ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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STRUCTURE FUNCTION APPROACH IN QED FOR HIGH-ENERGY PROCESSES

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The structure function method is considered, based on the renormalization group approach: when combined with exact calculation at the lowest order of perturbation theory it allows one to calculate the differential cross sections in leading and next-to-leading order approximations, again providing the thousands accuracy.

The implementation of this method to calculation of radiative corrections is also done for some processes on colliders. Among them are radiative corrections to DVCS tensor, π_{e2} decay, deep inelasic scattering, muon decay, Bhabha scattering, electron–positron annihilation and others.

In some cases the explicit expression of nonleading terms (the so-called K factor) is given or numerical estimation of it is done.

Рассматривается метод структурных функций, основанный на использовании ренормгруппового подхода, позволяющий рассчитывать радиационные поправки в лидирующем и следующем за лидирующим приближении на основе расчета в низшем порядке теории возмущений.

Приведены примеры расчетов процессов для физически интересных случаев, а именно радиационные поправки к глубоконеупругому виртуальному комптон-тензору, глубоконеупругому рассеянию, распаду мюона, баба-рассеянию, электрон-позитронной аннигиляции и мн. др.

В большинстве случаев оценка нелидирующих вкладов дана в явной аналитической форме.

PACS: 12.15.Lk; 12.20.-m; 12.20.Ds

To the blessed memory of Teachers Alexandr Ilich Akhiezer, Sergej Semenovich Sannikov, and Vladimir Naumovich Gribov

INTRODUCTION

The problem of infrared divergences (related to the zero-mass limit of the «photon mass», λ) as well as the problem of the extraction of the electron mass singularities are considered in this review. In the early sixties, it was understood that the physical quantities, the measurable observables such as cross sections and decay widths are finite in the photon zero-mass limit, but contain an auxiliary parameter, ϵ, the maximum energy carried by soft photons, which are emitted and escape the detection. The ϵ dependence turns out to have a universal exponent form, associated to each charged particle.

The problem of the extraction of the main contributions to the cross section of radiative processes, associated to the large logarithm L was solved in the Born approximation, with the development of the QREM (see references).

Combining soft photon emission with QREM, the universal evolution equations for probability densities were built in the seventies by L. Lipatov in QED, and in 1977 by G. Altarelli and G. Parisi for the QCD case.

These quantities — probability density to find a parton of a given kind inside the initial one (considering electrons and photons as a parton) can be classified by twist expansion. The lowest ones, of twist two, are named structure functions.

It was realized that all the QED processes can be described in the frame of SF approach as cross section or widths written in the form of Drell–Yan processes. This formalism manifests the factorization of long- and short-distance contributions. From the formal point of view, it is the conversion of universal SF (long distances) with the cross section in the Born approximation. This form guarantees that RC are taken into account in the so-called Leading Logarithmic Approximation:

$$\frac{\alpha}{\pi} \ll 1, \quad \frac{\alpha L}{\pi} \sim 1.$$

To increase the accuracy of these parton-type formulas, one introduces the so-called K factor:

$$K(\alpha, y) = 1 + \frac{\alpha}{\pi} K(y)$$

as additional factor in the Drell–Yan formulae. The calculation of the definite expression of the K factor for the specific processes is the object of several Sections below.

The relevant quantities can be constructed for light charged bosons (pions). When being applied to the description of heavy hadron decay, the logarithm of mass ratio of heavy hadron and pion plays the role of «large logarithm».

The application of SF approach to the analysis of DIS and to DVCS turns out to be successful as well as of the annihilation through one virtual photon in presence of narrow resonances with $\mathcal{J}^C = 1^-$ such as ω, ϕ, and J/ψ.

Throughout our paper we use the next designations:

DIS — deep inelastic scattering

DVCS — deep virtual Compton scattering

FD — Feynman diagram

LBL — light-by-light

LLA — leading logarithmic approximation

LSF — lepton structure functions

NLO — next-to-leading order

QCD — quantum chromodynamics

QED — quantum electrodynamics
QREM — quasi-real electron method
RC — radiative corrections
SF — structure function.

1. STRUCTURE FUNCTION APPROACH

1.1. Process of Electron–Positron Annihilation into Hadrons through One-Photon Exchange at High Energy as a Drell–Yan Process. The process of electron–positron annihilation into hadrons through one-photon exchange can be considered as the crossed process of Drell–Yan annihilation of hadrons into lepton pairs at high energies.

The methods developed in QCD based on factorization of mass singularities and on the renormalization group can be applied [10].

Let us consider electron, positron, and photon as a parton A, or $\bar{A}$ in Drell–Yan approach, and calculate the cross section taking into account the emission of any number of real and virtual photons from the initial leptons.

