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THE QUANTUM NUMBER *COLOR*, *COLORED QUARKS* AND QCD (Dedicated to the 40th anniversary of the discovery of *color*)

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A brief review is given of the priority works which were mainly carried out at the Laboratory of Theoretical Physics, JINR, and devoted to the introduction to hadron physics of the concept of *color and colored quarks*, and to the description of hadrons in the framework of the model of *quasi-free quarks*. These ideas play a key role in the modern theory of strong interactions — quantum chromodynamics.

Дается краткий обзор приоритетных работ, выполненных в основном в Лаборатории теоретической физики ОИЯИ и посвященных введению в физику адронов концепции *цвета*, *цветных кварков* и описанию адронов в рамках модели *квазисвободных кварков*. Эти идеи играют ключевую роль в современной теории сильных взаимодействий — квантовой хромодинамике.

INTRODUCTION

The idea of colored quarks — fundamental fermions possessing a specific quantum number, color, and representing elementary constituents of matter on an equal footing with leptons — underlies modern concepts of elementary particles and of the microcosm.

In 1964, when the hypothesis of quarks — hypothetical particles composing all the observed particles undergoing strong interactions, i.e., mesons and baryons, — was put forward by Gell-Mann [1] and Zweig [2], quarks were only considered to be mathematical objects, in terms of which it was possible, in a most simple and elegant way, to describe the properties, already revealed by that time, of the approximate unitary $SU(3)$ symmetry of strong interactions [3]. At the beginning, these particles, exhibiting fractional charges and not observable in a free state, were not given the necessary physical interpretation.

First of all, making up hadrons of quarks, possessing spin 1/2, led to a contradiction with the Pauli principle and the Fermi–Dirac statistics for systems composed of particles of semiinteger spin.

The problem of the quark statistics was not, however, the sole obstacle in the path of theory. No answer existed to the following question: why were only systems consisting of three quarks and quark–antiquark pairs realized in Nature, and why were there no indications of the existence of other multiquark states?

Especially important was the issue of the possible existence of quarks in a free state (the problem of quark confinement).

In 1965, analysis of these problems led N. Bogolubov, B. Struminsky and A. Tavkhelidze [4], as well as Y. Nambu and M. Han [5], and Y. Miyamoto [6], to the cardinal idea of quarks exhibiting a new, hitherto unknown, quantum number, subsequently termed *color* [7].

For already 40 years this idea underlies the physics of elementary particles. It has permitted one to deal with colored quarks like with real fundamental constituents of matter; the hypothesis of colored quarks possessing color charge subsequently led to the creation of quantum chromodynamics — the gauge theory of strong interactions, it resulted in the origination of numerous versions of the «grand unification» theory.

An essential step toward the development of the dynamical theory of hadrons was made by Nambu [8], who was the first, on the basis of $SU(3)$ symmetry requirements relative to the new quantum number (color), to consider eight vector fields, carriers of the interaction between quarks, which were the prototype of the gluon fields of quantum chromodynamics.

Thus, quantum chromodynamics (QCD) [14] resulted from unification of the hypothesized existence of a new quantum number (color), colored quarks, colored $SU(3)$ symmetry, the Yang–Mills local invariance principle, and the quantization of Yang–Mills fields [9].

In this anniversary article considered are the principal aspects of the early development of the theory of colored quarks, mainly implemented in Dubna at the JINR Laboratory of Theoretical Physics under the ideological influence and in collaboration with N. Bogolubov*. In the article, particular attention is paid to the creation of the relativistically invariant dynamic model of quasi-free quarks, within the framework of which the results obtained reflect most adequately the essence of the quark structure of hadrons. The research mentioned also contributed to the development of the quark-bag model [10] and the quark-parton model.

1. COLORED QUARKS AND HADRON DYNAMICS

The Hypothesis of Colored Quarks [4, 11]. Creation of the relativistically invariant dynamic quark model of hadrons was based, first of all, on the assumption of quarks representing real physical objects determining the structure of hadrons.

*A more detailed discussion of these issues can be found in the review by N. Bogolubov, V. Matveev and A. Tavkhelidze [25].

To make it possible for quarks to be considered fundamental physical particles, the hypothesis was proposed that the quarks, introduced by Gell-Mann and Zweig, should possess an additional quantum number and that quarks of each kind may exist in three (unitary) equivalent states $q \equiv (q^1, q^2, q^3)$ differing in values of the new quantum number, subsequently termed *color**. Since at the time when the new quantum number was introduced, only three kinds of quarks were known (u, d, s), the quark model with an additional quantum number was termed the three-triplet model.

With introduction of the new quantum number, color, the question naturally arised of the possible appearance of hadrons possessing color, which, however, have not been observed. From the assumption that quarks are physical objects, while the hadron world is degenerate in the quantum number, or, as one may customarily say, it is colorless, it followed that solutions of the dynamic equations for baryons and mesons in the s-state should be neutral in the color quantum numbers.

The wave function of the observed hadron family in the ground state, described by the totally symmetric 56-component tensor $\Phi_{abc}(x_1, x_2, x_3)$ in the approximation of spin-unitary symmetry, was assumed to be totally antisymmetric in the color variables of the three constituent quarks,

$$\Psi_{ABC}(x_1, x_2, x_3) = 1/6\varepsilon_{\alpha\beta\gamma}\Phi_{abc}(x_1, x_2, x_3), \quad \alpha, \beta, \gamma = 1, 2, 3,$$

where $A = (a, \alpha)$, $B = (b, \beta)$, $C = (c, \gamma)$, a, b, c are unitary quantum numbers, α, β, γ are color quantum numbers.

