{47}$$

and

$$G^{II}_{2+2}(\mu\nu\sigma\rho, \mu'\nu'\sigma'\rho', \Omega_4) = G^I_{2+2}(\sigma\nu\mu\rho, \sigma'\nu'\mu'\rho', \Omega_4), \tag{48}$$

and G_4^{0-1} describes the amputation of the four free quark propagators represented by the crosses in Fig. 3. Neglecting Fermi–Dirac occupation probabilities, G_4^{0-1} is given by

$$G_4^{0-1}(\mu\nu\sigma\rho, \mu'\nu'\sigma'\rho', \Omega_4) = \delta_{\mu\mu'}\delta_{\nu\nu'}\delta_{\sigma\sigma'}\delta_{\rho\rho'}\ (\Omega_4 - E_\mu - E_\nu - E_\sigma - E_\rho)\,. \tag{49}$$

$\widetilde{G}_4^{0-1}$ is the antisymmetrized form of G_4^0

$$\widetilde{G}_4^0(\mu\nu\sigma\rho, \mu'\nu'\sigma'\rho', \Omega_4) =$$

$$= \frac{\delta_{\mu\mu'}\delta_{\nu\nu'}\delta_{\sigma\sigma'}\delta_{\rho\rho'} - \delta_{\mu\sigma'}\delta_{\nu\nu'}\delta_{\sigma\nu'}\delta_{\rho\rho'} - \delta_{\mu\mu'}\delta_{\nu\rho'}\delta_{\sigma\sigma'}\delta_{\rho\nu'} + \delta_{\mu\sigma'}\delta_{\nu\rho'}\delta_{\sigma\mu'}\delta_{\rho\nu'}}{\Omega_4 - E_\mu - E_\nu - E_\sigma - E_\rho}. \tag{50}$$

Introducing the pair-flip potential

$$U_{2+2}(\mu\nu\sigma\rho, \mu'\nu'\sigma'\rho', \Omega_4) = -\left(\Omega_4 - E_\mu - E_\nu - E_\sigma - E_\rho\right)\, \delta_{\mu\sigma'}\delta_{\nu\nu'}\delta_{\sigma\mu'}\delta_{\rho\rho'}, \tag{51}$$

represented in Fig.4, all the sequences as in Eq. (46) can be broken up as

$$G_4^L = G_4^I + G_4^{II} - \widetilde{G}_4^0, \tag{52}$$
$$G_4^I = \widetilde{G}_4^0 + G_{2+2}^I\, U_{2+2}\, G_4^I, \tag{53}$$
$$G_4^{II} = \widetilde{G}_4^0 + G_{2+2}^{II}\, U_{2+2}\, G_4^{II}. \tag{54}$$

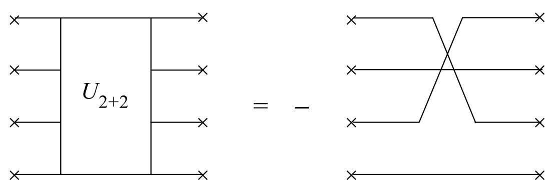

Fig. 4. The pair-flip potential defined in Eq. (51)

Let us consider Eq. (53). For the repeated sequence of this, $G_{2+2}^I\, U_{2+2}$, it is not difficult to show [5] that

$$G_{2+2}^I(\mu\nu\sigma\rho, \mu''\nu''\sigma''\rho'', \Omega)\, U_{2+2}(\mu''\nu''\sigma''\rho'', \mu'\nu'\sigma'\rho', \Omega) =$$

$$= -\frac{\Phi_{\alpha_1}^{*\mu\nu}\, \Phi_{\alpha_2}^{*\sigma\rho}\, \Phi_{\alpha_2'}^{\sigma''\rho''}\, \Phi_{\alpha_1'}^{\mu''\nu''}}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}} \times$$
$$\times \left[V_{q\bar{q}}(\mu''\rho'', \mu'\rho')\delta_{\nu''\nu'}\delta_{\sigma''\sigma'} + V_{q\bar{q}}(\sigma''\nu'', \sigma'\nu')\delta_{\mu''\mu'}\delta_{\rho''\rho'}\right]. \tag{55}$$

From the iteration of this, $G_{2+2}^I\, U_{2+2} G_{2+2}^I\, U_{2+2}$, one can read off the effective meson–meson potential

$$G_{2+2}^I\, U_{2+2} G_{2+2}^I\, U_{2+2} \sim \sum_{\alpha\beta\cdots} \Phi_\alpha^{\mu\nu} \Phi_\beta^{\sigma\rho}\, \mathcal{G}_{2,\mathrm{meson}}(\alpha, \beta, \Omega_4) \times$$

$$\times V_{mm}(\alpha\beta, \gamma\delta)\, \mathcal{G}_{2,\mathrm{meson}}(\gamma, \delta, \Omega_4) \Phi_\gamma^{*\mu''\nu''}\, \Phi_\delta^{\sigma''\rho''} \ldots \tag{56}$$

with

$$V_{mm}(\alpha\beta,\gamma\delta) = -\Phi_\alpha^{*\mu\nu}\,\Phi_\beta^{*\sigma\rho}\times$$
$$\times\Big[V_{q\bar{q}}(\mu\rho,\mu'\rho')\delta_{\nu\nu'}\delta_{\sigma\sigma'} + V_{q\bar{q}}(\sigma\nu,\sigma'\nu')\delta_{\mu\mu'}\delta_{\rho\rho'}\Big]\Phi_\gamma^{*\mu'\rho'}\,\Phi_\delta^{*\sigma'\nu'}, \qquad (57)$$

and

$$\mathcal{G}_{2,\mathrm{meson}} = \frac{1}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}}. \qquad (58)$$

This last equation is obtained with the neglect of Bose–Einstein occupation factors.

Notice that the effective meson–meson potential in Eq. (57) is precisely equal (after reshuffling indices) to 1/2 of the second line of Eq. (27). However, notice also that the first line of Eq. (27) has the same quark indices as Eq. (57), but the interaction indices are contracted with the indices of the mesons in the final states. We come back to this point in Section 4.