In the LLA

$$\frac{\alpha}{\pi}L \sim 1, \quad \frac{\alpha}{\pi} \ll 1, \quad L = \ln\frac{s}{m_e^2} \tag{1.1}$$

the radiative corrected cross section $\sigma_{\rm RC}^{e^+e^-\to\text{hadrons}}(s)$ can be written in the form

$$\sigma_{\rm RC}^{e^+e^-\to\text{hadrons}}(s) = \int\limits_{x_1^{\min}}^{1} dx_1 \int\limits_{x_2^{\min}}^{1} dx_2 \Theta\left(x_1 + x_2 - 2 + \frac{\Delta E}{E}\right) \times \\ \times \sum_{A=e^+,e^-,\gamma} D_{e^-}^{A}(x_1, s) D_{e^+}^{\bar{A}}(x_2, s) \sigma_B^{A\bar{A}\to\text{hadrons}}(s x_1 x_2), \tag{1.2}$$

where the sum runs over the partons, $D_{e^-}^{A}(x_1, s)$ is the distribution function of the energy fraction x_1 of a parton of type A with momentum squared up to s in the initial electron, $\sigma_B(s x_1 x_2)$ is the «shifted» cross section in the Born approximation, without taking into account radiative corrections to the initial lepton's state.

This formula was derived in the frames of QCD for the Drell–Yan process for creation of muon pair in hadron collisions. In QED the role of partons is played by leptons and photons.

The distribution functions $D(x, s)$ in LLA satisfy the Lipatov evolution [11] equations known in QCD as Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [7, 8] which describe their dependence on the energy fraction and the momentum squared

$$D_e^e(x,s) = \delta(1-x) + \int\limits_{m^2}^{s} \frac{dt\alpha(t)}{2\pi t} \left[\int\limits_x^1 \frac{dy}{y} D_e^e(y,t) P_e^e\left(\frac{x}{y}\right) + \right.$$
$$\left. + \int\limits_x^1 \frac{dy}{y} D_e^\gamma(y,t) P_\gamma^e\left(\frac{x}{y}\right) \right],$$

$$D_e^{\bar{e}}(x,s) = \int\limits_{m^2}^{s} \frac{dt\alpha(t)}{2\pi t} \left[\int\limits_x^1 \frac{dy}{y} D_e^{\bar{e}}(y,t) P_{\bar{e}}^{\bar{e}}\left(\frac{x}{y}\right) + \int\limits_x^1 \frac{dy}{y} D_e^\gamma(y,t) P_\gamma^{\bar{e}}\left(\frac{x}{y}\right) \right], \tag{1.3}$$

$$D_e^\gamma(x,s) = -\frac{2}{3} \int\limits_{m^2}^{s} \frac{dt\alpha(t)}{2\pi t} + \int\limits_{m^2}^{s} \frac{dt\alpha(t)}{2\pi t} \left[\int\limits_x^1 \frac{dy}{y} D_e^e(y,t) P_e^\gamma\left(\frac{x}{y}\right) + \right.$$
$$\left. + \int\limits_x^1 \frac{dy}{y} D_e^{\bar{e}}(y,t) P_{\bar{e}}^\gamma\left(\frac{x}{y}\right) \right],$$

$$\alpha(t) = \frac{\alpha}{1 - \frac{\alpha}{3\pi} \ln \frac{t}{m^2}},$$

where t is the square of parton momentum; $\alpha(t)$ is the «running» coupling constant. The quantity

$$\frac{\alpha(t)}{2\pi} P_A^B\left(\frac{x}{y}\right) \frac{dx\,dt}{yt}$$

is the differential probability of the decay of the parton A with energy fraction y, into a pair of partons, provided that for one of them, the parton B, the energy fraction lies in the interval $x+dx, x$ with $x<y$, and the momentum squared lies in the interval $t+dt, t$. The kernels of these integral equations are

$$P_e^e(z) = P_{\bar{e}}^{\bar{e}}(z) = \lim_{\Delta\to 0} \left[\frac{1+z^2}{1-z} \theta(1-z-\Delta) + \right.$$
$$\left. + \delta(1-z)\left(\frac{3}{2} + 2\ln\Delta\right) \right] = \left[\frac{1+z^2}{1-z}\right]_+,$$

$$P_\gamma^e(z) = P_\gamma^{\bar{e}}(z) = z^2 + (1-z)^2, \tag{1.4}$$
$$P_e^\gamma(z) = P_{\bar{e}}^\gamma(z) = \frac{1}{z}(1 + (1-z)^2).$$

The singularities in $P_e^e(z)$ are cancelled out when integrating with any flat function. This property of cancellation has the same origin as the cancellation of infrared divergences from the contribution of virtual and soft real photon emission to the observable cross section (Blokh–Nordsic theorem). For instance, the moments of the kernel $P_e^e(z) = P(z)$ are

$$C_n = \int\limits_0^1 z^n P(z) dz = \int\limits_0^1 \frac{1+z^2}{1-z}(z^n - 1) = \frac{3}{2} - 2 \sum_{k=1}^{k=n-1} \frac{1}{k} + \frac{1}{(n+1)(n+2)}. \tag{1.5}$$

