

# MULTIPARTICLE PRODUCTION

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# MULTIPARTICLE PRODUCTION\*

## 1. INTRODUCTION

### 1.1. Some Empirical Regularities in the Processes of High-Energy Multiparticle Production of Hadrons

The problem of multiparticle production is one of the central problems in elementary particle physics. For a long time its study was possible only in cosmic rays. In spite of great experimental difficulties, connected with considerable errors, cosmic ray physics laid down the foundations of our notions about multiparticle production.

Modern accelerators have made possible the intensive and detailed investigations of multiparticle production in a large energy interval ( $10$ – $10^3$  GeV). But no reasons so far exist to consider that we have a complete and clear description of phenomena.

At the same time a number of fundamental regularities and specific properties have been established for such processes.

1. We should note that the prediction by Wataghin (1934) concerning the increase of the relative number of inelastic channels at high energies is confirmed. The data from ISR are as follows:

$$(E \sim 10^3 \text{ GeV}) : \frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \sim 0.175.$$

It means that under the hadron–hadron collision additional particles fail to be produced in only 17 cases out of a hundred. Thus the hadron–hadron collisions are mainly inelastic. The elastic ones obviously show themselves as a shadow of inelastic channels. This fact received an obvious interpretation in the Logunov–Tavkhelidze quasi-potential approach.

It is interesting to note that the hypothesis of Wataghin anticipated the prediction of  $\pi$  meson by Yukawa (1935).

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It was proved later that most of the secondaries are pions (at ISR-energies):

$$\frac{\langle n_\pi \rangle}{\langle n_{\text{sec}} \rangle} \sim 0.8.$$

Their relative number in inelastic processes somewhat decreases with energy. For example, with  $E \sim 20$  GeV

$$\frac{\langle n_\pi \rangle}{\langle n_{\text{sec}} \rangle} \sim 0.9.$$

2. Another important property of inelastic collisions at high energies is the smallness of the momentum transfer or the transverse momentum of secondaries (see Fig. 1.1).

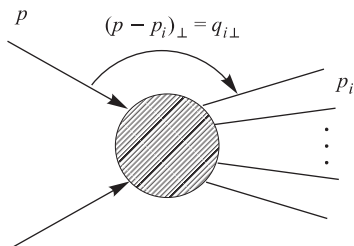


Fig. 1.1.

At the available energies the average value of the transverse momentum of secondaries does not depend on the interval

$$\langle p_{i\perp} \rangle \sim 0.2 \div 0.4 \text{ GeV}/c.$$

This empirical fact is closely related to the existence of the leading particle effect. This energy depends on  $s$  rather weakly. It is limited by the notion that appeared and was effectively used in cosmic rays. By a leading particle we conditionally mean one of the colliding hadrons which loses a negligible part of its momentum under the interaction. Thus, the particles produced in the collisions have mainly small momentum, compared with that of the incoming hadron.

3. The total cross sections have been actively investigated since the accelerators in Serpukhov, Batavia and CERN were put into operation. Measurement of this quantity is the simplest multiparticle experiment, since it is extremely critical to the theoretical models.

First unexpected results concerning the dependence of the total cross sections on energy were obtained in 1971 at the Serpukhov accelerator in the energy interval from 30 to 70 GeV. The decrease in cross sections, determined at lower energies, became slower and approached a constant value in most of the hadron-hadron processes. In the case of  $K^+p$  collisions an increase in the total cross sections was found. This phenomenon, comprising the change of cross section with increasing energy, was called the Serpukhov effect. Later the total cross sections were also studied at ISR (1973) for the proton-proton collisions in the 300 to 2000 GeV range and at the accelerator in Batavia (1974) for all the hadron reactions at energies up to 200 GeV.

The new data confirmed the Serpukhov effect and also showed that it may start a new phenomenon in high energy physics: rapid and, perhaps, unlimited increase of this quantity. It is so far difficult to determine an

analytical function which would describe the increase of  $\sigma_{\text{tot}}$ . We can make use of all the increasing functions up to the upper bound of possible increase of the total cross sections, determined by Froissart (1961) from the general principles of quantum field theory

$$(\sigma_{\text{tot}} \leq A \ln^2 s).$$

4. Another rather general feature of inelastic processes is the average multiplicity. Most of the theoretical models predict its increase with energy. The models of a statistical type give us the power dependence

$$\langle n \rangle = as^b.$$

Multiperipheral, parton and a number of other models predict the logarithmic increase:

$$\langle n \rangle = a \ln ns + b.$$

It should be noticed that the maximum number of particles (pions), permitted by the energy-momentum conservation law, is written in the form

$$n_{\text{max}} = \frac{\sqrt{s} - 2m_p}{m_\pi}.$$

The observed multiplicity increases more slowly than predicted by the former equation; i.e., it is extremely small in comparison with what is kinematically allowed (see Fig. 1.2).

Unfortunately, the comparison of the models with the experiment does not allow one to give preference to the logarithmic or power dependence of the average multiplicity on energies. One may only state the increase to be moderate.

5. Another important feature of the processes of high-energy

multiparticle production is deviation of the multiplicity distributions (or the topological cross sections) from the simple one.

Poisson law is determined at energies higher than 25 GeV (see Fig. 1.3).

The topological cross section is that with the given number of charged particles and arbitrary number of neutral particles in the final state. If the production of particles in the given collision is considered to be of a

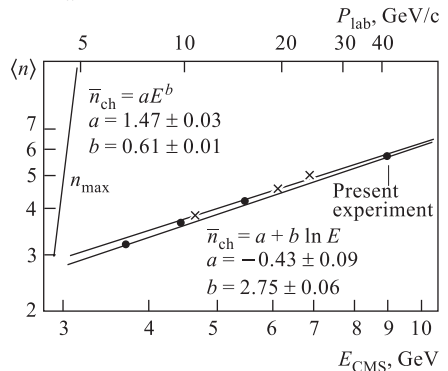


Fig. 1.2. Increase in multiplicity with  $E_{\text{CMS}}$  (2-meter propane chamber collaboration JINR-IHEP, U-70, Protvino)

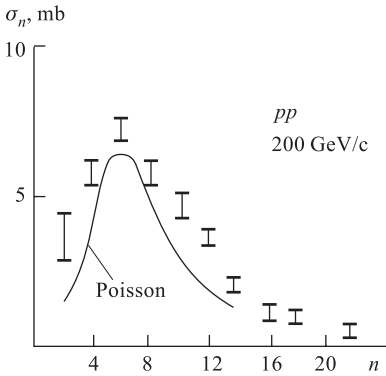


Fig. 1.3. Deviation of  $\sigma_n$  from the simple Poisson law

random nature, the distribution naturally assumes the Poisson form:

$$\sigma_n = \sigma_{\text{inel}} e^{-\nu} \frac{\nu^n}{n!}.$$

This distribution has the following properties:

$$\langle n \rangle \equiv \sum_{n=0}^{\infty} n P_n(\nu) = \nu,$$

$$\langle n^2 \rangle \equiv \sum_{n=0}^{\infty} n^2 P_n(\nu) = \nu^2 + \nu.$$

Thus, for the Poisson distribution the correlation function  $f_2$  is equal to zero:

$$f_2 \equiv \langle n(n-1) \rangle - \langle n \rangle^2, \quad f_2^{\text{Poisson}} = 0.$$

However, the experimental data obtained at the accelerators in Serpukhov and Batavia show that the multiplicity distributions are broader than the Poisson distributions. And the quantity differs considerably from zero:

$$f_2 = 7.44 \pm 0.72 \quad (\text{at } P_{\text{LAB}} \sim 200 \text{ GeV}/c).$$

This fact shows that the production of secondaries at high energies cannot be considered as statistically independent process. Satisfactory distributions are obtained by the approaches based on consideration of production of whole hadron associations (or clusters). The models based on the accounting for two (or more) mechanisms of the hadron production, leading to the multicomponent description of distributions, are more successful in the description of experimental data. The possibility of extracting the contributions of various mechanisms (the ranges of the  $n$ -particle phase-space volume) to the cross sections of multiparticle processes was first pointed out by Logunov and collaborators.

6. The idea of two production mechanisms gives wide possibilities for the theoretical description of the correlation phenomena.

Already for the simplest distribution which is the topological cross section (depending on  $n_{\text{ch}}$ ), one could see that the secondaries are not independent but correlate with each other. Then the question arises about the sensitivity of neutral particles to the charged hadron production (i.e., charge-neutral correlations); i.e., whether the particles «feel» that the momentum has a produced «near» or a «distant» (in the momentum scale) neighbour (short-range and long-range momentum correlations).

As we shall have the opportunity to touch upon this question later, we should only note that the latest experiments at high energies gave a

number of qualitatively new results. We have reference to the detection of linear dependence of an average number of neutral particles on a number of prongs (see Fig. 1.4). Such a correlation has not been observed at energies up to 20 GeV.

7. A large number of empirical facts on the dynamics of multiparticle processes make it possible to interpret experimentally observed scaling regularities of strong interaction characteristics. These regularities are the display of a rather general principle of automodelity characteristics of a number of physical problems. Here we mean, roughly speaking, the decrease in the number of independent variables

of the studied physical quantity connected with definite similarity properties and symmetry of the problem (in the space of the given independent variables). The principle of automodelity was first suggested for the lepton-hadron and hadron-hadron processes by Matveev, Muradyan and Tavkhelidze. They point out the analogy of these processes with an explosion in gas dynamics.

Among the scaling regularities, studied in strong interactions, the hypothesis by Feynman on the decrease of the number of variables of the invariant differential cross sections when  $s \rightarrow \infty$  is widely used:

$$E \frac{d\sigma}{d\mathbf{p}} = f(s, p_z, p_\perp) \xrightarrow{s \rightarrow \infty, x \text{ fixed}} f\left(x \equiv \frac{2p_z}{\sqrt{s}}, p_\perp\right).$$

Scaling regularities suggested by Koba, Nielsen, Olesen are of great use in the multiplicity distributions:

$$\langle n \rangle \frac{\sigma_n}{\sigma_{\text{inel}}} = \psi\left(\frac{n}{\langle n \rangle}\right),$$

where  $\sigma_n$  is the topological cross section,  $\sigma_{\text{inel}}$  is the total inelastic cross section,  $\langle n \rangle$  the average multiplicity.

These and a whole number of scaling laws approximately satisfied at the available energies make it possible to assume that the strong interactions have some definite symmetry (perhaps not only one).

The facts known at present about the hadron-hadron processes are not limited to the above-listed properties. However, these properties reflect the basic characteristics of multiple production in strong interactions. These properties are constantly exploited by theoreticians, no matter which way they go. Those advocating phenomenological schemes and

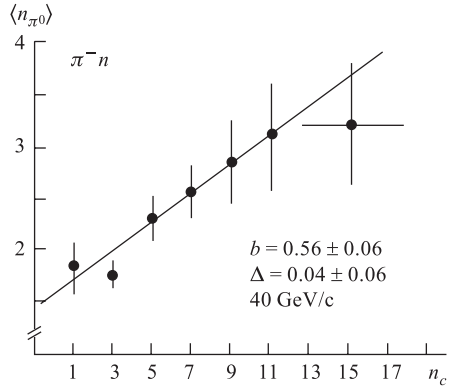


Fig. 1.4. Charge-neutral correlations

empirical formulae seek to use these properties in constructing the models. Others who keep to field-theoretical approaches verify consistency of these properties with the basic axioms of quantum field theory and develop approximations adequate for the general properties found.

It may be hoped that these two approaches, studying the same phenomena from different viewpoints, after being united, will provide a closer description of high-energy multiparticle production.

## 1.2. Basic Definitions

The analysis of multiparticle production processes is very difficult both from the technical aspect and from the viewpoint of kinematical description. Therefore, it is especially important to obtain information in the language of inclusive reactions. We mean here the processes where only some of the secondaries are detected. The consideration of such reactions was first proposed by Logunov et al. in 1967.

It is customary to write the inclusive  $n$ -particle reaction in the form

$$a + b \rightarrow p_1 + p_2 + \dots + p_n + X, \quad (1.1)$$

where  $X$  stands for «anything», i.e., all possible particles which are not subjected to observation in a given experiment. Unlike the inclusive case, the reaction

$$a + b \rightarrow p_1 + p_2 + \dots + p_{n'} \quad (1.2)$$

when all the particles in a final state are detected, is characterized by the differential (exclusive) production cross section\*

$$\frac{d\sigma_{n'}}{d\mathbf{p}_1 \dots d\mathbf{p}_{n'}} = |T(ab \rightarrow p_1 \dots p_{n'})|^2 \delta(P - \sum p_{n'}), \quad (1.3)$$

where  $T(ab \rightarrow p_1 \dots p_{n'})$  is the amplitude of transition of two particles  $a$ ,  $b$  into  $n'$  particles with momenta  $p_1, \dots, p_{n'}$ .

The transition from (1.3) to the inclusive distribution of the process, where only  $n$  of  $n'$  particles are identified, is achieved by integrating over

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\*Later we shall use for different forms of the phase volume

$$d\mathbf{p} = \frac{d^3p}{E} = \frac{2\pi d^2p_{\perp} dp_{\parallel}}{E} \approx 2\pi d^2p_{\perp} \frac{dx}{x} = 2\pi d^2p_{\perp} dy = s dt dM^2,$$

where the variables are

$$\mathbf{p} = (p_{\parallel}, \mathbf{p}_{\perp}), \quad E = \sqrt{\mathbf{p}^2 + m^2}, \quad x = \frac{2p_{\parallel}}{\sqrt{s}}, \quad y = \frac{1}{2} \ln \frac{E + p_{\parallel}}{E - p_{\parallel}}.$$

momenta of the undetected particles:

$$\frac{d\sigma_{n'}}{d\mathbf{p}_1 \dots d\mathbf{p}_n} = c \int |T(ab \rightarrow p_1 \dots p_{n'})|^2 \delta(P - \sum p_{n'}) d\mathbf{p}_{n+1} \dots d\mathbf{p}_{n'}. \quad (1.4)$$

If we sum over all the channels with  $n$  particles of reaction (1.1), we arrive at the so-called  $n$ -particle inclusive distribution:

$$\frac{d\sigma}{d\mathbf{p}_1 \dots d\mathbf{p}_n} = c \sum_{n'} \frac{d\sigma_{n'}}{d\mathbf{p}_1 \dots d\mathbf{p}_n} = \sum_{n'} \int d\mathbf{p}_{n+1} \dots d\mathbf{p}_{n'} \frac{d\sigma_{n'}}{d\mathbf{p}_1 \dots d\mathbf{p}_{n'}}. \quad (1.5)$$

If differential cross sections were known for all the exclusive channels, we could construct, by means of this formula, all the inclusive distributions.

And, conversely, knowing all the inclusive distributions, one could reproduce the cross sections of the channels. Thus, in principle, both the descriptions contain complete information on the two-hadron collision process.

As long as we dealt with a small number of secondaries it was more convenient to employ the exclusive description. At higher energies, when ten and more particles are produced, it is better to keep the inclusive consideration.

**1.2.1. The One-Particle Distribution.** Consider an example of the inclusive reaction with only one detected particle

$$a + b \rightarrow p_1 + \overbrace{p_2 + \dots + p_{n'}}^X. \quad (1.6)$$

The one-particle cross section with a fixed multiplicity is defined as follows:

$$\frac{d\sigma_{n'}}{d\mathbf{p}_1} = \frac{1}{(n-1)!} \int \frac{d\sigma_{n'}}{d\mathbf{p}_1 \dots d\mathbf{p}_{n'}} \prod_{i=2}^{n'} d\mathbf{p}_i. \quad (1.7)$$

Knowing this cross section, one can easily go over to the topological cross section of particle production

$$\sigma_{n_i} = \frac{1}{n'} \int \frac{d\sigma_{n'}}{d\mathbf{p}_1} d\mathbf{p}_1. \quad (1.8)$$

Here  $\sum_{n'} \sigma_{n'} = \sigma_{\text{inel}}$  is the total inelastic cross section.

As is seen from the previous definitions, summing (1.7) over  $n'$ , we get the one-particle inclusive distribution

$$\frac{d\sigma}{d\mathbf{p}_1} = \sum_{n'} \frac{d\sigma_{n'}}{d\mathbf{p}_1} \quad (1.9)$$



with the normalization

$$\int \frac{d\sigma}{d\mathbf{p}_1} d\mathbf{p}_1 = \sum_{n'} n' \sigma_{n'} \equiv \langle n' \rangle \sigma_{\text{inel}}. \quad (1.10)$$

From the sum rule

$$\int \frac{d\sigma}{d\mathbf{p}} p_\mu d\mathbf{p} = \sigma \langle p_\mu \rangle$$

it is easy to get definitions of average momenta  $\langle p_\perp \rangle$  at  $\mu = 1, 2$  and  $\langle p_\parallel \rangle$  at  $\mu = 3$ .

Notice that relation (1.10) is the definition of mean multiplicity  $\langle n \rangle$  of secondaries.

**1.2.2. The Two-Particle Distribution.** Analogously, one can consider the inclusive reaction with identification of two particles

$$a + b \rightarrow p_1 + p_2 + \overbrace{p_3 + \dots + p_{n'}}^X. \quad (1.11)$$

The two-particle inclusive distributions arising here define a series of widely used average quantities:

$$\begin{aligned} \int \frac{d^2\sigma}{d\mathbf{p}_1 d\mathbf{p}_2} d\mathbf{p}_1 &= \langle n(p_1) \rangle \frac{d\sigma}{d\mathbf{p}_1}, \\ \int \frac{d^2\sigma}{d\mathbf{p}_1 d\mathbf{p}_2} d\mathbf{p}_1 d\mathbf{p}_2 &= \sum n(n-1) \sigma_n = \langle n(n-1) \rangle \sigma_{\text{inel}}. \end{aligned} \quad (1.12)$$

The quantity  $\langle n(p_1) \rangle$ , in particular, is called the associated multiplicity.

Making use of the two- and one-particle inclusive cross sections, one can construct the two-particle correlation function

$$C_2 = \frac{1}{\sigma_{\text{inel}}} \frac{d^2\sigma}{d\mathbf{p}_1 d\mathbf{p}_2} - \frac{1}{\sigma_{\text{inel}}^2} \frac{d\sigma}{d\mathbf{p}_1} \frac{d\sigma}{d\mathbf{p}_2} \quad (1.13)$$

and the corresponding moment of distribution

$$f_2(s) = \int C_2(p_1, p_2) d\mathbf{p}_1 d\mathbf{p}_2 = \langle n(n-1) \rangle - \langle n \rangle^2 = D^2 - \langle n \rangle, \quad (1.14)$$

where  $D \equiv \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$  is the dispersion.

Higher correlation functions and moments  $C_3, f_3, \dots, C_n, f_n$  are defined analogously. It is natural in the case of independent production of particles for all the  $C_n$  and  $f_n$  to be zero. This has been demonstrated above when considering the Poisson law.

## REFERENCES

1. *Wataghin G.* Symposium on Cosmic Rays, Acad. Bras. de Cienoisas Rio, 1941; Comptes Rend. 1938. V.207. P.358, 421; Z. Physik. 1934. V.88. P.92.
2. *Yukawa H.* // Proc. Phys. Math. Soc. Japan. 1935. V.17. P.48.
3. *Giacomelli G.* Rapporteur talk at the Batavia Conference, 1972.
4. *Zalewsky K.* Rapporteur talk at the London Conference, 1974.
5. *Logunov A. A., Mestvirishvili M. A.* CERH, TH-1707. Geneva, 1973.
6. *Logunov A. A., Mestvirishvili M. A., Nguyen van Hieu.* // Phys. Lett. B. 1967. V.25. P.617.
7. *Bogolubov N. N., Vladimirov V. S., Tavkhelidze A. N.* // TMP (in Russian). 1972. V.12. P.305.
8. *Matveev V. A., Muradyan R. M., Tavkhelidze A. N.* // Part. Nucl. 1970. V.2. Part I. P.3-32.
9. *Feinberg E. L.* // Physics Reports C. 1972. V.5, No.5.
10. *Slansky R.* // Physics Reports C. 1974. V.11, No.3.
11. *Zinoviev G. M.* JINR 2-8551. Dubna, 1975.
12. *Jacob M.* // Proc. CERN-JINR School of Physics, Ebeltoft, Denmark, 1973.
13. *Muradyan R.* // Proc. CERN-JINR School of Physics, Ebeltoft, Denmark, 1973.
14. *Logunov A. A., Tavkhelidze A. N.* // Nuovo Cimento. 1963. V.29. P.38.
15. *Garsevanishvili V. R., Matveev V. A., Slepchenko L. A., Tavkhelidze A. N.* // Phys. Rev. D. 1971. V.4. P.849.
16. *Feynman R. P.* // Phys. Rev. Lett. 1969. V.23. P.1415.
17. *Benecke J., Chou T. T., Yang C. N., Yen E.* // Phys. Rev. 1969. V.188. P.1259.
18. *Koba Z., Nielsen H. B., Olesen P.* // Nucl. Phys. 1972. V.1340. P.317.
19. *Van Hove L.* // The IV Int. Symp. on Multiparticle Hadrodynamics, Pavia, 1973.
20. *Slepchenko L.* JINR P2-6867. Dubna, 1972. P.91.
21. *Sissakian A.* JINR P1,2-8529. Dubna, 1974.

