УДК 539.12.01

APPLICATIONS OF DEFORMED FERMION REALIZATIONS OF $sp(4)$

K. D. Sviratcheva, J. P. Draayer,

Louisiana State University, Department of Physics and Astronomy,
Baton Rouge, Louisiana, USA

A. I. Georgieva

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia

A fermion representation of the compact symplectic $sp(4)$ algebra provides a natural description of the pairing interaction in nuclei. In the nondeformed and deformed cases the reduction chains to different realization of $u(2)$ and $u_q(2)$ are explored for multiple orbits. One of the realizations is associated with the SU(2) group of the valene isospin. The other reductions describe pairing between identical fermions or proton-neutron configuratrions. Microscopic nondeformed and deformed Hamiltonians are expressed in terms of the generators of $sp(4)$ and $sp_q(4)$. In both cases eigenvalues of the Hamiltonian are fit to experimental ground state energies which allows the role of the deformation to be investigated. The q-deformation parameter varies the pairing strength, thereby providing for a nonlinear expansion of the nuclear collective motion.

INTRODUCTION

The pairing problem in nuclear physics was originally investigated [1] as a means for describing binding energies of nuclei and their low-lying vibrational spectra [2]. Common solutions invoked group theory which is a powerful tool for exploring symmetries: ($SU(2)$ model) [3–5], ($SO(5)$ model) [6], ($SO(8)$ model) [7], (IBM) [8]. The importance of isovector pairing [9] leads naturally to the $SO(5)$ seniority model [10, 11], which introduces a relation between like-particles and proton–neutron (pn) isovector pairing modes. Recently, there has been renewed interest in this problem through studies of exotic nuclei with proton excess or with $N \approx Z$ [12].

Based on a fermion realization of $sp(4)$ (isomorphic to $so(5)$), our aim is to investigate the properties of the pairing interaction by considering the symplectic algebra to be a dynamical symmetry algebra. This yields an isospin breaking phenomenological Hamiltonian written in terms of the group generators. The limiting cases of $sp(4)$ correspond to different reductions to $u(2)$ and reveal the properties of different coupling modes of the isovector pairing interaction. A generalization to multiple shells provides a classification scheme for nuclear

ground states when the valence nucleons occupy more than a single orbit. This introduces shell structure into the theory and allows for an investigation of the dependence of pairing correlations on the dimensionality of the model space.

A q-deformation of the classical algebraic structure is introduced in order to provide for novel, richer and more exact reproduction of pairing features in nuclei, including nonlinearity of the interactions and the respective changes of the pairing strength parameters. An analysis of the results, obtained by the fitting of the model parameters to experimental data in both the deformed and nondeformed cases, provides for a reliable prediction of the binding energies of nuclei, some of which are unknown and of contemporary interest.

1. GENERALIZED FERMION REPRESENTATION OF $sp(4)$ ALGEBRA AND ITS DEFORMATION

The $sp(4)$ algebra is realized in terms of fermion creation (annihilation) operators $c^\dagger_{j,m,\sigma}$ ($c_{j,m,\sigma}$), $-j \leq m \leq j$, $\sigma = \pm 1$, where these operators create (annihilate) a particle of type σ in a state of total angular momentum $j = (2k+1)/2$, $k = 0, 1, 2, \ldots,$ with projection m on the z axis [13]. They satisfy Fermi anti-commutation relations $\{c_{j',m',\sigma'}, c^\dagger_{j,m,\sigma}\} = \delta_{j',j}\delta_{m',m}\delta_{\sigma',\sigma}$, $\{c^\dagger_{j',m',\sigma'}, c^\dagger_{j,m,\sigma}\} = \{c_{j',m',\sigma'}, c_{j,m,\sigma}\} = 0$, and the Hermitian conjugation relation: $(c^\dagger_{j,m,\sigma})^* = c_{j,m,\sigma}$. The usual single-orbit fermion realization of the $sp(4)$ algebra can be extended very easily to a multiple-orbit theory [5]. For a given σ, the dimension of the fermion space is $2\Omega = \sum_j 2\Omega_j = \sum_j (2j + 1)$, with the convention that $\sum_j$ is a sum over the number of orbitals p. A pair of fermions can be created or annihilated by the operators

$$A_{\sigma,\sigma'} = \xi \sum_{j,m} (-)^{j-m} c^\dagger_{j,m,\sigma} c^\dagger_{j,-m,\sigma'}, \ B_{\sigma,\sigma'} = \xi \sum_{j,m} (-)^{j-m} c_{j,-m,\sigma} c_{j,m,\sigma'},$$