It is convenient to distinguish the singlet and nonsinglet contributions to the structure function D_e^e [10, 12]

$$D_e^e = D^S + D^{\mathrm{NS}}, \quad D^S = D_e^{\bar{e}}. \tag{1.6}$$

D^{NS} corresponds to Feynman diagrams in which the electron may emit a photon, but the electron line arrives at the point of annihilation, D^S, to the diagrams where the electron is converted into a photon and then back into an electron before reaching the annihilation point. The nonsinglet term satisfies the equation

$$D^{\mathrm{NS}}(x, \beta) = \delta(x-1) + \int\limits_{m^2}^{s} \frac{\alpha(t)}{2\pi} \frac{dt}{t} \int\limits_x^1 \frac{dz}{z} P(z) D^{\mathrm{NS}}\left(\frac{x}{z}, \beta_t\right).$$

$$\beta_t = \frac{\alpha}{2\pi}\left(\ln \frac{t}{m^2} - 1\right). \tag{1.7}$$

In order to satisfy the requirement of accuracy at the level 0.1% (we note that in reality the expansion parameter $\frac{\alpha}{\pi} L, L = \ln s/m_e^2$ is small. For LEP facility it takes the value 0.05, with $L \approx 20$), it is sufficient to retain in D^S the leading term of the expansion

$$D^S(x, \beta) = \frac{1}{8}\left(\frac{\alpha L}{\pi}\right)^2 F_S(x),$$
$$F_S(x) = \int\limits_x^1 \frac{dy}{y} P_e^\gamma(y) P_\gamma^e\left(\frac{x}{y}\right) = \frac{(1-x)}{3x}(4 + 7x + 4x^2) + 2(1+x)\ln x. \tag{1.8}$$

Before analyzing the single-photon annihilation channel, let us remind some properties of the nonsinglet structure function. Equation (1.7) is solved through an iteration procedure which leads to the expansion

$$
\begin{gathered}
D^{\mathrm{NS}}(x,\beta) = \delta(1-x) + \sum_{k=1}^{k=\infty} \frac{1}{k!}\beta^k P^{(k)}(x), \\
P^{(1)}(x) = P_e^e(x) = P(x), \quad P^{(2)}(x) = \int\limits_x^1 \frac{dy}{y} P(y) P\left(\frac{x}{y}\right), \\
P^{(k)}(x) = \int\limits_x^1 \frac{dy}{y} P^{(k-1)}(y) P\left(\frac{x}{y}\right), \quad \beta = \frac{\alpha}{2\pi}(L-1),
\end{gathered}
\tag{1.9}
$$

where $P(x) = P^{(1)}(x)$ was given in (1.4). The explicit form of the higher iterations $P^{(k)}(x)$ can be found in [12].

The smoothed form of $D(x,\beta)$ is

$$
D(x,\beta) = 2\beta(1-x)^{2\beta-1}\left(1+\frac{3}{2}\beta\right) - \beta(1+x) + \mathcal{O}(\beta^2). \tag{1.10}
$$

The distribution functions D^{NS} obey the following properties:

$$
\begin{gathered}
\int\limits_0^1 D^{\mathrm{NS}}(x,\beta)dx = 1, \quad \int\limits_0^1 P^{(k)}(x)dx = 0, \quad k = 1,2,\ldots, \\
\int\limits_x^1 \frac{dy}{y} D^{\mathrm{NS}}(y,\beta_1) D^{\mathrm{NS}}\left(\frac{x}{y},\beta_2\right) = D^{\mathrm{NS}}(x,\beta_1+\beta_2).
\end{gathered}
\tag{1.11}
$$

We note that the result for the cross section (1.2) is valid for the case $\Delta E \sim E$ as well. When $\Delta E \ll E$, the conditions $x_1 + x_2 > 2 - \Delta E/E$ and $x_1 x_2 > 1 - \Delta E/E$ are equivalent. When the ratio $\Delta E/E$ is small, the contribution of D^S is negligible. For smooth functions as $\sigma(s)/|1-\Pi(s)|^2$ one has

$$
\begin{gathered}
\sigma_{\mathrm{RC}}(s) = \frac{\sigma_0(s)}{|1-\Pi(s)|^2} R\left(1 - \frac{\Delta E}{E}, s\right), \\
R(x,s) = \int\limits_x^1 dx_1 \int\limits_{x/x_1}^1 dx_2\, D^{\mathrm{NS}}(x_1,\beta) D^{\mathrm{NS}}(x_2,\beta).
\end{gathered}
\tag{1.12}
$$

The quantity $R(z,s)$ satisfies the integral equation

$$
R(z,s) = 1 + \int\limits_{m^2}^{s} \frac{\alpha(t)dt}{\pi t} \int\limits_z^1 dy\, P(y) R\left(\frac{z}{y}, t\right). \tag{1.13}
$$