It was verified that for meson form factors, found without account of the quark colors, to remain intact it sufficed to assume the wave function of the mesons in the ground state to have the form

$$\Psi_A^B(x_1, x_2) = 1/\sqrt{3}\delta_\alpha^\beta\Phi_a^b(x_1, x_2).$$

Such solutions select mesons for which the states Ψ_1^1, Ψ_2^2 and Ψ_3^3 appear with the same weight.

The choice of baryon and meson wave functions, proposed above, leads to the conclusion that the observed mesons and baryons are neutral with respect to the color quantum number and that the known mesons and baryons are composed of colored quarks and antiquarks as follows:

$$\bar{q}^\alpha(1)q_\alpha(2) \text{ — mesons,}$$
$$\varepsilon^{\alpha\beta\gamma}q_\alpha(1)q_\beta(2)q_\gamma(3) \text{ — baryons.}$$

*We shall further use the term *color* for the new quantum number.

In principle, colored quarks could have integer, as well as fractional, electric charges, if the charge operator is assumed to act not only on unitary indices, related to the quark flavors, but on color indices as well.

We define the charge operator as follows:

$$e_A^{A'} = e_a^{a'} + e_\alpha^{\alpha'}.$$

The operator $e_a^{a'}$ acts on the unitary flavor indices and has the form

$$e_a^{a'} = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}.$$

The operator $e_\alpha^{\alpha'}$ acts on the additional quantum numbers of quarks (the color charge indices) and has the form

$$e_\alpha^{\alpha'} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & -2/3 \end{pmatrix}.$$

Consequently, colored quarks turn out to exhibit integer electric charges,

a/α	1	2	3
1	1	1	0
2	0	0	–1
3	0	0	–1

Taking into account that $e_\alpha^{\alpha'}$ is a diagonal operator, we obtain

$$\varepsilon^{\alpha\beta\gamma} e_\alpha^{\alpha'} \varepsilon_{\alpha'\beta\gamma} = 0, \quad \delta_\beta^\alpha e_\alpha^{\alpha'} \delta_{\alpha'}^\beta = 0.$$

Hence we have

$$e_a^{a'} = \frac{1}{6}\,\varepsilon^{\alpha\beta\gamma}(e_a^{a'} + e_\alpha^{\alpha'})\varepsilon_{\alpha'\beta\gamma} = 0,$$
$$e_a^{a'} = \frac{1}{3}\,\delta_\beta^\alpha(e_a^{a'} + e_\alpha^{\alpha'})\delta_{\alpha'}^\beta = 0.$$

With the aid of these equalities, relativistic hadron form factors, calculated in the lowest order of perturbation theory in the electric charge, have also been shown to remain unaltered for colored quarks with integer charges.

Dynamic Quark Models and Hadron Form Factors [4, 11–13]. The introduction of colored Fermi quarks, representing physical fundamental particles, paved the way for the dynamic description of hadrons.

The main obstacle, here, was the absence of quarks in a free state. Although it was evident that the issue of confinement could be ultimately settled only by experiments, a series of attempts were undertaken to provide a logically non-contradictory explanation for the «eternal confinement» of quarks inside hadrons; thus, for example, the «quark-bag» model was proposed — the Dubna quark bag represented a preliminary version, followed by the improved MIT model [10].

The dynamic relativistic quark model, the development of which was initiated in Dubna in 1964, was based on the assumption of quarks being extremely heavy objects bound in hadrons by enormous forces, which on the one hand provide for the large quark mass defect in hadrons and on the other hand impede their leaving the hadron interior.

The dynamic equations were required to have solutions for hadrons inside which quarks are in a quasi-free state, resulting in the property of approximate additivity, inherent in non-relativistic quark models, being conserved in the calculations of various physical quantities.

The quasi-free quark model depends essentially on the satisfactory explanation of the magnetic moment enhancement of a heavy quark bound in a hadron. This effect can be demonstrated in a most simple way applying the example of the Dirac equation for a quark bound by a scalar field, described by a rectangular potential well $U(r) = -U_0\theta(r_0 - r)$, in the presence of a magnetic field H:

$$[E + i\boldsymbol{\alpha}(\boldsymbol{\nabla} + ie\mathbf{A})]\Psi = \beta M^*\Psi; \quad r \leqslant r_0,$$

where $M^* = M_q - U_0$, $\mathbf{H} = \operatorname{rot}\mathbf{A}$.

Resolution of this equation in the limit of an infinite mass of the free quark, M_q, and given a fixed value of the effective mass M^*, which we further set equal to zero, results in the following magnetic moment of the bound quark in the ground state:

$$\mu_q = \frac{e}{2E_q}(1 - \delta),$$

where E_q is the energy of the bound quark state, and δ is the mean component of its orbital momentum:

$$\delta = \langle\uparrow |L_z| \uparrow\rangle,$$

in the state in which the total quark momentum $J_z = 1/2$.

We stress that the magnetic moment of an infinitely heavy bound quark being finite is a consequence of the binding potential assumed to exhibit a scalar character and is not fulfilled, for instance, in the vector case [15].

In the model of quasi-bound quarks the constituent quarks of the hadron move independently in a certain self-consistent scalar field $U(r)$, the relation with which leads to compensation of their large mass*.

*Abdus Salam figuratively termed such a picture of the hadron «Archimedes' bath».

In the model of quasi-free quarks the meson wave function represents a second-order mixed spinor $\Phi_A^B(p)$ that satisfies the equations

$$(\hat{p} - m_q)_A^{A'} \Phi_{A'}^B(p) = 0 \text{ for the constituent quark,}$$
$$(\hat{p} + m_q)_B^{B'} \Phi_A^{B'}(p) = 0 \text{ for the antiquark,}$$

m_q is the effective mass of the quark (antiquark) in the meson, which is renormalized owing to compensation of the large quark (antiquark) mass by the strong scalar field. $A = (\alpha, a)$, $B = (\beta, b)$ are the color and unitary quantum numbers of the quark and antiquark, respectively.

