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NEW INSIGHTS IN PARTICLE DYNAMICS FROM GROUP COHOMOLOGY*

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The dynamics of a particle, moving in background electromagnetic and gravitational fields, is revisited from a Lie group cohomological perspective. Physical constants characterising the particle appear as central extension parameters of a group which is obtained from a previously extended kinematical group (Poincaré or Galilei) by making local some subgroup. The corresponding dynamics is generated by a vector field inside the kernel of a presymplectic form, which is derived from the canonical left-invariant one-form on the extended group. The nonrelativistic (Newtonian) limit is derived from the geodesic motion via an Inönü–Wigner contraction. A deeper analysis of the cohomological structure reveals the possibility of a new force associated with a nontrivial mixing of gravity and electromagnetism leading to testable predictions, such as a mass difference between charged particles and antiparticles.

1. GENERAL SETTING

The spirit of the work, we are presenting in this talk, is that of clarifying the underlying algebraic structure behind the dynamics of a particle, moving inside a background field. We will see how the constants characterising the properties of the particle and its couplings can be understood in terms of the parameters associated with the central extensions of certain groups, thus bringing into scene group-cohomological concepts.

In order to motivate the role of central extensions, let us recall some basic and well-known facts with the help of an example which probably represents the simplest physical system one can imagine: the free particle with Galilei symmetry. In order to define the system we make use of the Poincaré–Cartan form with support on what we can call *evolution* space (obtained from phase

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space by adding time, (x, p, t)):

$$\Theta_{\mathrm{PC}} = pdx - \frac{p^2}{2m}dt.$$

A realization of the Galilei group on evolution space is the following:

$$(*) \qquad \begin{array}{ll} x' = x + A + Vt, & X_A = \dfrac{\partial}{\partial x}, \\ t' = t + B, & X_B = \dfrac{\partial}{\partial t}, \\ p' = p + mV, & X_V = m\dfrac{\partial}{\partial p} + t\dfrac{\partial}{\partial x}. \end{array}$$

However when we check the invariance of the Poincaré–Cartan form under the Galilei group, for instance from the infinitesimal point of view, we realize that its variation under the action of boosts is not zero but, rather, a total differential:

$$L_{X_V}\Theta_{\mathrm{PC}} = d(mx) \neq 0,$$

thus leading to the idea of semi-invariance. Of course, this is not a problem at the classic level, since the classic equations of motion are not sensitive to a variation of the Lagrangian by a total derivative (the Lagrangian is closely related to the Poincaré–Cartan form, in such a way that the former can be obtained by evaluating the latter on the trajectories of the particle). Despite it is not an unavoidable need, let us raise to the level of a postulate the claim for strict invariance and let us consider what consequences we can derive from this assumption.

In order to implement such a strict invariance, let us extend the evolution space with a new variable $\zeta = \mathrm{e}^{i\phi}$ which transforms under the Galilei group in such a way that the variation of its total differential inside a modified one-form, $\Theta \equiv \Theta_{\mathrm{PC}} + d\phi$, cancels the term $d(mx)$. That is,

$$d\phi' = d\phi - d(mx).$$

The finite action of the Galilei group on this new variable is:

$$\zeta' = \zeta \exp\left(-i[(1/2)mV^2 t + mVx + \varphi]\right),$$

(where φ is a new group parameter needed for consistency of the group law) which together with the action on the rest of the variables of the extended evolution space $(*)$, allow us to compute the infinitesimal generators of the symmetry, whose commutators are the following:

$$[\tilde{X}_B, \tilde{X}_A] = 0, \;\; [\tilde{X}_B, \tilde{X}_V] = \tilde{X}_A, \;\; [\tilde{X}_A, \tilde{X}_V] = m\tilde{X}_\varphi.$$

We notice the appearance of a central term in the last commutator. The claim for strict invariance thus leads to the centrally extended Galilei group.