The expression of Blaschke and Röpke is identical to the expression obtained by Barnes and Swanson using the Quark-Born-Diagram method [3]. As mentioned in the Introduction, the QBD is similar to the «constituent exchange» of Gunion, Brodsky and Blankenbecler [4] for high-energy hadron scattering. For high-energy processes, there is strong experimental evidence for the «exchange force» from large momentum transfer processes [31]. For low-energy processes, the situation is not so clear in view of the model dependence of the microscopic interactions, whose connection to QCD is not yet understood, as discussed in Section 2.

The way to obtain the effective hadron–hadron interaction in the QBD method is as follows. Initially a generic scattering diagram with initial and final hadron–hadron states is drawn. Then initial and final quark lines are connected in all possible ways consistent with flavor conservation. The next step consists in inserting interaction lines (e.g., one-gluon-exchange interactions) between all pairs of initial quarks in different initial baryons. Naturally many diagrams are trivially zero because of color symmetry, and the potential can be read-off immediately. Care must be exercised with combinatorial factors, i.e., the number of ways that quark lines can be connected. Although the applications of Barnes, Swanson and collaborators [3] for scattering cross sections were done in the Born approximation, there is no reason for not using the effective potential in an integral equation for obtaining the scattering amplitude.

In the next section we discuss the Fock–Tani method and discuss further comparisons with the methods discussed here.

4. THE FOCK–TANI REPRESENTATION AND EFFECTIVE HADRON HAMILTONIANS

In this section we summarize the basic features of the Fock–Tani representation. We use a simple example to explain the formalism, but it should become clear that the applicability of the method is not restricted to this example. For the purposes of illustration, we consider the representation for mesons, considered as a bound-state of a quark and an antiquark as in Section 2. The Hamiltonian is taken as in Eq. (16). We note that a great variety of quark-model Hamiltonians used in the literature can be written in such a form. However, at this point of the discussion we have not included in Eq. (16) terms such as pair-creation, which are of the form $\bar{q}^\dagger q^\dagger q^\dagger q$, as discussed in Section 2. However, it will become clear from the discussion in the next sections such terms are treated without difficulties.

The change to the FT representation is implemented by means of a unitary transformation U, such that a *single* composite meson state $|\alpha\rangle = M^\dagger_\alpha|0\rangle$ is transformed into a *single* ideal-meson state $|\alpha) = m^\dagger_\alpha|0) \equiv U^{-1}|\alpha\rangle$, where U is of the general form

$$U = \exp(-\pi/2F), \quad F = \sum_\alpha \left(m^\dagger_\alpha O_\alpha - O^\dagger_\alpha m_\alpha\right). \tag{59}$$

The $m^\dagger_\alpha$ and m_α are the ideal-meson creation and annihilation operators and the $O^\dagger_\alpha$ and O_α operators are functionals of the $M^\dagger_\alpha$, M_α and $\Delta_{\alpha\beta}$. By definition, the m's and O's satisfy canonical commutation relations

$$[m_\alpha, m^\dagger_\beta] = [O_\alpha, O^\dagger_\beta] = \delta_{\alpha\beta},$$

$$[m_\alpha, m_\beta] = [m^\dagger_\alpha, m^\dagger_\beta] = [O_\alpha, O_\beta] = [O^\dagger_\alpha, O^\dagger_\beta] = 0, \tag{60}$$

and, by definition, the $m^\dagger$ and m commute with the quark and antiquark operators.

The operator U acts on an enlarged Fock space $\mathcal{I}$, which is the graded direct product of $\mathcal{F}$ and an *ideal* state space $\mathcal{M}$, the space with the new degrees of freedom described by the ideal meson operators $m^\dagger_\alpha$ and m_α. The vacuum state of $\mathcal{M}$ is denoted by $|0\rangle_\mathcal{M}$ and so, the vacuum state of $\mathcal{I}$ is

$$|0) = |0\rangle \times |0\rangle_\mathcal{M}. \tag{61}$$

In $\mathcal{I}$ the physical states, $|\psi\rangle$, constitute a subspace $\mathcal{I}_0$ isomorphic to $\mathcal{F}$ and satisfy the constraint equation

$$m_\alpha|\psi\rangle = 0. \tag{62}$$

Now, the new degrees of freedom acquire physical content when the unitary operator U transforms the physical states $|\psi\rangle$ of $\mathcal{I}_0$ to states $|\psi) = U^{-1}|\psi\rangle$. The image states $|\psi)$ span the FT space $\mathcal{F}_{FT} = U^{-1}\mathcal{I}_0$, and satisfy the transformed constraint equation

$$U^{-1} m_\alpha U|\psi) = O_\alpha|\psi) = 0. \tag{63}$$

Although the physical content of the Fock spaces $\mathcal{F}$ and $\mathcal{F}_{FT}$ is the same, the mathematical representation of states and operators in $\mathcal{F}_{FT}$ involves only canonical field operators. A more detailed discussion of these and other formal aspects of the mapping procedure can be found in [18].

The operators $O_\alpha^\dagger$ and O_α are constructed by an iterative procedure as a power series in the Φ's

$$O_\alpha = \sum_n O_\alpha^{(n)}, \tag{64}$$

where n identifies the power of Φ in the expansion. The expansion starts at zeroth order with

$$O_\alpha^{(0)} = M_\alpha. \tag{65}$$

The construction of the higher order terms $O_\alpha^{(n)}$, $n \geq 1$, involves addition of a series of counterterms such that commutation relations of $O^\dagger$ and O are satisfied order by order. Since at zeroth order one has

$$[O_\alpha^{(0)}, O_\beta^{(0)\dagger}] = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \tag{66}$$

and $\Delta_{\alpha\beta}$ is of second order [see Eq. (9)], one has that

$$O_\alpha^{(1)} = 0. \tag{67}$$

The next nonzero term is then of order $n = 2$. It is not difficult to show that the second order counterterm that has to be added to $O_\alpha^{(0)}$ to cancel the $\Delta_{\alpha\beta}$ in $[O_\alpha^{(0)}, O_\beta^{(0)\dagger}]$ is equal to

$$\frac{1}{2}\Delta_{\alpha\beta} M_\beta. \tag{68}$$

Then, up to $n = 2$,

$$O_\alpha = B_\alpha + \frac{1}{2}\Delta_{\alpha\beta} B_\beta, \tag{69}$$

and one obtains

$$\begin{aligned}[O_\alpha, O_\beta^\dagger] &= \delta_{\alpha\beta} - \frac{1}{2}[\Delta_{\alpha\gamma}, M_\beta] M_\gamma - \frac{1}{2} M^\dagger [M_\alpha, \Delta_{\gamma\beta}] = \\ &= \delta_{\alpha\beta} + \mathcal{O}(\Phi^3).\end{aligned} \tag{70}$$