The wave function of a baryon composed of three quasi-free quarks represents a third-order mixed spinor $\Phi_{ABC}(p)$ that satisfies the equations

$$(\hat{p} - M_q)_A^{A'} \Phi_{A'BC}(p) = 0, \quad (\hat{p} + M_q)_C^{C'} \Phi_{ABC'}(p) = 0,$$

where M_q is the renormalized mass of the heavy quark in the hadron; $A = (\alpha, a)$, $B = (\beta, b)$, $C(\gamma, c)$ are the color and unitary indices of the constituent quarks.

Baryons Φ_{ABC} and mesons Φ_A^B are represented by a superposition of all admissible states over the quantum numbers (A, B, C) and (A, B), satisfying the requirements of $SU(6)$ symmetry, of quark statistics in hadrons and of hadron neutrality in the color quantum number.

The dynamic composite quasi-free quark model has made possible the systematic description of both the statically observed characteristics of hadrons $(\mu, g_A/g_V$, etc.), and their form factors. We introduce weak and electromagnetic interactions in a minimal manner,

$$i\partial_\mu \to i\partial_\mu + \begin{cases} eA_\mu \text{ — electromagnetic interaction,} \\ \dfrac{G}{\sqrt{2}}\tau^\pm \gamma_5 e_\mu^\pm \text{ — weak interaction,} \end{cases}$$

where A_μ is the electromagnetic potential; $e_\mu^\pm$ represent charged lepton weak currents; G is the Fermi weak interaction constant. For the ratio g_A/g_V of the axial and vector weak interaction constants and for the magnetic moment of, say, the proton we obtain

$$g_A/g_V \simeq -5/3(1 - 2\delta),$$
$$\mu_p \simeq \frac{e}{2E_q}(1 - \delta),$$

where the parameter

$$\delta = \langle \uparrow | L_z | \uparrow \rangle = -i \int d^3 r \Psi^*(r) [\mathbf{r} \times \boldsymbol{\nabla}]_z \Psi(r).$$

Here L_z and E_q are, respectively, the orbital momentum and the energy of a quark bound in the nucleon with the projection of its total angular momentum equal to 1/2; the energy of a bound quark is approximately equal to one third of the nucleon mass, $E_q \cong M_p/3$.

Note that the quantity δ characterizes the magnitude of relativistic corrections. In the ultrarelativistic case, when $\langle q \rangle / E_q^2 \sim 1$, and the value obtained of $\delta \sim 1/6$, the resulting correction for the ratio g_A/g_V is of the order of 30%. This example shows how significant the effect can turn out to be of relativistic corrections to predictions of the non-relativistic quark model.

The quasi-free quark model has permitted one to explain the lepton decays of pseudoscalar π and K mesons and, also, the electromagnetic decays of the vector mesons ρ^0, ω^0 and Φ^0 into electron–positron pairs as annihilation of quark–antiquark pairs bound in the mesons [16].

Analysis of the data on the widths of these decays resulted in a conclusion on the dependence of the scales of distances (effective sizes) on the quantum numbers of a bound system, for example,

$$\frac{|\Psi_k(0)|^2}{|\Psi_\pi(0)|^2} \cong \frac{m_k}{m_\pi}.$$

In the case of the decay $\pi^0 \to 2\gamma$, determined by the triangular anomaly [17] of the axial current, the annihilation model points to the width of this decay being proportional to the number of different quark colors [16].

2. THE MODEL OF QUASI-FREE QUARKS AND THE LAWS OF SCALING AT HIGH ENERGIES [18, 19]

Experiments in which inclusive reactions [20] were studied at high energies and momentum transfers, and the scaling regularities revealed, as well as their theoretical investigation, have extended our comprehension of the nature of strong interactions and have given an impetus to further development of the theory of hadron quark structure.

Here, of essential significance was the investigation of deep inelastic processes in the inclusive scattering of electrons off nucleons, performed at the Stanford Center (SLAC, Stanford), which in 1968 resulted in observation of the scaling properties of reactions: Bjorken scaling indicating the existence of a «rigid» pointlike nucleonic structure [21].

Experiments carried out at other major accelerators for investigation of the scaling properties of inclusive hadron reactions (IHEP, Protvino) [40], and also of processes of deep-inelastic neutrino and antineutrino interactions with nucleons (CERN, Geneva; FNAL, Batavia) confirmed the idea of the pointlike behavior of the nucleon. In other words, the effective size of the nucleon seems to vanish in such interactions, when all the revealed reaction channels are taken into account*.

In 1969, on the basis of the quasi-free quark model, the assumption was put forward [18] that the scaling properties of electron–nucleon interaction processes, revealed in experiments, are common for all deep inelastic lepton–hadron processes and that they can be derived in a model-independent manner on the basis of the automodelling principle**, or the principle of self-similarity.

The essence of the self-similarity principle consists in the assumption that in the asymptotic limit of high energies and large momentum transfers form factors and other measurable quantities of deep inelastic processes are independent of any dimensional parameters (such as particle masses, the strong interaction radius, etc.), which may set the scale of measurement of lengths or momenta. Thus, the form factors of deep inelastic processes turn out to be homogeneous functions of relativistically invariant kinematic variables, the degree of homogeneity of which is determined by analysis of the dimensionality.