The crucial feature about the requirement of a central extension to achieve strict invariance is that it is not linked to a particular realization of the group, but it is a consequence of its intrinsic algebraic structure, in fact, its group-cohomology. This suggests to consider the group itself as the starting point in the definition of the dynamics of a physical system.

Basically this is the aim of the so-called Group Approach to Quantization (GAQ) (see [1] and references therein) which tries to derive the dynamics directly in a quantum setting by starting from the symmetry of the corresponding physical system, so that group-cohomology plays a central role in the process. Even though the main stress of the approach leans on its quantum aspects, it also has nontrivial implications at the (semi-)classic level, which is the one we are going to emphasize here.

The extended phase space and the Poincaré–Cartan form are generalized by objects that can be recovered from the centrally extended symmetry group. The latter has the structure of a principal fibre bundle $\tilde{G}$ with base the nonextended group G and fibre the $U(1)$ group of phase invariance of Quantum Mechanics. The relevant cohomology in the construction is that of G.

The object generalizing the Poincaré–Cartan form is that component of the left-invariant canonical one-form on the group which is dual to the vertical (or central) vector field, $\Theta = \theta^{L^{(\zeta)}}$. By its own construction, this Θ generalizing $\Theta_{\rm PC} + d\phi$ (to be called quantization one-form) is invariant under the left action of the extended group (meanwhile $\Theta_{\rm PC}$ is only semi-invariant under $\tilde{G}/U(1)$).

For the sake of completeness, let us briefly present the fundamentals of GAQ. The main goal is to construct a unitary and irreducible representation of the basic symmetry in such a way that the generators become hermitian operators. It relies on the rather basic observation on the possibility of constructing two different actions for the group by making use of the group law:

$$g'' = g' * g = L_{g'}g = R_g g'.$$

This two actions do commute, which at the infinitesimal level is expressed in the trivial commutators,

$$[\tilde{X}^L_a, \tilde{X}^R_b] = 0 \; \forall a, b.$$

We can choose, for instance, the right-invariant vector fields to represent the operators corresponding to the infinitesimal generators of the group ($\hat{a} \approx \tilde{X}^R_a$), acting on the complex $U(1)$-functions defined over the group. This representation, usually named prequantization, is highly reducible as can be directly seen from the fact that there is a whole set of nontrivial vector fields commuting with the representation of the group (the generators of the other action indeed). A way of reducing the representation would be achieved by imposing a trivial action of these left-invariant vector fields, but this is not allowed due to the presence of central terms. Thus, we are forced to look for a maximal set which can be trivialized in a consistent way. This generalizes that in the language of geometrical quantization is known as a polarization. Once one imposes this polarization as a set of conditions on the wave functions ($\tilde{X}^L_a\Psi = 0$), we obtain the reduction leading to the quantum representation.

Regarding classical dynamics, the form Θ can be seen as the potential of a presymplectic form, in such a way that the solution space (i. e., the phase space) is obtained from the group by getting rid of those variables inside the kernel of $d\Theta$. In this way, the vector fields inside this kernel can be seen as generalized equations of motion [2]. In principle, there is a certain ambiguity in choosing the Hamiltonian vector field, but this problem will not arise in the systems we shall consider here.

Let us review the free Galilean particle in order to illustrate this technique. From the realization of the extended Galilei group on the extended phase we presented above, we can derive a suitable group law and use it to compute the right- and left-invariant vector fields (we identify A with x, V with v, B with t and φ with ϕ)

$$
\begin{aligned}
\tilde{X}^L_t &= \frac{\partial}{\partial t} + v\frac{\partial}{\partial x} + \frac{1}{2}mv^2\frac{\partial}{\partial \phi}, & \tilde{X}^R_t &= \frac{\partial}{\partial t}, \\
\tilde{X}^L_x &= \frac{\partial}{\partial x}, & \tilde{X}^R_x &= \frac{\partial}{\partial x} + mv\frac{\partial}{\partial \phi}, \\
\tilde{X}^L_v &= \frac{\partial}{\partial v} + mx\frac{\partial}{\partial \phi}, & \tilde{X}^R_v &= \frac{\partial}{\partial v} + t\frac{\partial}{\partial x} + mtv\frac{\partial}{\partial \phi}, \\
\tilde{X}^L_\phi &= \frac{\partial}{\partial \phi}, & \tilde{X}^R_\phi &= \frac{\partial}{\partial \phi}
\end{aligned}
$$

