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A HOMOGENEOUS STATIC GRAVITATIONAL FIELD AND THE PRINCIPLE OF EQUIVALENCE

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In this paper any gravitational field (both in the Einsteinian case and in the Newtonian case) is described by the connection, called gravitational. A homogeneous static gravitational field is considered in the four-dimensional area $z > 0$ of a space-time with Cartesian coordinates x, y, z, and t. Such field can be created by masses, disposed outside the area $z > 0$ with a density distribution independent of x, y, and t. Remarkably, in the four-dimensional area $z > 0$, together with the primitive background connection, the primitive gravitational connection has been derived. In concordance with the Principle of Equivalence all components of such gravitational connection are equal to zero in the uniformly accelerated frame system, in which the gravitational force of attraction is balanced by the inertial force. However, all components of such background connection are equal to zero in the resting frame system, but not in the accelerated frame system.

Гравитационное поле (как эйнштейновское, так и ньютоновское) описывается здесь связностью, называемой гравитационной. Однородное статическое гравитационное поле рассматривается в четырехмерной области $z > 0$ пространства-времени с декартовыми координатами x, y, z и t. Такое поле порождается массами, отсутствующими в области $z > 0$, а вне этой области распределенными независимо от x, y и t. Замечательно, что в четырехмерной области $z > 0$ вместе с примитивной фоновой связностью гравитационная связность получается тоже примитивной. В соответствии с принципом эквивалентности все компоненты такой гравитационной связности равняются нулю в равномерно ускоренной системе отсчета, в которой гравитационная сила уравновешивается силой инерции. Компоненты же фоновой связности равны нулю не в ускоренной, а в покоящейся системе отсчета.

1. FORMULATION OF THE PROBLEM

Let us determine the space-time M as a direct product $M = E \times T$ of the Euclidean space E and the Euclidean line T.

Choosing the length unit (for example, the centimetre), we define a metric in E. This means that the square length between two points in E, marked by the indices 1 and 2, in Cartesian coordinates x, y, z (according to Phifagor's theorem) equals

$$s^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2. \tag{1}$$

Choosing the time unit (for example, the second), we define a metric in T. This means, that the time τ between two instants in T, marked by the indices 1 and 2, in Cartesian coordinate t equals

$$\tau = t_2 - t_1. \tag{2}$$

The set Ξ of coordinates $x^1 = x,\ x^2 = y,\ x^3 = z,\ x^4 = t$ is a simple coordinate map in the product $E \times T$, mutually unique to its covering, and so the space-time M turns out to be a simple four-dimensional manifold.

Supplying the space-time M with the coordinate map Ξ, we will call the pair (M, Ξ) the resting frame system. In order to justify this name, we will consider M as a four-dimensional affine space, in which the coordinates of the map Ξ turn out to be affine coordinates. The world trajectory of a particle, freely moving by inertia in the absence of an outside force, is represented in M as an affine straight line. In the particular case the world trajectory of a particle «at rest» is represented in the reference frame (M, Ξ) as a line $x = x_0$, $y = y_0$, $z = z_0$.

Defining M as a four-dimensional affine space, we have in this way introduced in the space-time M a symmetric affine connection. Let us call it a background one and denote it by $\check{\Gamma}$.

In Newton's theory of gravity the background connection $\check{\Gamma}$ is present in an explicit form. About the necessity of restoring this connection in Einstein's theory of gravity see Ref. 1.

Let us note the following four properties of the background connection $\check{\Gamma}$ in M.

1. In the map Ξ all its components $\check{\Gamma}^a_{mn}$ equal zero.
2. Its curvature tensor $\check{R}^a_{mnk}$ equals zero.
3. The equality $\check{\Gamma}^a_{mn} = 0$ is preserved under the transition from the map Ξ to another affine map in M.
4. The equality $\check{R}^a_{mnk} = 0$ preserves under the transition from the map Ξ to any other coordinate map in M.

Each symmetric connection, the curvature tensor of which is equal to zero, shall be called a primitive one. In a suitable coordinate map all the components of the primitive connection are zero.

In Newton's and Einstein's theories of gravity the background connection is a primitive one. In other theories of gravity the background connection may be a nonprimitive one. For example, in Lobachevky's theory of gravity the background connection is a nonprimitive one.