Now, consider a deep inelastic interaction process, in which the large momentum q is transferred from leptons to hadrons with the momenta p_i. In the Bjorken limit $\nu_i \sim s_{ij} \sim |q^2| \gg p_i^2 = m_i^2$, for fixed values of the dimensionless ratios of large kinematic invariants ν_i/q^2, s_{ij}/q^2, where $\nu_i = qp_i$, $s_{ij} = p_i p_j$ $(i \neq j)$, the observable physical characteristic of the studied process $F(q,p)$ under scaling transformations of the momenta

$$q_\mu \to \lambda q_\mu, \quad (p_i)_\mu \to \lambda (p_i)_\mu$$

transforms, in accordance with the self-similarity principle, like a homogeneous function of the order $2k$

$$F(q,p_i) \Rightarrow F(\lambda q, \lambda p_i) = \lambda^{2k} F(q,p_i),$$

where $2k$ represents the physical dimensionality of the quantity $F(q,p_i)$. Consequently, the most general form of the form factor satisfying these requirements is

$$F(q,p_i) = (q^2)^k f(\nu_i/q^2, s_{ij}/q^2),$$

where function f depends only on dimensionless ratios of large kinematic variables, which remain finite in the Bjorken limit.

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*In 1964 the idea of the pointlike behavior of the total lepton–nucleon interaction cross sections was put forward by M. Markov on the basis of purely theoretical arguments concerning the dominant role of the newly revealed channels as compared to the suppression factor due to hadron form factors.

**Automodelling behavior in high energy physics is closely analogous to the property of self-similarity in problems related to gases and gas dynamics, from which the term «automodelling» was adopted.

In the case of deep inelastic electron–nucleon scattering, the self-similarity principle for the structure functions $W_1(q^2, \nu)$ and $W_2(q^2, \nu)$ leads to a scaling-invariant behavior of functions $W_{1,2}$, first found by Bjorken:

$$\nu W_2(q^2, \nu) = F_2(q^2/\nu), \quad \nu W_1(q^2, \nu) = F_1(q^2/\nu),$$

since

$$[W_1] = m^0, \quad [W_2] = m^{-2}.$$

Application of the self-similarity principle resulted in the scaling law being found for the first time, which describes the mass spectrum of muon pairs, produced in proton collisions at high energies, $p + p \to \mu^+ + \mu^- +$ hadrons, namely [19],

$$\frac{d\sigma}{dM^2} \sim \frac{1}{M^4} \Psi\left(\frac{M}{E}\right),$$

where M is the effective mass of the muon pair, and E is the initial energy of the colliding particles. Experimental studies of this process, initiated in 1970 by the group of L. Lederman at Brookhaven [22], confirmed this scaling law, and it was precisely in these processes that a new class of hadrons, the J/ψ particles, was subsequently observed.

Quark Counting Rules [23]. Especially interesting and important consequences that originated from the idea of quasi-free quarks are related to the field of deep inelastic or exclusive hadron interactions, for example, the investigation of binary large-angle hadron scattering reactions at high energies. In this kinematic region all momentum and energy transfers are large; so, consequently, we deal with interaction processes mainly restricted to the region of small distances and time intervals, where the «rigid» pointlike quark structure of hadrons should be directly manifested.

In 1973, in Refs. 23, a general formula, based on the self-similarity principle and the quasi-free quark model, was established, which determines the character of the energy dependence of the differential cross section of any arbitrary binary large-angle scattering reaction at high energies $E = \sqrt{s}$ as well as the asymptote of form factors at large momentum transfers $Q = \sqrt{-t}$:

$$(d\sigma/dt)_{ab \to cd} \sim S^{-(n_a+n_b+n_c+n_d-2)},$$
$$F(t) \sim t^{-(n_a-1)},$$

where $n_{i=a,b,c,d}$ represent the amounts of elementary constituents participating in the hadron reaction.

These formulae, known as the quark counting formulae, establish a direct relationship between the power decrease rate of the differential cross section of a binary large-angle scattering reaction with the increase of energy and the degree of complexity of the particles participating in this process, i.e., the number of their elementary constituents.

The discovery of the quark counting formulae opened extensive possibilities of experimental investigation of the quark structure of hadrons and light atomic nuclei [23, 24].

We shall briefly dwell upon derivation of the quark counting formulae on the basis of an analysis of dimensionalities («dimensional quark counting»).

Consider a binary reaction of the general form $a+b \to c+d$. Assume particle a to behave like a composite system composed of n_a pointlike constituents — quarks. The state vector of such a system can be written in the form

$$|a\rangle = \hat{N}_a |n_a \text{ quarks}\rangle,$$

where the symbol $\hat{N}_a$ stands for the operation yielding the product of the state vector of free quarks and the appropriate wave function of the system and integration (summation) over the quark variables.

The differential cross section of the binary reaction can, correspondingly, be represented in the form

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} = \mathrm{Tr}\left(\prod_{i=a,b,c,d} \hat{\rho}_i \frac{d\hat{\sigma}}{dt}\right),$$

where

$$\begin{aligned} \hat{\rho}_i &\equiv \hat{N}_i \times N_i^+, \\ \frac{d\hat{\sigma}}{dt} &\equiv \frac{1}{s^2} |\langle n_a n_b | T | n_c n_d \rangle|^2. \end{aligned}$$

The dimension of a one-particle state, normalized in a relativistically invariant manner, is known to be

$$[|\text{one-particle}\rangle] = m^{-1}.$$

Hence follow the dimensionalities of the factors $\hat{\rho}_i$ and $d\hat{\sigma}/dt$.

Considering, in accordance with the self-similarity principle, the interaction between quarks to be scaling invariant at small distances, i.e., not to depend on dimensional dynamic parameters, we arrive at the conclusion that the quantities $\hat{\rho}_i$ and $d\hat{\sigma}/dt$ exhibit a power behavior as the energy and momentum transfers increase, as does the differential cross section of the exclusive reaction:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \to \left(\frac{1}{s}\right)^{n-2} f\left(\frac{t}{s}\right), \quad n = n_a + n_b + n_c + n_d.$$

The function $f(t/s)$, which only depends on the relation between large kinematic variables or, which is the same, on the scattering angle, is by itself a dimensional quantity, while a natural scale is represented, here, by the effective size of the particles. Thus, the asymptotic power law points to factorization of the effects of large and small distances.