A third order counterterm has to be added such that the $\mathcal{O}(\Phi^3)$ piece cancels, and so on to higher orders. However, for our purposes here one needs O_α up to $n = 3$ only

$$O_\alpha = M_\alpha + \frac{1}{2}\Delta_{\alpha\beta} M_\beta - \frac{1}{2} M_\beta^\dagger [\Delta_{\beta\gamma}, M_\alpha] M_\gamma\ . \tag{71}$$

The transformation of the Hamiltonian is made by transforming initially the quark and antiquark operators. Since the O operators are given by a power series, the transformed quark operators are also obtained as a power series, which can be obtained by expanding the exponential in Eq. (59) to the desired order or, equivalently, by means of the «equation of motion» technique [19, 21]. Up to third order, one obtains [18]

$$
\begin{aligned}
&q_\mu^{(0)} = q_\mu, \quad \bar{q}_\nu^{(0)} = \bar{q}_\nu, \\
&q_\mu^{(1)} = \Phi_\alpha^{\mu\nu_1} \bar{q}_{\nu_1}^\dagger \left(m_\alpha - M_\alpha\right), \quad \bar{q}_\nu^{(1)} = \Phi_\alpha^{\mu_1\nu} q_{\mu_1}^\dagger \left(M_\alpha - m_\alpha\right), \\
&q_\mu^{(2)} = -\tfrac{1}{2} \Phi_\alpha^{*\mu_2\nu_1} \Phi_\beta^{\mu\nu_1} \left(m_\alpha^\dagger m_\beta + M_\alpha^\dagger M_\beta + 2M_\alpha^\dagger m_\beta\right) q_{\mu_2}, \\
&\bar{q}_\nu^{(2)} = -\tfrac{1}{2} \Phi_\alpha^{*\mu_1\nu_2} \Phi_\beta^{\mu_1\nu} \left(m_\alpha^\dagger m_\beta + M_\alpha^\dagger M_\beta + 2M_\alpha^\dagger m_\beta\right) \bar{q}_{\nu_2}, \\
&q_\mu^{(3)} = \tfrac{1}{2} \Phi_\alpha^{*\rho\sigma} \Big[\Phi_\beta^{\mu\sigma} \Phi_\gamma^{\rho\sigma_1} \bar{q}_{\sigma_1}^\dagger \left(-m_\alpha^\dagger m_\beta m_\gamma - M_\alpha^\dagger m_\beta M_\gamma + m_\alpha^\dagger m_\beta M_\gamma + \right. \\
&\left. +M_\alpha^\dagger M_\beta M_\gamma\right) + \Phi_\alpha^{\mu\nu_1} \Big(\Phi_\beta^{\rho\sigma_1} \bar{q}_{\nu_1}^\dagger \bar{q}_{\sigma_1}^\dagger \bar{q}_\sigma + \Phi_\beta^{\rho_1\sigma} \bar{q}_{\nu_1}^\dagger q_{\rho_1}^\dagger q_\rho\Big) \left(m_\beta - 2M_\beta\right)\Big], \\
&\bar{q}_\nu^{(3)} = -\tfrac{1}{2} \Phi_\alpha^{*\rho\sigma} \Big[\Phi_\beta^{\rho\nu} \Phi_\gamma^{\rho_1\sigma} q_{\rho_1}^\dagger \left(-m_\alpha^\dagger m_\beta m_\gamma - M_\alpha^\dagger m_\beta M_\gamma + m_\alpha^\dagger m_\beta M_\gamma + \right. \\
&\left. +M_\alpha^\dagger M_\beta M_\gamma\right) + \Phi_\alpha^{\mu_1\nu} \Big(\Phi_\beta^{\rho\sigma_1} q_{\mu_1}^\dagger \bar{q}_{\sigma_1}^\dagger \bar{q}_\sigma + \Phi_\alpha^{\mu_1\nu} \Phi_\beta^{\rho_1\sigma} q_{\mu_1}^\dagger q_{\rho_1}^\dagger q_\rho\Big) \left(m_\beta - 2M_\beta\right)\Big].
\end{aligned}
\tag{72}
$$

The transformation of the microscopic Hamiltonian is obtained by using the transformed quark operators of Eq. (72) in Eq. (16). This is done by considering all possible combinations of the form $T(\mu) q_\mu^{(n)\dagger} q_\mu^{(m)}$, $V_{qq}(\mu\nu;\sigma\rho) q_\mu^{(n)\dagger} q_\nu^{(m)\dagger} q_\rho^{(k)} q_\sigma^{(l)}$, etc., where $n, m, k, l = 1, 2, 3$. One obtains that the general structure of the transformed Hamiltonian is

$$H_{\rm FT} = H_q + H_m + H_{mq}, \tag{73}$$

where the subindices identify the operator content of each term. The quark Hamiltonian H_q has an identical structure to the one of the microscopic quark Hamiltonian of Eq. (16), except that the term corresponding to the quark–antiquark interaction is modified to

$$
\begin{aligned}
V_{q\bar{q}}(\mu\nu;\sigma\rho) \to \Big[&V_{q\bar{q}}(\mu\nu;\sigma\rho) - \Delta(\mu\nu;\mu'\nu') H(\mu'\nu';\sigma\rho) - H(\mu\nu;\sigma'\rho') \times \\
&\times \Delta(\sigma'\rho';\sigma\rho) + \Delta(\mu\nu;\mu'\nu') H(\mu'\nu';\sigma'\rho') \Delta(\sigma'\rho';\sigma\rho)\Big] \, q_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho q_\sigma,
\end{aligned}
\tag{74}
$$

where $\Delta\left(\mu\nu;\mu'\nu'\right) = \Phi_\alpha^{\mu\nu} \Phi_\alpha^{*\mu'\nu'}$ is the «bound state kernel". When Φ is an eigenstate of the microscopic Hamiltonian, Eq. (17), the quark–antiquark interaction is then modified to

$$V_{q\bar{q}}(\mu\nu;\sigma\rho) \to \left[V_{q\bar{q}}(\mu\nu;\sigma\rho) - E_\alpha \Phi_\alpha^{\mu\nu} \Phi_\alpha^{*\sigma\rho}\right] \, q_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho q_\sigma. \tag{75}$$

