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HIGH ENERGY SCATTERING IN THE BRANE-WORLD AND BLACK-HOLE PRODUCTION

I.Ya.Aref'eva

V.A.Steklov Mathematical Institute, Russian Academy of Sciences, 117966, Moscow, Russia

Black-hole production in the collision of ultra-relativistic particles in the brane-world approach is considered. In particular, stability of the brane under collision with ultrarelativistic particles is discussed. As a toy model we consider the 3-dimensional version of the Randall and Sundrum solution and show that stability of the brane depends on a choice of continuation of the solution across the horizon. In the unstable case black holes can be produced in the collision of a particle with the brane.

1. INTRODUCTION

Main physical questions which are addressed in this letter are the following:
- Can two ultrarelativistic particles produce a black hole? If the answer is «yes», then the following question arises
- Is a black-hole production observable in theories with low scale gravity and large extra dimensions?

The first question has been already discussed in the literature [1–4]. In a series of papers, Amati, Ciafaloni and Veneziano and 't Hooft conjectured that black holes occur in the collision of two light particles at planckian energies. It was argued [1, 2] that at extremely high energies interactions due to gravitational waves will dominate all other quantum field theoretic interactions. In [4] the following scenario for such a process was proposed. Each of the two ultra-relativistic particles generates a gravitational wave (GW) and the gravitational waves are considered as plane waves. Then these plane gravitational waves collide and they produce a singularity or a black hole.

$$\textbf{Particles} \;\rightarrow\; \textbf{GW} \;\sim\; \textbf{Plane GW} \;\rightarrow\; \textbf{Black Holes}. \tag{1}$$

To realize this scenario analytically the Chandrasekhar–Ferrari–Xanthopoulos duality between the Kerr black hole solution and colliding plane gravitational waves was used [4, 5].

A typical parameter for such a process is the Schwarzschild radius R_S of a body mass m, which is equal to the energy of colliding particles in the centre-of-mass frame. Since R_S in the 4-dimensional case is of the order of $M_{Pl,4}^{-1}$, these processes are out to be observable. However if we accept the brane-world scenario [6] where the fundamental higher dimensional Planck scale $M_{Pl,4+n}^{-1}(n > 2)$ can be in the TeV range, one can expect that such processes could lead to physical consequences.

For this scenario we need a solution describing colliding plane gravitational waves in higher dimensional space-time and we have to find a black-hole geometry in the collision domain. As a background geometry we consider brane-world geometry [7, 8] and the RS model [9] dealing with an infinite extra dimension. In this model we live on a brane (domain wall) inside AdS space and four-dimensional gravity is recovered on the brane. An analytical description of colliding plane gravitational waves in $n+4$-dimensional space-time especially in AdS background is unknown. We will consider as a toy example the collision of particles in the 3-dimensional version of the AdS background. The solution describing two colliding plane gravitational waves in the 3-dimensional AdS space-time was found in [10, 11].

We analyze how the presence of moving particle influences the brane stability. We find that a brane in AdS_3 due to gravitational interaction with a particle becomes unstable and it can split on disjoint branes or totally disappears. This takes place, of course, only in the case when a particle can be created. This case corresponds to a special continuation of the RS solution across the horizon. In this case due to the reflection symmetry of the RS solution the brane in some sense imitates the second particle and in accordance with [10, 11] a black hole can be created.

One can expect a similar picture in the higher dimensional case. To support this we use the one plane wave solution in AdS_d proposed in [12–15] and argue that to have a black hole production we have to use a solution that beyond the horizon is pure AdS with no brane present. If this black hole is created, it is a higher dimensional object. Phenomenological aspects of such objects have been discussed in [16]. Black-hole formation due to gravitational collapse of matter trapped on a brane has been studied recently in [17].

The paper is organized as following. In Section 2 we remind the scenario of the black-hole creation from [4, 5]. In Section 3 we discuss changes of geometry of AdS_3 in the presence of moving particles and the influence of these changes on the brane. In Section 4 some comments about a possible generalization of 3-dimensional picture to higher dimensional cases are presented.