and the quantization one-form

$$\Theta \equiv \theta^{L^\phi} = -mxdv - \frac{1}{2}mv^2dt + d\phi,$$

where the two first terms correspond to the original Poincaré–Cartan form (the apparent interchange between x and v can be restored by introducing a trivial coboundary into the cocycle of the extended group law).

A polarization suitable to obtain the quantum representation is spanned by $\tilde{X}_x, \tilde{X}_t$. After imposing the $U(1)$-function condition and the polarization we arrive at the Schrödinger equation in momentum space.

2. INTERACTIONS

2.1. Electromagnetism. Up to now we have sketched two alternative ways of considering the role of the symmetry group when dealing with the dynamics of a physical system. The first one makes use of a particular realization of the group on an extended phase space, meanwhile the second one (that of GAQ) emphasizes the singular role of the group, considering it as the departing point in the analysis. Now we are going to switch on interactions and consider the situation from both perspectives, beginning with the case of electromagnetism.

A natural question that one can formulate, when considering the centrally extended Galilei group, is what happens if we turn into local the $U(1)$ part of the symmetry [3]. The Lie algebra is then composed of the former Galilei generators realized on the extended phase space, together with the tensor product of local functions and the central term: $f(x,t) \otimes X_\phi$. But when we check the invariance of the modified Poincaré–Cartan form under the new generators, semi-invariance reappears into scene:

$$L_{f\otimes X_\phi}\Theta = df.$$

We follow here the same strategy as before, that is, we look for new variables extending the phase space and compensating the variation of a newly modified one-form under the symmetry group. Fortunately, in this case there are natural guesses and we are able to find new variables A_0, A_x, transforming in the desired way ($A' = A - df$). The realization of the fields in the newly extended phase space $(x, p, t, \phi, A_0, A_x)$ is

$$X_B = \frac{\partial}{\partial t},\;\; X_A = \frac{\partial}{\partial x},$$

$$X_V = t\frac{\partial}{\partial x} + m\frac{\partial}{\partial p} - mx\frac{\partial}{\partial \phi} + A_x\frac{\partial}{\partial A_0},$$

$$f\otimes X_\phi = -f\frac{\partial}{\partial \phi} - \frac{\partial f}{\partial x}\frac{\partial}{\partial A_x} + \frac{\partial f}{\partial t}\frac{\partial}{\partial A_0}.$$

The new strictly invariant one-form is:

$$\Theta = pdx - \frac{p^2}{2m}dt - A_x dX + A_0 dt + d\phi,$$

from which the Lorentz force felt by the particle can be derived [3].

As an alternative approach, we apply the techniques of GAQ to this problem, which results in a more automatic algorithm and a more general treatment. In fact, if we consider an arbitrary group $\tilde{G}$ whose infinitesimal generators are

$\{X_A\},\ (A = 1, ..., n)$ and an invariant subgroup ($\{X_a\},\ a = 1, ..., m < n$), thus satisfying,

$$[X_A, X_a] = C^b_{Aa} X_b,$$

we can make local this invariant subgroup, obtaining an algebra spanned by

$$\{f^a \otimes X_a, X_A\},$$

and whose new commutators are

$$\begin{aligned}[X_A, f^a \otimes X_a] = f^a \otimes [X_A, X_a] + L_{X_A} f^a \otimes X_a = \\ = f^a \otimes C^b_{Aa} X_b + L_{X_A} f^a \otimes X_a.\end{aligned}$$

One then applies the quantization program, consequently obtaining the quantization one-form Θ in the proccess.