(1)

where $\xi = 1/\sqrt{2\Omega(1 + \delta_{\sigma,\sigma'})}$ and $A_{\sigma,\sigma'} = (B_{\sigma,\sigma'})^*$, $A_{\sigma,\sigma'} = A_{\sigma',\sigma}$, $B_{\sigma,\sigma'} = B_{\sigma',\sigma}$. The number preserving Weyl generators are defined as

$$D_{\sigma,\sigma'} = \frac{1}{\sqrt{2\Omega}} \sum_{j,m} c^\dagger_{j,m,\sigma} c_{j,m,\sigma'}.$$

(2)

Based on different interpretations of the quantum number σ, the generators (1) and (2) have different physical meaning. When σ distinguishes between protons ($\sigma = 1$) and neutrons ($\sigma = -1$), the Cartan generators of the $Sp(4)$ group $N_\sigma \equiv \sqrt{2\Omega} D_{\sigma,\sigma}$ enter as the number of the valence protons and valence neutrons, respectively. The valence fermions are created (annihilated) by $c^\dagger_{j,m,\sigma}$ ($c_{j,m,\sigma}$)

above the pairing vacuum state of the nuclear system, which is a doubly-magic core.

In the nondeformed limit, the generators of the $Sp(4)$ group $\tau_{\pm 1} \equiv D_{\pm 1, \mp 1}$, $\tau_0 = (N_1 - N_{-1})/2$ and $N = N_{+1} + N_{-1}$ close on the $u^\tau(2)$ subalgebra. In this case the creation (annihilation) operator $A_{(\sigma + \sigma')/2} \equiv A_{\sigma, \sigma'}$ $(B_{(\sigma + \sigma')/2} \equiv B_{\sigma, \sigma'})$, $\sigma, \sigma' = \pm 1$, is a tensor of the first rank $\{A\}_{0, \pm 1}$ $(\{B\}_{0, \pm 1})$ with respect to the $SU^\tau(2)$ subgroup. The operators $A_{0, \pm 1}$ $(B_{0, \pm 1})$ create (annihilate) a pair of fermions coupled to total angular momentum and parity $J^\pi = 0^+$ and constitute boson-like objects. Other two realizations of the $u(2)$ subalgebra include the following generators: A_0, B_0, $D_0 \equiv N/2 - \Omega$ and τ_0 of $u^0(2)$; $A_{\pm 1}$, $B_{\pm 1}$, $D_{\pm 1} \equiv (N_{\pm 1}/2) - (\Omega/2)$ and $N_{\mp 1}$ of $u^\pm(2)$.

The deformation of the $sp_q(4)$ algebra can be introduced in terms of q-deformed creation (annihilation) operators $\alpha^\dagger_{j,m,\sigma}$ $(\alpha_{j,m,\sigma})$, $(\alpha^\dagger_{j,m,\sigma})^* = \alpha_{j,m,\sigma}$, assuming that $\alpha^{(\dagger)}_{j,m,\sigma} \to c^{(\dagger)}_{j,m,\sigma}$ in the limit $q \to 1$. The deformed single-particle operators are defined through their anticommutation relation in the form $\{\alpha_{j,m,\sigma}, \alpha^\dagger_{j,m',\sigma}\}_{q^{\pm 1}} = q^{\pm N_\sigma / 2\Omega} \delta_{m,m'}$, and through the action of the «classical» operators of the number of fermions of each kind, $[N_\sigma, \alpha^\dagger_{j,m,\sigma'}] = \delta_{\sigma,\sigma'} \alpha^\dagger_{j,m,\sigma'}$ and $[N_\sigma, \alpha_{j,m,\sigma'}] = -\delta_{\sigma,\sigma'} \alpha_{j,m,\sigma'}$ $(\sigma, \sigma' = \pm 1)$. Two of the generators of the respective deformed $Sp_q(4)$ group remain nondeformed, $N_{\pm 1}$. The rest of the generators are given in terms of the q-deformed fermion operators:

$$F_{\sigma, \sigma'} = \xi \sum_{j,m} (-)^{j-m} \alpha^\dagger_{j,m,\sigma} \alpha^\dagger_{j,-m,\sigma'}, \ \ G_{\sigma, \sigma'} = \xi \sum_{j,m} (-)^{j-m} \alpha_{j,-m,\sigma} \alpha_{j,m,\sigma'},$$