To solve this equation let us apply Mellin transform and define the moments:

$$R_n(s) = \int\limits_0^1 dx\, x^{n-1} R(x,s). \tag{1.14}$$

In terms of moments we have

$$R_n(s) = \frac{1}{n} + C_n \int\limits_{m^2}^{s} \frac{dt}{t}\frac{\alpha(t)}{\pi} R_n(t), \tag{1.15}$$

with C_n — the moments of the kernel defined in Eq. (1.5). Using the inverse Mellin transform we get

$$R(x,s) = \frac{1}{2\pi i}\int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{dn}{n} x^{-n} \exp\left[C_n \int\limits_{m^2}^{s} \frac{dt}{t}\frac{\alpha(t)}{\pi}\right]. \tag{1.16}$$

One can obtain an analytical expression for the case $1 - x + \Delta E/E \ll 1$, where large values of n are essential. Using the asymptotic representation for C_n at large n

$$C_n \approx \frac{3}{2} - 2C - 2\ln n + O\left(\frac{1}{n}\right)$$

(here $C = 0.577\ldots$ is the Euler constant), we obtain

$$\begin{gathered} R(x,s) = \exp\left(\frac{3}{2} - 2C\right)\xi \frac{1}{2\pi i}\int\limits_{-i\infty}^{i\infty} \frac{dn}{n} n^{-2\xi}\ \mathrm{e}^{-n\ln x}, \\ \xi = \int\limits_{m^2}^{s} \frac{dt}{t}\frac{\alpha(t)}{\pi} = -3\ln\left(1 - \frac{\alpha}{3\pi}L\right). \end{gathered} \tag{1.17}$$

The final expression for R is

$$R\left(1 - \frac{\Delta E}{E}, s\right) = \left(\frac{\Delta E}{E}\right)^{\xi} \mathrm{e}^{\frac{3}{4}\xi} \frac{\mathrm{e}^{-C\xi}}{\Gamma(1+\xi)}\left(1 + \mathcal{O}\left(\frac{\Delta E}{E}\right)\right). \tag{1.18}$$

Expansion in LLA parameter $\alpha L/\pi$ has the form:

$$\begin{aligned} R\left(1 - \frac{\Delta E}{E}, s\right) \approx 1 &+ \frac{2\alpha L}{\pi}\left(\ln\frac{\Delta E}{E} + \frac{3}{4}\right) + \\ &+ \left(\frac{\alpha L}{\pi}\right)^2\left[2\ln^2\frac{\Delta E}{E} + \frac{10}{3}\ln\frac{\Delta E}{E} + \frac{11}{8} - \frac{\pi^2}{3}\right]. \end{aligned} \tag{1.19}$$

This result can be compared with the one obtained by Yu. Tsai [13]:

$$R_{\text{Tsai}} = \left(1 - \frac{\alpha}{3\pi} L\right)^{6 \ln \frac{\Delta E}{E} - \frac{9}{2}} \approx 1 + \frac{2\alpha L}{\pi} \left(\ln \frac{\Delta E}{E} + \frac{3}{4}\right) + \\ + \left(\frac{\alpha L}{\pi}\right)^2 \left[2 \ln^2 \frac{\Delta E}{E} + \frac{10}{3} \ln \frac{\Delta E}{E} + \frac{11}{8}\right], \quad (1.20)$$

which differs from Eq. (1.19) by the term $-(\pi^2/3)$. The reason is that in [13] the renormalization group arguments were applied to the total cross section — the quantity which has mass singularities, instead of those to the amplitudes. Explicit calculations allow one to take into account the nonleading terms of sort $\alpha(\alpha L/\pi)^n$. It turns out that the real expansion parameter is

$$\beta = \frac{\alpha}{2\pi}(L - 1). \quad (1.21)$$

Next-to-leading order terms can be taken into account introducing a K factor in the integrand of the Drell–Yan formula (1.2):

$$1 + \frac{\alpha}{\pi} K = 1 + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2}\right). \quad (1.22)$$

So the final result can be expressed as (for simplicity $D^{\text{NS}} = D$)

$$\sigma_{\text{RC}}(s) = \int\limits_{x_1^{\min}}^{1} dx_1 \int\limits_{x_2^{\min}}^{1} dx_2 \, D(x_1, \beta) D(x_2, \beta) \frac{\sigma_0(s x_1 x_2)}{|1 - \Pi(s x_1 x_2)|^2} \left(1 + \frac{\alpha}{\pi} K\right), \quad (1.23)$$

where $D = D^\gamma + D^\pm$ in the case when pair production is detected by the experiment and $D = D^\gamma$ if pairs are not detected. The explicit expressions for expansion of $D^\gamma, D^\pm$, which provides the precision of the theoretical formulae on the level 0.1%, are [10]

$$D^{\text{NS}}(x, \beta) = D^\gamma(x, \beta) = \\ = 2\beta(1 - x)^{2\beta - 1} \left[1 + \frac{3}{2}\beta - \frac{1}{3}\beta^2 \left(\frac{1}{3} L + \pi^2 - \frac{47}{8}\right)\right] - \beta(1 + x) + \\ + \frac{1}{2}\beta^2 \left[4(1 + x) \ln \frac{1}{1 - x} + \frac{1 + 3x^2}{1 - x} \ln \frac{1}{x} - 5 - x\right] + \mathcal{O}(\beta^3),$$