## 2. MULTIPLICITY DISTRIBUTIONS

### 2.1. Multicomponent Descriptions of Multiparticle Production

The multiplicity distributions or the topological cross sections are related to a number of the simplest characteristics of the processes of multiparticle production. These are determined by the number of events with a given number of secondaries. As a rule, the charged secondary

particles are taken into consideration. In the high energy range up to 20 GeV the experimental topological cross sections have been very well described by a series of theoretical models and phenomenological formulae.

Firstly, one may successfully use the usual Poisson formula

$$P(n_{\pm}) = \frac{\langle n_{\pm} \rangle^{n_{\pm}}}{n_{\pm}!} \exp[-\langle n_{\pm} \rangle]$$

describing an independent production of particles. Two models applying to the description of charged distributions were suggested by Wang.

The first one started from the assumption of uncorrelated production of the hadron pairs  $\pi^+\pi^-$ . In this case the multiplicity distribution has a simple quasi-Poisson formula:

$$P(n_{\pm}) = \frac{\left(\frac{1}{2}\langle n_{\pm} - \alpha \rangle\right)^{(1/2)(n_{\pm} - \alpha)}}{\left[\frac{1}{2}(n_{\pm} - \alpha)\right]!} \exp\left[-\frac{1}{2}(n_{\pm} - \alpha)\right],$$

where  $\alpha$  is the number of charged particles in the initial state.

The second one, suggested by Wang, led to the Poisson distribution for the charged secondary particles subtracting the leading particles:

$$P(n_{\pm}) = \frac{(\langle n_{\pm} - \alpha \rangle)^{n_{\pm} - \alpha}}{(n_{\pm} - \alpha)!} \exp(-\langle n_{\pm} - \alpha \rangle).$$

It was assumed in the Chow and Pignotti multiperipheral model that the Poisson dependence describes the distribution over a number of secondary pions excluding events of the purely neutral particle production (0-prong events). Considerable deviations of the topological cross sections from the Poisson law are observed at energies  $\sim 25$  GeV (see Fig. 2.1). This testifies to failure of the model based on the assumption of uncorrelated production of single particles. The experiments performed in Serpukhov on the 2-meter propane chamber, irradiated with  $\pi^-$  mesons, when  $p = 40$  GeV proved to be especially critical to the multiplicity distributions.

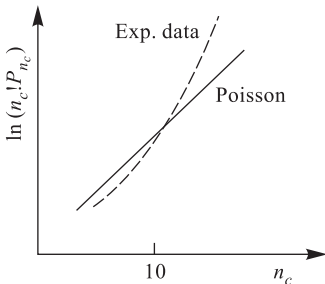


Fig. 2.1. Qualitative variation of  $\ln(n_c! P_{n_c})$  with  $n_c$

If at an energy of  $\sim 25$  GeV the Wang and Chow-Pignotti models were in satisfactory agreement with the experiment, then the data from the 2-meter propane chamber, in combination with those from the recent experiments in Batavia and at ISR, give evidence in favour of the multicomponent description of multiparticle production.

Attempts to combine the two extreme approaches to multiparticle production at high energies became a starting point for the origin of the multicomponent description. One of them, the diffraction dissociation, proceeds from the assumption that the secondaries are produced due to the leading particle fragmentation (target particle and incoming particle). We may say that the secondaries have information about the colliding hadrons and they may be combined with one of the initial particles. Figuratively speaking, they remember their «parents». The diffraction dissociation approach leads to the topological cross sections of the type  $\sigma_n \sim n^{-2}$  which disagree with the recent experimental data as well as with the Poisson distributions.

The other approach deals with the secondaries which do not «remember» their origin from one or another initial particle. In this category we may refer to the models of independent emission, some of which have been discussed above. It is convenient to classify these approaches according to the correlations of the produced particles. The difference between the correlations depends on whether the secondaries are in the same (short-range) or different (long-range) ranges of the phase space volume of  $n$  particles. If

$$y_1, y_2 \left( y = \frac{1}{2} \ln \frac{E + p_{\parallel}}{E - p_{\parallel}} \right)$$

are the rapidities of the secondaries, then

1) short-range (SR) correlations exist between the particles produced with approximately equal rapidities and with increasing  $|y_1 - y_2|$  tend to zero, as

$$C_2(y_1, y_2) \sim e^{-\gamma|y_1 - y_2|}$$

when  $|y_1 - y_2| \gg 1/\gamma$ ,  $\gamma \neq 0$ ;

2) long-range correlations (LR) exist between particles produced in distant ranges of  $y$  space, i.e., for

$$|y_1 - y_2| \gg \frac{1}{\gamma},$$

and the two-particle function of the distribution increases rapidly when both particles come from one «cluster».

In other words, when observing the particle with  $y_1$  the information about the possible presence of another secondary with any admissible rapidity is the LR effect. And conversely information about the probability of the presence of another particle with similar rapidity is the SR effect.

In the diffraction dissociation approach there are strong LR correlations. Concrete realizations of the second approach, i.e., the models of independent emission, are characterized by either absence of correlations (Poisson,  $C_2 = 0$ ) or presence of small SR correlations.

We should note that the possibility of extraction of contributions of various mechanisms (ranges of phase volume of  $n$  particles) to multiple cross sections was first pointed out by Logunov and collaborators.

In this connection, in recent years there have been changes in the philosophy of approach to the mechanism of high-energy multiparticle production. Wilson and Feynman proposed the two-component model. The simplest version of this model is based on the multiplicity distribution, written in the form of the sum:

$$\sigma_n = \alpha n^{-2} + \beta P(n)$$

with the chosen contributions of each component; in particular, the parameters may be so chosen that there is left only a term corresponding to one of the approaches.

If both components are present at all the energies, the first moments  $\langle n^n \rangle$  of the distribution are of the form

$$\begin{aligned} \langle n \rangle &= a + (\alpha_1 + \beta_1) \ln s, \\ \langle n^2 \rangle &= b + \alpha_2 \sqrt{s} + \beta_2 (\ln^2 s + 2 \ln s - 4), \\ \langle n^3 \rangle &= c + \alpha_3 s + \beta (\ln^3 s + \dots), \end{aligned}$$

where the contributions with the coefficient  $\alpha_i$  are consistent with the first component, and those with the coefficient  $\beta_i$  are consistent with the second one.

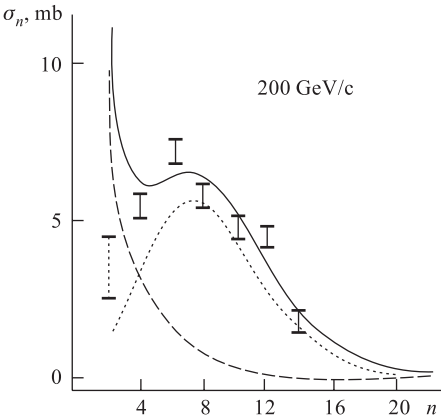


Fig. 2.2. Description of topological cross section with the help of two-component model:  $\sigma_n = \alpha n^{-1} + \beta P_n(\bar{n})$

It follows that the first component (the component of the diffraction dissociation) dominates beginning from the second-order moment at high energies.

It should be noted that in the given approach the «play» of these two components leads to the existence of a weak dip in the multiplicity distribution. The dip becomes more noticeable with increasing energy (see Fig. 2.2).

In spite of a number of virtues (for example, the increase of the second correlation parameter  $f_2$  is in excellent agreement with the experiment), the two-component model results in discrepancies between the higher correlation moments and the experimental data. Note that such

discrepancies cannot be eliminated in models with a large ( $n > 2$ ) number of components.

## 2.2. Model of «Two Mechanisms»

Let us consider the multicomponent description of multiparticle production, resulting from the phenomenological model of «two mechanisms», suggested by the Dubna group (Matveev, Kuleshov, Sissakian, Grishin, Jancso) in 1972.

The IMP model appeared as a concrete phenomenological scheme on the basis of the study of the processes of multiparticle production in the framework of the straight-line path approximation (SLPA) in quantum field theory. The physical essence of SLPA is the following: At high energies the main contribution to the process amplitude in the form of the Bogolubov–Feynman functional integral over the particle paths gives trajectories which are nearly straight lines having the same direction as the momentum vectors of the leading particles before and after the correlation. In the field-theoretical language, SLPA rests on the assumption of a leading particle. For the most important results of SLPA we should refer to the generalized Poisson law for the topological cross sections, the automodel or point-like behaviour of the cross sections and prediction about the dependence of average multiplicities on a transverse momentum of an extracted particle. We shall refer to some of these results when considering the picture of multiparticle production.

The main point of the TMP model is the hypothesis of the existence of two mechanisms for production of secondaries:

1) There exist the leading particles, dissociating with the local conservation of isospin.

2) In the process of interaction in a statistically independent way, there likewise appear the hadron associations of clusters which then decay into mesons.

It is natural to suppose that the average numbers of these associations at high energies are independent of the types of the colliding particles.

According to these assumptions, one can see that in the TMP model the probability of production of clusters at the given dissociation channels of the leading particles ( $i, j$ ) takes the form

$$W_{n_1, n_2, \dots}^{i, j} = \alpha_i \beta_j P_{n_1}(\langle n_1 \rangle) P_{n_2}(\langle n_2 \rangle) \dots, \quad (2.1)$$

where  $\alpha_i, \beta_j$  are the probabilities of the dissociation channels;  $n_1, n_2 \dots$  are the numbers of clusters produced according to the Poisson law. Thus, the distribution over the number of secondaries in a given model has the form of superposition of the Poisson factors. Multicomponent character appears as a result of summation over a number of channels of the leading particle dissociation. Now consider a concrete example of description by the TMP model of the charged distribution in  $\pi^- p$  and  $\pi^- n$  interactions.

In this case it is sufficient to consider only the simplest channels of dissociation of the colliding particles and hadron clusters with isospin  $I = 0$ . Thus, we consider dissociation of the leading nucleon in the following scheme:

1.  $N \rightarrow N$  with the probability of channel  $\alpha_1$ ;
2.  $N \rightarrow N\pi^0$  with the probability of channel  $\alpha_2$ ;
3.  $N \rightarrow N'\pi^\pm$  with the probability of channel  $\alpha_3$ ,

where  $\sum_{i=1}^3 \alpha_i = 1$  and  $\alpha_3 = 2\alpha_2$  by the assumption on local isospin conservation. As another source for secondary particle production we introduce the  $\sigma$  and  $\omega$  associations, produced by the Poisson law, with isospin  $I = 0$  and parity  $G = \pm 1$ .

We confine ourselves to the main schemes for the decay of the  $\sigma$  and  $\omega$  associations:

1.  $\sigma \rightarrow \pi^+\pi^-, \pi^0\pi^0$ ,
2.  $\omega \rightarrow \pi^+\pi^-\pi^0$ .

In accordance with the assumptions of the TMP model, one can easily see from Eq. (2.1) that the production probability for the pion pairs ( $n_\pm$ ,  $n_0$ ) and the triplets of pions  $n_3$  at the given channel of the nucleon dissociation is defined by the expression

$$W_{n_\pm, n_0, n_3}^i = \alpha_i P_{n_\pm}(\alpha_\pm) P_{n_0}(\alpha_0) P_{n_3}(b), \quad (2.2)$$

where  $P_n(\langle n \rangle)$  is the Poisson factor;  $\alpha_\pm$ ,  $\alpha_0$ ,  $\alpha_3$  are the average numbers of pion pairs and of pion triplets, correspondingly.

From the condition that the pairs are produced with the isospin  $I = 0$ , it follows that

$$\alpha_\pm = 2\alpha_0 \equiv \alpha.$$

It is evident that the number of charged particles  $n_c$  and neutral pions  $n_{\pi_0}$  can be written as follows:

$$\begin{aligned} n_c^i &= 2n_\pm + 2n_3 + \ell_c^i, \\ n_{\pi_0}^i &= 2n_0 + n_3 + \ell_{\pi_0}^i, \end{aligned} \quad (2.3)$$

where  $\ell_c^i, \ell_{\pi_0}^i$  are, respectively, the numbers of charged particles and  $\pi^0$  mesons among the dissociation products of the leading particles in the  $i$ th dissociation channel (see Table 2.1).

From Eqs. (2.2) and (2.3) for distributions over the number of charged particles, it follows:

for the  $\pi^-p$  interaction

$$W_{n_c} = P_{(n_c-2)/2}(a'); \quad (2.4)$$

for the  $\pi^-n$  interaction

$$W_{n_c} = (1 - 2\alpha_2)P_{(n_c-1)/2}(a') + 2\alpha_2P_{(n_c-3)/2}(a'), \quad (2.5)$$

Table 2.1

	$i = 1$		$i = 2$		$i = 3$	
	$\pi^- p$	$\pi^- n$	$\pi^- p$	$\pi^- n$	$\pi^- p$	$\pi^- n$
$\ell_c$	2	1	2	1	2	3
$\ell_{\pi^0}$	0	0	1	1	0	0

where  $a' = a + b$  has the essence of the average number of pairs  $\pi^+\pi^-$  including the contribution from similar combinations among the pion triplets  $\pi^+\pi^-\pi^0$ .

Note that in this simple case of distribution over the number of charged particles only two components are important, each of which corresponds to the pair independent emission. However, it appears to be sufficient to describe the broadening of the distributions, which is characteristic of high energies.

Note also that, unlike the Wang-I model, the case of distributions of the type — superposition of the Poisson factors with the same number of parameters gives a good joint description for the  $\pi^- p$  and  $\pi^- n$  collisions (see Table 2.2) with the same average value of the  $\pi^+\pi^-$  combinations. It is consistent with a natural physical hypothesis of independence of particle production in the nondiffraction region of the type of colliding hadrons. Multicomponent structure of distributions arises also in this case if, under the same assumptions, heavy strange particles are taken into consideration.

Table 2.2

Type of interaction	Number of events	$\langle n \rangle$	$\sqrt{D}$	$\chi^2$ fit by Wang-I model	$\chi^2$ fit by suggested model	Degrees of freedom
$\pi^- p$	4400	$5.62 \pm 0.4$	2.75	8	8	8
$\pi^- n$	1860	$5.32 \pm 0.7$	2.82	13	8.5	7

The following additional channels of dissociation of nucleons are possible:

$$\begin{aligned}
 p &\rightarrow \Lambda^0 K^+, & n &\rightarrow \Lambda^0 K^0, \\
 p &\rightarrow \Sigma^0 K^+, & n &\rightarrow \Sigma^0 K^0, \\
 p &\rightarrow \Sigma^+ K^0, & n &\rightarrow \Sigma^- K^+.
 \end{aligned}$$



Under the given assumption, the pion dissociates with the most probability, according to the schemes

$$\begin{aligned}\pi^- &\rightarrow \pi^-, \\ \pi^- &\rightarrow 2\pi^-\pi^+, \\ \pi^- &\rightarrow 2\pi^0\pi^-, \end{aligned}$$

It is necessary to include independent production of heavy  $\Lambda$ -associations besides the pion pairs and pion triplets:

$$\Lambda \rightarrow K^+ + \bar{K}^-, \quad K^0 \bar{K}^0.$$

The scheme leads to the following distribution over the charged particles:

$$W_{n_c}^{\pi^- p} = \mu P_{(n_c-2)/2}(\alpha'') + \nu P_{(n_c-4)/2}(\alpha''),$$

$$W_{n_c}^{\pi^- n} = f_1 P_{(n_c-1)/2}(\alpha'') + f_2 P_{(n_c-3)/2}(\alpha'') + f_3 P_{(n_c-5)/2}(\alpha''), \quad (2.6)$$

where the parameters  $\mu$ ,  $\nu$ ,  $f_i$  are connected with probabilities of the channels of dissociation, and  $\alpha''$  is the average number of combinations, including charged pairs.

It is seen from the above consideration that the idea of joining two opposite viewpoints on the mechanism at secondary particle production, namely:

i) independent emission;

ii) dissociation (or fragmentation) of leading particles,

may turn out to be rather fruitful.

The simplicity of such a synthetic approach is very attractive. The assumption of uncorrelated production of associations (or clusters) makes it possible to combine the advantages of the models of independent emission with the possibility (it will be shown in the following section) of studying the correlation dependences. Apart from the suggested approach discussed above, still more models are available based on the idea of joining two mechanisms. Note that the old schemes are reconstructed in accordance with the new ideology. To explain, in the multi-Regge scheme, experimental data on charged distributions and correlation dependences, the assumption is made of the necessity of consideration of diagrams with a large number of showers (or clusters) at high energies. The latter is also equivalent to the multicomponent structure of multiplicity distributions.

### 2.3. Scaling Properties of Topological Cross Sections

As previously mentioned, one of the characteristic features of topological cross sections is «broadening» of distribution with increasing energy. Consideration of normalized topological cross sections

$$P(n, s) = \frac{\sigma_n}{\sum_n \sigma_n} \quad (2.7)$$

as a function of the number of particles and energy  $s$  shows that curves strongly change their form with increasing  $s$  (see Fig. 2.3).

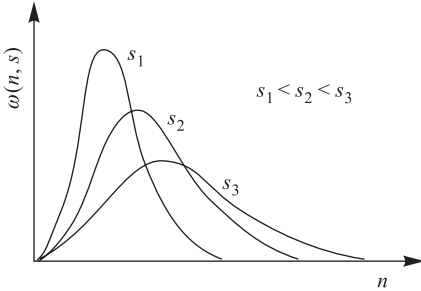


Fig. 2.3. Normalized topological cross section as a function of  $n$  and  $s$

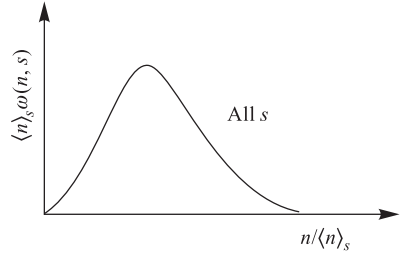


Fig. 2.4. Universal KNO curve

If one plots the function  $\langle n \rangle (\sigma_n / \sigma)$  in the scale  $n / \langle n \rangle$ , it appears that at high energies the family of distributions over multiplicity for various energies  $s$  will be on the same universal curve (see Fig. 2.4).

In fact, it means that the function  $\langle n \rangle (\sigma_n / \sigma)$  depends only on the ratio  $n / \langle n \rangle$ :

$$\langle n \rangle \frac{\sigma_n}{\sum_n \sigma_n} \xrightarrow{s \rightarrow \infty} \Psi \left( \frac{n}{\langle n \rangle} \right). \quad (2.8)$$

The existence of such a regularity was first pointed out by Koba, Nielsen and Olesen. Thus, it is called KNO scaling. The KNO scaling was obtained under the assumption of Feynman scaling, i.e., of scale properties with respect to  $x = 2p_{||} / \sqrt{s}$ . At present, this universal property is thoroughly confirmed by experiments for various types of particle interactions at the accelerators in Serpukhov and Batavia. This favours the statement that at high energies hadron-hadron collisions tend to be similar.