In field theories the gravitational field is described by the affine connection. Let us call it a gravitational one and denote it by Γ. In each coordinate map its components Γ^a_{mn} are functions of the coordinates, composing this map.

If the gravitational field is absent, then the gravitational connection coinsides with the background one. It may be said that the background connection describes the gravitational field in its vacuum state. Such a gravitational field is trivial.

If in an arbitrary space-time area the gravitational field is absent, then in this area the tensor field

$$P^a_{mn} = \check{\Gamma}^a_{mn} - \Gamma^a_{mn} \tag{3}$$

of the affine deformation equals zero.

If the tensor field of the affine deformation (3) in a certain space-time area does not equal zero, then in this area a nontrivial gravitational field is present.

The gravitational connection is an equiaffine one. In particular, the background connection is also an equiaffine one.

As proved in [1], in space-time area with no gravitational field sources, in the general case the gravitational connection is a solution of the equation

$$R_{mn} = \check{R}_{mn}. \tag{4}$$

In this paper it is required to find a nontrivial gravitational field, not depending on x, y, and t and satisfying in the area $z > 0$ of space-time M the equation $R_{mn} = 0$. Such field can be created by masses, disposed outside the area $z > 0$ with a density distribution independent of x, y, and t.

The following facts about the curvature tensor R^a_{kln} composed by means of the affine connection Γ^a_{mn} are necessary for the solution of the problem.

2. ALGEBRAIC PROPERTIES OF THE CURVATURE TENSOR

The following construction is called a curvature tensor of each affine connection Γ^a_{mn}:

$$R^a_{kln} = \partial_k \Gamma^a_{ln} - \partial_l \Gamma^a_{kn} + \Gamma^a_{ks}\Gamma^s_{ln} - \Gamma^a_{ls}\Gamma^s_{kn}. \tag{5}$$

Obviously

$$R^a_{kln} + R^a_{lkn} = 0. \tag{6}$$

That is why of interest are only the following contractions:

$$R_{ln} = R^a_{aln}, \quad \Omega_{kl} = R^a_{lla}. \tag{7}$$

It is not difficult to see that

$$\Omega_{kl} = R^a_{kla} = \partial_k \Gamma_l - \partial_l \Gamma_k, \tag{8}$$

where

$$\Gamma_m = \Gamma^a_{ma}. \tag{9}$$

The contraction (9) is called a contracted connection. The contraction (18) is called a curvature tensor of the contracted connection (9).

If

$$\Gamma^a_{nm} = \Gamma^a_{mn}, \tag{10}$$

then the connection is called a symmetric one. Such a connection is given by the geodesic equations:

$$\frac{d^2x^a}{d\tau^2} + \Gamma^a_{mn}\frac{dx^m}{d\tau}\frac{dx^n}{d\tau} = 0. \tag{11}$$

Together with (6), the curvature tensor of the symmetric connection has the following property:

$$R^a_{kln} + R^a_{nkl} + R^a_{lnk} = 0. \tag{12}$$

From here it follows that

$$\Omega_{kl} + R_{kl} = R_{lk}. \tag{13}$$

Theefore, in the case of a symmetric connection the equality

$$R_{kl} = R_{lk} \tag{14}$$

is equivalent to the equality

$$\Omega_{kl} = 0. \tag{15}$$

The symmetric connection, satisfying condition (15), is called an equiaffine one.

Consequence. Since the energy-momentum tensor is a symmetric one, each solution of the Einstein's equation represents an equiaffine connection.

3. METRIC OBJECTS IN M IN THE ABSENCE OF A GRAVITATIONAL FIELD

In the absence of a gravitational field the metric objects in M represent themselves in the map Ξ as differential forms with constant coefficients. In the group of affine transformations these objects single out in the first case — the nonhomogeneous Galilean group, in the second case — the nonhomogeneous Lorentz group (i.e., the Poincare group).