The power law of the decrease of the hadron form factor reflects a particular case of an exclusive reaction — the scattering of a structureless lepton on a hadron composed of n_a quarks.

The success of the quark counting formulae rendered their substantiation within the framework of quantum chromodynamics a problem of utmost importance, and a series of studies have been devoted to this problem. A review of these works can be found, for example, in [25, 42].

In a number of important works of the last years there was suggested an impressive non-perturbative derivation of the asymptotic power laws of the quark counting for form factors and the exclusive scattering cross sections of hadrons in the framework of the conformal versions of QCD dual to the string theory [41].

3. ON THE SCALING-INVARIANT ASYMPTOTICS OF QUANTUM FIELD THEORY [26]

As has been noted, the experimentally observed scaling properties of elementary particle interaction processes can be described in a unique manner on the basis of the self-similarity principle, proceeding from the laws of physical similarity and an analysis of dimensionalities.

At the same time, the question arises as to the degree of scaling-invariant behavior being consistent with the principal ideas and requirements of quantum field theory, such as the locality principle, microcausality and the spectral property.

The most complete investigation of this problem was presented in 1972 in the works of N. Bogolubov, V. Vladimirov and A. Tavkhelidze, in which the sufficient and, in certain cases (the free field case), the necessary conditions for the existence of scaling-invariant asymptotes in quantum field theory. One of the results of this approach is establishment of the exact relationship between the asymptotes of observed quantities — amplitudes and cross sections — and the interaction property in the vicinity of the light cone.

To this end, in the aforementioned work the process was considered of deep inelastic scattering of electrons on nucleons, studied earlier in 1968 by Bjorken, and under certain intuitively plausible assumptions asymptotic properties were established of the form factors $W_{1,2}$, which are widely known as Bjorken scaling.

Now, consider the deep inelastic process of electron scattering on nucleons. The cross section of deep inelastic scattering of an electron on a nucleon is determined with the aid of the Fourier image of the commutator

$$W_{\mu\nu}(q,p) = \frac{1}{8\pi}\sum_{\sigma}\int \langle p,\sigma|[j_\mu(x), j_\nu(0)]|p,\sigma\rangle\, \mathrm{e}^{iqx}dx =$$
$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(q,p) + \left(p_\mu - \frac{qp}{q^2}q_\mu\right)\left(p_\nu - \frac{qp}{q^2}q_\nu\right) W_2(q,p),$$

in which j_μ represents the components of the electromagnetic current; q is the four-momentum of the virtual photon; $q^2 < 0$, and the matrix element is calculated between identical one-nucleon states $|p,\sigma\rangle$ with the 4-momentum p, mass $p^2 = 1$ and spin $\sigma = \pm 1/2$.

The problem consists in finding the asymtotics of the form factors $W_{1,2}$ in the Bjorken region

$$|q^2| \to \infty, \quad \nu = 2pq \to \infty, \quad s = -q^2/2pq = \text{const}$$

($s > 0$ is the physical region) and relating it to the behavior of the current commutators in the vicinity of the light cone.

First of all, the form factors $W_{1,2}$ were shown to be causal*; i.e., their Fourier images turn to zero at $x^2 < 0$.

For investigation of the asymptotics of form factors it is convenient to introduce the invariant causal functions

$$F_1(p,q) = \Sigma g_{\mu\mu} g_{\nu\nu} p_\mu p_\nu W_{\mu\nu} \equiv p^\mu p^\nu W_{\mu\nu},$$
$$F_2(p,q) = F_1(p,q) - \Sigma g_{\mu\mu} W_{\mu\mu}, \quad g^{00} = 1.$$

In the laboratory system ($\mathbf{p} = 0$, $p = (1,0)$) the following notation is used:

$$F_i(q) \equiv F_i(q;1,0), \quad W_i \equiv W_i(q;1,0).$$

And below $F(x)$ will denote the Fourier image

$$F(q) \equiv \int F(x)\,\mathrm{e}^{iqx}dx.$$

From the definition of F_i it follows that these functions are odd with respect to q^0, radially symmetric, i.e., depend only on q^0, $|q|$ and turn to zero in the region $(-q^2/2|q^\nu|) > 1$ (the spectral property condition), and besides,

$$\varepsilon(q^0)F_i(q) \geqslant 0, \text{ if } q^2 < 0,$$
$$\varepsilon(q^\nu)W_i(q) \geqslant 0, \text{ if } q^2 < 0.$$

*The previously existing proof [27] was not convincing, since used were commutator relations between currents, which did not follow from the general principles of quantum field theory.

Bjorken made the assumption that for currents of interacting fields, like in the case of free fields, the quantities F_1 and $F_2 - F_1$ in the physical region tend toward finite and differing from zero limit distributions. Then, in the Bjorken region we obtain the asymptotic relations

$$W_1 \sim \frac{1}{2}(F_2 - F_1), \quad W_2 \sim \frac{2\zeta}{\nu}(F_2 - F_1), \quad W_1 \sim \frac{\nu}{4\zeta} W_2,$$

analogous to the exact equalities of free fields (see below). Denoting in the Bjorken region

$$F_i(\zeta, \nu) \sim F_1(\zeta),$$

we obtain the scaling-invariant Bjorken formulae in the physical region:

$$W_1(\zeta, \nu) \sim f_1(\zeta), \quad \frac{\nu}{4} W_2(\zeta, \nu) \sim f_2(\zeta),$$

$$f_1(\zeta) = \frac{1}{2}[F_2 - F_1], \quad f_2(\zeta) = \zeta f_1(\zeta) > 0.$$

Thus, for such asymptotic formulae it suffices to require, as proposed by Bjorken, the functions F_1 and $F_2 - F_1$ to tend in the physical region toward finite, differing from zero, limits. Therefore, the question naturally arises as to what additional requirements, following from the dynamics of the process, must be imposed on the functions F_1 and F_2, so as to provide for the scaling-invariant Bjorken formulae to be valid.