2. COLLIDING PLANE GRAVITATIONAL WAVES AND BLACK HOLES

Two main assumptions of mechanism of black-hole creation (1) are [4, 5]:

- The transition amplitude for the process of creation of black hole in the collision of two particles is determined by the semiclassical transition amplitude for the

process of creation of black hole in the collision of two gravitational waves.
• Gravitational waves produced by ultrarelativistic particles are considered as plane waves.

Saying shortly, the mechanism (1) means that ultrarelativistic particles generate plane gravitational waves, then these plane gravitational waves collide and produce a singularity or a black hole. This mechanism uses an idealized picture that plane gravitational waves already have been produced by ultrarelativistic particles. This idealization is based on the fact that ultrarelativistic particles generate gravitational waves and any gravitational wave far away from sources can be considered as a plane wave. We used this idealized picture because it is difficult to perform calculations in the realistic situation. We assume that plane waves already have been produced by ultrarelativistic particles and then consider analytically the process of black hole formation when the waves collide.

We discuss the process of creation of black hole in the collision of two plane waves in the semiclassical approximation. Note that black holes cannot be incorporated into the theory if we consider quantum field theory in Minkowski space-time.

There exists a well known class of plane-fronted gravitational waves with the metric

$$ds^2 = 2dudv + h(u, X, Y)du^2 - dX^2 - dY^2, \tag{2}$$

where u and v are null coordinates. In particular the gravitational field of a particle moving with the speed of light is given by the Aichelburg–Sexl solution [22]. The metric has the form

$$ds^2 = 2dudv + E\log(X^2 + Y^2)\delta(u)du^2 - dX^2 - dY^2 \tag{3}$$

and describes a shock wave. It is difficult to find a solution which describes two sources, except the 3-dimensional case [10, 11]. An approximate solution of Einstein equation for two particles as the sum of one particle solutions describes well the scattering amplitude for large impact parameter, but does not describe nonlinear interaction of shock waves which is dominant in the region of small impact parameter. To analyze nonlinear effects we took, instead of dealing with shock wave, plane gravitational waves. In some respect one can consider plane wave as an approximation to more complicated gravitational waves, in particular shock waves. This solution in some sense can be interpreted as an approximation for a solution of Einstein equation in the presence of two moving particles.

A particular class of plane waves is defined to be plane-fronted waves in which the field components are the same at every point of the wave surface. This condition requires that $h(u, X, Y)$ is a function with a quadratic dependence on X and Y. One can then remove the dependence of h on X and Y altogether by a coordinate change.

Classical collision of plane gravitational waves has been the subject of numerous investigations, see for example [18–21], and it has a remarkably rich structure. In [4] was used the Chandrasekhar–Ferrari–Xanthopoulos duality between colliding plane gravitational waves and the Kerr black hole solution.

Let us present a metric corresponding to the collisions of two plane waves with zero impact parameter. In this case the colliding plane gravitational waves produce in the interaction region a space-time that is isometric to the interior of the Schwarzschild solution. The metric is given by

$$ds^2 = 4m^2[1+\sin(u\theta(u))+v\theta(v)]dudv$$

$$-[1-\sin(u\theta(u))+v\theta(v)][1+\sin(u\theta(u))+v\theta(v)]^{-1}dx^2 \qquad (4)$$

$$-[1+\sin(u\theta(u))+v\theta(v)]^2\cos^2(u\theta(u))-v\theta(v))dy^2,$$

where $u<\pi/2, \quad v<\pi/2, \quad v+u<\pi/2$.