In the Galilean case the time t is absolute in the Newtonian meaning, and therefore in M the linear form

$$d\tau = dt \tag{16}$$

is given. Moreover, in the Galilean case the degenerate cometric is determined:

$$\frac{\partial}{\partial x} \otimes \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \otimes \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \otimes \frac{\partial}{\partial z}. \tag{17}$$

In the Lorentz case in M the metric

$$-\frac{1}{c^2}(dx \otimes dx + dy \otimes dy + dz \otimes dz) + dt \otimes dt) \tag{18}$$

and the cometric

$$\frac{\partial}{\partial x} \otimes \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \otimes \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \otimes \frac{\partial}{\partial z} - \frac{1}{c^2} \frac{\partial}{\partial t} \otimes \frac{\partial}{\partial t}. \tag{19}$$

are given.

Here and further (in the theory of gravitation) c denotes the light velocity — the known parameter in the Lorentz transformation

$$\acute{x} = x, \quad \acute{y} = y, \quad \acute{z} = \frac{z - vt}{\sqrt{1 - (\frac{v}{c})^2}}, \quad c^2 \acute{t} = \frac{c^2 t - vz}{\sqrt{1 - (\frac{v}{c})^2}}. \tag{20}$$

The Newtonian case of the theory of gravitation we will denote by the sign $c = \infty$ of infiniteness of the light velocity.

The Einsteinian case of the theory of gravitation we will denote by the sign $c < \infty$, meaning the finiteness of the light velocity.

In the limit $c \to \infty$ the case $c < \infty$ transforms into the case $c = \infty$.

For example, the Lorentz transformation (20) in the limit $c \to \infty$ transforms into the Galilean transformation

$$\acute{x} = x, \quad \acute{y} = y, \quad \acute{x} = x - vt, \quad \acute{t} = t. \tag{21}$$

It is interesting that background connection does not depend on the light velocity c.

4. SOLUTION OF THE PROBLEM IN THE CASE OF $c = \infty$

In the case of $c = \infty$ the gravitational field is described by the affine connection with the help of Newton's potential U and the equations of motion of a material point. In our problem potential $U = U(z)$ depends only on z. Therefore we have the Newton's equations in the form

$$\frac{d^2x}{dt^2} = 0, \quad \frac{d^2y}{dt^2} = 0, \quad \frac{d^2z}{dt^2} = -\frac{dU}{dz}. \tag{22}$$

In the case $c = \infty$ the time t is always absolute in the Newtonian meaning, and the linear form (16) in the presence of the gravitational field is conserved. Consequently we must put

$$\frac{dt}{d\tau} = 1. \tag{23}$$

From (22) and (23) the equations of the type (11) follow, namely

$$\frac{d^2x}{d\tau^2} = 0, \quad \frac{d^2y}{d\tau^2} = 0, \quad \frac{d^2z}{d\tau^2} + \frac{dU}{dz}\frac{dt}{d\tau}\frac{dt}{d\tau} = 0, \frac{d^2t}{d\tau^2} = 0. \tag{24}$$

That is why in the case of $c = \infty$ the following component of the gravitational connection equals

$$\Gamma^3_{44} = \frac{dU}{dz} \tag{25}$$

and the others are zero. Now we find the following components

$$R^3_{344} = \frac{d^2U}{dz^2} = -R^3_{434} = R_{44} \tag{26}$$

of the gravitational curvature tensor and its contraction; the others are equal to zero.

Therefore in this case the equation $R_{mn} = 0$ is equivalent to the equation $U'' = 0$ and to the equation $R^a_{kln} = 0$. Consequenly, in the problem solved here, the gravitational connection should be a primitive one and the potential U should obey the equation $U'' = 0$ in the area $z > 0$. Thus, $U(z)$ is a linear function: $U = A + Bz$, if $z > 0$.

Let us note that the equations of motion (22) have the integral of energy

$$E = \frac{1}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] + U(z). \tag{27}$$

Assuming that $E = 0$ when a material point is at rest on the plane $z = 0$, we find $A = 0$.

Accorging to (22), the constant B equals the acceleration of a freely falling material point on the plane $z = 0$. We may put $B = g$, where g is the acceleration of a freely falling body, measured in the town Dubna.

Thus we find that in the area $z > 0$ the gravitational potential equals $U = gz$, where g is a positive constant.