It is not difficult to show (see Appendix C of Ref. 21) that this modified interaction does not produce the quark–antiquark bound states. This feature leads to the same effect of curing the bound state divergences of the Born series as in Weinberg's quasiparticle method [26] discussed in the Introduction: the modified quark–antiquark interaction is unable to form mesons, the mesons are redescribed by the H_m part of the effective Hamiltonian.

H_{mq} describes quark–meson processes as meson breakup into a quark–antiquark pair, meson–quark scattering, meson–meson total breakup into two quark–antiquark pairs, etc. In models where quarks are truly confined, these terms contribute to free-space meson–meson processes as intermediate states only. However, in high temperature and/or density systems hadrons and quarks can co-exist and the breakup and recombination processes can play important role.

The term involving only ideal meson operators has a component that represents an effective meson–meson interaction. This meson–meson interaction is of the general form

$$H_m = E_\alpha m^\dagger_\alpha m_\alpha + \frac{1}{2} V_{mm}(\alpha\beta;\gamma\delta)\, m^\dagger_\alpha m^\dagger_\beta m_\delta m_\gamma \ , \tag{76}$$

where the effective meson–meson potential V_{mm} can be divided into a sum of *direct*, *exchange*, and *intra-exchange* parts, as given by Eqs. (25)-(28). The higher order terms by Φ which are neglected from these expression give rise to many-meson (higher than two-meson) forces, and also introduce orthogonality corrections. The orthogonality corrections are precisely of the same nature of the higher-order terms of the expansion of the square-root of the normalization kernel of the RG method, Eq. (33). As seen in the last section, the cancellation of the intra-exchange terms in lowest order is the dominant effect of the orthogonalization terms and higher order corrections are in general small.

The technique can be applied in a straightforward way to baryon bound states of three constituent quarks as in Eq. (11). For a Hamiltonian as given in Eq.(16), the effective baryon–baryon Hamiltonian consistent with the lowest-order orthogonality corrections is [18]

$$H_b = \Psi^{*\mu\nu\lambda}_\alpha H(\mu\nu;\sigma\rho) \Psi^{\sigma\rho\lambda}_\beta \, b^\dagger_\alpha b_\beta + \frac{1}{2}\, V_{bb}(\alpha\beta;\delta\gamma)\, b^\dagger_\alpha b^\dagger_\beta b_\gamma b_\delta, \tag{77}$$

where b_α and $b^\dagger_\alpha$ are the ideal baryon operators, and V_{bb} is the effective baryon–baryon potential. V_{bb} is given as a sum of five terms

$$V_{bb}(\alpha\beta;\gamma\delta) \quad = \quad \sum_{n=1}^{5} V_n(\alpha\beta;\gamma\delta), \tag{78}$$

where the V_n's are given in terms of the baryon amplitudes Ψ as

$$
\begin{aligned}
V_1(\alpha\beta;\gamma\delta) &= +9V_{qq}(\mu\nu;\sigma\rho)\,\Psi^{*\mu\mu_2\mu_3}_{\alpha}\Psi^{*\nu\nu_2\nu_3}_{\beta}\Psi^{\rho\nu_2\nu_3}_{\gamma}\Psi^{\sigma\mu_2\mu_3}_{\delta},\\
V_2(\alpha\beta;\gamma\delta) &= -36V_{qq}(\mu\nu;\sigma\rho)\,\Psi^{*\mu\mu_2\mu_3}_{\alpha}\Psi^{*\nu\nu_2\nu_3}_{\beta}\Psi^{\rho\nu_2\mu_3}_{\gamma}\Psi^{\sigma\mu_2\nu_3}_{\delta},\\
V_3(\alpha\beta;\gamma\delta) &= -9V_{qq}(\mu\nu;\sigma\rho)\,\Psi^{*\mu\mu_2\mu_3}_{\alpha}\Psi^{*\nu\nu_2\nu_3}_{\beta}\Psi^{\sigma\nu_2\nu_3}_{\gamma}\Psi^{\rho\mu_2\mu_3}_{\delta},\\
V_4(\alpha\beta;\gamma\delta) &= -18V_{qq}(\mu\nu;\sigma\rho)\,\Psi^{*\mu_1\mu\nu}_{\alpha}\Psi^{*\nu_1\nu_2\nu_3}_{\beta}\Psi^{\nu_1\nu_2\rho}_{\gamma}\Psi^{\mu_1\sigma\nu_3}_{\delta},\\
V_5(\alpha\beta;\gamma\delta) &= -18V_{qq}(\mu\nu;\sigma\rho)\,\Psi^{*\mu\mu_2\mu_3}_{\alpha}\Psi^{*\nu_1\nu_2\nu}_{\beta}\Psi^{\nu_1\nu_2\mu_3}_{\gamma}\Psi^{\sigma\mu_2\rho}_{\delta}.
\end{aligned} \tag{79}
$$

In the next two figures we show a graphical representation of the different contributions V_n, n=$1,\cdots,5$ to V_{bb}. The qualitative difference between V_1 and the $V_2\ \cdots\ 5$ is that the latter involve quark interchange between the two colliding nucleons.

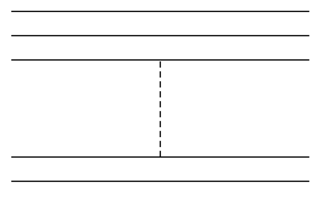

Fig. 5. Graphical representation of V_1

Note that this effective baryon–baryon interaction is completely general, it depends only on the fact that the baryons are three-quark bound-states, and that quarks interact through two-body forces. The method however can handle more complicated Fock-space amplitudes and more complicated microscopic interactions. The necessary extension of the formalism to the more general situation can be found in Ref. 17.