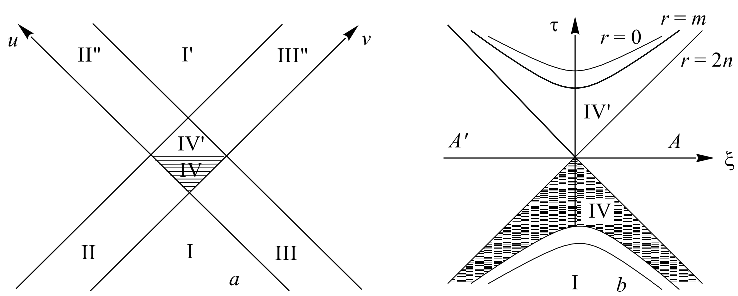

Fig. 1. *a*) Plane coordinates, *b*) Kruskal coordinates

Figure 1,*a* illustrates this solution of the vacuum Einstein equations. The background region I describes a region of space-time before the arrival of gravitational waves and it is Minkowskian. Two plane waves propagate from opposite directions along the z-axis. Regions II and III contain the approaching plane waves. In the region IV the metric (4) is isomorphic to the Schwarzschild metric. To see this one can make the following change of variables from the plane-waves coordinates to the Schwarzschild coordinates, $(u,v,x,y)\rightarrow(t,r,\theta,\phi)$ defined by,

$$r=m[1+\sin(u+v)], \quad t=x, \quad \theta=\pi/2+u-v, \quad \phi=y/m, \qquad (5)$$

or to Kruskal coordinates τ,ζ,θ,ϕ

$$\tau=-a(r)\cosh t/4m, \quad \zeta=-a(r)\sinh t/4m, \quad a(r)=(1-r/2m)^{1/2}e^{r/4m}. \qquad (6)$$

Then one gets

$$ds^2 = \frac{32m^3}{r} e^{-r/2m}(d\tau^2 - d\zeta^2) - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

The section of the region IV bounded by $x = 0,\ y = 0$ corresponds to segment in the Kruskal diagram and the section of the region IV by the plane $x = x_0$, $y_0 = 0$ corresponds to the hatched region in the Kruskal diagram in Fig. 2. The lines corresponding to $r = 2m$ (horizon) apart from the point ($\tau = 0,\ \zeta = 0$) correspond to the infinite value of x-plane wave coordinate.

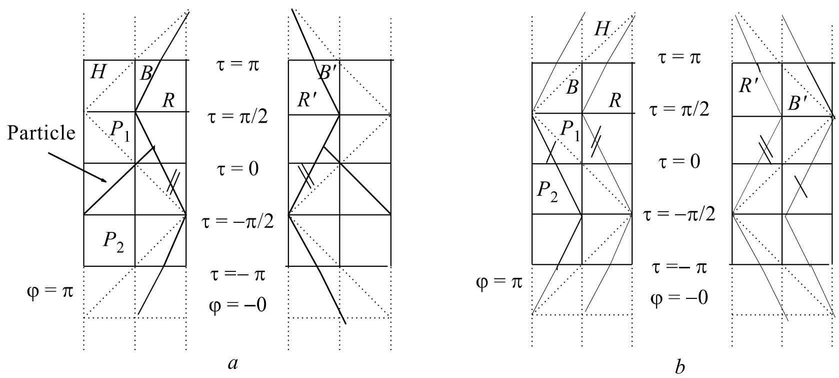

Fig. 2. Penrose diagrams for AdS_3 with a brane. A brane B is located in the region P_1. Diagrams *a*) and *b*) show the different continuations across the horizon H. *a*) there are no branes in the region P_2; *b*) there is a brane B in the region P_2. Identifications are shown by "/" and "//". R and R' denote the removed regions

The above metric in the (u, v) plane can be extended beyond the event horizon $u + v = \pi/2$ in one of two ways.

The first possibility, shown in Fig. 1 consists in reflecting along the line $u + v = \pi/2$. The second one involves in gluing to the horizon the whole upper-half part of the Kruskal diagram.

Both extensions are solutions of Einstein equations. There is *a priori* nonzero probability to get a finite state corresponding to a black hole or two outgoing plane waves. Calculations of the probabilities for these processes in the semiclassical approximation are performed in [4].

3. BRANE-PARTICLE COLLISION IN AdS_3

In this section we consider the solution of Einstein equations describing interaction of brane and particle in the AdS_3 space-time.