(3)

$$E_{1,-1} = \frac{1}{\sqrt{2\Omega}} \sum_{j,m} \alpha^\dagger_{j,m,1} \alpha_{j,m,-1}, \quad E_{-1,1} = \frac{1}{\sqrt{2\Omega}} \sum_{j,m} \alpha^\dagger_{j,m,-1} \alpha_{j,m,1}, \quad (4)$$

where $F_{\sigma, \sigma'} = F_{\sigma', \sigma} = (G_{\sigma, \sigma'})^*$. In the deformed case, the three different realizations of $sp_q(4)$ are given in terms of the following deformed generators: $T_\pm \equiv E_{\pm 1, \mp 1}$, $T_0 \equiv \tau_0$ and N of $u^\tau_q(2)$; F_0, G_0, $K_0 \equiv (N/2) - \Omega$ and T_0 of $u^0_q(2)$; $F_{\pm 1}, G_{\pm 1}, K_{\pm 1} = (N_{\pm 1}/2) - (\Omega/2)$ and $N_{\mp 1}$ of $u^\pm_q(2)$, where the deformed creation (annihilation) operators $F_{(\sigma + \sigma')/2} \equiv F_{\sigma, \sigma'}$ $(G_{(\sigma + \sigma')/2} \equiv G_{\sigma, \sigma'})$, $\sigma, \sigma' = \pm 1$, are components of a vector $F_{0, \pm 1}$ $(G_{0, \pm 1})$ with respect to the $SU^\tau_q(2)$ subgroup. In the first realization of the unitary subalgebra, $su^\tau(2)$, the generators τ are associated with the isospin of the valence particles. The other three limits describe pairing between particles of different types $(SU^0(2): pn)$ and coupling between identical particles $(SU^\pm(2): pp$ or $nn)$. The commutation relations and all consecutive formulae remain the same as derived in [13], but with $\Omega_j \to \Omega$.

The nondeformed (deformed) fermion operators act in finite space, with a vacuum $|0\rangle$ $(\langle 0|0 \rangle = 1)$ defined by $c_{j,m,\sigma}|0\rangle = 0$ $(\alpha_{j,m,\sigma}|0\rangle = 0)$ for each shell j. The q-deformed states in general differ from the classical ones but coincide

in the limit $q \to 1$. The nondeformed and q-deformed basis sets are obtained by orthonormalization of vectors, which can be expressed in terms of the «boson-like creation operators», acting on the vacuum state [14]:

$$|\Omega; n_1, n_0, n_{-1}) = (A_1)^{n_1} (A_0)^{n_0} (A_{-1})^{n_{-1}} |0\rangle, \qquad (5)$$

$$|\Omega; n_1, n_0, n_{-1})_q = (F_1)^{n_1} (F_0)^{n_0} (F_{-1})^{n_{-1}} |0\rangle, \qquad (6)$$

where n_1, n_0, n_{-1} are the total number of pairs of each kind, pp, pn, nn, respectively. A symmetric representation of $Sp(4)$ is labelled by one quantum number Ω, which is related to the eigenvalue of the second-order Casimir operator of the $Sp(4)$ group [15] and which is dropped from the labelling of the states within a representation. The states (5) and (6) are eigenvectors of the number operators $N_{\pm 1}$, N and τ_0 with eigenvalues $N_\pm = 2n_{\pm 1} + n_0$, $n = 2(n_1 + n_{-1} + n_0)$ and $i = n_1 - n_{-1}$, respectively.

In Table 1, each of the $u_q(2)$ realizations is given along with the invariant operator of first order, the eigenvalues of the second-order Casimir invariant of $su_q(2)$, and the corresponding eigenvectors (basis states) where by definition $[X]_k = (q^{kX} - q^{-kX})/(q^k - q^{-k})$ and $\rho_\pm = (q^{\pm 1} + q^{\pm (1/2\Omega)})/2$. In each limit, $\nu_0 = n_1 + n_{-1}$ and $\nu_1 = n_0$ are the respective seniority quantum numbers that count the number of remaining pairs that can be formed after coupling the fermions in the primary pairing mode. The eigenvalues of the Casimir invariant of $SU_q^+(2)$ and $SU_q^-(2)$ differ within the coefficient $\rho_\pm$, which sets the difference between proton pairs and neutron pairs. All the q formulae revert back to the «classical» ones in the limit when $q \to 1$.

Table 1. **Reduction limits of $Sp_q(4)$**

$u_q^\mu(2)$	$\mathbf{C}_1(U_q^\mu(2))$	Eigenvalues of $\mathbf{C}_2(SU_q^\mu(2))$	Basis states
$u_q^\tau(2)$	N	$2\Omega \left[\dfrac{1}{2\Omega}\right] [T]_{\frac{1}{2\Omega}} [T+1]_{\frac{1}{2\Omega}}$	$\|n, \tau, i\rangle$
$u_q^0(2)$	T_0	$2\Omega \left[\dfrac{1}{2\Omega}\right] \left[\dfrac{2\Omega - 2\nu_0}{2}\right]_{\frac{1}{2\Omega}} \left[\left(\dfrac{2\Omega - 2\nu_0}{2} + 1\right)\right]_{\frac{1}{2\Omega}}$	$\|n_1, n_0, 0)$ $\|0, n_0, n_{-1})$
$u_q^\pm(2)$	$N_\mp$	$\rho_\pm\, \Omega \left[\dfrac{1}{\Omega}\right] \left[\dfrac{\Omega - \nu_1}{2}\right]_{\frac{1}{\Omega}} \left[\dfrac{\Omega - \nu_1}{2} + 1\right]_{\frac{1}{\Omega}}$	$\|n_1, 0, n_{-1})$ $\|n_1, 1, n_{-1})$