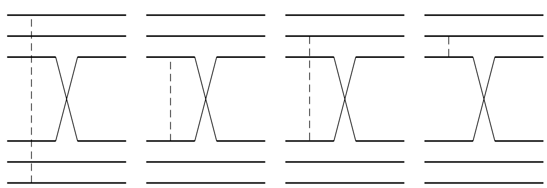

Fig. 6. Graphical representation of $V_2\cdots V_5$

One particularly important property of the effective hadron–hadron interactions in the Fock–Tani representation is that they lead to scattering T-matrices that are post-prior symmetrical [32]. That is, the scattering matrix is symmetric under exchange of initial and final states. The lack of this symmetry is of no importance for the case of «symmetric» initial and final states, as in processes like $\pi+\pi\to\pi+\pi$. However, it is of importance [18] for asymmetric cases like $J/\Psi+\pi\to D$-mesons [33]. The different position of the quark–antiquark interaction and the factors of 1/2 in the effective meson–meson interaction of Blaschke and Röpke as compared to the corresponding Fock–Tani (or RGM) interaction are the cause [18] of the breaking of the post-prior symmetry in the calculation of the charmonium dissociation in Ref. 33.

In the next section we specialize to the case of the nucleon–nucleon (NN) interaction, and obtain an effective NN interaction which will be used in Section 5 in a nuclear matter calculation.

5. SHORT-RANGE PART OF THE NN INTERACTION FROM QUARK-PION EXCHANGE

The nucleon–nucleon interaction exhibits a strongly repulsive short-distance core which is attributed to the exchange of the ω-meson (and ρ-exchange). Since nucleons have a radii of about 0.8 fm and the range of the meson exchange force is $1/m_\omega \approx 0.2$ fm, it is natural to expect that the nucleon substructure will play a role at such short distances. The replacement of vector-meson exchange as the main source of the short-range part of the NN interaction is one of our main motivations in this section. Motivated by the Manohar and Georgi model for the low-energy structure of the nucleon, we consider the lowest-order three-level one-pion exchange between constituent quarks, as given in Eq. (2). We then apply the Fock–Tani transformation to this interaction and obtain an effective NN potential, and compare this potential to the short-range part of the Bonn potential.

For later convenience, we start rewriting Eq. (2) as

$$V_{\pi q} = -\left(\frac{1}{f_\pi}\right)^2 t^{a(1)} t^{a(2)} \, \frac{\boldsymbol{\sigma}^{(1)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(2)}\cdot\mathbf{q}}{\mathbf{q}^2 + m_\pi^2} =$$

$$= -\left(\frac{1}{f_\pi}\right)^2 t^{a(1)} t^{a(2)} \, \frac{1}{3}\left[\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} \frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + m_\pi^2} + \frac{S_{12}}{\boldsymbol{q}^2 + m_\pi^2}\right] =$$

$$= -\left(\frac{1}{f_\pi}\right)^2 t^{a(1)} t^{a(2)} \, \frac{1}{3}\left[\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} \frac{m_\pi^2}{\boldsymbol{q}^2 + m_\pi^2} + \frac{S_{12}}{\boldsymbol{q}^2 + m_\pi^2}\right]. \quad (80)$$

This shows clearly the usual pieces of the OPE interaction, a short-range spin-spin interaction (a delta function in coordinate space) and long-range spin-spin and tensor interactions.

The Fock-space amplitude Ψ for the nucleon can be written as

$$\Psi_\alpha^{\mu_1\mu_2\mu_3} = \frac{\epsilon^{c_1c_2c_3}}{\sqrt{3!}} \, \frac{\chi_\lambda^{m_1m_2m_3}}{\sqrt{18}} \, \delta(\boldsymbol{p} - \boldsymbol{k}_1 - \boldsymbol{k}_2 - \boldsymbol{k}_3)\, \Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3), \quad (81)$$

where $\boldsymbol{p}$ is the c.m. momentum of the nucleon, the Φ is the momentum-dependent amplitude, the $\epsilon^{c_1c_2c_3}$ is the color antisymmetric tensor and $\chi_\lambda^{m_1m_2m_3}$ is the Clebsch–Gordan coefficient of spin-isospin, where $m_1 = \{s_1, f_1\} \cdots$ denote the spin-flavor of a quark.

Using this in the expression for the effective NN interaction given in Eq. (79), one obtains

$$V_{NN} = \frac{1}{2}\int \frac{d\boldsymbol{Q}\; d\boldsymbol{Q}'\, d\boldsymbol{p}\, d\boldsymbol{p}'}{(2\pi)^3}\, \delta(\boldsymbol{Q}' - \boldsymbol{Q})\, \langle\lambda_1\lambda_2|V_{NN}(\boldsymbol{\sigma},\boldsymbol{\tau},\boldsymbol{p}',\boldsymbol{p})|\lambda_3\lambda_4\rangle \times$$

$$\times b^\dagger_{\lambda_1}(\boldsymbol{p}' + \boldsymbol{Q}'/2)\, b^\dagger_{\lambda_2}(-\boldsymbol{p}' + \boldsymbol{Q}'/2)\, b_{\lambda_4}(-\boldsymbol{p} + \boldsymbol{Q}/2)\, b_{\lambda_3}(\boldsymbol{p} + \boldsymbol{Q}/2), \tag{82}$$

with

$$V_{NN}(\boldsymbol{\sigma},\boldsymbol{\tau},\boldsymbol{p}',\boldsymbol{p}) = \sum_{n=1}^{5} \mathcal{O}^{ij}_n(\boldsymbol{\sigma},\boldsymbol{\tau})\, u^{ij}_n(\boldsymbol{\sigma},\boldsymbol{\tau},\boldsymbol{p}',\boldsymbol{p}), \tag{83}$$

where the $\mathcal{O}_n$ contain the spin-isospin dependence; and u_n, the momentum dependence of the potential. The spin-isospin factors can be written as the product

$$\mathcal{O}_n = z_n\, C_n\, \Lambda_n, \tag{84}$$

where z_n is the overall numerical factor (including the sign) in front of each of the V_n in Eqs. (79), C_n is the result of the summation over the color indices, and Λ_n is the result of the summation over the spin-flavor indices of the quarks.