Note that at asymptotically high energies

$$\begin{aligned} \sum_n n^q \frac{\sigma_n(s)}{\sigma} &\sim \int dnn^q \frac{\sigma_n(s)}{\sigma} \approx_{s \rightarrow \infty} \int dnn^q \frac{1}{\langle n \rangle} \psi \left( \frac{n}{\langle n \rangle} \right) = \\ &= \langle n \rangle^q \int dz z^q \psi(z), \quad q \ll \langle n \rangle; \end{aligned}$$

i.e., dependence (2.8) may be given in the form

$$\langle n^q \rangle \xrightarrow{s \rightarrow \infty} C_q \langle n \rangle^q.$$

Thus, universality of Eq. (2.8) is equivalent to the statement that the ratio of moments  $C_q = \langle n^q \rangle / \langle n \rangle^q$ ,  $q = 1, 2, \dots$  does not depend on energy. Such a dependence, as was mentioned above, yields in models of independent

emission (i.e., at SR correlations), where

$$\langle n \rangle \sim \ln s, \quad \langle n^2 \rangle \sim \ln^2 s \dots$$

However, the KNO scaling describes such processes in which one cannot fail to take into consideration the LR correlations. Most probably, a mechanism leading to the KNO behaviour of distributions unites many components, which leads to a nontrivial disappearance of the dependence  $C_q = \langle n^q \rangle / \langle n \rangle^q$  at sufficiently high energies. Various modifications of the KNO scaling, derivation of this regularity from different approaches, and fit by empirical functions are intensively discussed at present in many theoretical and experimental works. We shall return to some of these questions when considering inclusive or semi-inclusive reactions.

Note that the KNO scaling is one of the most interesting examples of a general principle of automodelity in hadron-hadron interactions at high energies. Further investigation of this regularity and of divergence from it makes it possible to understand the dynamics of multiparticle processes more profoundly.

#### REFERENCES

1. *Wroblewski A.* Rapporteurs talk at the Kiev Conference, 1970.
2. *Wang C. P.* // *Nuovo Cimento A.* 1969. V. 64. P. 546; *Phys. Rev.* 1969. V. 180. P. 1463; *Phys. Lett. B.* 1969. V. 30. P. 115.
3. *Chew G. F., Pignotti A.* // *Phys. Rev.* 1968. V. 176. P. 2112.
4. *Feynman R. P.* // *Rev. Mod. Phys.* 1947. V. 20. P. 376.
5. *Bogolubov N. N.* // *Dokl. Akad. Nauk SSSR* (in Russian). 1954. V. 99. P. 225.
6. *Bogolubov N. N., Shirkov D. V.* Introduction to the Theory of Quantized Fields. N. Y.: Interscience, 1959; M.: Nauka, 1973.
7. *Barbashov B. M., Kuleshov S. P., Matveev V. A., Pervushin V. N., Sissakian A. N., Tavkhelidze A. N.* // *Phys. Lett. B.* 1970. V. 33. P. 484.
8. *Sissakian A. N.* Preprint, Research Institute for Theoretical Physics. University of Helsinki, 1974.
9. *Kuleshov S. P., Matveev V. A., Sissakian A. N.* // *Fizika* (Zagreb). 1973. No. 5. P. 67; IRB preprint. Zagreb, 1972.
10. *Grishin V. G., Jancso O., Kuleshov S. P., Matveev V. A., Sissakian A. N.* JINR, E2-6596. Dubna, 1972; *Nuovo Cimento Lett.* 1973. V. 8. P. 590.
11. *Amaglobeli N. S., Mitryushkin V. K., Sissakian A. N., Tsvitvadze E. T.* JINR, P2-7752. Dubna, 1974.
12. *Dubna-Budapest-Bucharest-Warsaw-Cracow-Serpukhov-Sofia-Tbilisi-Ulan-Bator-Hanoi Collaboration.* JINR, P1-6491. Dubna, 1972; *Yad. Fiz.* (in Russian). 1972. V. 16. P. 989; JINR, P1-6928, P1-7267, P1-7268. Dubna, 1973.
13. *Dushutin N. K., Maltsev V. M.* JINR, P2-7676. Dubna, 1974; JINR, E2-7276. Dubna, 1973.

14. *Roy D. P.* Talk at the IV Int. Symp. on Multiparticle Hydrodynamics, Pavia, 1973.
15. *Koba Z., Nielsen H. B., Olesen P.* // Nucl. Phys. B. 1972. V. 40. P. 317; Phys. Lett. B. 1972. V. 38. P. 25.
16. *Olesen P.* Talk at the IV Int. Symp. on Multiparticle Hydrodynamics, Pavia, 1973.
17. *Slepchenko L. A.* Lecture at Tbilisi School on Elementary Particle Physics, 1973.
18. *Markov M. A.* Neutrino (in Russian). M.: Nauka, 1967.
19. *Matveev V. A., Muradyan R. M., Tavkhelidze A. N.* JINR, P2-4572. Dubna, 1969.
20. *Muradyan R. M.* Automodelity in Inclusive Reactions. P2-6762. Dubna, 1972.
21. *Arestov Yu. L., Moiseev A. M.* IHEP SPC 74-72. Serpukhov, 1974.

### 3. CORRELATION DEPENDENCES

#### 3.1. The Problem of Correlations

Correlation dependences in multiparticle production processes can be conditionally separated into two groups. The first group consists of correlations between the different parameters of a single particle. For instance, the dependence of  $p_{\perp}$  on  $p_{\parallel}(x)$ .

The second group of correlation effects arises in studying the two-particle distributions in the inclusive experiments. To this group one can relate the dependences between different particles; for instance, the correlations between neutral and charged particles, found only at  $E \geq 25$  GeV, the problem of factorization in the distributions of different particle contributions (it is usually connected with the problem of deviation from independent emission) and a number of other effects. Two-particle, three-particle, ... correlations are considered. There exist many reasons as to the appearance of correlations. Among them, the production of associations, clusters and other dynamics phenomena are important. There may exist other less evident reasons.

In the present section we consider mainly the problem of two-particle correlations, and especially the relationship between charged and neutral particles, since these effects occur in the latest investigations of modern accelerators. We shall try to interpret these phenomena from the viewpoint of multicomponent description of multiparticle processes, since this makes it possible to understand the nature of such correlations from the viewpoint of an important hypothesis of clusterization in multiparticle production.

### 3.2. Two-Particle Correlations

If one considers an arbitrary multicomponent reaction

$$a + b \rightarrow p_1 + p_2 + \dots + p_n, \quad (3.1)$$

then the invariant  $n$ -particle cross section can be written in the form

$$f_n(s, \mathbf{p}_1, \dots, \mathbf{p}_n) = \left( \prod_{i=1}^n E_i \right) \frac{d^{3n} \sigma_n}{\prod_{i=1}^n d^3 p_i}, \quad (3.2)$$

where  $E_i$ ,  $\mathbf{p}_i$  are the energy and three-dimensional momentum of  $i$ th secondary particle, respectively, and  $s = (p_a + p_b)^2$  the familiar Mandelstam variable. The corresponding distribution density may be obtained by separating  $f_n$  into total inelastic cross sections  $\sigma_{\text{inel}}$ :

$$\rho_n(s, \mathbf{p}_1, \dots, \mathbf{p}_n) = \frac{1}{\sigma_{\text{inel}}} f_n(s, \mathbf{p}_1, \dots, \mathbf{p}_n). \quad (3.3)$$

In the present section we consider only two-particle distributions.

If all the particles are independent, then the  $\rho_2$  distribution is simply connected with the one-particle distribution:

$$\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1) \rho_1(\mathbf{p}_2). \quad (3.4)$$

However, if here are correlations between particles 1 and 2, then simple factorization is not present; i.e., it becomes necessary to introduce the correlation term:

$$\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1) \rho_1(\mathbf{p}_2) + C_2(\mathbf{p}_1, \mathbf{p}_2), \quad (3.5)$$

where  $C_2$  is the two-particle correlation function. The meaning of  $C_2$  is that it is a measure of the influence of particle 1 (with momentum  $\mathbf{p}_1$ ) on the probability that another particle 2 has a momentum  $\mathbf{p}_2$  for any distribution over momenta of the remaining particles.

The correlation function determined in the rapidity space

$$C_2(y_1, y_2) = \frac{1}{\sigma} \frac{d\sigma}{dy_1 dy_2} - \frac{1}{\sigma^2} \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2} \quad (3.6)$$

is widely used, where  $\sigma$  is the cross section for the given class of events. Sometimes it is convenient to consider the given correlation function

$$R_2(y_1, y_2) = C_2(y_1, y_2) \sigma^2 \left/ \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2} \right. \quad (3.7)$$

The two-particle correlation function  $C_2$  is simply related to  $f_2$ , i.e.,

$$f_2 = \int C_2 \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} = \begin{cases} \langle n(n-1) \rangle - \langle n \rangle^2, \\ \langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle, \end{cases} \quad (3.8)$$

where we have considered the distributions over multiplicity; i.e.,  $f_2$  is the completely integrable correlation function  $C_2$ .

Note that the models of a multiperipheral type (i.e., the models with the SR correlations) predict a logarithmic dependence of the function  $s$  on  $f_2$ :

$$f_2 \sim a \ln s$$

and the diffraction dissociation approach (where the LR correlations are taken into account) gives the power dependence

$$f_2 \sim As^{1/2}.$$

In the multicomponent description, we separate the contributions of different mechanisms into multiparticle cross sections. In this case the behaviour of correlation functions is determined by superposition of the correlators, corresponding to each of the mechanisms. Their concrete form depends on the method of realization of the multicomponent approach. In particular, one may consider decomposition of the type

$$\sigma_n = \sum_a \sigma_n^{(a)}, \quad (3.9)$$

where

$$\sum_n \sigma_n^{(a)} = C_a \sigma, \quad a = 1, 2, \dots$$

Contributions to the average multiplicity and higher distribution moments are received for different mechanisms separately:

$$\langle n \rangle_a = \frac{\sum_n n \sigma_n^{(a)}}{\sum_n \sigma_n^{(a)}}, \quad f_2 = \langle n(n-1) \rangle_a - \langle n \rangle_a^2,$$

where  $a = 1, 2, \dots$ , and the total (observed) quantities are correspondingly equal to

$$\begin{aligned} \langle n \rangle &= \sum C_a \langle n \rangle_a, \\ f_2 &= C_1 f_2^{(1)} + C_2 f_2^{(2)} + \dots + C_1 C_2 (\langle n \rangle_1 - \langle n \rangle_2)^2 + \dots, \end{aligned} \quad (3.10)$$

where  $\sum_a C_a = 1$ .

The <sup>a</sup> formula

$$C_2 = a_d C_2^d + a_\pi C_2^\pi + \frac{a_d}{a_\pi} \left[ \frac{1}{\sigma} \frac{d\sigma}{dy_1} - \frac{1}{\sigma_d} \frac{d\sigma_d}{dy_1} \right] \left[ \frac{1}{\sigma} \frac{d\sigma}{dy_2} - \frac{1}{\sigma_d} \frac{d\sigma}{dy_2} \right] \quad (3.11)$$

is widely used for the two-particle correlation function ( $a_\pi + a_d = 1$ ).

This formula shows the character of the correlation function in the case of the two-component description, i.e., when the inelastic collisions may be described by the fraction  $a_d$  of the diffraction dissociation processes and

the fraction  $a_\pi$  of the pionization process (processes with SR correlations are often so designated).

Note that in this case, as is seen from (3.11), the resulting two-particle correlation function  $C_2$  is not an average quantity of  $C_2^\pi, C_2^d$  calculated for each of the components. It may be sufficiently large even if the two-particle correlation functions are very small for each of the components taken separately. It is sufficient to assume the one-particle distributions to be different for both components in order that the last term in (3.11) should be large.

Note that a large number of correlation functions and parameters have been proposed for consideration. We shall determine only the widely accepted ones.

We wish to point out some experimental data. The experimental values for the function  $f_2$  obtained in the  $pp$  interaction are shown in Fig. 3.1.

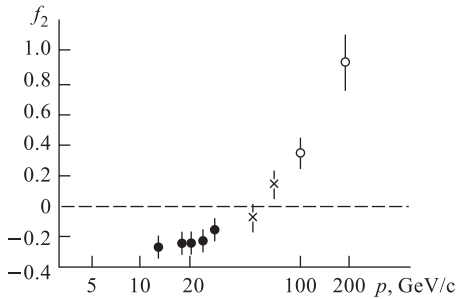


Fig. 3.1. Experimental values of  $f_2$  as a function of energy (for  $pp$  interaction)

This curve has a rather characteristic trend showing that independent emission in  $pp$  collisions (the Poisson distribution over multiplicity) occurs only at  $E_0 \sim 50$  GeV. Here  $f_2(s) = 0$ . The integrated correlation function has small but negative values, corresponding to approximately independent emission (for instance, the uncorrelated jet model) at  $E_0 < 50$  GeV. At energies larger than 50 GeV, the two-particle

correlation parameter  $f_2$  increases rather rapidly. Note that multicomponent description will be most useful in describing this energy region (see, for instance, (3.11)). In 1972 Ganguli and Malhotra considered the dependence of the two-particle structure function  $\langle n(n-1) \rangle$  on  $s$ . They compared experimental data in a large energy interval with the predictions of the limiting fragmentation model (Benecke, Chou, Yang, Yen) and the multiperipheral model (Horn, 1972). The first model predicts the dependence  $\langle n(n-1) \rangle \sim \sqrt{s}$ , the second one predicts  $\sim \ln s$  or  $\sim \ln^2 s$ . If one assumes  $\langle n \rangle \sim \ln s$ , then in the asymptotic region the quantity  $\langle n \rangle / D$  is expected to be proportional to  $\ln s / s^{1/4}$  in the limiting fragmentation model and to  $(\ln s)^{-1}$  in the multiperipheral model. The authors concluded that the multiperipheral model fits better the dependence of  $\langle n(n-1) \rangle$  and  $\langle n \rangle / D$  on  $s$ .

In the range of energies with data available from the bubble chamber there is little difference between the two models, and the conclusion is based on the data from cosmic ray research at  $E \sim 2 \cdot 10^4$  GeV.

However, this argument for the model with SR correlations is not essential due to a weak sensitivity of the studied dependences of the function  $\langle n(n-1) \rangle$  to experiment. The dependence of the dispersion  $D$  on  $\langle n \rangle$  is more critical in this relation.

Consider the following example.

If the SR correlations dominate and do not depend on energy, then  $f_2$  determined according to (3.8) by the integral

$$f_2 = \int C_2 dy_1 dy_2 \quad (3.12)$$

receives its main contribution in the integration region from diagonal  $y_1 \approx y_2$  and has an order  $f_2 \sim \ln s$  due to the fact that the surface of kinematic region in the  $y_1, y_2$  plane broadens with energy proportional to  $(\ln s)^2$ . As has already been mentioned, in the models with SR correlations an average multiplicity increases with energy in proportion to  $\ln s$ . Thus, for the integrated two-particle function we have  $f_2 \sim \langle n \rangle$ .

Correlation (3.12) leads to the following dependence of  $D$  on  $\langle n \rangle$ :

$$D^2 = f_2 + \langle n \rangle \sim \langle n \rangle \quad (3.13)$$

However, this dependence is not confirmed experimentally. This may be seen in Fig. 3.2.

Linear dependence between  $D$  and  $\langle n \rangle$  holds for all currently available  $pp$  data. It is confirmed, though with different slopes, in the data on meson-proton collisions (see Fig. 3.3).

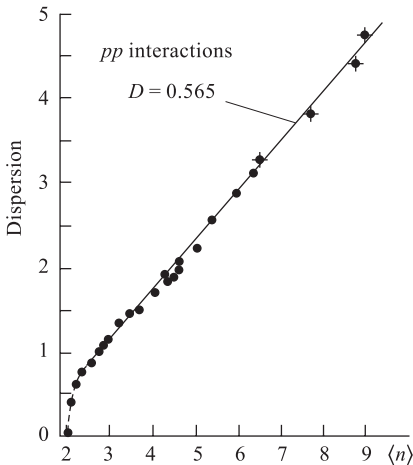


Fig. 3.2. Dependence of  $D$  on  $\langle n \rangle$  for  $pp$  interactions

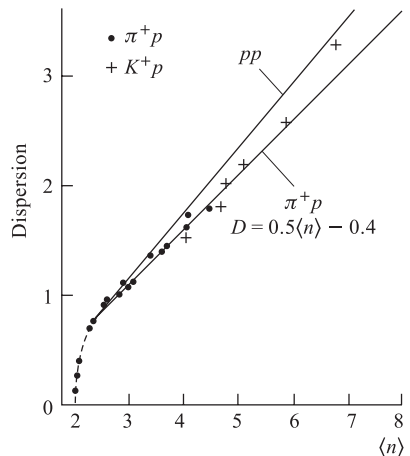


Fig. 3.3. Dependence of  $D$  on  $\langle n \rangle$  for  $\pi p$  and  $K p$  interactions



Note, however, that the assumption of energy dependence of SR correlations allowed Bialas (1973) to construct a model where linear dependence between  $D$  and  $\langle n \rangle$  holds approximately in some region, including almost ISR energies (see Fig. 3.4).

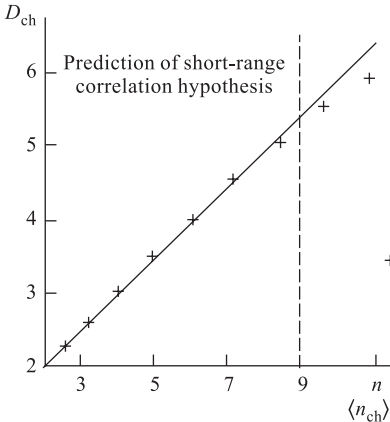


Fig. 3.4. Prediction of the hypothesis on energy dependence of SR correlation (by Bialas)

When considering the SR correlations it is convenient to use the concept of clusters, which appeared in the gas theory. If there is SR interaction between the particles, then it is natural to consider grouping of particles into clusters, i.e., into the particle associations which are sufficiently close to permit them to interact. We can explain this by the picture shown in Fig. 3.5.

Here, particles 1, 2, 3 make one cluster, 4, 5, 6, 7 make another cluster, and particle 8 alone makes a cluster. If particle 8 is moved to the position indicated by the arrow, it will interact with particles 1 and 4 and we shall have one

large cluster. As a matter of fact, we have introduced SR interactions between clusters. Thus, if the clusters interact in the SR way, they merge into one large cluster. Clusters in the above sense may be introduced at the SR interactions. For elementary particles there are SR interactions in the rapidity space, thus consideration of clusters is justified.

To do this in terms of associations of clusters and concrete dependences in multiparticle production, it is necessary to make some additional assumptions concerning the character of their production and their quantum numbers. Concrete of the notion of hadron associations has been considered above in the multicomponent model of two mechanisms.

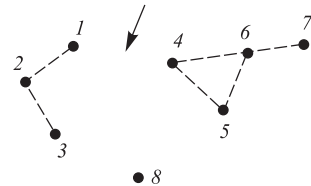


Fig. 3.5.

### 3.3. Neutral-Charged Correlations

Consider the charged-neutral correlations between secondaries making use of the model of two mechanisms.

Now we turn to the model of two mechanisms. For simplicity, we do not take into account strange particles. As has already been mentioned, the initial assumptions of the model are:

i) Dissociation of the leading particles with local conservation of isospin, and

ii) Independent production of associations (see Fig. 3.6) lead in the given case to the distribution

$$W_{n_{\pm}, n_0, n_3}^i = a_i P_{n_{\pm}}(a_{\pm}) P_{n_0}(a_0) P_{n_3}(b), \quad a_{\pm} = 2a_0 \equiv a. \quad (3.14)$$

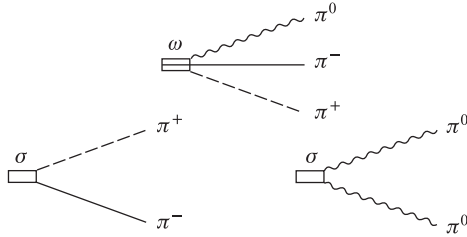


Fig. 3.6.

Taking into account that a number of neutral pions may be presented in the following way (see formula 2.6 and Table 2.1):

$$n_{\pi^0}^i = 2n_0 + n_3 + \ell_{\pi^0}^i,$$

one can easily obtain the average number of neutral pions

$$\langle n_{\pi^0} \rangle_{n_c} = \frac{2 \langle n_0 \rangle_{n_c} + \langle n_3 \rangle_{n_c} + \langle \ell_{\pi^0} \rangle_{n_c}}{W_{n_c}}. \quad (3.15)$$

Formulae (3.15) and (3.14) lead to a linear correlation between the average number of neutral particles and the number of charged particles:

$$\langle n_{\pi^0} \rangle_{n_c} = k_1 + k_2(n_c - \bar{n}_c), \quad (3.16)$$

where

$$k_1 = a + b + a_2, \quad k_2 = \frac{b}{2(a+b)}$$

and an average number of charged particles

$$\bar{n}_c = \begin{cases} 2(a+b) + 2 & (\text{for } \pi^- p \text{ collisions}), \\ 2(a+b+a_2) + 1 & (\text{for } \pi^- n \text{ collisions}). \end{cases}$$

The case of  $\pi^- N$  interactions is particularly considered here, in order to illustrate quantitative comparison of the model with experimental data, obtained at Serpukhov with a two-meter propane chamber irradiated with 40 GeV  $\pi^-$  mesons. The results are given in Fig. 3.7.