2. APPLICATIONS TO NUCLEAR STRUCTURES

In our approach, we use a phenomenological Hamiltonian of a system with symplectic dynamical symmetry, expressed through the generators of the $Sp_{(q)}(4)$

group [14],

$$H = -\epsilon N - GA_0 B_0 - F(A_{+1}B_{+1} + A_{-1}B_{-1}) -$$
$$- C\frac{N(N-1)}{2} - D\left(\tau_0^2 - \frac{N}{4}\right), \quad (7)$$

in the nondeformed case, and in the q-deformed case

$$H = -\epsilon N - G_q F_0 G_0 - F_q(F_{+1}G_{+1} + F_{-1}G_{-1}) -$$
$$- C_q\frac{N(N-1)}{2} - D_q\left(\tau_0^2 - \frac{N}{4}\right), \quad (8)$$

where ϵ is a Fermi energy, and $G_{(q)}$, $F_{(q)}$, $C_{(q)}$ and $D_{(q)}$ are constant interaction strength parameters ($G_{(q)} \geq 0$, $F_{(q)} \geq 0$). The G and F terms account for pairing between nonidentical and identical particles, respectively, and give the ground state pairing energy $\varepsilon_{\text{pair}}$. The last two terms ($C$, D) arise naturally from the microscopic picture of the interaction Hamiltonian [4] and can be written through the other two diagonal operators N_{+1} and N_{-1}. In this way, the energy operator (7) contains the quantity $N_+ N_-$, which is connected to the deformation of the nuclei [16]. This means the Hamiltonian is applicable in the whole Ω space including regions of deformed nuclei. As a consequence of the Pauli principle, the particle-hole description enters naturally in the pairing terms only and gives the decrease in energy with respect to a ground state with no pairing [17].

An important feature of the phenomenological Hamiltonian (7) is that it breaks the isospin symmetry ($F \neq G$), which allows distinct isospin values to contribute to the ground state of a nucleus with a given n and i in unique ways [18]. This is different from other applications of nondeformed and deformed $sp(4)$ or $o(5)$ algebras with isospin invariant Hamiltonians [11, 12, 19].

The eigenvalue of the deformed pairing Hamiltonian can be expanded in orders of $\varkappa$ ($q = \exp(\varkappa)$) in each limit:

$$\varepsilon_{\text{pair}}^q\Big|_{SU^0(2)} = -G_q\varepsilon_{pn}\left\{1 + \frac{\varkappa^2}{24\Omega^2}\left\{(n_0^2 - 4\Omega^2 - 1) + \frac{4\Omega^2}{n_0^2}\varepsilon_{pn}^2\right\} + O(\varkappa^4)\right\}, \quad (9)$$

$$\varepsilon_{\text{pair}}^q\Big|_{SU^+(2)\otimes SU^-(2)} = -F_q\Bigg\{\varepsilon_{pp}\Bigg(1 + \varkappa\frac{1+2\Omega}{4\Omega} +$$
$$+ \frac{\varkappa^2}{6\Omega^2}\left\{\left(n_1^2 - \frac{\Omega^2}{2} - \frac{5}{8}\right) + \frac{\Omega^2}{n_1^2}\varepsilon_{pp}^2\right\}\Bigg) + \varepsilon_{nn}\Bigg(1 - \varkappa\frac{1+2\Omega}{4\Omega} +$$
$$+ \frac{\varkappa^2}{6\Omega^2}\left\{\left(n_{-1}^2 - \frac{\Omega^2}{2} - \frac{5}{8}\right) + \frac{\Omega^2}{n_{-1}^2}\varepsilon_{nn}^2\right\}\Bigg) + O(\varkappa^3)\Bigg\}, \quad (10)$$

where the nondeformed energies $\varepsilon_{pn} = \dfrac{G}{\Omega} n_0 \dfrac{2\Omega - N + n_0 + 1}{2}$, $\varepsilon_{pp(nn)} = \dfrac{F}{\Omega} n_{\pm 1}$ $(\Omega - n_{\pm 1} - n_0 + 1)$ are the zeroth order approximation of the corresponding deformed pairing energies. While the proton–neutron interaction is even with respect to the deformation parameter $\varkappa$, the identical particle pairing includes also odd terms as a consequence from the introduced coefficient $\rho_\pm$.

Eigenvalue of Hamiltonians (7) and (8) gives a phenomenological formula for estimating the ground state energy of nuclei in a (N, τ_0) classification scheme. Its positive value is defined to be the binding energy of the system, $|BE|$, which can be fit to the measured binding energy. The latter needs to be corrected for the Coulomb energy since it is not accounted for by the model, $|H| = |BE_{\exp}| + V_{\text{Coul}}$, where we use the Coulomb potential from [20].