Inspection of Eq. (79) reveals that

$$z_1 = +9, \qquad z_2 = -36, \qquad z_3 = -9, \qquad z_4 = z_5 = -18. \tag{85}$$

The color coefficients are given by $C_1 = 1$ and $C_i = 1/3$ for $i = 2, 3, 4, 5$. The spin-flavor coefficients are most easily evaluated making use of the «substitution rules» of Holinde [34] and Liu, Swift, Thomas and Holinde [35]. These are rules to transcribe spin-flavor operators at the quark level to the nucleon level. The spin-flavor dependence of the quark-pion interaction is of the form $\boldsymbol{\tau}^{(1)}_q\cdot\boldsymbol{\tau}^{(2)}_q\, \sigma^{(1)i}_q\sigma^{(2)j}_q$ and the substitution rules lead to

$$\begin{aligned}
\Lambda^{ij}_1 &= \frac{25}{81}\boldsymbol{\tau}^{(1)}_N\cdot\boldsymbol{\tau}^{(2)}_N\, \sigma^{(1)i}_N\sigma^{(2)j}_N, \\
\Lambda^{ij}_2 &= \frac{1}{36}\left\{\delta^{ij}\left[\frac{25}{3} + \frac{1}{9}\left(1 + 18\boldsymbol{\sigma}^{(1)}_N\cdot\boldsymbol{\sigma}^{(2)}_N\right)\boldsymbol{\tau}^{(1)}_N\cdot\boldsymbol{\tau}^{(2)}_N\right] + \right. \\
&+ \left.\frac{1}{3}\left(1 + \frac{7}{3}\boldsymbol{\tau}^{(1)}_N\cdot\boldsymbol{\tau}^{(2)}_N\right)\sigma^{(1)i}_N\sigma^{(1)j}_N\right\}, \\
\Lambda^{ij}_3 &= \frac{1}{36}\left\{\delta^{ij}\left[27 - 3\boldsymbol{\sigma}^{(1)}_N\cdot\boldsymbol{\sigma}^{(2)}_N - \left(1 - \frac{25}{9}\boldsymbol{\sigma}^{(1)}_N\cdot\boldsymbol{\sigma}^{(2)}_N\right)\boldsymbol{\tau}^{(1)}_N\cdot\boldsymbol{\tau}^{(2)}_N\right] + \right. \\
&+ \left.\left(6 - \frac{50}{9}\boldsymbol{\tau}^{(1)}_N\cdot\boldsymbol{\tau}^{(2)}_N\right)\sigma^{(1)i}_N\sigma^{(2)j}_N\right\},
\end{aligned}$$

$$
\begin{aligned}
\Lambda_4^{ij} &= \frac{1}{36}\left\{\delta^{ij}\left[15+\frac{1}{3}\left(1+10\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}\right)\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}\right]+\right.\\
&+\left.\left(1-\frac{5}{9}\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}\right)\sigma_N^{(1)i}\sigma_N^{(2)j}\right\},\\
\Lambda_5^{ij} &= \Lambda_4^{ij}.
\end{aligned}
\tag{86}
$$

The momentum-dependent functions u^{ij} cannot in general be evaluated in closed form because of the multidimensional integrals over the quark momenta. However, for a Fock-space amplitude $\Phi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ of gaussian form

$$
\Phi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)=\left(\frac{3b^4}{\pi^2}\right)^{3/4}\exp\left(-\frac{b^2}{6}\sum_{i<j}^{3}(\boldsymbol{k}_i-\boldsymbol{k}_j)^2\right),
\tag{87}
$$

where b is the r.m.s. radius of the nucleon, almost all the integrals over the quark coordinates can be performed analytically.

The most important contribution to the NN potential at short distances comes, as expected, from the delta-function piece of the quark–pion interaction. This component of the NN potential can be calculated in a closed form. The result is

$$
\begin{aligned}
V_{NN}=-\frac{1}{3}\frac{1}{4f_\pi^2}\Bigg[&\frac{25}{9}\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}\,\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}\,v_1(\boldsymbol{p}',\boldsymbol{p})-\\
&-\frac{1}{3}\left(25+\frac{1}{3}\,\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}+\frac{1}{3}\,\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}+\frac{61}{9}\,\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}\,\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}\right)v_2(\boldsymbol{p}',\boldsymbol{p})-\\
&-\frac{1}{4}\left(27-\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}-\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}+\frac{25}{27}\,\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}\,\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}\right)v_3(\boldsymbol{p}',\boldsymbol{p})-\\
&-\frac{1}{6}\left(45+\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}+\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}+\frac{85}{9}\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}\,\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}\right)v_4(\boldsymbol{p}',\boldsymbol{p})-\\
&-\frac{1}{6}\left(45+\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}+\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}+\frac{85}{9}\boldsymbol{\tau}_N^{(1)}\cdot\boldsymbol{\tau}_N^{(2)}\,\boldsymbol{\sigma}_N^{(1)}\cdot\boldsymbol{\sigma}_N^{(2)}\right)v_5(\boldsymbol{p}',\boldsymbol{p})\Bigg],
\end{aligned}
\tag{88}
$$

where

$$
v_1(\boldsymbol{p}',\boldsymbol{p}) = e^{-b^2/3\,(\boldsymbol{p}'-\boldsymbol{p})^2},
\tag{89}
$$

$$
v_2(\boldsymbol{p}',\boldsymbol{p}) = \left(\frac{3}{4}\right)^{3/2}e^{-b^2/6\,(\boldsymbol{p}'^2+\boldsymbol{p}^2)},
\tag{90}
$$

$$v_3(\boldsymbol{p}',\boldsymbol{p}) = e^{-b^2/3\,(\boldsymbol{p}'-\boldsymbol{p})^2}, \tag{91}$$

$$v_4(\boldsymbol{p}',\boldsymbol{p}) = \left(\frac{12}{11}\right)^{3/2} e^{-2b^2/11\,(\boldsymbol{p}-\boldsymbol{p}')^2-b^2/33\,(\boldsymbol{p}'^2+7\boldsymbol{p}^2)}, \tag{92}$$

$$v_5(\boldsymbol{p}',\boldsymbol{p}) = \left(\frac{12}{11}\right)^{3/2} e^{-2b^2/11\,(\boldsymbol{p}-\boldsymbol{p}')^2-b^2/33\,(\boldsymbol{p}^2+7\boldsymbol{p}'^2)}. \tag{93}$$