Therefore, in the given case in the map Ξ only one component of the gravitational connection is not equal to zero, namely

$$\Gamma^3_{44} = g. \tag{28}$$

As a plane $z = 0$, one can choose the floor, on which in Dubna the world ex-champion stands. In the year 1952, it was accelerating protons up to 480 MeV. In that time the concurrent rival in Berkeley (USA) was accelerating protons up to 340 MeV.

5. SOLUTION OF THE PROBLEM IN THE CASE $c < \infty$

In the case $c < \infty$ the gravitational field is described by the Christoffel's connection for the metric

$$-\frac{1}{c^2}(dx \otimes dx + dy \otimes dy + dz \otimes dz) + V^2 dt \otimes dt, \tag{29}$$

where $V = V(z)$ is a function only of one coordinate z. From the geodesic equations

$$\frac{d^2x}{d\tau^2} = 0, \quad \frac{d^2y}{d\tau^2} = 0,$$

$$\frac{d^2z}{d\tau^2} + c^2 V \frac{dV}{dz}\Big(\frac{dt}{d\tau}\Big)^2 = 0\,, \quad \frac{d^2t}{d\tau^2} + \frac{2}{V}\frac{dV}{dz}\frac{dz}{d\tau}\frac{dt}{d\tau} = 0\,, \tag{30}$$

for the metric (29) we find following three componets of this connection

$$\Gamma^3_{44} = c^2 V V', \quad \Gamma^4_{34} = \frac{V'}{V} = \Gamma^4_{43} \tag{31}$$

and the others are zero.

Let us consider the curvature tensor of this connection. If one of the indices of the component R^a_{kln} (see (5)) of the curvature tensor or of the component R_{ln} (see (7)) of the contracted curvature tensor equals 1 or 2, then, as can be easily seen, such a component equals zero. Together with these, owing to (6), the components R^a_{33n} and R^a_{44n} equal zero.

It remains to consider the four components

$$R_{33}\,, \quad R_{44}\,, \quad R_{34} = R_{43} \tag{32}$$

of the tensor R_{ln} and the eight pair-opposite components

$$R^3_{343} = -R^3_{433}\,, \quad R^3_{344} = -R^3_{434}\,,$$
$$R^4_{343} = -R^4_{433}\,, \quad R^4_{344} = -R^4_{434} \tag{33}$$

of the tensor R^a_{kln}. The following relation holds between them:

$$R_{33} = R^4_{433}\,, \quad R_{34} = R^4_{434}\,,$$
$$R_{43} = R^3_{343}\,, \quad R_{44} = R^3_{344}\,. \tag{34}$$

Let us calculate these components:

$$R_{33} = -\frac{d}{dz}\Gamma^4_{43} - \Gamma^4_{34}\Gamma^4_{43} = -\frac{V''}{V}\,, \quad R_{34} = 0\,,$$
$$R_{43} = 0\,, \quad R_{44} = \frac{d}{dz}\Gamma^3_{44} - \Gamma^3_{44}\Gamma^4_{43} = c^2 V V''\,. \tag{35}$$

Therefore in this case the equation $R_{mn} = 0$ is equivalent to the equation $V'' = 0$ and to the equation $R^a_{kln} = 0$. Consequenly, in the problem solved here, gravitational connection should be a primitive one and the function V should obey the equation $V'' = 0$ in the area $z > 0$. Thus, $V(z)$ is a linear function: $V = a + bz$ if $z > 0$.

Since on the plane $z = 0$ the metric (29) should coincide with the unperturbed metric (18), we have $a = 1$. Moreover, instead of (23) we have the equality

$$c^2V^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dx}{d\tau}\right)^2 - \left(\frac{dy}{d\tau}\right)^2 - \left(\frac{dz}{d\tau}\right)^2 = c^2. \tag{36}$$

From (36) we find

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{V^2 - v^2/c^2}}\ , \tag{37}$$

where

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2. \tag{38}$$

The integral of energy in the case $c < \infty$ has the form

$$E = \left[V^2\frac{dt}{d\tau} - 1\right]c^2. \tag{39}$$

From (37) and (39) we find

$$E = \left(\frac{V^2}{\sqrt{V^2 - v^2/c^2}} - 1\right)c^2. \tag{40}$$

From (27) and(40) in the case $v^2 = 0$ we receive $E = U(z) = gz$ and $E = [V(z) - 1)c^2 = bc^2z$. Consequently, $bc^2 = g$ and

$$V = V(z) = 1 + \frac{gz}{c^2}. \tag{41}$$

In the limit $c \to \infty$ (40) transforms into (27).