To describe a moving particle in AdS_3 it is convenient to use the global coordinate system (τ, r, ϕ) and the matrix representation

$$\mathbf{x} = \mathbf{x}_{-1}1 + \mathbf{x}^a\gamma_a, \quad \det\mathbf{x} = 1, \tag{7}$$

where r is a radial coordinate $0 < r \leq 1$, $0 \leq \varphi < 2\pi$ is an angular coordinate, τ is a real time coordinate and

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{8}$$

The metric is

$$ds^2 = \frac{1}{2}\mathrm{Tr}\,(\mathbf{x}^{-1}d\mathbf{x}\;\mathbf{x}^{-1}d\mathbf{x}) = \Big(\frac{2}{1-r^2}\Big)^2\left(dr^2 + r^2\; d\varphi^2\right) - \Big(\frac{1+r^2}{1-r^2}\Big)^2\; d\tau^2. \tag{9}$$

AdS_3 space can be considered as a Poincaré disc evolving in time. To construct a space-time containing a point particle according to [10, 11] one has to fix the holonomy, say

$$\mathbf{u} = 1 + \tan\epsilon\,(\gamma_0 + \gamma_1), \qquad 0 < \epsilon < \pi/2, \tag{10}$$

and find a curve $\mathbf{w}_-$ in the τ-plane such that its image $\mathbf{w}_+$ under a spatial isometry, $\mathbf{u}\mathbf{w}_+\mathbf{u}^{-1} = \mathbf{w}_-$ lies in the same τ-plane. Then one has to cut out the wedge between these lines and to identify the faces according to the isometry. A world line of the particle is the set of fixed points of the isometry. The resulting space-time manifold has a constant curvature everywhere except on the world line. The curves $\mathbf{w}_\pm$ are given by

$$\frac{2r}{1+r^2}\sin(\epsilon \pm \varphi) = \sin\tau\sin\epsilon. \tag{11}$$

Let us now consider the three-dimensional version of the Randall and Sundrum (RS) model [9] which deals with a brane located at $y = 1$, where y is one of the sets of the Poincare coordinates describing AdS_3. RS slice AdS along the surface $y = 1$, remove the portion $0 < y < 1$ and assume Z_2 reflection symmetry at the boundary surface. In the global coordinate system (t, r, ϕ) the 2-dimensional surface $y = 1$ is described by the equation

$$\cos\phi = \frac{1 - r^2 - (1+r^2)\sin\tau}{2r}. \tag{12}$$

Let us note that there are several ways to analytically continue the RS solution across the horizon (see [23] and references therein). Two obvious choices of

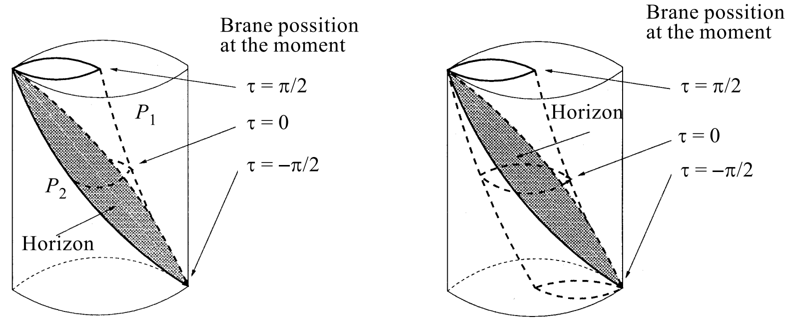

Fig. 3. Three-dimensional picture of the brane. AdS_3 is displayed as the interior of the cylinder. The horizon H (the shared cut) divides AdS_3 into two regions P_1 and P_2, each of which is covered by a set of Poincare coordinates. *a*) and *b*) show the different continuations across the horizon