In the case of a particle inside the electromagnetic field, we are dealing with the Galilei group extended by $U(1)(\mathbf{x}, t)$, that is, $\phi = \phi(\mathbf{x}, t)$. In order to parametrize properly the quantization group we make the formal splitting between the rigid and the local part,

$$\begin{aligned}\phi(\mathbf{x}, t) &= \phi(0, 0) + \phi_\mu(\mathbf{x}, t) x^\mu \\ &\equiv \phi + A_\mu(\mathbf{x}, t) x^\mu.\end{aligned}$$

We apply then the previous rule for computing the commutators, thus producing an infinite-dimensional algebra. The recipe of GAQ suggests us to centrally extend this algebra. Even though the algebra is infinite-dimensional, an analysis of the cohomology reveals that the relevant part for the dynamics of a particle inside the background field is the following finite-dimensional subalgebra:

$$\begin{aligned}
&[\tilde{X}^L_t, \tilde{X}^L_x] = 0, \quad [\tilde{X}^L_t, \tilde{X}^L_v] = -\tilde{X}^L_x, \quad [\tilde{X}^L_t, \tilde{X}^L_{A_x}] = 0, \\
&[\tilde{X}^L_t, \tilde{X}^L_{A_0}] = -q\tilde{X}^L_\phi, \quad [\tilde{X}^L_x, \tilde{X}^L_v] = m\tilde{X}^L_\phi, \quad [\tilde{X}^L_x, \tilde{X}^L_{A_x}] = q\tilde{X}^L_\phi, \\
&[\tilde{X}^L_x, \tilde{X}^L_{A_0}] = 0, \quad [\tilde{X}^L_v, \tilde{X}^L_{A_x}] = \tilde{X}^L_{A_0}, \quad [\tilde{X}^L_v, \tilde{X}^L_{A_0}] = 0,
\end{aligned}$$

where, apart from the central term related to the inertial mass, m, a new central parameter, q, appears being eventually interpreted with the aid of motion equations as the charge of the particle.

After the exponentiation of the group, we compute the quantization one-form which turns out to be

$$\Theta = -m\mathbf{x} \cdot d\mathbf{v} - q\mathbf{x} \cdot d\mathbf{A} - \left(\frac{1}{2} m\mathbf{v}^2 + qA_0\right) dt + d\phi,$$

i. e., the Poincaré–Cartan one-form of a particle inside an electromagnetic field plus the differential of the central parameter, $d\phi$.

In order to derive the equations of motion of the particle we have to force the functions A_i to depend on the position of the particle, $A = A(x_{\text{particle}})$. The vector field X in the kernel of Θ, that is, satisfying $i_X d\Theta = 0$ is

$$X = \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} - \frac{q}{m}\left[\left(\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}\right)v^j + \frac{\partial A_0}{\partial x^i} + \frac{\partial A_i}{\partial t}\right]\frac{\partial}{\partial v_i},$$

and its trajectories are governed by the following equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}, \quad m\frac{d\mathbf{v}}{dt} = q\left[\mathbf{v} \wedge (\boldsymbol{\nabla} \wedge \mathbf{A}) - \boldsymbol{\nabla} A_0 - \frac{\partial \mathbf{A}}{\partial t}\right].$$

If we define

$$\boldsymbol{\nabla} \wedge \mathbf{A} \equiv \mathbf{B}, \quad -\boldsymbol{\nabla} A_0 - \frac{\partial \mathbf{A}}{\partial t} \equiv \mathbf{E},$$

we obtain the standard expression for the Lorentz force,

$$m\frac{d\mathbf{v}}{dt} = q[\mathbf{E} + \mathbf{v} \wedge \mathbf{B}].$$

We have studied this electromagnetic example in the Galilean scheme for pedagogical reasons, but we must point out that everything can be reproduced in the relativistic case, starting from the Poincaré group and resulting in the corresponding final expression.