In order to obtain insight on the range and strength of the potential, we make a local approximation as suggested by Barnes, Capstick, Kovarik and Swanson (the last reference in Ref. 3), and perform a Fourier transform to coordinate space. The spin-flavor part of the potential is of course unaffected by this, and the radial part becomes

$$v_1(r) = v_3(r) = \left(\frac{3\pi}{b^2}\right)^{3/2} e^{-3/4\,(r^2/b^2)}, \tag{94}$$

$$v_2(r) = \left(\frac{9\pi}{b^2}\right)^{3/2} e^{-3\,(r^2/b^2)}, \tag{95}$$

$$v_4(r) = v_5(r) = \left(\frac{9\pi}{2b^2}\right)^{3/2} e^{-33/32\,(r^2/b^2)}. \tag{96}$$

This potential is obviously of short range, because we are considering only the δ-function part of the pion–quark interaction. Also, the quark-exchange contributions are typically of the order of the nucleon size. In Fig. 7 the quark–pion exchange potential of Eq. (96), for $b = 0.6$ fm, is compared with the one-boson-exchange Bonn potential for ω and ρ exchanges. These are the main sources of repulsion in the NN interaction. Because of the Yukawa form of the meson-exchange potentials, we have multiplied the potentials by r^2. In this way, we obtain a better understanding of the relevant contributions to observables. The corresponding expressions of the Bonn potential are given by Eqs. (A-19) and (A-28) of Ref. 36. The parameters are given in Table A.3 of the same reference.

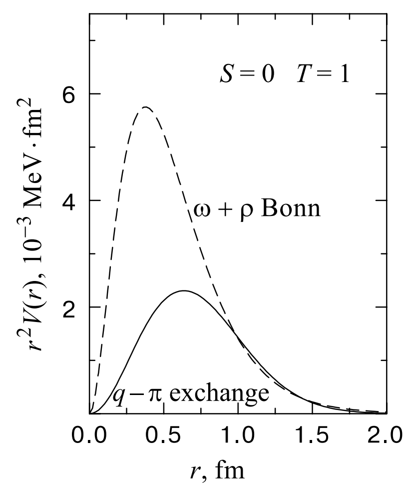

Fig. 7. The quark–π (solid) and $\omega + \rho$ (dashed) Bonn potentials

It is seen that the two potentials have roughly the same ranges, but have very different strengths (volume). Note however that in order to have a better assessment of the ranges of the potentials (for distances larger than 1 fm), for consistency a gaussian form factor for the vector-mesons should be used, instead of the dipole form factor of Ref 36.

In the next section, when we consider the nuclear matter problem, we will show that although the quark–pion exchange interaction provides a large fraction of the required repulsion to stabilize nuclear matter against collapse, extra repulsion is needed to saturate nuclear matter at the right density.

6. FOCK–TANI REPRESENTATION FOR NUCLEAR MATTER

In the limit that the quark cores of the nucleons do not overlap, effects from the Pauli principle at the quark level can be neglected, and the anticommutation relations of creation and annihilation operators of composite nucleons are simply the ones of elementary particles, as discussed previously. For field operators that satisfy canonical (anti)commutation relations, the coupled-cluster expansion (or e^S-formalism) is a very powerful formalism for treating many-body problems. This is a formalism that treats short-range correlations induced by strong short-range repulsion, as is the case of the NN interaction, and allows for systematic improvement as the density of the system increases [37, 38]. This formalism seems to be particularly appropriate also for the case of composite nucleons when used in connection with the Fock–Tani representation.

The idea is to implement the e^S-formalism in the ideal space. In the Fock–Tani space $\mathcal{F}_{FT}$, the Hamiltonian can be split as

$$H_{\rm FT} = H_q + H_b + H_m + H_{bm} + H_{qb} + H_{qm} \ , \tag{97}$$

where each component has obvious meaning. When the Hamiltonian is truncated to involve only ideal nucleons, in analogy with the point nucleon case, the wave function of nuclear matter can be written as

$$|\Psi >= e^S \, |\Phi >, \tag{98}$$

where $|\Phi >$ is a Fermi-gas state of *ideal* nucleon states

$$|\Phi >= \lim_{N\to\infty} b^\dagger_{\alpha_1} b^\dagger_{\alpha_2} \cdots b^\dagger_{\alpha_N} |0), \tag{99}$$

and S is the operator that creates *ideal nucleon* particle-hole states on the top of the ideal Fermi-gas state

$$S = \sum_{n>1} s_n, \tag{100}$$

with

$$s_n = \frac{1}{(n!)^2} \sum_{\alpha > k_F} \sum_{\beta < k_F} s_n(\alpha_1 \cdots \alpha_n; \beta_1 \cdots \beta_n) \, b^\dagger_{\alpha_1} \cdots b^\dagger_{\alpha_n} b_{\beta_1} \cdots b_{\beta_n}. \tag{101}$$

The form of the functions $s_n(\alpha_1 \cdots \alpha_n; \beta_1 \cdots \beta_n)$ are in general chosen such as to minimize the energy density of nuclear matter [37, 38]. Explicit *ideal* meson degrees can be incorporated with no extra conceptual difficulties, again using the analogy with point hadrons [39].

With increasing density, the terms H_{qb} and H_{qm} of the effective Hamiltonian in $\mathcal{F}_{\mathrm{FT}}$ cannot be neglected. These terms describe the possibility of hadrons breaking up into quarks that can propagate outside the confining region within hadrons. The incorporation of such effects within the e^S-formalism seems to be very natural. The functions s_n can be generalized such as to describe the deconfining effects.