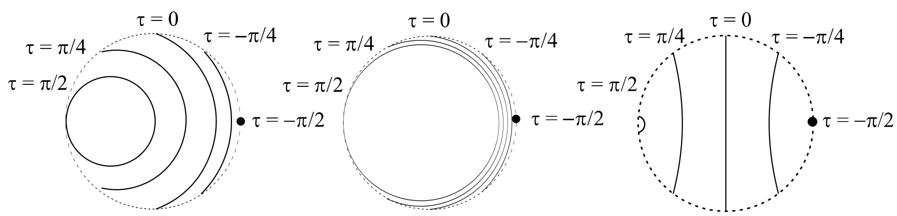

Fig. 4. Brane positions on the Poincare disk in times $-\pi/2 \leq \tau \leq \pi/2$: *a*) $a = 1$, *b*) $a \ll 1$; *c*) $a \gg 1$

continuation are shown in Fig. 2. For the first continuation (Fig. 2,*a*) there is no brane beyond the horizon. For the second one (Fig. 2,*b*) there is a brane beyond the horizon.

In Fig. 4 the brane positions under the assumption of the global structure presented in Figs. 2,*a* and 3,*a* are shown for different values of τ. We consider here the brane positions for the time between $\tau = -\pi/2$ and $\tau = \pi/2$. We see that the brane at the initial moment is just a point on the Poincare disk and becomes a circle of 1/2 radius in the last moment.

One can consider the brane located at $y = a$

$$\cos\phi = \frac{(1-r^2)a^{-1} - (1+r^2)\sin\tau}{2r}. \tag{13}$$

In the case of $a \neq 1$ the brane positions between $\tau = -\pi/2$ and $\tau = \pi/2$ are shown in Figs. 4,*b* and 4,*c*. For all values of a at the initial moment the brane is just a point on the Poincare disk. For a small enough it becomes a closed curve located near the boundary of the Poincare disk in the last moment. For large a and $\tau = \pi/2$ the brane is a curve located near $\varphi = \pi$.

Let us consider the collision of the brane and the particle. We assume that at the initial moment $\tau = -\pi/2$ the particle is at the point $\varphi = \pi,\ r = 1$ (the case of the symmetric initial position of the particle and the brane). The pictures of positions of the brane and the wedge in subsequent moments of time $-\pi/2 \geq \tau \geq -\pi/2$ are presented in Figs. 5 and 6. In dependence of holonomy (10) characterizing the moving particle, there are two different cases. Both of them schematically are presented in Figs. 5 and 6. In the first case the «removing» part of the Poincare disk at the moment $\tau = \tau_c$ is large enough to place there the domain with $y \geq 1$. Note that since the boundaries of wedge are unidentified (points A and A' as well as B and B' are identified) the brane in Fig. 5 is in fact connected. At the moment $\tau = \tau_c$ the brane is totally in a region that we have to cut out and the brane disappears. To have this picture we have to take ϵ near to $\pi/2$. In the second case the brane does not disappear between $\tau = -\pi/2$ and $\tau = \pi/2$. This picture takes place for ϵ near to 0.

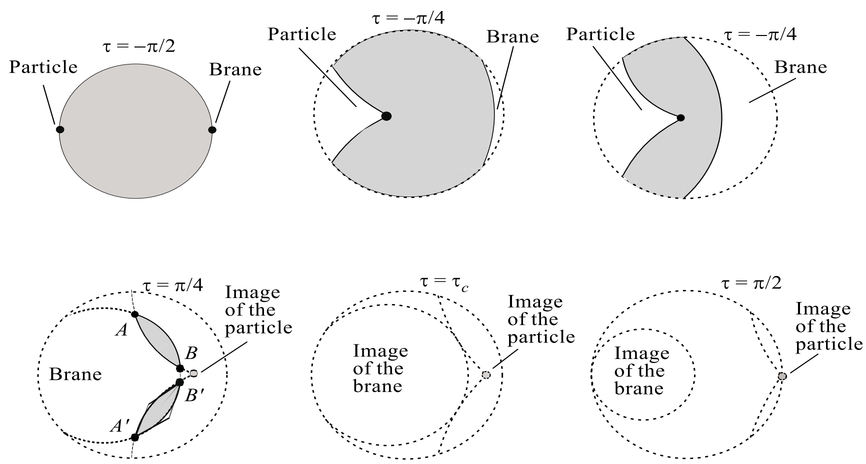

Fig. 5. Brane positions on the Poincare disk with wedge cuts ($\epsilon = \pi/4$) in times $-\pi/2 \leq \tau \leq \pi/2$

A case of a nonsymmetric position of the particle and the brane in the initial moment $\tau = -\pi/2$ is presented in Fig. 7. Note that here at the moment $t = \pi/4$ the brane is composed of two pieces CA and $B'D$ (point A is identified with A' but not with B'). At the moment $\tau = \tau_0$ the word line of the particle crosses the brane and the brane splits on three pieces. Later one of these pieces disappears. Two pieces of the brane are pasted to one brane at the moment $\tau = \pi/2$.