In both the nondeformed and deformed cases, the ground state energies (eigenvalues of (7) and (8)) are fit to the experimental binding energies [21] of several groups of nuclei: (I) $\Omega = 2$ $(1d_{3/2})$ with a $^{32}_{16}$S core; (II) $\Omega = 4$ $(1f_{7/2})$ with

Fig. 1. Coulomb corrected binding energy (in MeV) versus τ_0 for isotopes of nuclei with $Z = 20$ to $Z = 28$ in the $1f_{7/2}$ $(\Omega = 4)$ level

a $^{40}_{20}$Ca core; (III) $\Omega = 11$ $(2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2})$ with a $^{56}_{28}$Ni core; and (IV) $\Omega = 16$ $(1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 1h_{11/2}, 3s_{1/2})$ with a $^{100}_{50}$Sn core. For the nondeformed case, the fitting procedure parameters and statistics are shown in Table 2. In these cases the Coulomb corrected experimental values were used:

$$E_B^{\exp}(N_+, N_-) = |BE_{\exp}(N_+, N_-)| - |BE_{\exp}|_{\text{core}} + V_{\text{Coul}}(N_+, N_-),$$

where $|BE_{\exp}|_{\text{core}}$ is the binding energy of the core. The single-particle energies were considered fitting parameters in all cases but Sn, for which the values were taken from theoretical calculations [22].

In Table 2, $S \equiv \left(|E_B^{\text{th}}| - |E_B^{\exp}|\right)^2$ is the residual sum of squares and the statistical factor $\chi \equiv \sqrt{S/(N_d - n_p)}$ defines the goodness of the fit, where N_d is the number of data in the statistics and n_p is the number of fitting parameters. In all cases there is a good agreement with the experiment (small χ), as shown in Table 2 as well as in Fig. 1 for the second case.

In general, the pairing strength decreases as the nuclear mass increases. This fact is well known, but only for the identical particle case [2]. The theoretical model with a $sp(4)$ dynamical symmetry algebra reproduces the properties of identical nucleon pairing $(\varepsilon_{pp} + \varepsilon_{nn})$ [3–5, 17] which has its maximum at half
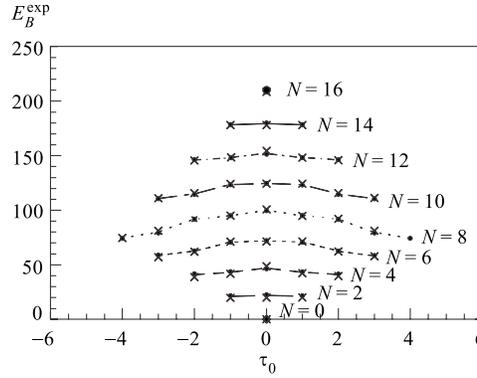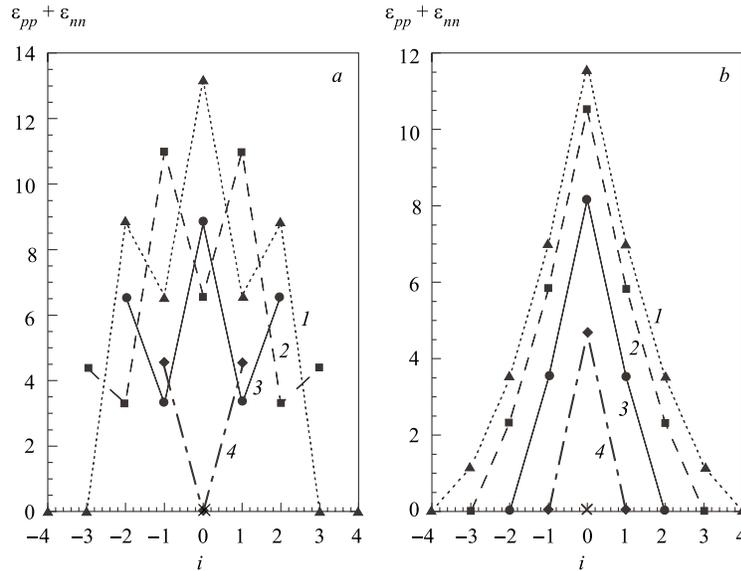
Fig. 2. Identical and nonidentical particle energies (in MeV) versus τ_0 for the isotopes of nuclei with $Z = 20$ to $Z = 28$ in $1f_{7/2}$: *1*, ▲ — $N = 8$; *2*, ■ — $N = \{6, 10\}$; *3*, ● — $N = \{4, 12\}$; *4*, ♦ — $N = \{2, 14\}$; × — $N = \{0, 16\}$

shell (Fig. 2, *a*). The proton–neutron coupling (ε_{pn}) (Fig. 2, *b*) has its maximum when $N_+ = N_-$ ($\tau_0 = 0$), which is consistent with α-clustering theories [23] and the charge independence in the region of light nuclei when protons and neutrons fill the same shells [9, 24]. In that region, the nonidentical particle energy is bigger than the identical particle energy for odd-odd nuclei and are of the same order as for even–even nuclei (Fig. 3), which is consistent with good isospin symmetry in even–even nuclei.