Good agreement with experiment ( $\chi \cong 0.5$  on one degree of freedom) confirms the prediction of the model about the linear form of correlation

$$\langle n_{\pi^0} \rangle_{n_c} = A + B n_c. \quad (3.17)$$

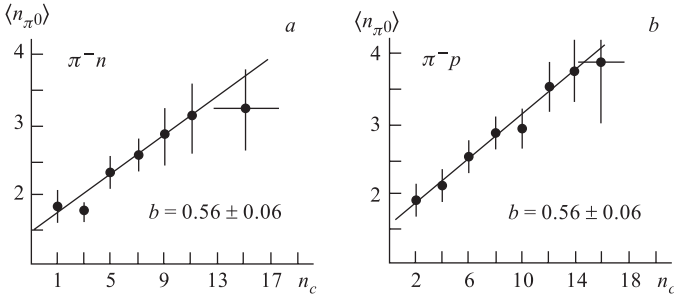


Fig. 3.7. Comparison of the TMP model with the experimental data at  $p = 40 \text{ GeV}/c$

One of the conclusions of the model is that the slope does not depend on the type of colliding particles ( $B_{\pi^- n} = 0.16 \pm 0.02$ ,  $B_{\pi^- p} = 0.15 \pm 0.02$ ). It is seen from (3.16) that the slope is expressed in terms of the parameters of independently produced clusters. If one assumes that the probability of production of multiparticle clusters increases with increasing energy, then one obtains increase in the slope with energy.

Indeed, the slope (see formula (3.16)) is connected with the relation of average numbers of the hadron associations (clusters):

$$B = \frac{1}{2} \frac{\bar{N}(\omega \rightarrow \pi^+ \pi^- \pi^0)}{\bar{N}(\sigma \rightarrow \pi^+ \pi^-) + \bar{N}(\sigma \rightarrow \pi^+ \pi^- \pi^0)}. \quad (3.18)$$

As extreme cases (in the given assumption), from (3.18) it follows:

1) at  $\bar{N}(\omega) \gg \bar{N}(\sigma)$ ,  $s \gg s_{\text{threshold}}$

$$B \rightarrow \frac{1}{2};$$

2) at  $\bar{N}(\omega) \ll \bar{N}(\sigma)$ ,  $s \ll s_{\text{threshold}}$

$$B \rightarrow 0.$$

These conclusions are consistent with experimental data. The experimental data at ISR in  $pp$  collisions ( $E \sim 2000 \text{ GeV}$ ) also demonstrate a dependence of the type (3.17) with the slope  $B \sim 1/2$ . The absence of such a correlation at low energies means that  $B \sim 0$ . By using the multicomponent distribution (2.6) the scheme presented above can easily be extended to the case of multiparticle production involving strange particles.

The model of two mechanisms in this case gives a distribution over the number of charged particles in the form of the superposition of Poisson functions, and predicts correlations between multiplicities  $K^+$  and  $K^-$  as well as between  $K^0$  and  $\bar{K}^0$  mesons. The average number of  $K^0$ ,  $\Lambda^0$

and  $\Sigma^0$  in the cases considered below (when the production of clusters off three heavy strange particles is hardly probable) is independent of the number of charged particles in  $\pi^-p$  collisions and reaches its constant value at sufficiently large number of charged particles in  $\pi^-n$  collisions. The processing of results from the two-meter propane chamber has produced good agreement of the model with experiment (see Figs. 3.8 and 3.9).

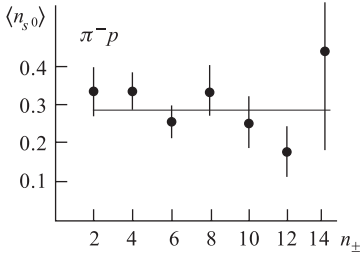


Fig. 3.8. Correlation between  $\langle n_{s^0} \rangle$  and  $n_c$  for  $\pi^-p$  interaction at 40 GeV/c

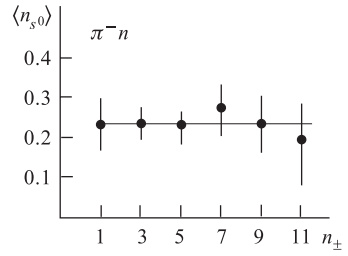


Fig. 3.9. Correlation between  $\langle n_{s^0} \rangle$  and  $n_c$  for  $\pi^-n$  interaction at 40 GeV/c

The model of two mechanisms, realizing the idea of multicomponent description, and its comparison with the results of experiments for the  $\pi^-n$  and  $\pi^-p$  interactions show that at high energies many characteristics of multiparticle processes for various collisions have a tendency to be similar. Such a tendency is observed experimentally. In the spirit of the TMP model this looks quite natural: dissociation gives relatively little contribution multiplicity, at increasing energies multiple characteristics are determined by increasing number of clusters (with the tendency to increase weight), which are produced independently of each other and of the leading particles.

Indeed, everything may be more complicated. Most probably, small distinctions in the characteristics of different types of interactions ( $pp$ ,  $\pi p$ ,  $Kp$ ,  $\pi n$ ,  $\bar{p}p$ , ...) will provide a better description of multiparticle production. However, one may hope that the rough scheme being observed at modern energies, as well as its simple and obvious realizations, will provide a convenient framework for future theories.

Note in conclusion that now there are many new approaches to the correlation problem.

In this connection I should like to mention that it was proposed at the same School in 1970 by El. Mihul to use a new variable which is available exclusively for multiparticle production. It is the determinant of the matrix formed from the components of four 4-momenta. Consider the process

$$a + b \rightarrow a_1 + a_2 + a_3 + a_4$$

and

$$\Delta = \det \{P_i^k\}, \quad i = 1, 2, 3, 4; \quad k = 0, 1, 2, 3$$

as only one variable built with the particles of final state. In the center-of-mass system of colliding particles

$$\Delta = \sqrt{s}(\mathbf{p}_1 \times \mathbf{p}_2)\mathbf{p}_3 = \sqrt{s}(\mathbf{p}_1 \times \mathbf{p}_3)\mathbf{p}_4 = \sqrt{s}(\mathbf{p}_2 \times \mathbf{p}_3)\mathbf{p}_1,$$

where  $\sqrt{s}$  is c.m.s. energy. For a fixed  $s$  the variable we consider is the measure of the volume of the parallelepiped of any three 3-vectors of four. The experimental distributions on  $\Delta$  for  $p + p \rightarrow p + p + \pi^+ + \pi^-$  have been performed for ten values of energy between 4.0 and 24.8 GeV. A strong shrinkage with respect to the energy is obtained when they are compared with the phase space distributions.  $\Delta$  equal to zero defines the singular domain of the physical region. So for increasing energy this region becomes dominant.

In connection with this approach it is interesting to find from experimental data the answer to the following questions:

a) Do  $\Delta$  distributions as functions of energy, i.e., depend on the nature of colliding particles or final particles (neutrino production, photon production, etc.)?

b) For four inclusive reactions (four prong events) one can divide the interval of the energy of the four particles in their center-of-mass system  $(p_1 + p_2 + p_3 + p_4)^2$  into intervals of the «fixed» energy. Will the shrinkage be the same with respect to energy for the events corresponding to every certain energy to get the  $\Delta$  histogram?

c) It is important to know from the reactions with more than four particles in the final state for which a few independent determinants exist if they are simultaneously going to zero for a given event. There are  $n(n-1)(n-2)(n-3)/24$  determinants, but not all are independent since

$$\Delta_{i_1, i_2, i_3, i_4} \Delta_{k_1, k_2, k_3, k_4} = \det \{P_i P_k\}_{i=i_1, i_2, i_3, i_4; k=k_1, k_2, k_3, k_4}.$$

d) Finally, since  $\Delta$  is a pseudoscalar, it is interesting to investigate if there exists any asymmetry in the  $\Delta$  distribution with respect to  $\Delta = 0$ . It is possible to perform it for the channels where the four particles in the final state are different; hence they can be uniquely labelled, and the ordering of them permits one to introduce the orientation of the space.

## REFERENCES

1. *Zalewski K.* Rapporteurs talk at the London Conference, 1974.
2. *Murzin V. S., Sarycheva L. I.* Multiple Processes at High Energies (in Russian). M.: Atomizdat, 1974.
3. *Bialas A.* Invited talk at IV International Symposium on Multiparticle Hadrodynamics. Pavia, 1973.

4. *Slepchenko L. A.* Lecture at Tbilisi School on Elementary Particle Physics, 1973.
5. *Ganguli S. N., Malhotra P. K.* Preprint TIFR-BC-72-68. Bombay, 1972.
6. *Benecke J. et al.* // Phys. Rev. 1969. V. 188. P. 02159.
7. *Chou T., Yang C. N.* // Phys. Lett. 1970. V. 25. P. 1072.
8. *Horn D.* // Phys. Rep. C. 1972. V. 4. P. 1.
9. *Ranft J.* Rapporteurs talk at the Leipzig Conference on Multi-particle Hydrodynamics, 1974.
10. *Pokorski S., Van Hove L.* // Nucl. Phys. B. 1973. V. 60. P. 49.
11. *Kuleshov S. P., Matveev V. A., Sissakian A. N.* IRB-TP-72-3. Zagreb, 1972; Fizika (Zagreb). 1973. V. K 5. P. 67.
12. *Grishin V. G., Kuleshov S. P., Matveev V. A., Sissakian A. N., Jančso G.* JINR, E2-6596, F2-6950, D2-7180. Dubna, 1972–1973; Yad. Fiz. (in Russian). 1973. V. 17. P. 1281; Nuovo Cimento Lett. 1973. V. 8. P. 290.
13. *Amaglobeli N., Mitrjushkin V., Sissakian A., Tsvitsivadze E.* JINR, P2-7752, 1974.
14. *Berger E. L., Horn D., Thomas G. H.* NAL, Argonne, 1972.
15. *Horn D., Schwimer A.* CALT, California, 1972.
16. *Mihul El. A.* Lecture at the CERN–JINR School, Loma-Koli, 1970.
17. *Laurikainen K. V., Mihul El. A.* JINR, E2-7819. Dubna, 1974.

## 4. INCLUSIVE AND SEMI-INCLUSIVE PROCESSES

### 4.1. The Problems of Description of Multiparticle Processes

The analysis of the processes of multiparticle production is important for understanding the nature of hadron interactions at high energies. It has considerable difficulties both from the technical point of view and from the viewpoint of kinematical description.

It is necessary to find integral characteristics of inelastic processes which give sufficiently complete information on the hadron interactions at high energies and at the same time are rather simple both for theoretical and for experimental analysis.

Characteristics of such a type were first introduced in 1967 (Logunov, Mestvirishvili, Nguyen Van Hieu). Later a set of processes contributing to these characteristics was called «the inclusive processes». Thus, the first stage of the experiments is mainly concentrated on measurements of the most direct quantities: the inclusive and topological characteristics of the particle production spectra.

It was first experimentally determined at the Serpukhov accelerator (Bushnain et al.) that the ratio of the production probabilities of  $K$  mesons

and antiprotons to the production probabilities of  $\pi$  mesons depends only on the ratio of momenta  $P/P_{\max}$ . The experimental consideration of the scaling invariance at high energies, as well as the difficulties of microscopic description of multiparticle processes (first of all the absence of strict mathematical apparatus), leads to the appearance of phenomenological approaches and models (the parton model, «droplet» model) and, based on them, to the appearance of the hypothesis of limiting fragmentation and scaling (Feynman, Yang).

The latter determine a number of limiting relations and restrictions for the cross sections of inclusive processes, correlations, and other characteristics. The principle of automodelity (Matveev, Muradyan, Tavkhelidze) on the basis of a generalized dimensional analysis makes it possible to classify the scaling relations at high energies.

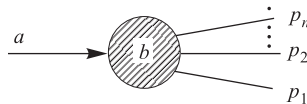
In the present lectures we shall not dwell on the problem of strong interactions, but only would like to remind of the reviews by Jacob; Logunov and Mestvirishvili, and Muradyan (see References).

## 4.2. Semi-inclusive Processes and Their Characteristics

The one-particle inclusive reactions have a number of practical advantages: they are easily obtained experimentally, the study of average values by the particles not fixed in the reaction clears up the collective properties of the system of secondaries. On the other hand, they represent a limited part of the dynamics, as the one-particle characteristics have been integrated over particles and summed over all the inclusive channels. In fact, in the inclusive approach various mechanisms of particle production, responsible for the different phenomena, are missed altogether. There arises the question of explaining the dependence of these effects on the multiplicity. To solve such problems we can make use of the so-called semi-inclusive processes of multiparticle production, i.e., of the characteristics of reactions with fixed multiplicity (topology) without averaging of the inclusive approach, and thus evidently take into account the contributions of different multiplicities to the physical effects.

**4.2.1. Basic Definitions.** Consider the process of particle production as a result of collision at high energies

$$a + b \rightarrow p_1 + p_2 + \dots + p_n + \dots$$

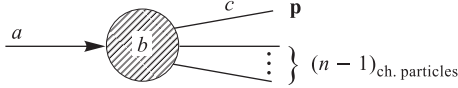


Denote the differential cross section of production of  $n$  charged particles (with a different number of neutral ones) through

$$\begin{aligned} \frac{d\sigma}{d\mathbf{p}_1 d\mathbf{p}_2 \dots d\mathbf{p}_n} &= \\ &= \sum_{k=n+1} \frac{1}{(k-n)!} \int \frac{d\sigma}{d\mathbf{p}_1 d\mathbf{p}_2 \dots d\mathbf{p}_n \dots d\mathbf{p}_k} \prod_{j=n+1}^k d\mathbf{p}_j. \end{aligned} \quad (4.1)$$

Then the semi-inclusive cross section of particle  $C(\mathbf{p})$  production with a given  $(n-1)$  number of charged particles

$$a + b \rightarrow c(\mathbf{p}) + \underbrace{\dots}_{(n-1)\text{ch. particles}} + \dots$$



will be of the form

$$\frac{d\sigma_n^c}{d\mathbf{p}} = \frac{1}{(n-1)!} \int \frac{d\sigma}{d\mathbf{p} \dots d\mathbf{p}_n} \prod_{i=2}^n d\mathbf{p}_i \quad (4.2)$$

with the normalizations (see Sec. 1)

$$\begin{aligned} \frac{1}{n} \int \frac{d\sigma_n^c}{d\mathbf{p}} d\mathbf{p} &= \sigma_n^c, \quad \sum_{n=2} \frac{d\sigma_n^c}{d\mathbf{p}} = \frac{d\sigma^c}{d\mathbf{p}}, \\ \int \frac{d\sigma^c}{d\mathbf{p}} d\mathbf{p} &= \langle n \rangle \sigma, \quad \langle n \rangle \sigma \equiv \sum n \sigma_n, \end{aligned} \quad (4.3)$$

where  $\sigma_n, \sigma$  are the partial (topological) and total (inelastic) cross sections of interaction ( $ab \rightarrow \dots$ ), correspondingly;  $\langle n \rangle$  is the average multiplicity of final particles.

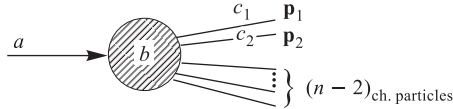
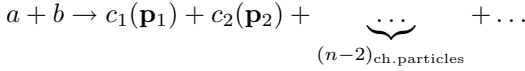
Define now the first moment of the semi-inclusive distribution (4.2)

$$\langle n(\mathbf{p}) \rangle = \left( \sum_{n=2}^{N(s)} (n-1) \frac{d\sigma_n^c}{d\mathbf{p}} \right) / \sum_n \frac{d\sigma_n^c}{d\mathbf{p}}. \quad (4.4)$$

Equation (4.4) defines the average multiplicity of charged particles, produced together (in association) with an extracted fixed particle « $c$ » with momentum  $\mathbf{p}$  and is called the associated multiplicity of charged particles.



Note the correlation character of the introduced value (4.4). In this connection consider the reaction with two (inclusively) extracted particles



and determine the corresponding two-particle spectra: the semi-inclusive distributions with fixed multiplicity

$$\frac{d\sigma_n^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} = \frac{1}{(n-2)!} \int \prod_{i=3}^n d\mathbf{p}_i \frac{d\sigma}{d\mathbf{p}_1 \dots d\mathbf{p}_n} \quad (4.5)$$

and the corresponding two-particle inclusive spectrum

$$\frac{d\sigma^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} = \sum_{n=3} \frac{d\sigma_n^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} \quad (4.6)$$

with the normalizations

$$\begin{aligned} \frac{1}{(n-1)n} \int \frac{d\sigma_n^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} d\mathbf{p}_1 d\mathbf{p}_2 &= \sigma_n, \quad \frac{1}{n-1} \int \frac{d\sigma_n^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} d\mathbf{p}_2 = \frac{d\sigma_n^{c_1}}{d\mathbf{p}_1}, \\ \int \frac{d\sigma^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} d\mathbf{p}_1 d\mathbf{p}_2 &= \sum n(n-1)\sigma_n = \langle n(n-1) \rangle \sigma. \end{aligned} \quad (4.7)$$

Having partially integrated (4.5) over the phase volume of the particle  $c_2(\mathbf{p}_2)$  taking into account (4.7) and

$$\sum_n \int \frac{d\sigma_n^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} d\mathbf{p}_2 = \sum \frac{d\sigma_n^{c_1}}{d\mathbf{p}_1} (n-1) = \int \frac{d\sigma^{c_1, c_2}}{d\mathbf{p}_1 d\mathbf{p}_2} d\mathbf{p}_2 = \langle n(p_1) \rangle \frac{d\sigma^{c_1}}{d\mathbf{p}_1},$$

and using the definition of the two-particle correlation function  $C_2(\mathbf{p}_1, \mathbf{p}_2)$ , we obtain the necessary relation

$$\langle n(\mathbf{p}_1) \rangle = \left( 1 / \frac{1}{\sigma} \frac{d\sigma}{d\mathbf{p}_1} \right) \int d\mathbf{p}_2 C_2(\mathbf{p}_1, \mathbf{p}_2) + \langle n \rangle, \quad (4.8)$$

i.e., in the absence of correlations between the particles  $c_1$  and  $c_2$  the associated average multiplicity does not depend on the momentum  $\mathbf{p}_2$  and

$$\langle n(\mathbf{p}_1) \rangle = \langle n \rangle - 1.$$

Let us also simplify the formula determining the semi-inclusive two-particle correlations, defining as

$$\rho_n(\mathbf{p}) = \frac{1}{\sigma_n} \frac{d\sigma_n}{d\mathbf{p}},$$

$$C_n^{(2)}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{\sigma_n} \frac{d^2\sigma_n}{d\mathbf{p}_1 d\mathbf{p}_2} - \rho_n(\mathbf{p}_1)\rho_n(\mathbf{p}_2), \quad (4.9)$$

$$R_n^{(2)}(\mathbf{p}_1, \mathbf{p}_2) = \frac{C_n^{(2)}(\mathbf{p}_1, \mathbf{p}_2)}{\rho_n(\mathbf{p}_1)\rho_n(\mathbf{p}_2)} = \frac{\sigma_n \cdot d^2\sigma_n/d\mathbf{p}_1 d\mathbf{p}_2}{d\sigma_n/d\mathbf{p}_1 \cdot d\sigma_n/d\mathbf{p}_2} - 1.$$

### 4.3. The Experimental Situation

Now we shall make use of the experimental data on semi-inclusive distributions and give a brief classification of the basic facts.