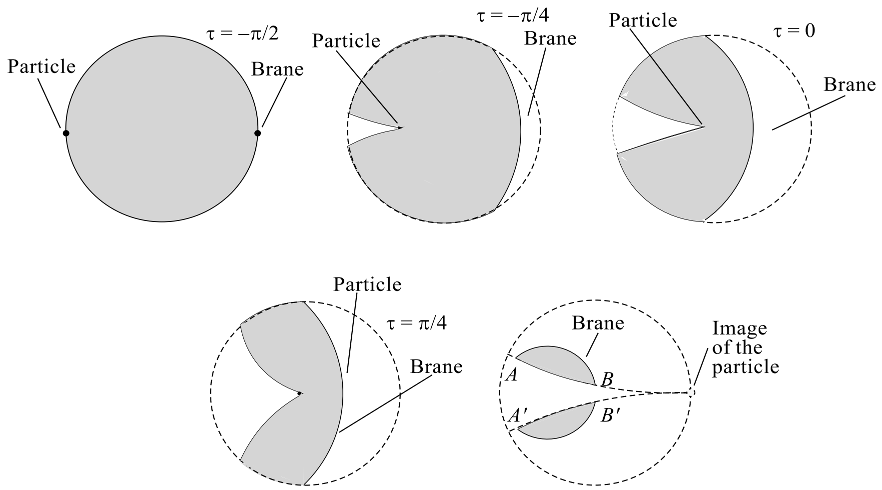

Fig. 6. Brane positions on the Poincare disk with wedge cuts ($\epsilon = 1/12\pi$) in times $-\pi/2 \leq \tau \leq \pi/2$

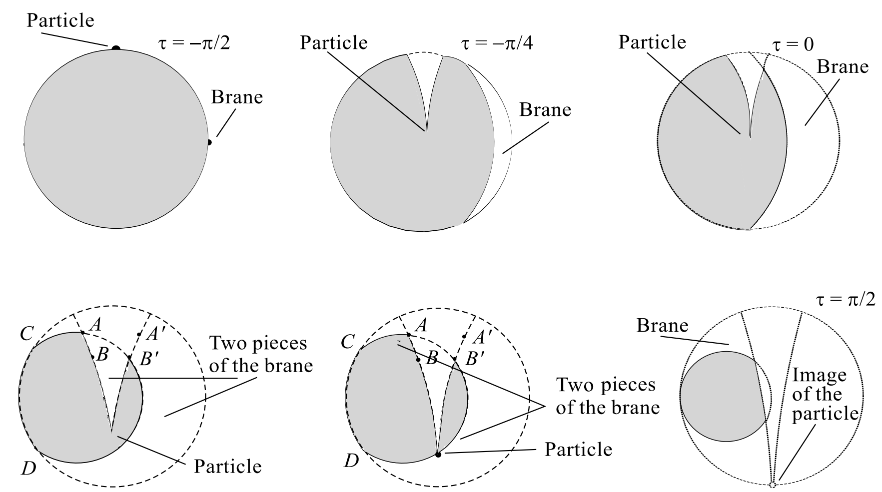

Fig. 7. Brane positions on the Poincare disk with wedge cuts ($\epsilon = 1/12\pi$) in times $-\pi/2 \leq \tau \leq \pi/2$

The above consideration demonstrates that the brane configuration is unstable under possible collisions with particles. Note that this consideration assumes the global structure 2,*a*.