For multiple orbits the pairing correlations and coupling modes depend on the dimensionality of the space. We investigated a possible sub-shell closure

Table 2. **Parameters and statistics,** $q = 1$

	(I)	(II)	(III)	(IV)
G/Ω, MeV	2.13	1.17	0.54	0.52
F/Ω, MeV	2.10	1.10	0.45	0.36
C, MeV	1.42	0.64	0.23	0.19
D, MeV	−0.02	−0.16	−0.18	−0.09
ϵ, MeV	7.19	8.25	8.84	9.41
S, MeV^2	0.03	34.53	511.5	809.5
χ, MeV	0.08	0.99	1.85	2.04

for nuclei above the ^{56}Ni core. Closing the shell at ^{80}Zr (Fig. 4, *a*, *c*) or ^{100}Sn (Fig. 4, *b*, *d*) changes not only the fitting parameters but also the mode and magnitude of coupling. When $N_+ = N_-$, the expected behavior is obtained for $\Omega = 11$, which is similar to the behavior of the nuclei above the ^{40}Ca core (Fig. 3). Also, there is a significant difference in the dominant mode of the isotopes (an example is $N_+ = 8$, Fig. 4): in the region around $N_+ = N_-$ with $\Omega = 11$ the *pn* pairing dominates, and only further away from that region the like-particle mode is dominant. In comparison with the single level cases, the closing at ^{100}Sn can be regarded as the correct behavior (Fig. 4, *b*, *d*).

The fitting procedure for (8) was performed for all cases, (I) to (IV), with *q*-deformation included (Table 3). The fit for nuclei in multiple orbits, (III)* and (IV)*, includes only isotopes with
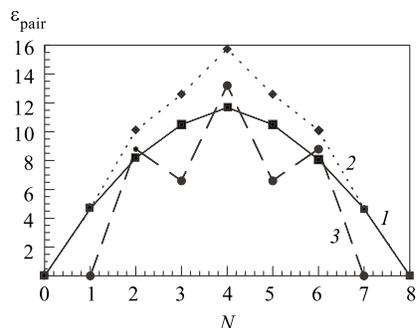

Fig. 3. Nonidentical and identical particles energies (in MeV) and total pairing energy (in MeV) vs. N_- when $N_+ = N_-$ for the nuclei in $1f_{7/2}$: *1*, ■ — ε_{pn}; *2*, ♦ — $\varepsilon_{\text{pair}}$; *3*, ● — $\varepsilon_{pp} + \varepsilon_{nn}$
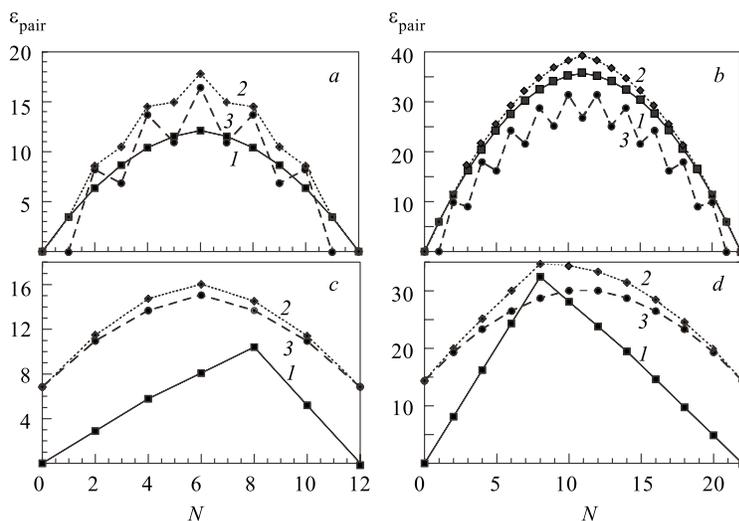

Fig. 4. Nonidentical and identical particles energies (in MeV) and total pairing energy (in MeV) vs. N_- when $N_+ = N_-$ (*a*, *b*) and $N_+ = 8$ (*c*, *d*) for nuclei above ^{56}Ni: *a*, *c*) $\Omega = 6$; *b*, *d*) $\Omega = 11$. *1*, ■ — ε_{pn}; *2*, ♦ — $\varepsilon_{\text{pair}}$; *3*, ● — $\varepsilon_{pp} + \varepsilon_{nn}$