#### 4.3.1. The One-Particle Spectra with Fixed Multiplicity.

1. The linear growth of semi-inclusive one-particle densities  $\rho_n(\mathbf{p})$  for the fixed  $y(p_\perp)$

(IHEP, FNAL, ISR)

$$\rho_n(y) = \frac{1}{\sigma_n} \frac{d\sigma_n}{dy} \Bigg|_{y \text{ - central}} = A + Bn \quad : \quad pp \rightarrow \pi + X_N; \quad \pi p \rightarrow \pi + X_N$$

see, e.g., Fig. 4.1 (IHEP)

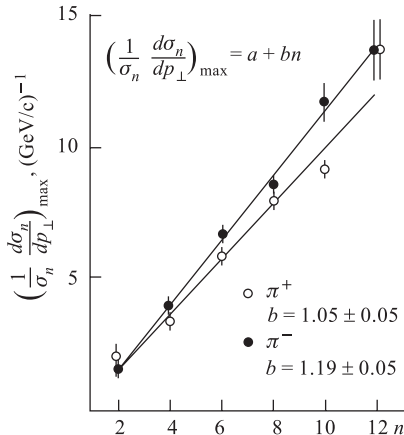


Fig. 4.1.  $\left(\frac{1}{\sigma_n}\right)\left(\frac{d\sigma_n}{dp_\perp}\right)_{\max}$  is distribution over  $n$  (for  $\pi^+\pi^-$ ) in  $\pi^-p$  collision at  $p_\perp = 40$  GeV/c (2-meter propane chamber collaboration JINR-IHEP, U-70 accelerator)

$$\rho_n(\mathbf{p}_\perp) = \frac{1}{\sigma_n} \frac{d\sigma_n}{d\mathbf{p}_\perp} = a + bn : \pi p \rightarrow \pi + X_N.$$

2. The shrinkage of semi-inclusive spectra with increasing multiplicity (FNAL, ISR)

$$\rho_n(y) = \frac{1}{\sigma_n} \frac{d\sigma_n}{dy} = \frac{n}{s_n \sqrt{2\pi}} e^{-y^2/2s_n^2} : pp \rightarrow \pi^- + X_N,$$

where  $s \sim 2/n$ .

(BNL)

$$\left. \begin{aligned} \frac{dN}{dp_\parallel} &= B_\parallel e^{-ap_\parallel}, & a &= (0.39n - 0.23) \text{ GeV}/c \\ \frac{dN}{dp_\perp} &= B_\perp p_\perp^{3/2} e^{-bp_\perp}, & b &= (0.31n + 5.36) \text{ GeV}/c \end{aligned} \right\} \pi^- p \rightarrow \pi + X_N$$

(BNL)

$$\left. \begin{aligned} \frac{dN}{dp_\parallel} &= N_\parallel a_\parallel e^{-a_\parallel p_\parallel}, & a_\parallel &\cong \frac{10 + 5n}{W_{\text{cm}}} \\ \frac{dN}{dp_\perp} &= N_\perp a_\perp^{5/2} p_\perp^{3/2} e^{-a_\perp p_\perp}, & a_\perp &\cong (b + 0.3n) + \frac{6(1+n)}{W_{\text{cm}}} \end{aligned} \right\} pp \rightarrow \pi + X_N$$

(IHEP)

$$\rho_n(y, p_\perp) = N e^{-anm_\perp \text{ch}(y-y')} : \pi^- p \rightarrow \pi^\pm + X_N.$$

#### 4.3.2. The Semi-inclusive Correlations.

1. The central region

$y_1 \sim y_2 \sim 0$  (FNAL, ISR)

$$R_n^{(2)}(0, 0) \sim \frac{1}{n} \quad pp \rightarrow \pi\pi$$

$$C_n^{(2)}(0, 0) \sim n.$$

2. In the range of large  $\Delta y = y_1 - y_2$  the correlations are maximum for small  $n$ . This effect increases with energy (long range) and

$$C_n^{(2)}(y_1, y_2) \sim -n, \quad R_n^{(2)}(y_1, y_2) \sim \frac{-1}{n}.$$

3. The associated multiplicities. The correlations between  $n$  and  $(y, p_\perp)$ .

The associated multiplicity as a function of various variables has been calculated in many experiments up to the ISR energies.

Consider the typical data.

a) There has not been found an essential dependence of  $\langle n(p_\perp) \rangle$  on the transverse momentum of secondaries  $(p, \pi, K, \Lambda)$  for  $p_\perp \leq 1 \text{ GeV}/c$  when  $p_\perp = 19 \text{ GeV}/c$  in the  $pp$  interaction (The Scandinavian group) as

well as in the  $\pi p$  interaction at  $p_{\perp} = 40$  GeV/c, though the same data in the same-opposite selection show an essential dependence on  $p_{\perp}$ . The value  $\langle n(x) \rangle$  ( $\langle n(y) \rangle$ ) is gradually decreasing with the growth of transverse momenta  $x(y)$ .

b) The BNL collaboration points out the increasing dependence of  $\langle n(p_{\perp}) \rangle$  on the transverse momentum of a leading proton in the range  $p_{\perp} \leq 2$  GeV/c and for different missing masses  $MM^*$ . The data from FNAL-ISR confirm this effect in a wide energy range and  $p_{\perp}$  (for a detailed discussion of the range of large  $p_{\perp}$ , see Sec. 5).

c) The associated multiplicity as a function of missing masses, produced with a leading particle, increases according to the same law as the average multiplicity as a function of  $\sqrt{s}$ .

#### 4.4. Theoretical Approaches

**4.4.1. Cluster Models.** Various experimental information on correlations, e.g., data on  $f_2(s)$ ,  $R^{(2)}(y_1 - y_2)$  for approximate validity of the KNO scaling for multiparticle distributions, etc., points out the fact that multiparticle production (most of it in any case) proceeds through multicluster intermediate states.

In particular, the assumption of the independent emission of isotropic clusters makes it possible to understand the positive short-range character of the completely inclusive two-particle correlation functions with respect to rapidities in the central region.

The central idea of this approach is that the hadron associations (clusters) are produced according to definite dynamics and that the secondaries observed are products of the decay of these clusters.

At present it is not yet clear whether clusters have intrinsic dynamics meaning or represent simply a phenomenological method, i.e., suitable initiation of more complicated dynamics.

The cluster models have been extensively studied recently (see review articles of Berger, Ranft) in connection with the experimental information of FNAL-ISR on correlations with respect to rapidities at multiplicity fixed (on semi-inclusive correlations).

We list here some model consequences:

1. It is convenient to split  $\sigma_n$  and  $d\sigma_n/dy$  into «diffractive» and «nondiffractive» parts;
2. The correlation length  $2\delta(C_n^{(2)} \sim e^{-(y_1-y_2)^2/4\delta^2})$  does not depend on  $n$  and  $s$ .
3. In the model of independent clusters:  $(1/\sigma_n)(d\sigma_n/dy) \approx n/y$ .

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\*Note that experimental observation of the linear relation between the average multiplicity and the transverse momentum of one of the final protons was first presented in the paper by Anderson and Collins (1967).

4. The behaviour of semi-inclusive correlations is consistent with experimental data of FNAL-ISR:

$$\begin{aligned} C_n^{(2)}(0,0) &\sim \frac{n}{\log s} f(\langle k \rangle), \\ R_n^{(2)}(0,0) &\sim \frac{\log s}{n} f(\langle k \rangle), \end{aligned} \quad (4.10)$$

where  $\langle k \rangle$  stands for the average number of hadrons in a cluster.

5. The  $n$ -dependence of the semi-inclusive correlation functions reflects the structure of multiparticle distribution inside a cluster.

In the concrete cluster model with diffractive excitation the one-particle distribution at a fixed multiplicity reduces to the following form:

$$\frac{d\sigma_n}{dy} = A_n \int^{N(s)} dM \rho(M) \delta \left( M - \frac{n}{\gamma} \right) \frac{e^{-\lambda^2 m^2 \text{sh}^2(y-\varphi)}}{\text{ch}^2(y-\varphi)}, \quad (4.11)$$

where  $M$  and  $y$  are respectively the cluster mass and rapidity.

In present versions of the cluster diffractive models, agreement of the slow decrease in topological cross sections  $\sigma_n$  and nondecreasing character of spectra relative to rapidities in the central region is achieved if one gives up the assumption of the isotropy of the cluster decay.

Note that if one takes as the cluster decay amplitude a modified distribution of the Bose gas, it is possible to avoid the artificial introduction of nonisotropy. In this case, in particular, observable properties of «shrinkage» of distributions are obtained and, unlike the standard models (DEM) resulting in a fall of the spectra in the central region, an increase in maximal values of distributions is obtained:

$$\begin{aligned} \rho_n(y) &\sim e^{-an \text{sh}^2(y/2)}, \quad \rho_n(y=0) \sim C \sqrt{n}(1+nm), \\ \rho_n(p_\perp) &\sim \frac{a+bp_\perp}{m_\perp^{1/2}} e^{-nb(m_\perp-m)}, \quad \rho_n(p_\perp = \text{max}) \sim cn. \end{aligned} \quad (4.12)$$

#### 4.4.2. Scaling in Semi-inclusive Distributions.

a) *Uncorrelated Production. KNO II.* Keeping to the same ideas that have resulted in the similarity law for multiparticle distributions (see Sec. 2), Koba, Nielsen and Olesen have obtained the law of automodel behaviour for semi-inclusive cross sections  $\rho_n(\mathbf{p})$ . Assuming the noncorrelated particle production (or weak short-range correlations) and the Feynman scaling for the one-particle spectral densities at a fixed multiplicity, they have found the asymptotical formula

$$\frac{1}{\sigma_n} \frac{d\sigma_n(\mathbf{p}, s)}{d\mathbf{p}} \xrightarrow{s \rightarrow \infty} h \left( \frac{n}{\langle n \rangle_s}, x, p_\perp \right) \left[ 1 + o\left(\frac{1}{\langle n \rangle}\right) \right] \quad (4.13)$$

for the reaction  $a + b \rightarrow c(\mathbf{p}) + (n-1)$  charged + anything neutral, where  $x = 2p_{\parallel}/\sqrt{s}$ ,  $\langle n \rangle_s$  is the mean multiplicity at energy  $s$ . Relation (4.13)

predicts that if one compares two or more semi-inclusive experiments at different (enough) energies taking the same values of  $n/(n)_s$ , then the distributions over momenta (in the variables  $x$  and  $p_\perp$ ) normalized to  $\sigma_n$  will be almost equal to each other; i.e., the cross sections  $d\sigma/dx dp_\perp$  for different  $s$  and different topologies but with the same ratio  $n/\langle n \rangle$  should be the same.

Due to the nonrigorous character of the arguments resulting in (4.13), it is interesting to check this relation with models. This has been done in the two cases: (1) the Feynman gas model (Olesen) and (2) the uncorrelated jet model.

In the first case, by using the method of generating functionals<sup>\*</sup>, the proper relations (4.13) are found for semi-inclusive cross sections and this is shown in the example of the reaction  $K^+ + p \rightarrow K^0 + n_{\text{ch}} + \text{anything neutral}$  at  $p_\perp = 5.82, 16 \text{ GeV}/c$ . The spectra are in qualitative agreement with the Feynman gas model within a good accuracy (except for boundary regions of phase space where effects of the energy-momentum conservation laws are important).

Since  $n$  is the discrete variable, a convenient way to check the prediction (4.13) is to obtain an analytical expression that then can be fitted to experimental data. Such an expression:

$$\frac{1}{\sigma_n} \frac{d\sigma_n}{d\mathbf{p}} = C \frac{n}{\langle n \rangle} (1-x)^{\lambda_1 \frac{n}{\langle n \rangle} + \lambda_2} \left[ 1 + O\left(\frac{1}{\langle n \rangle}\right) \right] \quad (4.14)$$

has been found in the uncorrelated jet model. Here  $\lambda_1, \lambda_2$  are constants. Formula (4.14) has been fitted to experimental data at  $p = 19 \text{ GeV}/c$  in the reaction  $pp \rightarrow \pi^+ + (n_\pi - 1)_c + \text{anything neutral}$ .

Applicability of the semi-inclusive scaling (4.13) (KNO II) to the one-particle spectra has been verified experimentally for the corresponding cross sections with pion production in  $pp$  collisions at  $205 \text{ GeV}/c$  (FNAL). A comparison has been made at fixed  $n/\langle n \rangle$  with data at low energies from 13 to  $28.4 \text{ GeV}/c$ . By relation (4.13), it follows that if one takes two energies  $s_1$  and  $s_2$  and two multiplicities  $n_1$  and  $n_2$ , then quantities  $(1/\sigma_n)(d\sigma_n/d\mathbf{p})$  should be equal if  $n_1/\langle n_1(s_1) \rangle = n_2/\langle n_2(s_2) \rangle$  (up to correction  $O(1/\langle n \rangle)$ ).

Though a qualitative agreement holds for such a behaviour (except for the region  $x \approx 0$ ), essential deviations are observed in data on the semi-inclusive scaling. These are considerably larger than for corresponding inclusive cross sections. Analogous results have been obtained for semi-inclusive distributions of pions in  $\pi^- p$  collisions at  $p = 40 \text{ GeV}/c$  (IHEP collaboration).

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<sup>\*</sup>The method of generating functional was first introduced in statistical physics by Bogolubov (1945).

Since  $\langle n(s) \rangle$  is a slowly varying function of energy and  $n/\langle n \rangle$  is roughly constant for a constant multiplicity  $n$  from (4.13), the usual inclusive scaling should be valid in a wide energy interval. This prediction was compared with experiment (Chliapnikov et al.). In the  $K^+p$  interaction at  $p = 5.82, 16 \text{ GeV}/c$  the quantity  $(1/\sigma_n)(d\sigma_n/d\mathbf{p})$  has been found to be independent of energy for  $n = 2, 4, 6$ , though the corresponding  $\langle n \rangle$  change considerably in this energy range.

*b) Strongly Correlated Production.* Experimental data on  $C_n^{(2)}, R_n^{(2)}$  and essential dependence of the associated momenta  $\langle n(\mathbf{p}) \rangle$  on  $\mathbf{p}$  point out considerable correlations in processes of the multiparticle production. Studies of the correlation dependences of average characteristics of hadron production processes can give evidence only of the existence of a certain relation between secondaries. In studying the semi-inelastic characteristics there arises the question: what restrictions on the shape and character of dependence of the one-particle distributions on  $n$  and  $\mathbf{p}$  do result from correlations between the average multiplicity and magnitude of the momentum or transfer momentum?

Consider a semi-inclusive reaction of the type  $a + b \rightarrow$  particle of large  $p_\perp + n_{\text{ch}} +$  anything neutral, where one of the secondaries which receives after interaction a large transverse momentum is produced inclusively.

When choosing a special form of the dependence of the average number on the transverse momentum, one should allow for considerations of a mechanism of multiparticle production.

Proceeding from the assumption of the coherent excitation of particles colliding at high energies (Matveev, Tavkhelidze), one may find that the average number of secondaries grows linearly with the squared transverse momentum transferred:

$$\langle n(p_\perp) \rangle = a + bp_\perp^2.$$

This result for the diffractive production of particles has been obtained in the framework of the straight-line path method. Such behaviour is in qualitative agreement with experimental data obtained in  $pp$  collisions at the laboratory momentum of incident proton  $p_{\text{lab}} \approx 30 \text{ GeV}/c$ .

Furthermore, retaining ideas of the physical similarity seen in a number of observed properties of particle interactions at high energies, we may assume that the shape of the dependence  $\langle n(\mathbf{p}_\perp) \rangle f(\mathbf{p}_\perp)$  will affect the character of asymptotic behaviour of cross sections of the semi-inclusive processes.

Let us assume, for instance, that the semi-inclusive cross sections obey the similarity relations:

$$\frac{d\sigma_n}{d\mathbf{p}_\perp} = A(p_\perp^2)\psi(n/f(\mathbf{p}_\perp)). \quad (4.15)$$

Substituting this relation into formula (4.4) for the associated multiplicity and changing the simulation by integration, we find

$$\langle n(\mathbf{p}_\perp) \rangle = \frac{\sum n F_n(\mathbf{p}_\perp, s)}{\sum F_n(\mathbf{p}_\perp, s)} = \frac{\int^{N_s} n dn \psi(n/f(\mathbf{p}_\perp))}{\int^{N_s} dn \psi(n/f(\mathbf{p}_\perp))} = f(\mathbf{p}_\perp) g(N_s/f(\mathbf{p}_\perp)), \quad (4.16)$$

where  $N_s \sim \sqrt{s}$ .

Thus, the function  $f(\mathbf{p})$  really represents the dependence of the associated multiplicity  $\langle n(\mathbf{p}) \rangle$  on momentum if  $g(N_s/f(\mathbf{p}_\perp)) \rightarrow 1$  for  $s \rightarrow \infty$  and fixed  $p_\perp$ . The deviation from this asymptotic limit may appear only in the region where  $f_{p_\perp}/\sqrt{s} \approx 1$ . If the function  $f_{p_\perp} \approx p_\perp$  has the power asymptotic behaviour, this condition corresponds to relatively small transverse momenta  $p_\perp \sim s^{1/2}$ , i.e., to values of the parameter  $x_\perp = 2p_\perp/\sqrt{s}$  tending to zero with increasing  $s$ .

Note further that the function  $A(p_\perp^2)$  defined by (4.15) can be related to the inclusive cross section

$$\frac{d\sigma}{d\mathbf{p}_\perp} = \sum_n \frac{d\sigma_n}{d\mathbf{p}_\perp} \sim A(p_\perp^2) f(\mathbf{p}_\perp). \quad (4.17)$$

Making use of formulae (4.15)–(4.17), one can easily establish the validity of the following relation (Matveev, Sissakian, Slepchenko):

$$\langle n(\mathbf{p}_\perp) \rangle \frac{d\sigma_n}{d\mathbf{p}_\perp} \bigg/ \frac{d\sigma^{\text{inel}}}{d\mathbf{p}_\perp} = \psi(n/\langle n(\mathbf{p}_\perp) \rangle). \quad (4.18)$$

We stress here that the similarity relation (4.18) analogous to the KNO scaling is based only on general ideas of the physical similarity and not in particular on the assumption of Feynman scaling.

As is known (see the review of experiment), to the decreasing character of the associative multiplicity there corresponds a «shrinkage» of semi-inclusive distributions; i.e. at small  $p_\perp$  the probabilities of production of a large number of particles drop much faster than those for small multiplicities. On the other hand, the growth of  $\langle n(p_\perp) \rangle \sim p_\perp$  corresponds to the transition to a new regime: at increasing  $p_\perp$  the cross sections with large  $n$  became smoother than for small multiplicities — the so-called «broadening» of distributions. Thus, the regions of small and large  $p_\perp$  are clearly separated by essentially different regimes of behaviour both for the inclusive and semi-inclusive cross sections and for the moments of these distributions.

The relation between the semi-inclusive distributions and associated multiplicities in definite combination (4.18) with an essentially different behaviour at small and large transverse momenta indicates a certain universality of the similarity law obtained (scaling law) for diffractive semi-inclusive spectra (4.18).



Thus, relation (4.18) can be considered as a particular manifestation of automodelity specific for a wide class of phenomena in particle interactions at high energies.

*c) Models with Weak Correlations.* We have already mentioned that within the framework of KNO scaling the result (4.13) is valid under the assumption of absence of correlations between the secondaries. The question arises: What will happen if we introduce the correlations? We have partly mentioned such examples when having considered the Feynman gas models and the uncorrelated jet model. Let us consider in more detail the Feynman gas model (Mueller, Olesen) in which only the two-particle correlations are taken into account. Define the function  $\tau_n^1(\mathbf{p})$  which determines the deviation from an uncorrelated case:

$$\tau_n^{(1)}(\mathbf{p}) = \frac{1}{\sigma_n} \frac{d\sigma_n}{d\mathbf{p}} (ab \rightarrow c(\mathbf{p}) + (n-1)_{\text{ch}} + \text{anything neutral}) - \frac{1}{\sigma} \frac{n}{\langle n \rangle} \frac{d\sigma^{\text{inel}}}{d\mathbf{p}} (ab \rightarrow c(\mathbf{p}) + \text{anything}). \quad (4.19)$$

It appears that the scaling law in the form (4.13) is valid for the considered model in the case of the short-range correlations. Then, the function is factorized with respect to the momentum and multiplicity:

$$\tau_n^{(1)}(\mathbf{p}) \simeq H(s, x, p_\perp) \psi(n, s).$$

In agreement with the scaling (4.13) at high energies

$$\begin{aligned} \lim_{s \rightarrow \infty} H(s, x, p_\perp) &= H(x, p_\perp), \\ \lim_{s \rightarrow \infty} \psi(n, s) &= \psi\left(\frac{n}{\langle n \rangle}\right), \quad \frac{n}{\langle n \rangle} \text{ is fixed.} \end{aligned} \quad (4.20)$$

This means the factorization of the semi-inclusive distribution. Note that these results can be obtained when considering the sum rules for the semi-inclusive cross sections and correlations. The factorization of semi-inclusive spectra in a general case can be written in the form

$$\frac{1}{\sigma_n} \frac{d\sigma_n}{d\mathbf{p}} = A(n) f(\mathbf{p}) [1 + \phi(n, \mathbf{p})], \quad (4.21)$$

where  $\phi(n, \mathbf{p})$  is the deviation measure (analogous to (4.19)). It can be written in the form

$$\phi(n, \mathbf{p}) = \left( \frac{1}{\sigma_n} \frac{d\sigma_n}{d\mathbf{p}} \bigg/ n \frac{d\sigma^{\text{inel}}}{d\mathbf{p}} \right) - 1. \quad (4.22)$$

Thus, the functions  $\tau_n(\mathbf{p})$  and  $\phi(n, \mathbf{p})$  may be considered analogous to the correlation functions  $C_n^{(2)}$  and  $R_n^{(2)}$ , respectively (see (4.10)).