Thus, we investigate the behavior of the distribution $F(q,p)$ (of its slow growth) in the Bjorken region, satisfying the following conditions:

$$\text{I. } F(q,p) = -F(-q,p),$$

$$\text{II. } F(q,p) = 0, \text{ if } -q^2/12pq > 1,$$

$$\text{III. } F(x,p) = 0, \text{ if } x^2 < 0,$$

$$\text{IV. } F(\Lambda p, \Lambda q) = F(p,q) \text{ for all } \Lambda \subset L_+^\uparrow.$$

By virtue of IV it suffices to consider the problem in the rest reference system $p = 0$. In this system functions $F(x)$ and $F(q)$ depend only on x^0, $|x|$ and $q^0|q|$, and, consequently, they satisfy the following conditions:

$$\text{I}'.\ F(q) = -F(-q),$$

$$\text{II}'.\ F(q) = 0, \text{ if } -q^2/|2q^0| > 1,$$

$$\text{III}'.\ F(x) = 0, \text{ if } x^2 < 0,$$

$$\text{IV}'.\ F(q) = F(q^0, |q|).$$

The asymptotic region assumes the form

$$|q^2| \to \infty, \quad \nu = 2q^0 \to \infty, \quad -q^2/2q^0 = \zeta, \quad q^0 \sim |q|.$$

For function $F(q)$, which satisfies conditions I'–IV', there exists a (unique) distribution $\Psi(|u|, \lambda^2)$ of slow growth, such that the integral Jost–Lehman–Dyson representation [28] holds valid:

$$F(\zeta, \nu) = \frac{2\pi}{\nu} \int_0^\infty d\lambda^2 \int_0^1 \rho^2 d\rho \Psi(\rho, \lambda^2) \int_{-1}^1 d\mu \delta \left(\zeta + \frac{\rho^2 + \lambda^2}{\nu} - \rho\mu \sqrt{1 + \frac{4\zeta}{\nu}} \right).$$

Here, the carrier Ψ is contained in the region

$$(u, \lambda^2) : |u| \leqslant 1, \quad \lambda^2 \geqslant (1 - \sqrt{1 - u^2})^2.$$

Note that from condition II it follows that $F(\zeta, \nu) = 0$, if $\zeta > 1$.

We stress that the properties of the weight function Ψ, listed above, generally speaking, do not provide for a definite asymptotic behavior of $F(\zeta, \nu)$. It is necessary to find the conditions that would provide for the scaling-invariant behavior of function F in the Bjorken region.

The asymptote of $F(\zeta, \nu)$ can be expressed in terms of the distribution over ζ. Therefore, in selecting the class of principal functions $f(\zeta)$, which are finite, infinitely differentiable, turn to zero at $\zeta < 0$, one can consider the following expression to serve as the definition of the distribution $F(\zeta, \nu)$ for $\nu > 0$:

$$\int F(\zeta, \nu) f(\zeta) d\zeta \quad =$$

$$= \frac{2\pi}{\nu} \int_0^\infty d\lambda^2 \int_0^1 \rho^2 d\rho \Psi(\rho, \lambda^2) \int_{-1}^1 d\mu \int d\zeta f(\zeta) \delta \left(\zeta + \frac{\rho^2 + \lambda^2}{\nu} - \rho\mu \sqrt{1 + \frac{4\zeta}{\nu}} \right).$$

Hence, in the limit of $\nu \to \infty$ the asymptote of the distribution $F(\zeta, \nu)$ within the physical region $0 \leqslant \zeta \leqslant 1$ is determined by the expression

$$\int F(\zeta, \nu) f(\zeta) d\zeta \to \frac{2\pi}{\nu} \int_0^\infty d\lambda^2 \int_0^1 \rho^2 d\rho \Psi(\rho, \lambda^2) \int_{-1}^1 d\mu f \left(-\frac{\lambda^2}{\nu} + \rho\nu \right).$$

Now, we shall restrict the class of weight functions Ψ to be considered by assuming that at sufficiently large λ^2 the distribution $\Psi(\rho, \lambda^2)$ is an ordinary function in λ^2 and that at certain $k > -1$ there exists a non-zero limit (in the sense of a distribution over ρ):

$$\lim_{\lambda^2 \to \infty} \frac{\Psi(\rho, \lambda^2)}{\lambda^{2k}} = \Psi_0(\rho) \neq 0$$

or

$$\Psi(\rho^2, \lambda^2) = \theta(\lambda^2)\lambda^{2k}\Psi_0(\rho) + \Psi_1(\rho, \lambda^2), \quad \Psi_1(\rho, \lambda^2)/\lambda^{2k} \to 0, \text{ if } \lambda^2 \to \infty.$$

Hence, in the Bjorken region for physical values of $\zeta^2 \geqslant 0$ follows the asymptotic equality

$$F(\zeta, \nu) \sim \nu^k F(\zeta), \quad F(\zeta) = \frac{2\pi}{k+1} \int\limits_{\zeta}^{1} \rho\Psi_0(\rho)(\rho - \zeta)^{k+1} d\zeta.$$

Thus, when the aforementioned restrictions are imposed on the weight function, the behavior of $F(x)$ in the physical region $0 \leqslant \zeta \leqslant 1$ is scaling-invariant. Note that for integer k the asymptotic formula represents an antiderivative of the order $k+1$.