$$D^{\pm}(x,\beta) = \left(\frac{\alpha}{2\pi}\right)^2 \left\{ \frac{1}{3(1-x)} \left(1 - x - \frac{4m}{\sqrt{s}}\right)^{2\beta} \left(L + 2\ln(1-x) - \frac{5}{3}\right)^2 \times \right.$$
$$\times \left[1 + x^2 + \frac{2}{3}\beta\left(L + 2\ln(1-x) - \frac{5}{3}\right)\right] +$$
$$\left. + L^2 \left[\frac{2}{3}\frac{1-x^3}{x} + \frac{1}{2}(1-x) + (1+x)\ln\frac{1}{x}\right]\right\} \theta\left(1 - x - \frac{4m}{\sqrt{s}}\right). \quad (1.24)$$

The cross section can be expressed in terms of one-fold integral, introducing a new variable $x_1 x_2 = 1 - x$ and performing the integration with the following result:

$$\sigma_{\mathrm{RC}}(s) = \int\limits_0^{x^{\max}} dx\, F(x,s) \frac{\sigma_0(s(1-x))}{|1 - \Pi(s(1-x))|^2}, \quad F = F^{\gamma} + F^{\pm}, \qquad (1.25)$$

where $x^{\max}$ is derived by the experimental constraint that $s(1-x^{\max}) > s_{\mathrm{threshold}} = 4m_\pi^2$. The explicit form of the functions F^{γ} and $F^{\pm}$ is

$$F^{\gamma}(x,\beta) = 4\beta x^{4\beta-1} \left[1 + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \right.$$
$$\left. + 3\beta - \frac{2}{3}\beta^2\left(\frac{1}{3}L + 2\pi^2 - \frac{37}{4}\right)\right] - 2\beta(2-x) +$$
$$+ 2\beta^2 \left[4(2-x)\ln\frac{1}{x} + \frac{1}{x}(1 + 3(1-x)^2)\ln\frac{1}{1-x} - 6 + x\right] + \mathcal{O}(\beta^3), \quad (1.26)$$

$$F^{\pm}(x,\beta) = \left(\frac{\alpha}{\pi}\right)^2 \left\{ \frac{1}{6x}\left(x - \frac{4m}{\sqrt{s}}\right)^{4\beta} \left[(2 - 2x + x^2)\left(L + 2\ln x - \frac{5}{3}\right)^2 + \right.\right.$$
$$\left. + \frac{4}{3}\beta\left(L + 2\ln x - \frac{5}{3}\right)^3\right] + \frac{1}{2}L^2 \left[\frac{2}{3}\frac{1-(1-x)^3}{1-x} + \right.$$
$$\left.\left. + (2-x)\ln(1-x) + \frac{1}{2}x\right]\right\} \theta\left(x - \frac{4m}{\sqrt{s}}\right).$$

1.2. Differential Cross Section in LLA. The differential distributions of process

$$A(p_A) + B(p_B) \to a'(p_{a'}) + b'(p_{b'}) + (X) \qquad (1.27)$$

as well can be expressed in terms of structure functions:

$$\frac{d\sigma_{A+B\to a+b+(X)}}{dO_a\,dy_a\,dy_b}(p_A,p_B)= \\ =\sum_{e,f,c,d}\int dx_c\int dx_d D_A^c(x_c,\beta)D_B^d(x_d,\beta)\frac{d\bar{\sigma}_{c+d\to e+f}}{dO_c}(p_Ax,p_By)\times \\ \times\frac{1}{x_a}\frac{1}{x_b}D_c^a\left(\frac{y_a}{x_c},\beta\right)D_f^b\left(\frac{y_b}{x_d},\beta\right). \quad (1.28)$$

Hard subprocess $cd\to ef$ is considered in the c.m.f. of initial particles:

$$p_A=\varepsilon(1,1,0,0),\quad p_B=\varepsilon(1,-1,0,0),\quad s=4\varepsilon^2\gg m^2. \quad (1.29)$$

The shifted kinematics is defined by conservation law

$$c(x_cp_A)+d(x_dp_B)\to e(p_e)+f(p_f),\quad p_e=\varepsilon x_e(1,\mathbf{n}_e),\quad p_f=\varepsilon x_f(1,\mathbf{n}_f), \quad (1.30)$$

with orts $\mathbf{n}_e$, $\mathbf{n}_f$ along the three-momentum of scattered particles and

$$x_c+x_d=x_e+x_f,\quad x_c-x_d=x_ec_e+x_fc_f,\quad x_e^2s_e^2=x_f^2s_f^2, \quad (1.31)$$

where $c_{e,f}=\cos(\widehat{\mathbf{p}_A\mathbf{n}}_{e,f})$, $s_{e,f}=\sin(\widehat{\mathbf{p}_A\mathbf{n}}_{e,f})$. For the energy fraction of the scattered particles of hard subprocess we can find

$$x_e=\frac{2x_cx_d}{a},\quad x_f=\frac{x_c^2+x_d^2+c_e(x_d^2-x_c^2)}{a}, \\ a=X+Y,\quad X=x_c(1-c_e),\quad Y=x_d(1+c_e). \quad (1.32)$$