Due to a weak decrease (constancy) in the associative multiplicity as a function of the transverse momentum of  $\pi$  mesons, we can come to a conclusion on the smallness of the transverse correlations of charged particles. It concerns the form  $d\sigma_n/dp_\perp$  of distributions. In particular, when analyzing the experimental data on the semi-inclusive distributions of  $\pi^+$  mesons in the  $\pi p$  interaction when  $p = 40$  GeV/c (IHEP accelerator; 2-meter propane chamber, JINR), it was found that these distributions as multiplicity functions are similar in form at different fixed values  $p_\perp$ ; i.e., the parametrization (4.21) holds. It follows that except for the range of small  $p_\perp$  ( $p_\perp \lesssim 0.2$ ) correlation  $\phi(n, p_\perp)$  is weak and holds, with good accuracy, the factorization of the  $n$  and  $p_\perp$  variables

$$\frac{d\sigma_n}{d\mathbf{p}} \simeq F(n)f(p_\perp), \quad F(n) = n\sigma_n, \quad f(p_\perp) = \frac{d\sigma^{\text{inel}}}{d\mathbf{p}}. \quad (4.23)$$

Note that for the semi-inclusive spectra with the noncorrelative  $n \leftrightarrow \mathbf{p}$  dependence (4.23) from the similarity law, there follows the relation of the KNO scaling for the multiplicity distributions

$$\langle n \rangle \sigma_n / \sigma = \psi(n / \langle n \rangle).$$

The relation of the moments of multiplicity distribution with the multiparticle inclusive spectra and the correlation functions made it possible to investigate automodelity properties of distributions over multiplicity to obtain in the case of weak (SR) correlations a number of rather interesting results for the inclusive and semi-inclusive reactions. In particular, for the process

$$a + b \rightarrow c(p_1) + c(p_2) + \dots + c(p_{k+1}) + \text{anything},$$

assuming the existence of scaling for inclusive multiparticle distributions (Chliapnikov, Gerdyukov, Manyukov, Minakata), there was obtained the asymptotic scaling behaviour of the associated moments like

$$\langle n^k(s, \mathbf{p}) \rangle_{s \rightarrow \infty} = a_k(x, \mathbf{p}_\perp^2) \ln^k(s) + O(\ln^{k-1}(s)). \quad (4.24)$$

Thus, for example, the average multiplicity of charged particles  $\langle n(M^2) \rangle$  in the reaction  $a + b \rightarrow c + x_m$  associated with the quantity  $M^2$  of the system  $x_m$  is

$$\langle n(M^2) \rangle = a(M^2/s) \ln s + b(M^2/s), \quad (4.25)$$

where  $a, b$  are the functions depending only on  $M^2/s$ . The experimental test of these relations is of interest.

The presence of weak correlations with the dependence of semi-inclusive spectra, both on multiplicity and on the secondaries momentum, made it possible to assume the existence (experimentally) of the so-called «scaling in the mean» (Dao et al.). The authors confirm that the forms of the  $p_\perp$  and  $p_\parallel$  spectra of produced particles are independent of the

multiplicity or the colliding particle momentum. The data on  $pp$  for different multiplicities between 13 and 300 GeV/c were analyzed. It was found that this hypothesis does not qualitatively contradict the production of  $\pi^-$  mesons. Thus, the cross sections, expressed in terms of normalization variables, must be of universal form:

$$\frac{\langle \nu \rangle_n}{\sigma_n} \frac{d\sigma_n}{dV} \sim \phi \left( \frac{V}{\langle \nu \rangle_n} \right), \quad (4.26)$$

where  $V$  is the transverse or longitudinal variable, and  $\phi(V/\langle \nu \rangle_n)$  is the universal function independent of  $s$  or the multiplicity. Though one has no grounds to consider that this behaviour has a quantitative support, it may serve as a useful approximate parametrization (see Figs. 4.2 and 4.3).

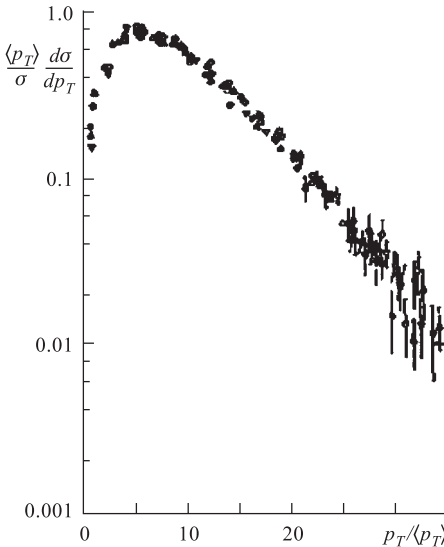


Fig. 4.2. «Scaling in the mean» for transverse variable

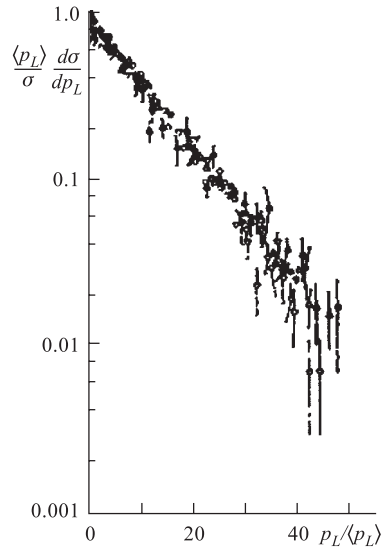


Fig. 4.3. «Scaling in the mean» for longitudinal variable

#### 4.5. Connection between Elastic and Inelastic Processes

It is convenient to study elastic and inelastic processes at high energies by making use of the approach based on the unitarity condition in quantum field theory. The unitarity equation of the amplitude of scattering of the 2-spinless particles has the form

$$\text{Im} T(s, t) = \int dw T(s, t') T^{-1}(s, t'') + F(s, t), \quad (4.27)$$

where

$$t = -(\mathbf{p} - \mathbf{k})^2, \quad t' = -(\mathbf{p} - \mathbf{q})^2, \quad t'' = -(\mathbf{q} - \mathbf{k})^2, \\ s = 4(m^2 + \mathbf{p}^2), \quad |\mathbf{p}| = |\mathbf{k}| = |\mathbf{q}|$$

and

$$dw = \frac{1}{8\pi^2} \frac{d\mathbf{q}d\mathbf{q}'}{2q_0 2q'_0} \delta(p + p' - q - q')$$

are connected with the two-particle phase-space volume. Condition (4.27) is graphically represented in Fig. 4.4.

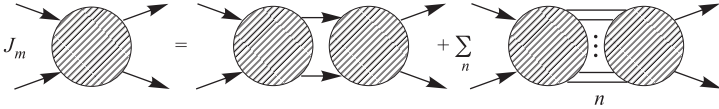


Fig. 4.4. The illustration of the  $s$ -channel unitarity condition

The value  $F_n(s, t) = \sum_n F_n(s, t)$ , which is called the Van Hove overlap function, is the contribution of inelastic (multiparticle) states to (4.27). According to this, the elastic amplitude at high energies is nothing else than the shadow of numbers of inelastic channels. Under definite assumptions on the character of scattering amplitude at high energies, one can obtain in the range  $s \rightarrow \infty$  and  $t/s \ll 1$  the known formula

$$\text{Im } T = \frac{1}{8\pi^2 s} T \cdot T^+ + F. \quad (4.28)$$

Representing the impact parameter, we rewrite (4.28) in the form

$$\text{Im } f(b, s) = \frac{1}{2} |f(b, s)|^2 + \rho(b, s), \quad (4.29)$$

where  $b$  is the impact parameter,  $f(b, s) \equiv T(s, b)$ .

Thus, an important result from the angular momentum conservation law is that the amplitude of elastic scattering with a given impact parameter  $\mathbf{b}$  is produced by absorption into inelastic channels with the same impact parameter. According to the definition of the overlap function

$$\rho(s, b) \equiv \sum_n |T_n(s, b)|^2 \equiv \frac{d\sigma_{\text{inel}}}{db^2}, \quad (4.30)$$

where  $T_n(s, b)$  is the production amplitude of the inelastic state with the  $n$  particles having impact parameter  $b$ . If the phase of elastic amplitude is known, we can solve equation (4.29). In particular, for the parametrization

$$f(b, s) = i(1 - e^{2i\delta(b, s)})$$

we obtain

$$\rho(b, s) = \frac{1}{2}(1 - e^{4\text{Im} \delta(b, s)}). \quad (4.31)$$

The value  $\rho(b, s) = 1/2$  corresponds to the unitarity limit reached in the case of a full absorption.

The approach to the diffraction scattering, considered as a shadow of inelastic processes, possesses a number of interesting questions: How close to the maximum absorption is  $\rho(s, b)$  when  $b = 0$ ? What is its form and average radius? How do individual  $n$ -particle amplitudes construct  $\rho(s, b)$ ? What processes (impact parameters) are responsible for growing cross sections with energy? And so on.

In the papers of the Serpukhov group (Khrustalev, Savrin, Semenov, Troshin, Tyurin) as well as of Bialas, Buras, Dias de Deus, Miettinen, some interesting similarity properties for  $\rho(s, b)$  are found and discussed. An analysis of the ISR experimental data on elastic  $pp$  scattering in the diffraction region makes it possible to draw several important conclusions about properties of inelastic channels.

In particular, the observed growth of the total inelastic cross section occurs, as the authors think, due to peripheral inelastic interactions, and in the energy region under consideration  $\rho(b, s)$  appears to depend only on the ratio  $\pi b^2/\sigma_{\text{inel}}$ . This relation is a manifestation of the geometrical similarity in inelastic processes at high energies:

$$\rho(b, s) \xrightarrow{s \rightarrow \infty} \left( \frac{b^2}{R(s)} \right). \quad (4.32)$$

All the approaches and models studying inelastic collisions in the language of impact parameters and also connections with the character of the behaviour of elastic collisions at high energies have been called geometrical approaches. These models accentuate the geometrical nature of collisions, an elementary act of collisions, productions being considered (in general of weakly correlated particles) to occur at a fixed impact parameter  $b$ . The total inelastic cross section is derived by integrating over all the impact parameters. Accordingly, inclusive (semi-inclusive) characteristics include mixtures of a large number of elementary components with given  $b$ . Following Van Hove, one has

$$F(s, t) = \int d^2b e^{i\Delta \cdot b} \sigma(b), \quad t = -\Delta^2, \quad (4.33)$$

where the total inelastic cross section with a given (4.30)

$$\sigma(b) = \sum_n \sigma_n(b) = \frac{d\sigma_{\text{inel}}}{db^2} \quad (4.34)$$

and the  $n$ -particle production (topological) cross section is written as a superposition of  $n$ -particle cross sections at fixed impact parameters  $b$ :

$$\sigma_n = \int d^2b \sigma_n(b) = \int d^2b \frac{d\sigma_n}{db^2}. \quad (4.35)$$

To obtain the multiplicity distributions we need  $\sigma(b)$ ,  $\sigma_n(b)$ . By using relation (4.33) and its partial analog overlap (semi-inclusive) function  $F_n(s, t)$

$$F_n(s, t) = \int d^2b e^{i\Delta \mathbf{b}} \sigma_n(b) \quad (4.36)$$

and also the corresponding formula for transformation (the Fourier–Bessel transformation), one may find relations between the functions  $\rho(s, b)$  and multiplicity distributions  $\sigma_n(s)$ . In particular, for the contribution of inelastic channels (4.36) obtained in the framework of a probability approach to the description of the scattering processes at high energies (Logunov, Khrustalev) it is shown, proceeding from the universality of function  $\rho(b, s)$  (4.32), that  $\rho(s, b)$  is connected by the Laplace transformation, with the function  $\psi(z, s)$  characterizing the multiplicity distribution in proton–proton collisions at high energies:

$$\psi(z) = \langle n \rangle \frac{\sigma_n}{\sigma}, \quad z = \frac{n}{\langle n \rangle}. \quad (4.37)$$

The three-component model for  $\psi(z)$  found and analyzed further describes well both the multiplicity distribution and the first ten moments of distribution (see discussion concerning difficulties of the two-component description, Sec. 2). It would be of interest to study what follows in the language of geometrical models, from separation of the mechanisms of multiparticle production into a sum of contributions from different components and in this connection to study the relation among such concepts as range of correlation, diffraction, fragmentation and independent emission of produced particles.

The existence of connection between elastic and inelastic processes following from the unitarity condition (4.27), (4.29) is supported also by the fact that the result of two-particle collision is defined by the internal structure of hadrons. The structure of interacting particles, displayed in the smoothness of an effective quasi-potential of interaction, defines also the multiparticle production processes. It is therefore natural to attempt to gain information on some simple characteristics of inelastic processes by using the quantity characterizing the elastic scattering.

Consider now several aspects of this problem.

*a) A connection of Parameters of the Elastic Scattering with Inclusive and Semi-inclusive Distributions.* In considering the model of independent emission of soft pions as a result of collisions of two scalar nucleons, the differential cross section of production of  $\mu$  mesons (semi-inclusive

distribution) can be written in the form (Khrustalev, Savrin, Semenov, Tyurin)

$$\frac{d\sigma_n}{d\mathbf{k}} = \frac{4\pi}{(n-1)!} \sum_{\ell} (2\ell+1) \int \prod_{i=2}^n d\mathbf{k}_i |f_{\ell}(\dots k_i, q)|^2. \quad (4.38)$$

If one introduces the density of meson distribution  $\rho(\dots k_i)$  and the corresponding quantity in  $r$  space, then the assumption on independent emission of mesons, together with partial unitarity, allows one to connect the quantity  $\rho_{\ell}(0)$  with the phase of elastic scattering of two nucleons:

$$\bar{\rho}_{\ell}(0) = 4 \operatorname{Im} \sigma_{\ell}, \quad (4.39)$$

and the cross section for  $n$ -meson production takes the form

$$\frac{d\sigma_n}{d\mathbf{k}} = \frac{\pi}{(2\pi)^3 q^2} \sum_{\ell} (2\ell+1) e^{-\bar{\rho}_{\ell}(0)} \frac{\rho_{\ell}^{n-1}(0)}{(n-1)!} \rho_{\ell}(\mathbf{k}); \quad (4.40)$$

i.e., the inclusive one-particle distribution is

$$\frac{d\sigma}{d\mathbf{k}} = \frac{\pi}{(2\pi)^3 q^2} \sum_{\ell} (2\ell+1) \rho_{\ell}(\mathbf{k}).$$

In the impact parameter representation one has

$$\frac{d\sigma}{d\mathbf{k}} = \frac{1}{(2\pi)^3} \int d^2 b \rho(\mathbf{k}, \mathbf{b}). \quad (4.41)$$

In this way, we arrive at the explicit relation between the inclusive distribution over transverse momentum and the imaginary part of the phase of elastic scattering of two nucleons (simultaneously with the spatial distribution of the hadron matter in the nucleon):

$$\begin{aligned} \frac{d\sigma}{d\mathbf{k}_{\perp}} &= \frac{1}{(2\pi)^3} g^2 \left( \frac{1}{2} \mathbf{k}_{\perp}, 0 \right), \\ g(\xi, 0) &= 2\sqrt{\operatorname{Im} \delta(2\xi)}. \end{aligned} \quad (4.42)$$

*b) Behaviour of the Associated Multiplicity as a Function of  $t = -\Delta^2$  and Elastic Rescattering on a Compound System.* Consider now, in the framework of eikonal approach and impact parameter representation, the interaction of a fast particle with a compound system, the target particle in the final state dissociating into  $n$  constituents. Consideration of the interaction with a compound system allows one (Kvinikhidze, Slepchenko) to obtain information on dependence of the one-particle distribution functions on the number of particles in the final state (on the number of constituents) and in this way to simulate the inclusive and semi-inclusive characteristics of multiparticle processes.

Consider the contribution of a multiple interaction to the one-particle distribution function of final particles. For  $n = 2$ , by definition, one has

$$\frac{d\sigma_{n=2}}{dx d\Delta^{-2}} \cong F^{-1} \int \prod_1^3 \frac{d^3 p_i}{2p_{oi}} \delta^n \left( Q - \sum_1^3 p_i \right) \delta(\Delta - (\mathbf{p}_3 - \mathbf{q}_3)^2) \times \\ \times \delta \left( X - \frac{p_{1z}}{p_{1z} + p_{2z}} \right) |M_{n=2}(x, p_{\perp}, \Delta)|^2, \quad (4.43)$$

where  $F = 2(2\pi)^2 \lambda^{1/2}(s, M_3^2, M^2)$  and  $M_n$  defines contributions of double interaction:

$$M_2 \sim \int d^2 b d^2 b_n e^{i\mathbf{b}\Delta_{\perp} + i\mathbf{p}_{2\perp}\mathbf{b}_{12}} x(\mathbf{b}_{12}, x) f_1(\mathbf{b} + x\mathbf{b}_{12}) f_2(\mathbf{b} - (1-x)\mathbf{b}_{12}), \quad (4.44)$$

where  $\mathbf{b}_{12} = \mathbf{b}_1 - \mathbf{b}_2$ ,  $\mathbf{b} = (1-x)\mathbf{b}_1 + x\mathbf{b}_2$  and  $\mathbf{b}_1, \mathbf{b}_2$  are individual impact parameters of interaction of the fast particle with constituents 1, 2. Substituting (4.44) into definition (4.43), we get

$$\frac{d\sigma_{n=2}}{dx d\Delta^2} = C \int d^2 b d^2 b' e^{i\Delta(\mathbf{b}-\mathbf{b}')} \int d^2 \mathbf{b}_{12} |\psi(b_{12}, x)|^2 \times \\ \times f_1^*(\mathbf{b}' + x\mathbf{b}_{12}) f_2(\mathbf{b}' - (1-x)\mathbf{b}_{12}) f_1(\mathbf{b} + x\mathbf{b}_{12}) f_2(\mathbf{b} - (1-x)\mathbf{b}_{12}), \quad (4.45)$$

where  $f_i(\mathbf{b}, x)$  are the two-particle elastic amplitudes and the wave function  $\psi(x, \mathbf{b}_{12})$  now plays the role of the probability amplitude of dissociation (fragmentation) of a compound system into constituents. From (4.45) one can easily see that the distribution over the squared momentum transfer is defined essentially by rescatterings of an incident particle with a compound system. On the other hand, the distribution over the relative momentum of particles composing a system

$$\frac{d\sigma_{n=2}}{dx dp_{\perp 1}^2} = C' \int d^2 b_{12} d^2 b'_{12} \psi^*(b'_{12}, x) \psi(\mathbf{b}_{12}, x) e^{i\mathbf{p}_{\perp}(\mathbf{b}_{12} - \mathbf{b}'_{12})} \times \\ \times \int d^2 b f_1^*(\mathbf{b} + x\mathbf{b}'_{12}) f_2^*(\mathbf{b} - (1-x)\mathbf{b}'_{12}) \times \\ \times f_1(\mathbf{b} + x\mathbf{b}_{12}) f_2(\mathbf{b} - (1-x)\mathbf{b}_{12}) \quad (4.46)$$

strongly depends on the character of the wave function (i.e., on properties of fragmentation of a target into constituents).