2.2. Electromagnetism and Gravity Mixing. We address now a more involved system and, in order to deal with it, we use the most algorithmic of the techniques we have presented so far, GAQ.

Starting from the (pseudo-)extended Poincaré group* we make local, instead of the central $U(1)$, the space-time translation subgroup. These local translations can be seen as local diffeomorphisms, thus suggesting the emergence of gravity notions into scene [4].

The $1+1$ (for simplicity) pseudoextended Poincaré–Lie algebra can be written as:

$$[P_0, P] = 0, \quad [P_0, K] = P, \quad [P, K] = -P_0 - mX_\phi.$$

An interesting phenomenon shows up when making the space-time translations local by introducing what we might call gravity generators, $f \otimes P$. In fact, the computation of the commutators with the general rule, brings about

$$[K, f \otimes P] = (L_X f) \otimes P + f \otimes P_0 + f \otimes X_\phi.$$

*Poincaré group with the time generator redefined by the central term $P_0 \to P_0 + mX_\phi$.

That is, by making local the translation subgroup in the (pseudo-)extended Poincaré group, the appearance of a local $U(1)$ symmetry is implied. This is a consequence of the fact that the space-time translations subgroup is no longer invariant, once the Poincaré group has been pseudoextended. As we saw in the previous subsection, making local the central term leads to the coupling of the particle with an electromagnetic force. Therefore, we find that introducing the gravitational field offers the possibility of an automatic (potential) coupling of it with an electromagnetic field.

In order to derive the corresponding dynamics, we undertake exactly the same path we followed in the case of pure electromagnetism. Thus, we make a formal splitting between the rigid translations and the local part,

$$f^{\mu} \otimes P_{\mu} = (f^{\mu}(0) + h^{\mu\sigma} x_{\sigma}) \otimes P_{\mu},$$

where the variables $h^{\mu\sigma}$ will represent the dynamical part of the metric. We can calculate the Lie algebra and study the possible extensions. In this case the cohomological structure is more complicated and, in fact, the algebra we are presenting below does not exhaust all the possibilities, even though it is rich enough to provide profitable insights. In its $1+1$ version (without rotations) is given by

$$\begin{gathered}
[\tilde{X}_v, \tilde{X}_x] = -\tilde{X}_t + (m+qk)c\tilde{X}_{\phi}, \quad [\tilde{X}_v, \tilde{X}_t] = -\tilde{X}_x, \\
[\tilde{X}_v, \tilde{X}_{h_{00}}] = -\tilde{X}_{h_{0x}} - \frac{1}{q}(mc-g)\tilde{X}_{A_x}, \\
[\tilde{X}_v, \tilde{X}_{h_{0x}}] = -\tilde{X}_{h_{00}} + \tilde{X}_{h_{xx}} - \left[c\left(k - \frac{2m}{q}\right) + \frac{2g}{q}\right] kc\tilde{X}_{A_0}, \\
[\tilde{X}_v, \tilde{X}_{h_{xx}}] = \tilde{X}_{h_{0x}} + kc\tilde{X}_{A_x}, \quad [\tilde{X}_v, \tilde{X}_{A_0}] = -\tilde{X}_{A_x}, \quad [\tilde{X}_v, \tilde{X}_{A_x}] = -\tilde{X}_{A_0}, \\
[\tilde{X}_x, \tilde{X}_{h_{0x}}] = -\tilde{X}_t - mc\tilde{X}_{\phi}, \quad [\tilde{X}_x, \tilde{X}_{h_{xx}}] = -\tilde{X}_x, \quad [\tilde{X}_x, \tilde{X}_{A_x}] = q\tilde{X}_{\phi}, \\
[\tilde{X}_t, \tilde{X}_{h_{00}}] = \tilde{X}_t + g\tilde{X}_{\phi}, \quad [\tilde{X}_t, \tilde{X}_{h_{0x}}] = \tilde{X}_x, \quad [\tilde{X}_t, \tilde{X}_{A_0}] = -q\tilde{X}_{\phi}, \\
[\tilde{X}_{h_{00}}, \tilde{X}_{h_{0x}}] = \tilde{X}_v + \frac{1}{q}(mc-g+qkc)\tilde{X}_{A_x}, \quad [\tilde{X}_{h_{00}}, \tilde{X}_{A_0}] = \tilde{X}_{A_0}, \\
[\tilde{X}_{h_{0x}}, \tilde{X}_{h_{xx}}] = \tilde{X}_v + kc\tilde{X}_{A_x}, \quad [\tilde{X}_{h_{0x}}, \tilde{X}_{A_0}] = \tilde{X}_{A_x}, \\
[\tilde{X}_{h_{0x}}, \tilde{X}_{A_x}] = -\tilde{X}_{A_0}, \quad [\tilde{X}_{h_{xx}}, \tilde{X}_{A_x}] = \tilde{X}_{A_x}.
\end{gathered}$$