The relevant invariants we could put in the form (we imply that all particles in the hard subprocess are massless):

$$\tilde{s}=(x_cp_A+x_dp_B)^2=sx_cx_d,\quad \tilde{t}=(x_cp_A-p_e)^2=-s\frac{x_c^2x_d(1-c_e)}{a}, \\ \tilde{u}=(x_cp_A-p_f)^2=-s\frac{x_cx_d^2(1+c_e)}{a},\quad \tilde{s}+\tilde{t}+\tilde{u}=0. \quad (1.33)$$

Phase volume of final particle has the form:

$$d\Gamma_2=\frac{1}{(2\pi)^2}\frac{d^3p_e}{2\varepsilon_e}\frac{d^3p_f}{2\varepsilon_f}\delta^4(x_cp_A+x_dp_B-p_e-p_f)= \\ =\frac{dO_E}{16\pi^2}\frac{x_e}{x_f}\delta(x_c+x_d-x_e-x_f)dx_e=\frac{x_cx_d}{8\pi^2a^2}dO_e. \quad (1.34)$$

The list of relevant cross sections of hard subprocess for $c, d, e, f = e^{\pm}, \gamma, \mu^{\pm}$, $\pi^{\pm}$ are

$$\begin{aligned}
\frac{d\sigma^{e^-e^+\to\gamma\gamma}}{dO_e} &= \frac{2\alpha^2}{s}\frac{X^2+Y^2}{a^2XY}, \\
\frac{d\sigma^{\gamma\gamma\to\pi^-\pi^+}}{dO_e} &= \frac{4\alpha^2}{s}\frac{1}{a^2}, \\
\frac{d\sigma^{\gamma e^{\pm}\to\gamma e^{\pm}}}{dO_e} &= \frac{2\alpha^2(a^2+Y^2)}{sYa^3}, \\
\frac{d\sigma^{\gamma\gamma\to e^-e^+}}{dO_e} &= \frac{2\alpha^2}{s}\frac{X^2+Y^2}{a^2XY},
\end{aligned} \tag{1.35}$$

with the vacuum polarization inclusion

$$\begin{aligned}
\frac{d\sigma^{e^-e^-\to e^-e^-}}{dO_e} &= \frac{4\alpha^2}{sa^2}\Bigg(\frac{a^2+Y^2}{2X^2}\frac{1}{(1-\Pi(\tilde{t}))^2} + \frac{a^2+X^2}{2Y^2}\frac{1}{(1-\Pi(\tilde{u}))^2} + \\
&\qquad + \frac{a^2}{XY}\frac{1}{(1-\Pi(\tilde{t}))(1-\Pi(\tilde{u}))}\Bigg), \\
\frac{d\sigma^{e^-e^+\to e^-e^+}}{dO_e} &= \frac{4\alpha^2}{sa^2}\Bigg(\frac{a^2+Y^2}{2X^2}\frac{1}{(1-\Pi(\tilde{t}))^2} + \frac{Y^2+X^2}{2a^2}\frac{1}{(1-\Pi(\tilde{s}))^2} - \\
&\qquad - \frac{Y^2}{Xa}\mathrm{Re}\left[\frac{1}{(1-\Pi(\tilde{t}))(1-\Pi(\tilde{s}))}\right]\Bigg),
\end{aligned} \tag{1.36}$$

$$\begin{aligned}
\frac{d\sigma^{e^-e^+\to\mu^-\mu^+}}{dO_e} &= \frac{2\alpha^2}{s}\frac{X^2+Y^2}{a^4}\frac{1}{|1-\Pi(\tilde{s})|^2}, \\
\frac{d\sigma^{e^-e^+\to\pi^-\pi^+}}{dO_e} &= \frac{2\alpha^2}{s}\frac{XY}{a^4}\frac{1}{|1-\Pi(\tilde{s})|^2}, \\
\frac{d\sigma^{e^-\mu^-\to e^-\mu^-}}{dO_e} &= \frac{2\alpha^2}{sa^2}\frac{a^2+Y^2}{X^2}\frac{1}{(1-\Pi(\tilde{t}))^2}
\end{aligned}$$

and without it (here we put the limit $\Pi(x) \to 0$):

$$\begin{aligned}
\frac{d\sigma^{e^-e^-\to e^-e^-}}{dO_e} &= \frac{4\alpha^2}{sX^2Y^2}\frac{(x^2+Y^2+XY)^2}{a^2}, \\
\frac{d\sigma^{e^-e^+\to e^-e^+}}{dO_e} &= \frac{4\alpha^2}{sX^2}\frac{(X^2+Y^2+XY)^2}{a^4}, \\
\frac{d\sigma^{e^-e^+\to\mu^-\mu^+}}{dO_e} &= \frac{2\alpha^2}{s}\frac{X^2+Y^2}{a^4},
\end{aligned} \tag{1.37}$$

$$\frac{d\sigma^{e^-e^+\to\pi^-\pi^+}}{dO_e} = \frac{2\alpha^2}{s}\frac{XY}{a^4},$$
$$\frac{d\sigma^{e^-\mu^-\to e^-\mu^-}}{dO_e} = \frac{2\alpha^2}{sa^2}\frac{a^2+Y^2}{X^2}.$$