Let us now study the asymptotic behavior of $F(x)$ in the vicinity of the light cone $x^2 \sim 0$:

$$F(x) = \frac{1}{(2\pi)^4} \int F(q)\, \mathrm{e}^{iqx} dx = -\frac{i}{2\pi} \int\limits_0^{\infty} D(x, \lambda^2)\Delta(|\mathbf{x}|, \lambda^2) d\lambda^2,$$

where $D(x, \lambda^2)$ is the commutator function for free scalar particles of mass λ:

$$D(x, \lambda^2) = \frac{i}{(2\pi)^3} \int \mathrm{e}^{-iqx} \varepsilon(q^0)\delta(q^2 - \lambda^2) dq,$$

while the spectral function $\Delta(|\mathbf{x}|, \lambda^2)$ is expressed by the formula

$$\Delta(r, \lambda^2) = 4\pi \int\limits_0^1 \Psi(\rho, \lambda^2) \frac{\sin \rho r}{r} \rho d\rho.$$

In an arbitrary reference system we have

$$F(x) = -\frac{i}{2\pi} \int\limits_0^{\infty} D(x, \lambda^2)\Delta\left(\sqrt{(p \cdot x)^2 - x^2}, \lambda^2\right) d\lambda^2.$$

If a spectral function satisfies the restrictions we adopted above, then we obtain the following formula for the asymptotic behavior of $F(x, p)$ in the vicinity of the light cone $x^2 \sim 0$:

$$F(x, p) \approx \frac{i}{\pi} G(p, x)\varepsilon(x^0)(-\Box)^k \delta'(x^2),$$

$$G(p \cdot x) = 4\pi \int\limits_0^1 \Psi_0(\rho) \frac{\sin\, \rho p \cdot x}{p \cdot x} \rho d\rho.$$

Thus, the coefficient $G(p \cdot x)$ at the principal singularity $F(x, p)$ in the vicinity of the light cone is expressed via the spectral function that determines the scaling-invariant behavior of function $F(q, p)$ in the Bjorken region.

In conclusion, we note that in the case of the free-field weight function $\Psi^0(\rho, \lambda^2)$ the restrictions $\lim \Psi^0(\rho, \lambda^2)/\lambda^{2k}$ are naturally satisfied at $k = 0$:

$$\Psi^0(\rho, \lambda^2) \equiv \Psi^0(\rho), \text{ if } \lambda^2 > 4,$$

and for $\Psi^0(\rho)$ we have an explicit expression [26]. The form factors of free nucleons, $W^0_{1,2}(\zeta, \nu)$, calculated with its aid, satisfy the following asymptotic relations exactly in the Bjorken region:

$$W^0_1 = \frac{1}{2}(F^0_2 - F^0_1), \quad W^0_2 = \frac{2\zeta}{\nu}(F^0_2 - F^0_1), \quad W^0_1 = \frac{\nu}{4\zeta}W^0_2.$$

Thus, the class of weight functions introduced above includes in a natural manner the weight functions of free fermion fields, which justifies application of the quasi-free quark model.

An important role in forming concepts of the quark structure of hadrons has been played by the investigation of sum rules, following from the algebra of local hadron currents [29]. Note in this connection that the form factors of mesons and baryons, constructed from equations of the model of quasi-independent colored quarks, reproduce these results to a significant degree [25, 39].

Note that the subsequent discovery of the phenomenon of asymptotic freedom [30] of an invariant charge [31] in quantum chromodynamics was an essentially important step in substantiation of the picture of quasi-free quarks in hadrons.

4. BROKEN COLOR SYMMETRY AND THE PROBLEM OF QUARK CHARGES [32, 33]

Colored quarks may have fractional or integer charges. The assumption that color symmetry exhibits an absolutely exact character is consistent only with colored quarks having fractional charges. However, in QCD, owing to quark confinement, quark charges can only be spoken of as effective constants characterizing the electromagnetic interaction between quarks at quite small distances.

In the case of integer quark charges color symmetry is not exact, and it is broken (locally or globally), at least in electromagnetic interactions.

Indeed, in models with integer quark charges the electromagnetic current is a sum of the singlet and octet terms in the $SU_c(3)$ group:

$$J^{\text{em}}_\mu = J^{c=0}_\mu(1) + J^{c\neq 0}_\mu(\underline{8}).$$

Thus, in the model of three color triplets

$$u \equiv (u^1, u^2, u^3), \quad d \equiv (d^1, d^2, d^3), \quad s \equiv (s^1, s^2, s^3)$$

selection of integer quark charges

$$Q_u = (1, 1, 0), \quad Q_d = Q_s = (0, 0, -1)$$

is implemented in accordance with the formula

$$Q_q = Q_0 + Q_c.$$

Here, Q_0 is the charge operator of $SU(3)$ quarks, and Q_c acts on color indices and is a generator of the color group:

$$Q_c = \frac{1}{\sqrt{3}}\lambda_8 = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right).$$

It is to be stressed that correspondence can be established between quarks with integer electric charges and integer baryon numbers, in accordance with the Nishijima–Gell-Mann formula

$$Q_q = T_3 + \frac{1}{2}B_q, \quad B_q = (1, 1, -1).$$

The dependence of quark charges on their color state, apparently, leads to violation of global color symmetry in electromagnetic interactions, which is manifested, for example, in the mass splitting of the color quark triplet and so on. It has, however, been shown that the neutrality of hadrons with respect to color, i.e., correspondence between the observed hadrons and singlet color wave functions, provides for the disappearance in all observed hadron characteristics (charges, magnetic moments, form factors, etc.) of any manifestation of the aforementioned violation of global color symmetry.

Quark charges and baryon numbers being integer provides the possibility for them to transform into leptons and other observable particles. As a result, we should arrive at the conclusion that quarks are not stable, which would explain the negative results of their searches. Subsequently, it was shown [34] that quark instability does not contradict the observed high stability of the nucleon and the extreme suppression of effects of non-conservation of the baryon number.