There are several comments regarding these commutators. Firstly, we notice a new central parameter g in the commutator between the time translation generator and the 00 component of the metric, which is essentially related to the coupling between the particle and the gravitational field, that is, to the gravitational mass. It is very important to realize that cohomology does not imply this dynamical mass to be related to the inertial one, m, both of them being, therefore, in principle independent. Secondly, it does appear a parameter k coupling the gravitational field and the electromagnetic field. This parameter implies a shift in the inertial mass by kq. On the other hand, a nondesired pathological behaviour arises for noncharged particles due to the presence of nonanalytical $1/q$ terms in the commutators.

Another strange feature in the previous relations is the fact that we expect to recover standard physics when switching off the k parameter. But even in this case a coupling between gravity and electromagnetism remains active, thus spoiling the consistency of the dynamics from a physical point of view. A way out of this situation is achieved by imposing $g = mc$.

That is, if we make the dynamical mass to be proportional to the inertial one, one can check that the conventional theory is recovered in the case $k = 0$. In a sense, this is a way of obtaining the equivalence principle in an algebraic language as a consistency condition. As a by-product the nonanalytical dependence in q disappears, thus making consistent the theory for noncharged particles.

Regarding the nonrelativistic limit of this system, this can be achieved via an Inönü–Wigner contraction with respect to the subalgebra spanned by $\tilde{X}_t, \tilde{X}_{A_x}$ (plus rotations in the three-dimensional case). The contracted algebra, with the $g = mc$ condition imposed, shows the role of the cohomological parameters in a more explicit way:

$$
\begin{gathered}
[\tilde{X}_v, \tilde{X}_x] = (m + kq)\tilde{X}_\phi, \quad [\tilde{X}_v, \tilde{X}_t] = -\tilde{X}_x, \quad [X_v, \tilde{X}_{h_{00}}] = 0, \\
[X_v, \tilde{X}_{h_{0x}}] = k\tilde{X}_{A_0}, \quad [X_v, \tilde{X}_{h_{xx}}] = 0, \quad [\tilde{X}_v, \tilde{X}_{A_0}] = 0, \\
[\tilde{X}_v, \tilde{X}_{A_x}] = -\tilde{X}_{A_0}, \quad [\tilde{X}_x, \tilde{X}_{h_{0x}}] = -m\tilde{X}_\phi, \quad [\tilde{X}_x, \tilde{X}_{h_{xx}}] = 0, \\
[\tilde{X}_x, \tilde{X}_{A_x}] = q\tilde{X}_\phi, \quad [\tilde{X}_t, \tilde{X}_{h_{00}}] = m\tilde{X}_\phi, \quad [\tilde{X}_t, \tilde{X}_{h_{0x}}] = \tilde{X}_x, \\
[\tilde{X}_t, \tilde{X}_{A_0}] = -q\tilde{X}_\phi, \quad [\tilde{X}_{h_{00}}, \tilde{X}_{h_{0x}}] = 0, \quad [\tilde{X}_{h_{00}}, \tilde{X}_{A_0}] = 0, \\
[\tilde{X}_{h_{0x}}, \tilde{X}_{h_{xx}}] = 0,\ [\tilde{X}_{h_{0x}}, \tilde{X}_{A_0}] = 0,\ [\tilde{X}_{h_{0x}}, \tilde{X}_{A_x}] = -\tilde{X}_{A_0},\ [\tilde{X}_{h_{xx}}, \tilde{X}_{A_x}] = 0.
\end{gathered}
$$