Substituting the (41) into (29) and (30), we receive

$$-(dx \otimes dx + dy \otimes dy + dz \otimes dz) + \left(c + \frac{gz}{c}\right)^2 dt \otimes dt \tag{42}$$

and

$$\frac{d^2x}{d\tau^2} = 0, \quad \frac{d^2y}{d\tau^2} = 0,$$
$$\frac{d^2z}{d\tau^2} + g\left(1 + \frac{gz}{c^2}\right)\left(\frac{dt}{d\tau}\right)^2 = 0\ , \quad \frac{d^2t}{d\tau^2} + \frac{2g}{c^2 + gz}\frac{dz}{d\tau}\frac{dt}{d\tau} = 0\ , \tag{43}$$

6. A NONINERTIAL REFERENCE FRAME WITHOUT ROTATION IN THE CASE $c = \infty$

Now consider in the case $c = \infty$ a homogeneous gravitational field, arbitrary depending on the time t. If only this field is acting on the material point, then its movement in the resting reference frame (M, Ξ) is described by the Newton equations

$$\frac{d^2x}{dt^2} = -g^1(t), \quad \frac{d^2y}{dt^2} = -g^2(t), \quad \frac{d^2z}{dt^2} = -g^3(t), \tag{44}$$

where $g^n(t), n = 1, 2, 3$, are integrable on Euclidean line T functions. In the noninertial reference frame $(M, \hat{\Xi})$ without rotation

$$\hat{x} = x + f^1(t), \quad \hat{y} = y + f^2(t), \quad \hat{x} = z + f^3(t), \quad \hat{t} = t, \tag{45}$$

where

$$f^n(t) = a^n + b^n t + \int_0^t d\eta \int_0^\eta g^n(\xi) d\xi, \tag{46}$$

$$a^n = f^n(0), b^n = \frac{df^n}{dt}(0), \quad n = 1, 2, 3,$$

the equations (44) acquire the simplest form

$$\frac{d^2\hat{x}}{d\hat{t}^2} = 0, \quad \frac{d^2\hat{y}}{d\hat{t}^2} = 0, \quad \frac{d^2\hat{z}}{d\hat{t}^2} = 0. \tag{47}$$

In such a way, in the mechanics of Newton in the given case the inertial force is balanced by the gravity force.

Taking into account (23), equations (44) can be written in the form of a geodesic equations, namely

$$\frac{d^2x^n}{d\tau^2} + g^n(t)\frac{dt}{d\tau}\frac{dt}{d\tau} = 0, \ n = 1, 2, 3, \quad \frac{d^2t}{d\tau^2} = 0. \tag{48}$$

Therefore, the considered field is described by a connection, three components of which in the map Ξ are equal to

$$\Gamma^n_{44} = g^n(t), \quad n = 1, 2, 3, \tag{49}$$

and the rest are equal to zero. This connection is a primitive one, since in the map $\hat{\Xi}$ all of its components are equal to zero and therefore its curvature tensor is equal to zero.

A partial case of the transformation (45) is the transition to a uniformly accelerated reference frame

$$\hat{x} = x, \quad \hat{y} = y, \quad \hat{z} = z + \frac{gt^2}{2}, \quad \hat{t} = t, \tag{50}$$

imitating a homogeneous static gravitational field in the case of $c = \infty$.

A resting reference frame with a homogeneous static gravitational field turns out to be equivalent to a uniformly accelerated (falling) frame of reference without a gravitational field not only in the case of $c = \infty$, but also in the case of $c < \infty$. This is the Principle of Equivalence, advanced by Einstein.