$N_+ = \{0, 1\}$ and isotones with $N_- = \{2\Omega - 1, 2\Omega\}$. The corresponding nondeformed limit is labeled by $q = 1$ and the deformation parameter is not a part of the fitting procedure in that case. The single-particle energies were kept fixed and their values taken from the nondeformed fit, or from theoretical calculations [22].
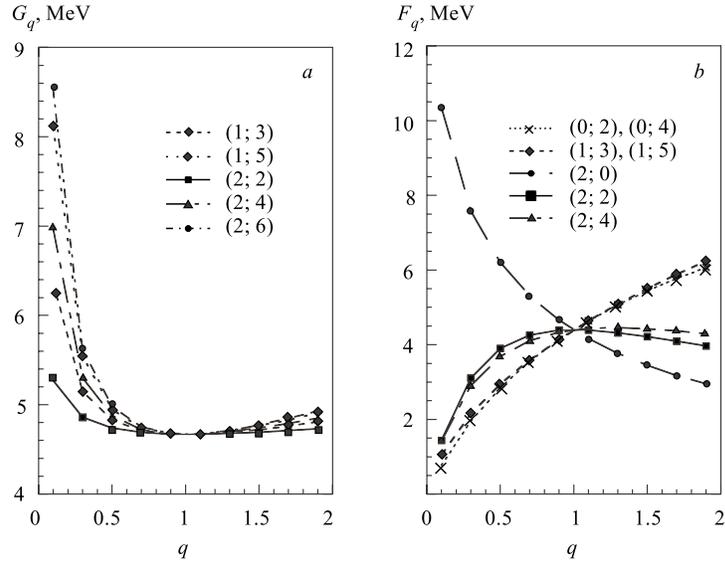

Fig. 5. pn pairing strength G_q (in MeV) (a) and identical particle pairing strength F_q (in MeV) (b) for (N_+, N_-) nuclei vs. q

The fits with and without a deformation can be compared by using the residual sum of squares (S) which is always smaller in the deformed case. Although the deformation does not change the parameters within the uncertainties in the case of a single level, for multiple orbits the role of the deformation parameter is significant when not all nuclei of a major shell are used in the fitting procedure. This turns out to be very important for fitting nuclei in the region where the binding energy of most of the proton-rich isotopes are not yet measured and therefore cannot be included in the fit. For these cases, the deformation varies the pairing strength parameters. This property of the q deformation is consistent with the change of the pairing strength with respect to q (Fig. 5). The change of the pn pairing strength G_q with changes in the deformation parameter is relatively small for q values around $q = 1$ but it increases as $q \neq 1$ (Fig. 5 and Table 3). The parameter F_q decreases monotonically with q only for nuclei without nn coupling and it increases for nuclei with a primary nn coupling. F_q is always smaller than

F ($q = 1$) when both the pp and nn coupling modes exist. Even though both $SU_q^{\pm}(2)$ groups are complementary, the different behavior of the multiplication constants $\rho_{\pm}$ is responsible for different changes of F_q in various isotopes, in contrast with the like-particle $SU_q(2)$ seniority model of [25].

Table 3. **Parameters and statistics. The $q = 1$ case for (I) and (II) is the same as in Table 2. Fits for (III)* and (IV)* include only isotopes with $N_+ = \{0,\ 1\}$ and isotones with $N_- = \{2\Omega - 1, 2\Omega\}$**

	(I)	(II)	(III)*		(IV)*	
q	0.99	1.04	1	1.62	1	1.60
G/Ω, MeV	2.13	1.17	0.41	0.48	0.51	0.59
F/Ω, MeV	2.10	1.10	0.42	0.43	0.30	0.37
C, MeV	1.42	0.64	0.23	0.23	0.19	0.18
D, MeV	–0.02	–0.16	–0.17	–0.17	0.09	–0.09
ϵ, MeV	7.19	8.25	8.84	8.84	9.41	9.41
S, MeV^2	0.03	34.25	294.2	90.59	226.47	93.55
χ, MeV	0.08	0.99	2.59	1.45	2.15	1.60

The fitting procedure not only estimates the magnitude of the pairing strength and describes the type of the dominant coupling mode but it also can be used to predict binding energies of nuclei that have not been measured. From the fit for the case of $1f_{7/2}$, the binding energy of the proton-rich nucleus of ^{48}Ni is estimated to be 347.98 MeV. A much more interesting region includes the nuclei above the core of ^{56}Ni. The neutron-rich isotopes are used in the fit in order to predict the ground state energy $E_B^{Sp(4)}$ of the nuclei on the proton-rich side. Several of them (for which data was available) are compared to [26], with the percent difference 8.78 % and the tendency the theoretically predicted binding energies $E_B^{Sp(4)}$ to be smaller than the semiempirical estimates [26].