The next step is exponentiating the algebra and constructing the quantization one-form from which the equations of motion can be derived. In this case, the more complicated structure of the algebra turns the exponentiation into a rather involved process and, therefore, we must follow a consistent order by order procedure which can be found in [5]. The equations of motion derived in this way correspond to the first nontrivial terms approximating the complete equations and have this form:

$$
\frac{d\mathbf{x}}{dt} = \mathbf{v}, \quad (m + kq)\frac{d\mathbf{v}}{dt} = q\left[\mathbf{v} \wedge (\boldsymbol{\nabla} \wedge \mathbf{A}^0) - \boldsymbol{\nabla} A^0 - \frac{\partial \mathbf{A}}{\partial t}\right] -
$$

$$- m\left[\mathbf{v}\wedge(\boldsymbol{\nabla}\wedge\mathbf{h}) - \boldsymbol{\nabla}h^{00} - \frac{\partial\mathbf{h}}{\partial t}\right] + \frac{m}{4}\nabla(\mathbf{h}\cdot\mathbf{h}) -$$
$$- \frac{kq}{2}\left[\mathbf{v}\wedge(\boldsymbol{\nabla}\wedge\mathbf{h}) - \frac{1}{4}\nabla(\mathbf{h}\cdot\mathbf{h}) - \frac{\partial\mathbf{h}}{\partial t}\right].$$

On the kinematical part, the left-hand side, we notice what was already foreseen at the Lie-algebraic level: the kinematical mass is corrected by a term proportional to k and the charge of the particle. On the dynamical side, the first line is again the expression of the Lorentz force, meanwhile the second line exactly corresponds to the geodesic motion in its first nontrivial perturbative expression (linearized gravity), which is the one obtained when working in the group law up to the third order in group variables as we have done, and which is also known as gravito-electromagnetism [6]. The last line is proportional to the mixing parameter k and shows the appearance of a new force of electromagnetic behaviour but of gravitational origin, consequence of this new possibility opened by the analysis of the cohomology of the underlying symmetry.

CONCLUSIONS

We have seen how the physical constants, characterizing the particle and its couplings, arise from the parameters associated with the cohomology of the symmetry underlying the physical system.

Known interactions, as electromagnetism and gravity, are derived by making local some invariant subgroups in the kinematical symmetry of the particle and, when exploring the possibilities that the Lie algebra offers, we have also found a new force as a consequence of the mixing of the standard previous interactions. A crucial observation in this process is underlining the relevance of making local the appropriate symmetry (translations) after the (pseudo-)extension of the group has taken place. This endows this new interaction with a quantum origin since the extended group is intrinsically tied to the quantum dynamics of the corresponding system [7].

An in principle testable consequence of this mixing between gravity and electromagnetism is a mass difference between charged particles and antiparticles by 2 kq. The current uncertainties in the values of such pairs (such as electron-positron) represent an upper limit for the constant k, implying a very small value for the latter (consistent with its quantum origin), even though with strong and fundamental implications, specially the violation of CPT symmetry*.

*We would like to thank S. Vinitsky for pointing out the potential relevance of these results in the context of Lamb-shift experiments in antihydrogen [8].

Finally, the presence of the new force could have important consequences in scenarios with extremely strong gravitational fields representing, for instance, a correction to the behaviour of collapsing matter.

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