In the general case of an arbitrary number ( $n$ ) of constituents one has

$$\begin{aligned}
 M_n(x_1 \dots x_n, \Delta_\perp, \mathbf{p}_{I\perp} \dots \mathbf{p}_{(n-1)\perp}; s) = \\
 = \int d^2b e^{i\mathbf{b}\Delta} \prod_{i=1}^n \left[ \int d^2b_i e^{i\mathbf{p}_{\perp i} \mathbf{b}_i} f_i(\mathbf{b}_1 + x_i \mathbf{b}) \right] \times \\
 \times \psi_{(n)}(\{x_i, b_i\}) \delta\left(\sum_1^n x_i b_i\right) \delta\left(\sum_i^n x_i - 1\right). \quad (4.47)
 \end{aligned}$$

As has been mentioned above, the distribution  $d\sigma_n/dx d\Delta^2$  corresponding to (4.47) is sensitive to the form of the two-particle amplitude of scattering on constituents  $f_i(b_i, x)$ .

Making different assumptions on the structure of the local two-particle quasi-potentials, one may obtain detailed information concerning the behaviour of a compound system.

In particular, let us assume that in the region of large  $\Delta^2$  the incident particle scatters on all  $n$  constituents of a target at least once. In this case, if the scattering angle is the same for each individual amplitude, then under rather general assumptions on the function  $\psi(\mathbf{b}, x)$  for  $x$  fixed one can show that

$$\frac{d\sigma^n}{d\Delta^2} = C(n, \dots) f^n((\Delta/n)^2), \quad (4.48)$$

where

$$f_i(\Delta) = f_2(\Delta) = \dots = f(\Delta);$$

i.e., (4.48) results in the so-called «broadening» of the effective slope of the  $\Delta^2$  distribution as a function of  $R$  (becomes smoother in the region of large  $\Delta$ ).

Composing the first moment (4.48), i.e., the corresponding associated multiplicity, under the assumptions made above, leading to the automodel behaviour of the dependence  $d\sigma_n/d\Delta^2 \rightarrow f(z)$  (see (4.18)), one may obtain the growing behaviour

$$\langle n(0) \rangle \sim C \Delta^2. \quad (4.49)$$

*c) A Relation between the Slopes of the Elastic Scattering Amplitude and Average Multiplicity of Secondaries.* Let us consider some results concerning multiparticle production in the framework of the straight-line path approximation (SLPA) in quantum field theory. As is known, this approximation has been suggested and developed by the Dubna group (Tavkhelidze, Barbashov, Matveev, Kuleshov, Pervushin, Sissakian) for high energies and fixed momentum transfers. This method leads to a number of interesting results for high-energy multiparticle production processes.

One of them is that the total differential cross section obtained by summing over the number of all emitted mesons is found to be independent of  $t$  in a certain range of secondary particle momenta:

$$\sum_n \frac{d\sigma^n}{dt} = \left( \frac{d\sigma^2}{dt} \right)_0 = \text{const.} \quad (4.50)$$

This is, in a certain sense, analogous to the point-like or automodel behaviour of the cross sections for deep inelastic hadron-lepton processes.

The real content of the result (4.50) consists of the fact that the total differential cross section can change noticeably only by changing  $\Delta t \sim t_{\text{eff}}$ , which greatly exceeds the sizes of the diffraction domain.

To estimate  $t_{\text{eff}}$ , we may make use of the unitarity condition which yields

$$-t_{\text{eff}} \leq \frac{8\pi}{\sigma^{\text{tot}}}. \quad (4.51)$$

This value of  $t_{\text{eff}}$  can be employed for estimating the average number of secondary particles  $\bar{n}_{\text{diff}}$  produced in the diffraction collisions of hadrons at high energies:

$$\bar{n}_{\text{diff}}(s) = \frac{1}{\sigma^{\text{tot}}} \int_0^{t_{\text{eff}}} \frac{d\sigma^{\text{tot}}}{dt} A(s) t dt \leq \frac{\text{const } A(s)}{\sigma^{\text{tot}}}.$$

Thus, the diffraction or peripheral part of the average multiplicity is defined by the parameters of the elastic zero-angle scattering amplitude. The conclusion about the behaviour of the total particle number  $\bar{n}(s)$  can be drawn only under definite assumptions about the contribution of small distances to high-energy multiple production processes. In particular, if the assumption about the disappearance of «pionization» effects at high energies, i.e., the production of secondaries with limited momenta in the c.m.s. of the colliding hadrons, is used, then relation (4.51) will define the behaviour of the total average multiplicity

$$\bar{n}(s) = \frac{\text{const } A(s)}{\sigma^{\text{tot}}} + \tilde{\nu}, \quad (4.52)$$

where  $\tilde{\nu}$  is the number of «leading» particles.

From the viewpoint of attempts to connect the regularities observable in multiple productions with the parameters of elastic scattering, this result can be treated as a contribution to the magnitude of the slope of the elastic scattering amplitude (this contribution is due to the diffraction mechanism). It is known that within the uncorrelated jet model very small values are obtained for the elastic slope, and a mechanism of the multiperipheral type gives very large values to the slope with increasing energy. In this respect it would be rather interesting to estimate the value of  $A(s)$  within the models allowing for the two mechanisms.

Using the well-known restriction on the asymptotic behaviour of the diffraction peak width in quantum field theory (Logunov et al., Eden) from Eq. (4.52), we get in the general case

$$\bar{n}(s) \leq \frac{\text{const}}{\sigma^{\text{tot}}} \ln^2 s. \quad (4.53)$$

This relation is an interesting interpretation of the increase in the strong interaction radius.

Indeed,  $A(s)$  is the «visible» hadron size,  $\sigma^{\text{tot}}$  defines the minimal distance  $R_0$  for which the automodel behaviour holds. One can see from Eq. (4.52) that

$$A(s) \sim R^2 = \pi R_0^2.$$

Thus, the strong interaction radius increases under the condition of the constant cross section, at the expense of the «swelling out» of hadrons associated with the «clouds» of secondary particles.

#### REFERENCES

1. *Logunov A. A., Mestvirishvily M. A.* CERN Preprint TH-1707. Geneva, 1973; Proc. Dubna School of Physics, Sukhumi, 1972. P2-6867. Dubna, 1972.
2. *Logunov A. A., Mestvirishvily M. A., Nguyen Van Hieu.* // Phys. Lett. B. 1967. V. 26. P. 611; Proc. Int. Conf. on Particles and Fields, New York, 1967.
3. *Bushnin Yu. B. et al.* // Yad. Fiz. (in Russian). 1969. V. 10. P. 585.
4. *Matveev V. A., Muradyan R. M., Tavkhelidze A. N.* JINR Preprint E2-6962. Dubna, 1971; Lett. Nuovo Cimento. 1972. V. 5. P. 907; JINR Preprint E2-6638. Dubna, 1972.
5. *Muradyan R. M.* Automodelity in Inclusive Reactions // Lectures delivered at Dubna School of Physics, Sukhumi, 1972. Dubna, 1972.
6. *Jacob M.* Multi-body Phenomena in Strong Interactions // Proc. CERN-JINR School of Physics, Ebeltoft, 1973.
7. *Feynman R. P.* // Phys. Rev. Lett. 1969. V. 23. P. 1415.
8. *Benecke J., Chou T. T., Tang C. N., Yen E.* // Phys. Rev. 1969. V. 188. P. 2159.
9. *Biswas N. N. et al.* // Phys. Rev. Lett. 1971. V. 26. P. 1589.
10. *Elbert J. W. et al.* // Phys. Rev. Lett. 1968. V. 20. P. 124.
11. *Smith D. B.* Preprint UCR2-20632. Berkeley, 1971.
12. *Smith D., Sprafka R., Anderson J.* // Phys. Rev. Lett. 1969. V. 23. P. 1064.
13. *Singer R. et al. (ANL-NAK-Stony Brook Collaboration).* Preprint ANL/MEP 7369.
14. *Singer R. et al.* // Phys. Lett. B. 1974. V. 49. P. 481.

15. *Belletini G. (Pisa–Stony Brook Collaboration)*. High Energy Collision 1973 (Stony Brook) AIP, NY, 1973; Proc. Pavia Conference, 1973. P. 140.
16. *Foa L.* Proc. Aix-en Provence Conf., 1973; J. de Phys. 1973. V. 34. P. 317.
17. *Abdurakhmanov E. et al.* // Nucl. Phys. B. 1974. V. 74. P. 1.
18. *Pratap M. et al.* // Phys. Rev. Lett. 1974. V. 33. P. 797.
19. *Baggild H. et al.* // Nucl. Phys. B. 1974. V. 72. P. 221.
20. *Anderson E. W., Collins G. B.* // Phys. Rev. Lett. 1967. V. 19. P. 201.
21. *Ramanauskas A. et al.* // Phys. Rev. Lett. 1974. V. 31. P. 1371.
22. *Bässer F. W. et al.* // Phys. Lett. B. 1974. V. 51. P. 306, 311.
23. *Anderson E. W.* Talk at the London Conference, 1974.
24. *Anderson E. W. et al.* // Phys. Rev. Lett. 1975. V. 34. P. 294.
25. *Berger E. L.* // Nucl. Phys. B. 1975. V. 85. P. 61.
26. *Berger E. L., Fox G. C.* // Phys. Lett. B. 1973. V. 47. P. 162.
27. *Berger E. L.* // Phys. Lett. B. 1974. V. 49. P. 369.
28. *Pirilä P., Pokorski S.* // Phys. Lett. B. 1973. V. 43. P. 502; Lett. Nuovo Cimento. 1973. V. 8. P. 141.
29. *Bialas A., Fialkowski K., Zalewski K.* // Phys. Lett. B. 1973. V. 45. P. 337.
30. *Hayot F., Morel A.* // Nucl. Phys. B. 1974. V. 68. P. 323.
31. *Quigg C., Thomas G. H.* // Phys. Rev. D. 1973. V. 7. P. 2752.
32. *Pokorski S., Van Hove L.* CERN Report ref TH-1772, 1973.
33. *Le Bellac M., Miettinen H., Roberts G.* // Phys. Lett. B. 1974. V. 48. P. 115.
34. *Chiu C. B., Wang K. H.* // Phys. Rev. D. 1973. V. 8. P. 2929.
35. *Ranft J., Ranft G.* // Phys. Lett. B. 1973. V. 45. P. 43.
36. *Ranft J.* Presented at the V Int. Symp. on Many Particle Hydrodynamics, Leipzig, 1974.
37. *Hayot F., Le Bellac M.* // Nucl. Phys. B. 1975. V. 86. P. 333.
38. *Bourdeau M. F., Salin Ph.* Bordeaux Report PTB-57, 1974.
39. *Darbaidze J., Slepchenko L.* // Bull. Acad. Sci. Georgian SSR. 1975. V. 78, No. 3.
40. *Koba Z., Nielsen H. B., Olesen P.* // Phys. Lett. B. 1972. V. 38. P. 25; Nucl. Phys. B. 1972. V. 43. P. 125.
41. *Ming Ma Z. et al.* // Phys. Rev. Lett. 1973. V. 31. P. 1320.
42. *Olesen P.* Pavia Conference, 1973.
43. *Olesen P.* // Nucl. Phys. B. 1972. V. 47. P. 157.
44. *de Groot E. H., Moller R., Olesen P.* Preprint Nbl-ME-72-7. Copenhagen, 1972.
45. *Bogohubov N. N.* // Selected Works, V. 2, Kiev: Naukova dumka, 1970.
46. *Mueller A. H.* // Phys. Rev. D. 1971. V. 4. P. 150.
47. *Chliapnikov P. V. et al.* // Phys. Lett. B. 1972. V. 39. P. 279.

48. *Matveev V. A., Tavkhelidze A. N.* JINR, E2-5141. Dubna, 1970.
49. *Kuleshov S. P. et al.* // Part. Nucl. 1974. V. 5, No. 1.
50. *Matveev V. A., Sissakian A. N., Slepchenko L. A.* JINR, P2-8670. Dubna, 1975.
51. *Finkelstein J., Kajantie K.* // Nucl. Phys. B. 1975. V. 85. P. 517.
52. *Kvinikhidze A. N., Slepchenko L. A.* JINR, P12-8529. Dubna, 1975.
53. *Slepchenko L. A.* JINR, P2-7042. Dubna, 1973.
54. *Driemin I. M.* // Yad. Fiz. (in Russian). 1973. V. 18. P. 617.
55. *Amati D., Fubini S., Stanghelini A.* // Nuovo Cimento. 1962. V. 26. P. 896.
56. *Amati D., Caneschi L., Testa M.* (See Ref. in Sec. 5).
57. *Chliapnikov P. V., Gerdyukov L. N., Manyukov P. A.* IHEP 74-77. Serpukhov, 1974.
58. *Minakata H.* // Nuovo Cimento Lett. 1974. V. 9. P. 411.
59. *Dao F. T. et al.* // Phys. Rev. Lett. 1974. V. 33. P. 389.
60. *Yang C.* // Nuovo Cimento Lett. 1972. V. 4. P. 352.
61. *Amati D., Cini M., Stanghelini A.* // Nuovo Cimento. 1963. V. 30. P. 193.
62. *Van Hove L.* Lectures given at the Cargese Summer School, 1963.
63. *Van Hove L.* // Rev. Mod. Phys. 1974. V. 36. P. 655; Nuovo Cimento. 1963. V. 28. P. 798.
64. *Garsevanishvili V. R., Matveev V. A., Slepchenko L. A., Tavkhelidze A. N.* // Phys. Rev. D. 1971. V. 4. P. 849.
65. *Barger V.* Report at the London Conference, 1974.
66. *Miettinen H. I.* CERN TH 1864. Geneva, 1974.
67. *Semenov S. V., Troshin S. M., Tyurin N. Ye., Khrustalev O. A.* IHEP Preprint. Serpukhov, 1974.
68. *Logunov A. A., Khrustalev O. A.* // Part. Nucl. 1971. V. 3.
69. *Bialas A., Bialas E.* Preprint CERN TH-1758, 1973.
70. *Khrustalev O. A., Savrin V. I., Semenov S. V., Tyurin N. Ye.* IHEP, 74-102. Serpukhov, 1974.
71. *Khrustalev O. A., Savrin V. I., Semenov S. V.* IHEP, 75-23. Serpukhov, 1975.
72. *Dias de Deus J.* // Nucl. Phys. B. 1973. V. 59. P. 231.
73. *Buras A. J., Dias de Deus J.* // Nucl. Phys. B. 1974. V. 71. P. 481; 1974. V. 78. P. 445.
74. *Sissakian A. N.* Preprint RITP. Helsinki University, 1974.
75. *Govorkov A. B.* JINR Preprint E2-7916. Dubna, 1974.

## 5. PHYSICS AT HIGH $p_{\perp}$

### 5.1. New Regularities in High-Energy Production

In this section we present a review of both experimental and theoretical results on large transverse momentum inclusive processes. An interest in these processes is due to the present experimental possibilities of getting large  $p_{\perp}$  or momentum transfers on new accelerators. On the other hand, there are some theoretical arguments that lead us to expect that the interaction mechanism at large transverse momentum differs essentially from that prevailing in the region of small transverse momentum.

Recent experiments on production of particles with large transverse momentum in hadron-hadron collisions at high energies have revealed definite changes in cross section behaviour compared with that in the small transverse momentum region. Some specific features of the processes in question are as follows: a steep decrease in the cross sections with growing  $p_{\perp}$  at fixed  $s$ , the increase in cross sections with energy at large fixed transverse momenta  $p_{\perp}$ , the appearance of appreciable correlations between particles with large  $p_{\perp}$  and other secondaries, etc. A general view of the behaviour of these processes as a result of analysis of experimental data is given in Table 5.1.

Table 5.1

	Small $p_{\perp}$	Large $p_{\perp}$
$s$ fixed $p_{\perp}$ increases	Rapid decrease of the cross sections with increasing $p_{\perp} \sim \exp(-ap_{\perp})$	Less rapid (less steep) decrease of the cross section with increasing $p_{\perp} \sim p_{\perp}^n$
$p_{\perp}$ fixed $s$ increases	Weak dependence of the cross sections on $s$	Growing cross section with increasing $s$
Particle ratios	Among secondaries the pions dominate $(k/\pi) \sim 10\%$ $\pi^+\pi^- \sim 1$	Heavy particles are produced relatively more copiously $pp$ (collisions) $\pi^+\pi^- > 1$
Associated multiplicity	Weak dependence of the associated multiplicity on $p_{\perp}$ $\langle n(p_{\perp}) \rangle \sim \text{const}$	Growth of the associated multiplicity with increasing $p_{\perp}$ $\langle n(p_{\perp}) \rangle \sim p_{\perp}^{\alpha}$
Correlations	Small	Large positive correlations between two large $p_{\perp}$ particles $C_2(p_{\perp 1}, p_{\perp 2})$

The first indication of surprisingly high cross sections at large  $p_{\perp}$  came from CERN ISR, where the cross section was found to be several orders

of magnitude higher than the extrapolation of an exponential fit to the invariant inclusive cross section found for  $p_{\perp} < 1$  GeV/c.

The data are consistent with a  $p_{\perp}$ -dependence given by

$$E \frac{d^3\sigma}{dp^3} = p_{\perp}^{-N} f(x_{\perp}) \text{ mb/GeV}^2, \quad (5.1)$$

where  $x_{\perp} = 2p_{\perp}/\sqrt{s}$  and  $N \approx 8$ ,  $f(x_{\perp}) \sim e^{-13x_{\perp}}$  for  $pp \rightarrow \pi^0(90^\circ) + \dots$ . The parametrization  $E(d^3\sigma/dp^3) = p_{\perp}^{-N} f(x_{\perp})$  with  $f(x_{\perp}) \sim e^{-ax_{\perp}}$  gives

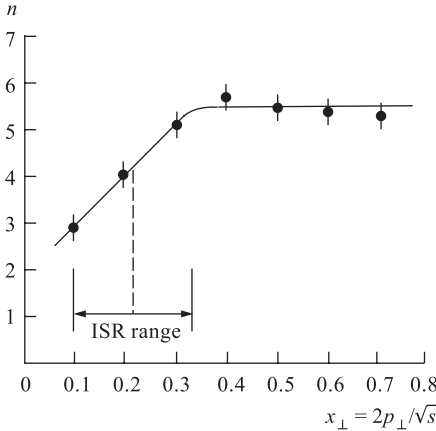


Fig. 5.1. Variation of exponent  $n$  in the parametrization  $E(d\sigma/d^3p) = p_{\perp}^{-2n}$  function of  $x_{\perp}$

a fair description of pion data at large  $p_{\perp}(x_{\perp})$ . However, with different values of the parameters  $N$  and depending on the region of  $p_{\perp}$  and  $s$  over which the fits are made, one can expect  $N \sim 8$  at the ISR for  $x_{\perp} \leq 0.5$  and  $N = 11$  at the larger values of  $x_{\perp}$  at FNAL. Figure 5.1 shows the variation of  $N$  as a function of  $x_{\perp}$  required to bring the charged pion data at different energies of the FNAL together.

Most of the experiments of production of particles with high transverse momenta are purely inclusive. They give only the  $p_{\perp}$ -distribution of secondaries of a given type without telling us what kind of collisions leads to the emission of high transverse momentum particles. A study of particle correlations in high-energy collisions leading to high transverse momentum of secondaries can provide further insight into the dynamics of these processes. Knowledge of the correlations between the high- $p_{\perp}$  particle and the other secondaries in an interaction is thus essential for a complete understanding of the production process at large transverse momentum. The experimental information presently available on such correlations at very high energy comes from ISR measurements involving photons and  $\pi^+$  mesons with large transverse momentum.

To make this problem clearer, the distributions of the charged particles emitted in proton-proton collisions in association with a photon of high transverse momentum were studied at the CERN ISR. The normalized total multiplicity, associated with the photon, is plotted in Fig. 5.2 as a function of  $p_{\perp}$  and for different c.m. energies. The multiplicity increases moderately with  $p_{\perp}$ , the growth being more pronounced at higher energies; above  $p_{\perp} \approx 3$  GeV/c the distribution is flattening. In order to understand such behaviour, the  $p_{\perp}$ -dependence was studied for the multiplicities



Fig. 5.2. Average total multiplicity of charged particles at  $\sqrt{s} = 23, 31, 45, 53$  and  $62$  GeV as a function of  $p_{\perp}$  of the photons detected at  $\Theta_{\text{cm}} = 90^{\circ}$

observed in the two hemispheres: towards the observed photon and away from it (or in the same and opposite directions). Figures 5.3, *a* and 5.3, *b* show the normalized hemisphere multiplicities as a function of  $p_{\perp}$  and for the same c.m. energies as in Fig. 5.2. The multiplicity away from the photon increases linearly with  $p_{\perp}$  and displays little  $s$ -dependence.