Let us now consider the second way of continuation across the horizon. The global structure of space time is shown in Fig. 2,*b*. In this case one deals with further branes beyond the horizon and there is no timelike infinity in the space time. Hence there is no «place» from which a particle can start its evolution. This shows a stability of the RS solution under assumption of many branes.

4. DISCUSSION AND CONCLUSION

In the 4-dimensional case the process of the black hole creation (1) takes place for the impact parameter b smaller than the Schwarzschild radius of the black hole with the mass equal to the energy of colliding particles in the centre-of-mass frame. Analogously we expect that the same process dominates in the $n+4$-dimensional case for

$$b < R_{S,n+4}, \tag{14}$$

where $R_{S,n+4}$ is the Schwarzschild radius of the $4+n$-dimensional black hole with mass m. The mass m is equal to the energy of colliding particle in the centre-of-mass frame. The Schwarzschild radius of the $4+n$-dimensional black hole of mass m is given by [24]

$$R_{S,n+4} = c_{n+4}\ (\kappa_{4+n} m)^{1/n+1}, \tag{15}$$

there $c_{4+n} = (\frac{8\Gamma((n+3)/2)}{(n+2)\pi^{n+1/2}})^{1/n+1}$. Using that $\kappa_{4+n} \sim M_{Pl,4+n}^{n+2}$ we have the bound

$$b < c_n M_{Pl,n+4}^{-1}\ (\frac{E}{M_{Pl,n+4}})^{1/n+1}, \tag{16}$$

where E is the energy of colliding particles in the centre-of-mass frame. Therefore, if one adapts the scenario of [6] and finds an analog of the metric (4), then one can conclude that the black hole production takes place at the TeV scale. This means that one has a very strong restriction according to which processes with transverse momenta larger than $R_{S,n+4}^{-1}$ should be completely absent. We see an interesting feature of the bound (16). Since we expect that n is large enough, say $n = 6$, the right-hand side of this inequality does not depend very much on the energy of colliding particles.

Let us note once again that to realize (1) in higher dimensional case one has to find a solution describing collision of gravitational waves. This is still an open problem for $n > 0$. Moreover, within the framework of low scale quantum gravity scenario one has to solve a problem of colliding waves in a particular compactified space. However, by analogy with the 3-dimensional case one can expect that the role of the second plane wave can be played by the brane within the RS scenario. The one plane wave in the AdS_d background was found for

$d = 4$ in [12, 13, 15] and for $d > 4$ in [14]. The metric has a simple form in plane wave coordinates

$$ds^2 = \frac{dUdV - \eta_{\alpha\beta}(dZ^\alpha + U\theta(U)H_{\alpha\gamma}dZ^\gamma)(dZ^\beta + U\theta(U)H_{\beta\gamma'}dZ^{\gamma'})}{[1-(UV - \eta_{\alpha\beta}Z^\alpha Z^\beta + U\theta(U)G]^2}, \quad (17)$$

$$\alpha, \beta = 1, ..d-2.$$

This metric for negative values of U reproduces the pure AdS_d metric. The plane wave coordinate U is related with the global coordinates via

$$U = \frac{\tan\rho\; n_1 - \sec\rho \sin\tau}{1 + \sec\rho\cos\tau}, \quad (18)$$

where n_1 is the first component of $d-2$-dimensional unit vector. This metric describes a metric in the presence of a massless particle which moves along the null geodesic $U = 0$ in AdS_d background. In the Penrose diagram (see Fig. 2,*a*) this looks like as the world line of the particle in the 3-dimensional case. This consideration supports an analogy with the 3-dimensional case, although to be sure a more detailed analysis is needed.

To summarize, in this letter we discussed the application of the mechanism of the black hole production from colliding plane gravitational waves in 4-dimensional space-time to the TeV energy scattering in the $n+4$-dimensional space-time in the presence of a brane. We have shown that the brane could be unstable in the presence of gravitational waves and the black hole can be formed. It was shown that the brane is stable within the many branes version of the RS solution in the 3-dimensional case. We also noted that a bound on transverse momenta of completely absent processes does not essentially depend on the energy of colliding particles.

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