Here in $d\sigma^{e^-e^+\to\gamma\gamma}$ when integrating over O_1, factor $1/2!$ should be included due to the two identical particles in the final state.

When integrating Eq. (1.28) over the final particle energy fraction, we obtain the differential distribution in the form:

$$\frac{d\sigma_{A+B\to a+b+(X)}}{dO_a}(p_A,p_B) = \sum_{c,d}\int dx_c \int dx_d D_A^c(x_c,\beta)\times \\ \times D_B^d(x_d,\beta)\frac{d\sigma_{c+d\to a+b}}{dO_a}(p_A x_c, p_B x_d). \quad (1.38)$$

1.3. Parton Picture of Electroweak Processes in High-Energy e^+e^- Collisions. Let us remind the known formula derived in [14] for the cross section of the process of creation of a system Φ of particles with invariant mass squared $s_1 \ll s = 2p_+p_- = 4E^2$, in the collisions of high-energy electrons and positrons through a two-photon production mechanism

$$\sigma(s)^{e^+e^-\to e'^+e'^-\Phi} = \beta^2\int\limits_{s_0}^{s}\frac{ds_1}{s_1}\sigma^{\gamma\gamma\to\Phi}(s_1)\phi_{\gamma\gamma}\left(\frac{s_1}{s}\right), \quad (1.39)$$

where $\sigma^{\gamma\gamma\to\Phi}(s_1)$ is the total cross section for the creation of a system Φ by two real photons; s_0 is the threshold value of invariant mass squared, and $\phi_{\gamma\gamma}(x)$ is the Brodsky–Kinoshita–Terazawa function

$$\phi_{\gamma\gamma}(x) = (2+x)^2\ln\frac{1}{x} - 2(3+x)(1-x) = x\int\limits_x^1\frac{dz}{z}P_e^\gamma(z)P_e^\gamma\left(\frac{x}{z}\right). \quad (1.40)$$

It is known, however, that this formula has a rather low accuracy, which is of the order of 20%. The differential invariant mass distribution has a much better accuracy for the case of a large invariant mass squared $s_1 \gg m^2$

$$\frac{d\sigma^{e^+e^-\to e'^+e'^-\Phi}(s_1,s)}{ds_1} = \frac{\beta^2}{s_1}\sigma^{\gamma\gamma\to\Phi}(s_1)\phi_{\gamma\gamma}\left(\frac{s_1}{s}\right), \quad s_1 \gg m^2. \quad (1.41)$$

This process can be considered a Drell–Yan process as well. Using the approximate solution of Lipatov's equations [12, 15]:

$$D^{\rm NS}(x,\beta) = 2\beta\left[\left(1+\frac{3}{2}\beta\right)(1-x)^{2\beta-1} - \frac{1}{2}(1+x)\right],$$

$$D^S(x,\beta) = \frac{1}{2}\beta^2\left[4x(1+x)\ln x + \frac{1}{3}(1-x)(4+7x+4x^2)\right], \tag{1.42}$$

$$D_e^\gamma(x,\beta) = \frac{\beta[1+(1-x)^2]}{x} + \frac{\beta^2}{4}\left[3+4\ln(1-x)\frac{1+(1-x)^2}{x} - \right.$$
$$\left. -2(2-x)\ln x + \frac{2}{x}(1-x)(2x-3)\right],$$

we can build the differential cross sections for different systems in the final state, created in e^+e^- collisions

$$\begin{aligned}
\frac{d\sigma^{e^+e^-\to(e\gamma)\Phi}(s_1,s)}{ds_1} &= \frac{1}{s_1}\sigma^{e\gamma\to\Phi}(s_1)\Phi_{e\gamma}\left(\frac{s_1}{s},\beta\right),\\
\frac{d\sigma^{e^+e^-\to(e'^+e'^-)\Phi}(s_1,s)}{ds_1} &= \frac{1}{s_1}\sigma^{\gamma\gamma\to\Phi}(s_1)\Phi_{\gamma\gamma}\left(\frac{s_1}{s},\beta\right),\\
\frac{d\sigma^{e^+e^-\to(\gamma\gamma)\Phi}(s_1,s)}{ds_1} &= \frac{1}{s}\sigma^{e^+e^-\to\Phi}(s_1)\frac{1}{|1-\Pi(s_1)|^2}\Phi_{e\bar{e}}\left(\frac{s_1}{s},\beta\right).
\end{aligned} \tag{1.43}$$