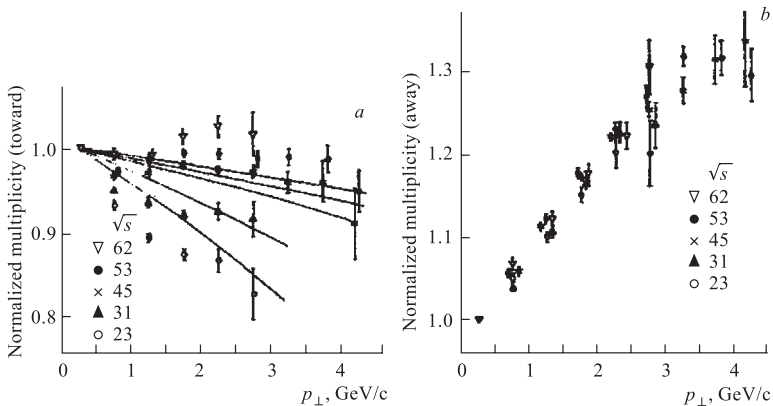


Fig. 5.3. Normalized partial multiplicities as a function of photon transverse momentum in the hemispheres: *a*) towards and *b*) away from the detected photon



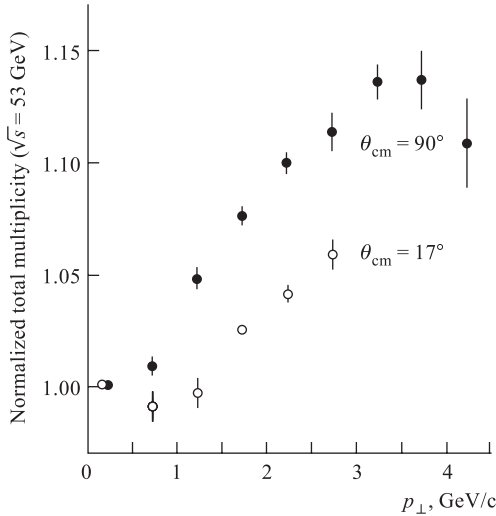


Fig. 5.4. Average total multiplicities of charged particles as a function of  $p_{\perp}$  of the detected photon for  $\Theta_{\text{cm}} = 90^{\circ}$  and  $17.5^{\circ}$ . (See Del Prete, 1974)

The dependence on energy seems to be entirely concentrated in the hemisphere towards the photon. Here the multiplicity decreases with  $p_{\perp}$  at the lowest c.m. energies, while only a slight increase is observed at the highest energy.

The following analysis has been repeated for photons emitted at  $\Theta = 17.5^{\circ}$  ( $y \approx 2$ ) and for c.m. energy  $\sqrt{s} = 53$  GeV. The data are compared with the corresponding  $90^{\circ}$  data at the same  $p_{\perp}$  value. The normalized total multiplicity, shown in Fig. 5.4, displays a rise with  $p_{\perp}$  which is more rapid than that observed at  $90^{\circ}$  and which begins at larger  $p_{\perp}$ .

One can summarize the relevant features of these data as follows:

1. The charged particle multiplicity increases with  $p_{\perp}$  in a wide cone opposite to the detected photon. The growth of multiplicity is roughly linear with  $p_{\perp}$  and energy-independent.

2. The mean multiplicity of charged particles, emitted in the same direction as the photon, generally decreases with increasing  $p_{\perp}$ ; only at the highest ISR energy a tiny rise is observed.

3. At small angles towards the beam directions the multiplicity decreases at all energies.

4. The forward photon data show also some observable increase in multiplicity in the «towards» hemisphere.

A similar effect has been obtained in a somewhat different type of high- $p_{\perp}$  correlation experiment which has been performed at BNL at a relatively low beam momentum of 28.5 GeV/c. In this experiment in the reaction as  $pp \rightarrow p(\pi) + MM$  the charged multiplicity of the fixed missing

mass ( $MM$ ) is measured as a function of the transverse momentum of the fast «towards» proton (pion). As is seen in Fig. 5.5, the multiplicity is roughly independent of  $p_{\perp}$  below 1 GeV/c but rises moderately as  $p_{\perp}$  increases from 1 to 2 GeV/c.

At the CERN ISR the measurements were also performed of  $\pi^0$  correlation as a function of transverse momentum when two neutral pions are detected at large angles on opposite sides of the ISR intersection. One finds that when a large- $p_{\perp}$  pion is detected on one side, the probability of having another  $\pi^0$  with large  $p_{\perp}$  on the opposite side is several orders of magnitude larger than would be expected from uncorrelated pion production.

Here by correlation function we mean

$$R(x_{1\perp}, x_{2\perp}) = \sigma_{\text{in}} \frac{d^6\sigma}{dp_1^3 dp_2^3} \bigg/ \frac{d^3\sigma}{dp_1^3} \frac{d^3\sigma}{dp_2^3}, \quad (5.2)$$

where  $x_{\perp} = 2p_{\perp}/\sqrt{s}$ .

The correlation is seen to increase with increasing  $x_{\perp}$  of either  $\pi^0$ , and  $R$  is as high as  $\sim 10^4$  for  $x_{1\perp} = x_{2\perp} = 0.2$ . This behaviour might be rather a consequence of momentum conservation; however, the function  $R$  for the same-side  $\pi^0$ 's is also positive and large ( $R \sim O(10)$  at  $x_{1\perp} = x_{2\perp} = 0.1$ ), an effect of which cannot be explained by kinematics.

## 5.2. Hadron Structure and High Transverse Momentum

A common view is that the collisions with small  $t = p_{\perp}^2$  are determined by a global structure of hadrons, for example, by the effective range of interactions of order 1 fermi which is related to the slope parameter of the cross section.

It is natural to expect that in collisions with extremely large transverse momentum (or momentum transfers)  $p_{\perp} \sim p_{\parallel} \sim E$ ,  $E \rightarrow \infty$ , an inner local structure of hadrons, which is presently assumed to have «hard» or «point-like» character, becomes more important. In inclusive reactions at large  $p_{\perp}$  the «hard» point-like structure of hadrons can be revealed. From the automodelity viewpoint, processes with large  $p_{\perp}$  are somewhat analogous to the phenomenon of point-like explosion and, therefore, they

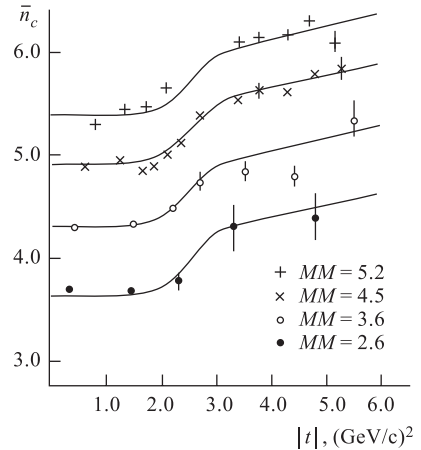


Fig. 5.5. Variation of average charged particle multiplicity,  $\bar{n}_c$ , with  $|t| = p_{\perp}^2$  for four intervals of  $MM$

must be described by the usual dimensional analysis. For large  $s$  by simple dimensional considerations, it follows that instead of the general form  $d\sigma/d\mathbf{p} = f(s, p_{\parallel}, p_{\perp})$  we may have the following asymptotic formula:

$$\frac{d\sigma}{d\mathbf{p}} = p_{\perp}^{-N} f(x, x_{\perp}).$$

In the framework of the quark model using the principle of automodelity it was shown (by Matveev, Muradyan and Tavkhelidze) that the above-mentioned power law at large angles depends essentially on the number of hadron constituents, i. e., on «a degree of complexity» of particles.

Attempts to derive the power character of the asymptotic behaviour of cross sections at large angles ( $p_{\perp}$ ) have been made in a number of recent works under the various model assumptions.

Recently various composite models, such as the quark model, parton model and others, have extensively been used in elementary particle theory. In this connection the problem of a self-consistent-relativistic description of interactions of composite particles is of much importance. An effective method of describing the properties of relativistic composite systems is the Logunov–Tavkhelidze quasi-potential approach in quantum field theory. This approach has turned out to be more suitable in explanation of general regularities of elastic and inelastic (inclusive) processes at high energy and transverse momentum. Quasi-potential formulation in terms of the light-front variables gives us in particular, within rather general assumptions about the behaviour of wave-functions of composite system, the intrinsic power dependence of measured quantities, e.g.,  $d\sigma/dp_{\perp} \sim \sim s^{-N} f(x_{\perp})$ , where  $f$  is a scaled function, in region of high  $p_{\perp} \sim \sqrt{s}$ ,  $s \rightarrow \infty$ . Such behaviour is obtained in the framework of various models in which a hadron is assumed to be a composite object with many point-like constituents. When these constituents are called partons (quarks), there are two possible mechanisms of the interaction of two colliding hadrons: parton–parton scattering and parton interchange. According to that, there exist, in fact, two parton models of the high- $p_{\perp}$  particle production. In the mechanisms of the parton–parton scattering discussed by Berman, Bjorken and Kogut, two colliding hadrons are considered as two colliding beams of partons. The interaction of hadrons occurs when a pair of partons interacts via a gluon exchange, scattered one against another. According to the parton model, the cross section for production of a high- $p_{\perp}$  particle is given by  $E(d\sigma/dp^3) \sim (1/p_{\perp}^4) f(p_{\perp}, s)$ , where factor  $p_{\perp}^{-4}$  comes from the vector gluon exchange in scattering of two partons. The function  $f(p_{\perp}, s)$  is determined by the probability that a parton has the momentum  $x$  and then one can obtain the particle with the transverse momentum  $p_{\perp}$ .

The second possible mechanism of the interaction of two hadrons was discussed by Blankenbecker, Brodsky and Gunion. They assumed that in a collision of two composite objects their constituents can be interchanged.

The probability of finding a parton with a large transverse momentum can be evaluated from the form factor of a hadron and gives the same power law for high- $p_{\perp}$  production  $E(d\sigma/dp) \sim p_{\perp}^{-N} G(p_{\perp}/\sqrt{s})$ , where exponent  $N$  can be calculated when we know the form factor of a pion.

The last group of models to be discussed are the cluster and multiperipheral approaches. Berger and Branson suggested that high- $p_{\perp}$  particles observed in high-energy collisions are the decay products of two clusters which decay anisotropically and their decay products are collimated along the line of flight of clusters. The cluster models predict that high transverse momentum particles are often accompanied by other particles with high transverse momenta, all of them being the decay products of the same cluster.

The production of high transverse momentum particles is strongly damped by the multiperipheral mechanism of particle production. In some recent versions of this model, attempts were made to describe high- $p_{\perp}$  data. A serious criticism of the model is the observed increase in the multiplicity associated with high- $p_{\perp}$  particles. The model requires that masses of many-particle systems should be small and, therefore, multiplicities of these systems to be low. Instead, the multiperipheral model describes correctly the increase in the heavier particle component at high  $p_{\perp}$ .

### 5.3. Associated Multiplicities

As was mentioned in this section, a dependence of the growth of average multiplicities on the transverse momentum was considered (by Matveev, Sissakian and Slepchenko) under the assumption of the auto-model character of the behaviour of semi-inclusive spectra. To demonstrate more clearly the correlation character of the associated multiplicity  $\langle n(p_{\perp}) \rangle$ , one can also introduce the equivalent to definition

$$\langle n(\mathbf{p}) \rangle = \left( 1 / \frac{1}{\sigma} \frac{d\sigma}{d\mathbf{p}_1} \right) \int C_2(\mathbf{p}_1, \mathbf{p}_2) d\mathbf{p}_2 + \langle n \rangle, \quad (5.3)$$

where  $C_2(p_1, p_2)$  is defined in Sec. 1. From (5.3) it is seen, in particular, that if there are no correlations between particles with momenta  $\mathbf{p}$  and  $\mathbf{q}$ , the associated multiplicity for the inclusive production of a particle with momentum  $\mathbf{q}$  does not depend on  $\mathbf{p}$ :

$$\langle n(\mathbf{p}) \rangle = \langle n \rangle_{\text{tot}} - 1.$$

Note that, in accordance with the total momentum conservation, the large transverse momentum  $p_{\perp}$  of the detected particle is balanced by the total transverse momentum of the group of other particles that causes a strong correlation between them.

When choosing a concrete form of dependence of the average number of particles on the transverse momentum, one should consider multiparticle production mechanisms. Proceeding from the assumption on coherent excitation of the particles colliding at high energies, one can obtain that the average number of secondaries increases linearly with the squared transfer momentum:

$$\langle n(p_{\perp}) \rangle = a + bp_{\perp}^2. \quad (5.4)$$

Within the framework of the straight-line path method, this result has been derived for the diffractive production of secondaries. Such behaviour is in qualitative agreement with the experimental data on  $pp$  collisions at the laboratory momentum of the incident proton  $p_{\text{lab}} \approx 30$  GeV/c (see Fig. 5.5).

An analogous phenomenon follows also from the hypothesis of limiting fragmentation, where the growth of  $\langle n \rangle$  with  $p_{\perp}$  arises due to the impossibility of giving large transverse momentum to a hadron without its break-up.

Note that in the multiperipheral model the mean multiplicity decreases logarithmically with growing  $p_{\perp}$ . This decrease is apparently a consequence of the fact that the multiperipheral model corresponds mainly to the mechanisms of secondary production connected with the appearance of hadron clusters in a central region, while the results of the coherent state model (Matveev, Tavkhelidze), the straight-line path method and fragmentation picture correspond to the mechanism of diffractive dissociation of colliding particles. The inclusive cross sections for a diffractive production of high- $p_{\perp}$  particle corresponding to topological (semi-inclusive) distribution, satisfying the differential scaling law, Eq. (4.18), are consistent with a power asymptotic behaviour of the form

$$\frac{d\sigma}{dp_{\perp}^2} \sim \frac{1}{(p_{\perp}^2)^{\alpha+2}} F\left(\frac{p_{\perp}^{2\alpha}}{\sqrt{s}}\right), \quad (5.5)$$

$$F(z) = e^{-2z} - e^{c\sqrt{s}/2}.$$

The associated multiplicity has approximately rising dependence on  $p_{\perp}$ :

$$\langle n(p_{\perp}) \rangle \sim (ap_{\perp})^{2\alpha}. \quad (5.6)$$

In this connection, note that the assumptions made in the framework of our consideration make it possible to establish a relation between the effective degree of fall for the inclusive cross sections at large  $p_{\perp}$  (taking into account the factor  $F(x_{\perp})$ ) and the increasing character of the associated multiplicity relative to  $p_{\perp}$ . This correlation depends on the range of  $x_{\perp}$ .

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\*Within the MP scheme it is possible to reproduce the growth of spectra with energy and their power decrease  $p_{\perp}^{-8}$  at large transverse momenta.

It is of interest to turn back to Fig. 5.1, where the correlation of such a type is drawn, i.e., an effective dependence of degree of power decreases on the interval of the variable  $x_{\perp}$ .

In particular it may serve as some evidence of the possibility to describe the inclusive spectra at large  $p_{\perp}$  not by a single term of the type (5.2), but by their superposition with various  $N$ . The value of  $N$  for the given region  $x_{\perp}$  decreases with increasing energy. Note that from the theoretical point of view the appearance of an effective dependence of the degree of the value may be interpreted as a result of the competition of several different dynamic mechanisms:

$$\begin{aligned}
 Mq &\rightarrow Mq & N = 2n_M = 4 \\
 q\bar{q} &\rightarrow M\bar{M} & \ll - \gg \\
 \bar{q}B &\rightarrow M2q & N = n_M + n_B = 5 \\
 q2q &\rightarrow MB & \ll - \gg \\
 \dots & & \dots
 \end{aligned}$$

In the language of quarks, the process of inclusive production of meson  $M$  with large  $p_{\perp}$  is determined by one of the exclusive interactions.

The extrapolation of the found dependence into the region gives  $n \approx 2$  that would correspond to the point-like behaviour of a cross section and could be defined by the elementary process  $qq \rightarrow qq$  with the subsequent fragmentation of quarks into real particles. A direct experimental examination of a dependence of the associated multiplicity on the particle transverse momentum is thus of great interest in the testing of theoretical models.

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#### REFERENCES

1. *Di Lella L.* Talk at the London Conference, 1974.
2. *Landshoff P. V.* Rapporteurs talk at the London Conference, 1974.
3. *Ellis S. D., Thun R.* CERN TH 1874. Geneva, 1974.
4. *Del Prete T.* Invited talk given at the 9th Balaton Symposium on Particle Physics, Symposium on High-Energy Hadron Interactions, 12–18 June, 1974.
5. *Sosnowski R.* JINR, D1,2-7721. Dubna, 1974.
6. *Matveev V. A.* // Proc. CERN–JINR School of Physics, Ebeltoft, Denmark, 1973.
7. *Muradyan R. M.* // Proc. CERN–JINR School of Physics, Ebeltoft, Denmark, 1973.

8. *Matveev V. A., Muradyan R. M., Tavkhelidze A. N.* // Lett. Nuovo Cimento. 1972. V. 5. P. 907; JINR, D2-7110. Dubna, 1973; Lett. Nuovo Cimento. 1973. V. 7. P. 719.
9. *Faustov R. N., Garsevanishvili V. R., Kvinikhidze A. N., Matveev V. A., Tavkhelidze A. N.* JINR, E2-8126. Dubna, 1974.
10. *Kuleshov S. P., Kvinikhidze A. N., Matveev V. A., Sissakian A. N., Slepchenko L. A.* JINR, E2-8128. Dubna, 1974.
11. *Faustov R. N., Garsevanishvili V. R., Kvinikhidze A. N., Matveev V. A.* JINR, E2-8600. Dubna, 1975.
12. *Büsser F. W. et al.* // Phys. Lett. B. 1973. V. 46. P. 471.
13. *Banner M. et al.* // Phys. Lett. B. 1973. V. 44. P. 531.
14. *Alper B. et al.* // Phys. Lett. B. 1973. V. 44. P. 521.
15. *Büsser F. W. et al.* // Phys. Lett. B. 1974. V. 51. P. 306, 311.
16. *Finnochiaro G. et al.* // Phys. Lett. B. 1974. V. 50. P. 396.
17. *Ramanauskas A. et al.* // Phys. Rev. Lett. 1974. V. 31. P. 1371.
18. *Anderson E. W. et al.* Talk at the London Conference, 1974.
19. *Turkot F. et al.* Talk at the London Conference, 1974.
20. *Matveev V. A., Sissakian A. N., Slepchenko L. A.* JINR, P2-8670. Dubna, 1975.
21. *Kvinikhidze A. N., Slepchenko L. A.* JINR, P2-8529. Dubna, 1975.
22. *Golosokov S. V., Kuleshov S. P., Matveev V. A., Smondyrev M. A.* JINR, P2-8337. Dubna, 1974.
23. *Kuleshov S. P., Matveev V. A., Sissakian A. N., Smondyrev M. A., Tavkhelidze A. N.* // Part. Nucl. 1974. V. 5, No. 1, P. 3–62.
24. *Berger E. L., Branson D.* // Phys. Lett. B. 1973. V. 45. P. 57.
25. *Berman S. M., Bjorken J. D., Kogut J. R.* // Phys. Rev. D. 1971. V. 4. P. 3388.
26. *Blankenbeckler R., Brodsky S. J., Gunion J. F.* // Phys. Lett. B. 1972. V. 39. P. 649.
27. *Amati D., Fubini S., Stanghellini A.* // Nuovo Cimento. 1962. V. 26. P. 896.
28. *Amati D., Caneschi L., Testa M.* CERN TH 1507. Geneva, 1973.
29. *Dremin I. M.* Yad. Fiz. (in Russian). 1973. V. 18. P. 617.
30. *Matveev V. A., Tavkhelidze A. N.* Lecture at High-Energy Summer School, Sochi, 1974.
31. *Matveev V. A.* Talk at the Kiev Conf., 1970.
32. *Landshoff P. V., Polkinghorne J. C.* // Phys. Lett. B. 1973. V. 44. P. 293; Phys. Rev. D. 1973. V. 8. P. 4157.
33. *Efremov A.* JINR, E2-7864. Dubna, 1974; JINR, P2-7167. Dubna, 1973.
34. *Abramovski V., Kancheli O. V., Matinyan S.* // Phys. Lett. B. 1971. V. 36. P. 565.
35. *Sissakian A. N., Slepchenko L. A.* Talk at IV Int. Seminar on Problems of High Energy Physics, Dubna, 5–11 June, 1975.