7. THE UNIFORMLY ACCELERATED REFERENCE FRAME IN THE CASE $c < \infty$

The transition from the resting reference frame (M, Ξ) to the uniformly accelerated reference frame $(M, \hat{\Xi})$ is represented by the following trasformation:

$$\hat{x} = x, \quad \hat{y} = y,$$

$$\hat{z} = \left(z + \frac{c^2}{g}\right) \cosh\frac{gt}{c} - \frac{c^2}{g},$$

$$c\hat{t} = \Big(z + \frac{c^2}{g}\Big) \sinh \frac{gt}{c}. \tag{51}$$

All the information about the homogeneous static field in the area $z > 0$ is contained in formulae (42) and (43). In its turn, the last ones are derived from formulae (51) as a result of simple differentiation. Since

$$-(d\hat{x} \otimes d\hat{x} + d\hat{y} \otimes d\hat{y}) = -(dx \otimes dx + dy \otimes dy) \tag{52}$$

and

$$\frac{d^2\hat{x}}{d\tau^2} = \frac{d^2x}{d\tau^2} , \quad \frac{d^2\hat{y}}{d\tau^2} = \frac{d^2y}{d\tau^2} , \tag{53}$$

it is enough to prove that

$$-d\hat{z} \otimes d\hat{z} + c^2 d\hat{t} \otimes d\hat{t} = -dz \otimes dz + \left(c + \frac{gz}{c}\right)^2 dt \otimes dt , \tag{54}$$

that

$$\begin{aligned} \frac{d^2\hat{z}}{d\tau^2} &= \left[\frac{d^2z}{d\tau^2} + g\Big(1 + \frac{gz}{c^2}\Big)\Big(\frac{dt}{d\tau}\Big)^2\right] \cosh\frac{gt}{c} + \\ &+ \left[\frac{d^2t}{d\tau^2} + \frac{2g}{c^2 + gz}\frac{dz}{d\tau}\frac{dt}{d\tau}\right] \Big(c + \frac{gz}{c}\Big) \sinh\frac{gt}{c} , \\ \frac{d^2\hat{t}}{d\tau^2} &= \left[\frac{d^2z}{d\tau^2} + g\Big(1 + \frac{gz}{c^2}\Big)\Big(\frac{dt}{d\tau}\Big)^2\right] \sinh\frac{gt}{c} + \\ &+ \left[\frac{d^2t}{d\tau^2} + \frac{2g}{c^2 + gz}\frac{dz}{d\tau}\frac{dt}{d\tau}\right] \Big(c + \frac{gz}{c}\Big) \cosh\frac{gt}{c} , \end{aligned} \tag{55}$$

and that the area

$$-\infty < x < \infty,\ -\infty < y < \infty,\ z > 0,\ -\infty < t < \infty \tag{56}$$

transforms to the area

$$\begin{gathered} -\infty < \hat{x} < \infty,\ -\infty < \hat{y} < \infty, \\ \hat{z} > \frac{c^2}{g}\left(\sqrt{1 + \Big(\frac{g\hat{t}}{c}\Big)^2} - 1\right),\ -\infty < \hat{t} < \infty . \end{gathered} \tag{57}$$

In view of (42), (43) and (52)–(57) with respect to the reference frame $(M, \breve{\Xi})$ in the four-dimensional space-time area (57) the gravitational field is absent and a material point moves according to the law

$$\frac{d^2\hat{x}}{d\tau^2} = 0,\ \frac{d^2\hat{y}}{d\tau^2} = 0,\ \frac{d^2\hat{z}}{d\tau^2} = 0,\ \frac{d^2\hat{t}}{d\tau^2} = 0, \tag{58}$$

i.e., on a straight line and uniformly (without acceleration).

In such a way, the following theorem has been proved: the transition from Cartesian coordinates $\hat{z}, \hat{t}$ on the pseudo-Euclidean plane to polar coordinates

$$r = z + \frac{c^2}{g}\,, \quad \phi = \frac{gt}{c} \tag{59}$$

is formally equivalent to a transition from the uniformly accelerated reference frame $(M, \hat{\Xi})$, in the four-dimensional area (57) in which there is no gravitational field, to the resting reference frame (M, Ξ) with the homogeneous static field in the four-dimensional area (56).

Footnote. According to (50) in the case $c = \infty$ instead of (57) we have the area

$$-\infty < \hat{x} < \infty,\ -\infty < \hat{y} < \infty,\ \hat{z} > \frac{g\hat{t}^2}{2},\ -\infty < \hat{t} < \infty. \tag{60}$$

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