CONCLUSION

A deformation of the $sp(4)$ algebra was obtained. Deformed subalgebras of $sp(4)$ were identified and the important reduction chains $Sp_q(4) \supset U(1) \otimes SU_q(2)$ constructed. A phenomenological Hamiltonian was written in terms of the generators of $Sp(4)$ and used to describe pairing correlations. The theory was tested by fitting calculated energies to experimental binding energies for single j level as well as for multiple orbits. In general, the fitting procedure yielded results that were in good agreement with the experiment. The results did not call for isospin invariance, which could be the case if the pairing parameters F and G turn out to be equal.

The theoretical model with $sp(4)$ dynamical symmetry algebra and its q-deformed version was used to investigate the properties of the pairing interaction. The proton–neutron interaction was found to be bigger than the identical particle interaction in odd–odd light nuclei and both interactions were found to be of the same order of magnitude in even–even nuclei. The dominant coupling mode and its strength were found to depend on the dimension of the occupied valence space.

The results show that the q-deformed case gives the best overall fit. It requires an increase in the coupling strength of the proton–neutron pairs. When $q > 1$, the neutron (proton) pairs are more strongly (weakly) bound and vice versa for $q < 1$.

The binding energy of nuclei in the proton-rich region were predicted using a simple microscopic model based on a symplectic symmetry. In doing this we were able to suggest a reliable form for the pairing interaction.

REFERENCES

1. *Racah G.* // Phys. Rev. 1942. V. 62. P. 438; 1943. V. 63. P. 367;
   Bohr A., Mottelson B. R., Pines D. // Phys. Rev. 1958. V. 110. P. 936;
   Belyaev S. T. // Mat. Fys. Medd. 1959. V. 31. P. 11.

2. *Lane A. M.* Nuclear Theory. W. A. Benjamin, Inc., 1964.

3. *Kerman A. K., Lawson R. D., Macfarlane M. H.* // Phys. Rev. 1961. V. 124. P. 162.

4. *Kerman A. K.* // Ann. Phys. 1961. V. 12. P. 300.

5. *Talmi I.* Simple Models of Complex Nuclei: The Shell Model and Interacting Boson Model. Harwood Academic Publishers GmbH, 1993.

6. *Parikh J. C.* // Nucl. Phys. 1965. V. 63. P. 214;
   Dussel G. G. et al. // Nucl. Phys. A. 1970. V. 153. P. 469.

7. *Pang S. C.* // Nucl. Phys. A. 1969. V. 128. P. 497;
   Engel J. et al. // Phys. Rev. C. 1997. V. 55. P. 1781.

8. *Van Isacker P., Warner D. D.* // Phys. Rev. Lett. 1997. V. 78. P. 3266.

9. *Macchiaveli A. O. et al.* // Phys. Lett. B. 2000. V. 480. P. 1.

10. *Ginocchio J. N.* // Nucl. Phys. 1965. V. 74. P. 321.

11. *Hecht K. T.* // Ibid. V. 63. P. 177; Phys. Rev. B. 1965. V. 139. P. 794; Nucl. Phys. A. 1967. V. 102. P. 11.

12. *Engel J., Langanke K., Vogel P.* // Phys. Lett. B. 1996. V. 389. P. 211.

13. *Sviratcheva K. D.* // J. Phys. A: Math. Gen. 2001. V. 34. P. 8365.

14. *Klein A., Marshalek E.* // Rev. Mod. Phys. 1991. V. 63. P. 375.

15. *Flowers B. H.* // Proc. Roy. Soc. A. (London). 1952. V. 212. P. 248.

16. *Casten R. F.* // Nucl. Phys. A. 1985. V. 443. P. 1.

17. *Heyde K. L. G.* The Nuclear Shell Model, Springer Series in Nuclear and Particle Physics. 1990; *Greiner W., Maruhn J. A.* Nuclear Models. Berlin; Heidelberg: Springer-Verlag, 1996.

18. *Civitarese O., Reboiro M., Vogel P.* // Phys. Rev. C. 1997. V. 56. P. 1840.

19. *Szpikowski S., Berej W., Prochniak L.* Symmetries in Science X. Plenum Press, 1997.

20. *Retamosa J. et al.* // Phys. Rev. C. 1997. V. 55. P. 1266.

21. *Audi G., Wapstra A. H.* // Nucl. Phys. A. 1995. V. 595. P. 409.

22. *Zhao Y. M. et al.* // Phys. Rev. C. 2000. V. 62. P. 014315.

23. *Hasegawa M., Kaneko K.* // Ibid. V. 61. P. 037306.

24. *Vogel P.* // Nucl. Phys. A. 2000. V. 662. P. 148.

25. *Shelly Sharma S., Sharma N. K.* // Phys. Rev. C. 2000. V. 62. P. 034314.

26. *Möller P., Nix J. R., Kratz K.-L.* Nuclear Properties.