«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
2000, ТОМ 31, ВЫП. 7А

УДК 536.75

EXACTLY SOLVABLE MODEL SYSTEMS AND THE HARTREE–FOCK–BOGOLUBOV APPROXIMATION FOR MODEL SYSTEMS WITH FOUR-FERMION INTERACTION

N.N.Bogolubov, Jr.

V.A.Steklov Mathematical Institute, Dept. of Statistical Mechanics, Moscow

An exactly solvable model system with pair interaction, which is of importance in superconductivity theory, is considered. The model reads as

$$H = H_0 + H_{\rm int}, \quad H_0 = \sum_{(p,s)} (E(p) - \mu) a^\dagger_{ps} a_{ps}.$$

$$H_{\rm int} = - \sum_{(p,p')} \frac{1}{V} J(p,p') a^t_{-p,-1/2} a^t_{p',1/2} a_{-p',-1/2},$$

where $a^\dagger_{p,\pm1/2}$, $a_{p,\pm1/2}$ are the Fermi operators and V is the volume of the system. The kernel $J(p,p')$ is assumed to be a real bounded function which vanishes in effect outside a certain region of its arguments. The summation in $H_{\rm int}$ is fulfilled over the momenta p and p' belonging to the energy layer $E_F - \omega < E(p) < E_F + \omega$.

It is shown how one can construct an asymptotically exact solution for this model using an approach based on the idea of the «approximating Hamiltonians». For a sufficiently general conditions imposed on the model system in question, such as the condition of separable interaction,

$$J(p,p') = \lambda(p)\lambda(p'),$$

a theorem is proved which allows rigorous calculation of the free energy per unit volume.

An approximate method, based on the same idea of the approximating Hamiltonian, is proposed that gives an opportunity to investigate dynamic properties of various models with four-fermion interaction of general type:

$$H = \sum_{(f,f')} \Omega(f',f) a^\dagger_f a_{f'} + \frac{1}{2} \sum_{(f_1,f_2,f_2',f_1')} U(f_1,f_2,f_2',f_1') a^\dagger_{f_1} a^\dagger_{f_2} a_{f_2'} a_{f_1'} +$$

$$+\frac{1}{2}\sum_{(f,f')} j_-(f',f,t)a_f^\dagger a_{f'}^\dagger + \frac{1}{2}\sum_{(f,f')} j_+(f',f,t)a_f a_{f'},$$

where $U(f_1,f_2,f_2',f_1')$ are symmetric functions with respect to simultaneous permutation of their arguments:

$$(1 \longleftrightarrow 2): \quad \{f_1 \longleftrightarrow f_2; f_1' \longleftrightarrow f_2'$$

and $\Omega(f',f)=\Omega_0(f',f)+j(f',f,t)$. This model includes the above-mentioned model as a particular case. The auxiliary source terms

$$\frac{1}{2}\sum_{(f,f')} j_-(f',f,t)a_f^\dagger a_{f'}^\dagger, \quad \frac{1}{2}\sum_{(f,f')} j_+(f',f,t)a_f a_{f'}, \quad \sum_{(f,f')} j_+(f',f,t)a_f a_{f'}$$

are chosen to ensure that the law of the total momentum conservation holds while the law of the number of particles conservation is broken.

This method unites the standard approximating Hamiltonian approach to models with separable interaction with an approximation scheme based on the ideas of self-consistency. The BCS model, which is of importance in the theory of superconductivity, is analyzed so as to demonstrate the usefulness of the proposed formalism.

REFERENCES

1. **Bogolubov N.N., Zubarev D.N., Tzercovnikov Yu.A.** — Dokl. Acad. Nauk SSSR, 1957, v.117, p.788 (Sov. Phys. Doclady, 1957, v.2, p.535).
2. **Bogolubov N.N., Zubarev D.N., Tzercovnikov Yu.A.** — Zh. Eksp. Teor. Fiz., 1960, v.39, p.120 (Sov. Phys. JETP, 1960, v.12, p.88).
3. **Bogolubov N.N.** — JINR Preprint P-511, Dubna, 1960.
4. **Bogolubov N.N., Jr.** — Physica, 1966, v.32. p.933; see also **Bogolubov N.N., Jr.** — Method for Studying Model Hamiltonians. Pergamon Press, 1972, v.43.
5. **Bogolubov N.N., Jr.** — Theor. Math. Phys., 1970, v.4, p.3.
6. **Bardeen J., Cooper L.N., Schriffer J.R.** — Phys. Rev., 1957, v.108, p.1117.
7. **Bogolubov N.N., Tolmachev V.V., Shirkov D.V.** — A New Method in the Theory of Superconductivity (1958); (English translation was published by Consultants Bureau, N.Y., 1959).
8. **Hertel P., Thirring W.** — Commun. Math. Phys., 1971, v.24. p.22.
9. **Bogolubov N.N., Jr., Petrina D.Ya.** — Theor. Math. Phys., 1977, v.33, p.2.
10. **Bogolubov N.N., Jr., Soldatov A.V.** — Int. J. Mod. Phys., 1996, v.B10, No.5, p.579.
11. **Bogolubov N.N., Bogolubov N.N., Jr.** — Introduction to Quantum Statistical Mechanics. M., Nauka, 1984, pp. 92–104.