In taking into account the quantum-chromodynamic interaction of quarks, the introduction of quarks with integer charge raises a fundamental problem. Straightforward selection of the electromagnetic current in the form of a sum of the singlet and octet terms in the $SU_c(3)$ group leads to the violation of local $SU_c(3)$ symmetry and, as a consequence, to the theory not being renormalizable.

The problem is removed if one considers spontaneously broken color $SU_c(3)$ symmetry, which requires introduction in the theory of new degrees of freedom, for example, of color scalar Higgs fields [32]. An essential consequence of such models consists, also, in the possible existence of a new family of hadrons, composed of quarks and Higgs color scalars, bound strongly by chromodynamic forces [33].

The issue of whether color symmetry is exact or broken cannot be resolved a priori. In other words, the ultimate answer to this question can only be given by experiments. A most complete analysis of theoretical models involving broken color symmetry can be found in the review [25].

5. PARASTATISTICS FOR QUARKS

The first attempt at resolving the problem of quark statistics, made by Greenberg in 1964, was based on the hypothesis of quarks being parafermions of rank three [35]. Within the framework of this approach an explanation was successfully given of the existence of baryons described by completely symmetric spin-unitary wave functions.

Although it was already stressed in early publications [36] that the application of para-Fermi statistics for quarks and the introduction of a new quark quantum number, color, and of color $SU_c(3)$ symmetry, corresponding to it, were two non-equivalent approaches in elementary particle theory that led, generally speaking, to different physical consequences, there, nevertheless, do appear publications in which the two are unfoundedly considered equal.

To clarify this important issue of principle, we shall briefly expound the main results presented in the works of A. Govorkov [37], in which the properties of gauge symmetry of the local interaction of parafermion and paraboson vector fields are studied and a proof is given that the use of parastatistics is not equivalent to the introduction of color and of the corresponding gauge $SU_c(3)$ symmetry, underlying quantum chromodynamics.

For particles with spin 1/2, the wave functions of which satisfy the Dirac equation, para-Fermi fields are represented in the form of a Green ansatz* [38]:

$$\Psi = \sum_{A=1}^{3} \Psi_A(x),$$
$$\left[\overline{\Psi}_A(x), \Psi_B(y)\right]_{2\delta_{AB-1}} = -i\delta_{AB}\hat{S}(y-x), \quad [\Psi_A(x), \Psi_B(y)]_{2\delta_{AB-1}} = 0.$$

*We recall that in applying the model with parafermion fields of rank 3, it only turns out to be possible to resolve the problem of baryon spectroscopy, which requires up to three identical quarks to be found in one and the same quantum state.

The brackets indicate an anticommutator or commutator for $A = B$ and $A \neq B$, respectively, $\hat{S}(x)$ is a singular function for the Dirac field.

In the case of vector particles the Green ansatz of the para-Bose field is of the form

$$B^\mu(x) = \sum_{A=1}^{3} B^\mu_A(x), \quad [B^\mu_A(x), B^\nu_B(y)]_{1-2\delta_{AB}} = -i\delta_{AB} D^{\mu\nu}(x-y),$$

$D^{\mu\nu}(x)$ is the commutator function for vector fields.

The Fermi and Bose parafields satisfy the trilinear Green relations. We note, however, that in [36] the proof is given that the representation of parafields in the form of a Green ansatz is only unique strictly in the case of free fields. In the same work it was shown that no additional restrictions arise in the theory if for the para-Bose and para-Fermi fields of the same rank, 3 in the case considered, the following relations of the paraboson type hold valid:

$$[\Psi_A(x), \Psi_B(x)]_{1-2\delta_{AB}} = \left[\overline{\Psi}_A(x), \Psi_B(y)\right]_{1-2\delta_{AB}} = 0,$$

which jointly provide the conditions necessary for construction of a local theory of interacting parafermion and boson fields.

The anomalous character of the commutator relations for Green components does not permit one to assign any physical meaning to them directly.

There exists, however, the Klein transformation, which is non-linear and non-local, and which permits one to reduce the commutator relations for Green components to the normal canonical form, both for para-Fermi and for para-Bose fields.

For these parafields, the components of which can be assigned a physical meaning, the Yang–Mills Lagrangian exhibits $SO(3)$ symmetry and is written as [37]

$$L(x) = -\frac{1}{4}\mathbf{F}^2_{\mu\nu}(x) + \overline{\mathbf{\Psi}}(x)[i\gamma_\mu\partial^\mu - m]\mathbf{\Psi}(x) - ig\mathbf{B}^\mu(x)[\,\overline{\mathbf{\Psi}}(x) \times \gamma^\mu \mathbf{\Psi}(x)],$$

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g\,[\mathbf{B}_\mu \times \mathbf{B}_\nu],$$

where

$$\mathbf{\Psi} \equiv (\Psi_1, \Psi_2, \Psi_3) \text{ and } \mathbf{B} = (B_1, B_2, B_3)$$

are vectors of the $SO(3)$ group.

Note that, unlike quantum chromodynamics, the theory with $SO(3)$ symmetry contains only three gluons, and in the particle spectrum there are to be found diquarks, fermions with quark-meson quantum numbers, and other exotic hadrons. Moreover, the theory with $SO(3)$ symmetry exhibits the property of asymptotic freedom only under the condition that there exist no more than two quark flavors.

Thus, the hypothesis of para-Fermi and para-Bose statistics is not equivalent to the introduction of color and of color $SU(3)$ symmetry, and it leads to results not confirmed experimentally.

It can be noted that many of the works of priority presented in this article were only published as preprints or in the proceedings of international conferences, in accordance with the opportunities that existed at the time.

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