One interesting term present in H_{qb} of Eq. (97) is [18]

$$V_{\mathrm{binary-break}} = \frac{3}{4} \Psi_\alpha^{\rho\nu_2\mu_3} \Psi_\beta^{\sigma\nu_2\nu_3} V_{qq}(\mu\nu; \sigma\rho)\, q_\mu^\dagger q_\nu^\dagger q_{\mu_2}^\dagger q_{\mu_3}^\dagger q_{\nu_2}^\dagger q_{\nu_3}^\dagger b_\alpha b_\beta \,. \tag{102}$$

This describes a process in which two nucleons collide and break up into six quarks. At high densities, such processes are expected to play an important role in the description of the equation of state of nuclear matter. Within the e^S-formalism, such processes can be taken into account by a term in the exponent S of the form

$$S_{\mathrm{binary-break}} = \sum_{\beta < k_F} \sum_{\mu_1 \cdots \mu_6} s(\mu_1 \cdots \mu_6; \beta_1 \beta_2)\, q_{\mu_6}^\dagger q_{\mu_5}^\dagger q_{\mu_4}^\dagger q_{\mu_3}^\dagger q_{\mu_2}^\dagger q_{\mu_1}^\dagger b_{\beta_2} b_{\beta_1} \,. \tag{103}$$

Other terms of the effective Hamiltonian, such as single-hadron breakup [18], can similarly be taken into account.

There are no numerical results of applications of this formalism. Of course, one technical problem is the large amount of algebraic manipulations necessary to obtain the relevant variational equations to be solved numerically. Another problem is the apparent necessity for a relativistic quark model, since it seems that a nonrelativistic model of the type used in the previous sections would not perform well in the high-density regime of nuclear matter.

To finalize, let us consider the Hartree–Fock approximation to the nuclear matter equation of state using the present formalism. This amounts to retaining only the part of the effective Hamiltonian that involves the ideal nucleon operators, and to neglecting the (interesting) correlations between nucleons, i.e., $S = 0$ in Eq. (98). We consider here a quark-meson-coupling model with constituent quarks [40], on the lines of the Guichon–Saito–Thomas model [41]. Ref. 40 considers a semi-relativistic quark model, where the massive constituent quarks are confined by a phenomenological nonrelativistic harmonic potential and interact via exchange of mesons. The meson exchanges are treated in a similar fashion to the traditional derivation of the one-gluon interaction [43], but the kinetic energy and the quark-meson interactions are taken to be relativistic.

The pion and a fictitious σ meson are coupled directly to the constituent quarks, as in the Guichon–Saito–Thomas model, but the ω meson is coupled to the nucleon core. The very short range part of the NN interaction is described by quark–pion exchange, while the ω meson is responsible for the outer part of the repulsion, since it is coupled to the nucleon with the form factor provided by the model. The rationale of such an idea is of course to replace the ω meson as the main source of the NN repulsion, as discussed above.

The «microscopic» quark–meson Hamiltonian is obtained from the Lagrangian density of the Walecka model [42]. In Ref. 40, the Fock–Tani representation was used to derive the effective NN interaction involving quark–pion exchanges of Fig. 6. For the effective NN interaction of Fig. 5, the Fock–Tani representation is of course not necessary. We present here only the contribution of the NN interaction of Eqs. (82)–(93) to the energy density of symmetrical nuclear matter [40]

$$\mathcal{V}_q^{exch} = \frac{1}{3f_\pi^2}\int_0^{k_F}\frac{d\boldsymbol{p}}{(2\pi)^3}\int_0^{k_F}\frac{d\boldsymbol{p}'}{(2\pi)^3}\Bigg[54 + 8\left(\frac{3}{4}\right)^{3/2} \mathrm{e}^{-1/12\,b^2\,(\boldsymbol{p}-\boldsymbol{p}')^2} + \\ +120\left(\frac{12}{11}\right)^{3/2} \mathrm{e}^{-2/33\,b^2\,(\boldsymbol{p}-\boldsymbol{p}')^2} - \frac{44}{3}\,\mathrm{e}^{-1/3\,b^2\,(\boldsymbol{p}-\boldsymbol{p}')^2} - \\ -\frac{272}{3}\left(\frac{12}{11}\right)^{3/2} \mathrm{e}^{-8/33\,b^2\,(\boldsymbol{p}-\boldsymbol{p}')^2}\Bigg]. \tag{104}$$

The contributions from nonquark-exchange graphs can be written down without difficulty and are given explicitly in Ref. 40.

The interesting result obtained in Ref. 40 is that the quark–pion exchange interaction does provide a large fraction of the required NN interaction to stabilize nuclear matter. Moreover, it turns out that the value of the $NN\omega$ coupling constant g_ω^2, adjusted to obtain a binding energy per nucleon $E/A - M_N \simeq -15.75$ MeV at $k_F \simeq 1.36\ \mathrm{fm}^{-1}$ is very close to the quark-model SU(6) symmetry prediction $g_\omega^2/4\pi \approx 9g_\rho^2/4\pi = 9\times 0.55$. Note that this value is a much smaller value than the ones used in one-boson-exchange models [36].

7. CONCLUSIONS AND FUTURE PERSPECTIVES

The traditional picture of the nucleus, which follows from a large body of experiments in the last 60 years, is that of a system of nucleons whose properties are not very different from free-space nucleons. This means that the explicit dynamics of the color degree of freedom must be limited to very short distances.

Therefore, any theoretical approach based on quark degrees of freedom that is intended to study low-energy properties of nuclei, should minimally deviate from, as well as contain in some limit, the traditional approach based on nucleon degrees of freedom. In this sense, the effective Hamiltonian of the Fock–Tani representation has a well-defined limit, since it explicitly describes the interactions among hadrons; quark–quark and quark-hadron interactions are treated separately as «residual» interactions that are expected to play an important role only at higher densities/temperatures.

For higher densities and/or temperatures, the Fock–Tani representation seems particularly useful when used in connection with the linked-cluster (or e^S) formalism. The Fock–Tani representation naturally leads to effective Hamiltonians that describe processes that are expected to be present in the system at the transition regime from a cold, low-density phase to a high density/temperature phase. The wave function of the system at this regime is naturally given by the linked-cluster formalism, where nucleon–nucleon correlations and other deconfining effects are built on the top of a Fermi-gas of confined, color-singlet clusters of quarks.

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