Here $\Pi(s) = \dfrac{\alpha}{3\pi}\left(\ln\dfrac{s}{m^2} - \dfrac{5}{3}\right)$ is the lepton contribution to the operator of vacuum polarization; the (un)detected final particles are putted in brackets in the left-hand side of the equations. The functions Φ_{ij} ($\Phi_{e\bar{e}}(x,\beta) = D^{\rm NS}(x,\beta)$ was considered above in Eq. (1.42)) are the following:

$$\Phi_{\gamma e}(x,\beta) = 2\beta(1+(1-x)^2)\left(1+\frac{3}{2}\beta\right)(1-x)^{2\beta} +$$
$$+ 2\beta^2[x(2-x)\ln x - (1-x)(3-2x)],$$

$$\Phi_{\gamma\gamma}(x,\beta) = \beta^2\phi_{\gamma\gamma}(x) + 8\beta^3\left[4(2+x)^2\left(\mathrm{Li}(x) - \frac{\pi^2}{6}\right) + \right.$$
$$+ x(4+x)\ln^2 x - x(12+5x)\ln x -$$
$$\left. - (1-x)(24+8x)\ln(1-x) + (1-x)(10+8x)\right]. \tag{1.44}$$

The Drell–Yan partons picture can be generalized for the case of colliding e^+e^- beams with definite chirality

$$\sigma_{\lambda_+\lambda_-}^{e^+e^-\to\Phi} = \sum_{A,\lambda_A}\sum_{B,\lambda_B}\int dx_1\int dx_2 D_{e^-\lambda_-}^{A,\lambda_A}(x_1,\beta)D_{e^+\lambda_+}^{B,\lambda_B}(x_2,\beta)\sigma_{AB\to\Phi}^{\lambda_A\lambda_B}(sx_1x_2), \tag{1.45}$$

Let us denote $D_\pm(G_\pm)$ — the distribution densities of probability to find an electron (photon) with chirality «$\pm$» in the initial electron with chirality «$+$». Using the QCD technique for building the kernels of the evolution equations and the chiral amplitudes [16, 17] one obtains (neglecting D^S):

$$D_+(x,\beta) = \delta(1-x) + \int\limits_0^L \frac{dt\,\alpha(t)}{2\pi} \int\limits_x^1 \frac{dy}{y} \left[D_+(y,\beta_t) P\left(\frac{x}{y}\right) + \right.$$
$$\left. + G_+(y,\beta_t) P^+_{\gamma+}\left(\frac{x}{y}\right) + G_-(y,\beta_t) P^+_{\gamma-}\left(\frac{x}{y}\right) \right],$$

$$D_-(x,\beta) = \int\limits_0^L \frac{dt\,\alpha(t)}{2\pi} \int\limits_x^1 \frac{dy}{y} \left[D_+(y,\beta_t) P\left(\frac{x}{y}\right) + \right.$$
$$\left. + G_-(y,\beta_t) P^-_{\gamma-}\left(\frac{x}{y}\right) + G_+(y,\beta_t) P^-_{\gamma+}\left(\frac{x}{y}\right) \right],$$

$$G_+(x,\beta) = -\frac{2}{3} \int\limits_0^L \frac{dt\,\alpha(t)}{2\pi} G_+(x,\beta_t) +$$
$$+ \int\limits_0^L \frac{dt\,\alpha(t)}{2\pi} \int\limits_x^1 \frac{dy}{y} \left[D_+(y,\beta_t) P^{\gamma+}_+\left(\frac{x}{y}\right) + D_-(y,\beta_t) P^{\gamma+}_-\left(\frac{x}{y}\right) \right],$$

$$G_-(x,\beta) = -\frac{2}{3} \int\limits_0^L \frac{dt\,\alpha(t)}{2\pi} G_-(x,\beta_t) +$$
$$+ \int\limits_0^L \frac{dt\,\alpha(t)}{2\pi} \int\limits_x^1 \frac{dy}{y} \left[D_-(y,\beta_t) P^{\gamma-}_-\left(\frac{x}{y}\right) + D_+(y,\beta_t) P^{\gamma+}_+\left(\frac{x}{y}\right) \right],$$

with D given in (1.10) and

$$\begin{aligned} &D_+ + D_- = D, && G_+ + G_- = G, \\ &P^{\gamma+}_+(z) = \frac{1}{z}, && P^{\gamma+}_-(z) = \frac{(1-z)^2}{z}, \\ &P^+_{\gamma+}(z) = z^2, && P^-_{\gamma+}(z) = (1-z)^2. \end{aligned}$$

Due to parity conservation in QED, the chiral amplitudes obey the following relations:

$$P^\lambda_{\gamma\lambda}(z) = P^{-\lambda}_{\gamma-\lambda}(z), \quad P^{\gamma\lambda}_\lambda(z) = P^{\gamma-\lambda}_{-\lambda}(z). \tag{1.46}$$

The approximate solutions of this system have the form:

$$\begin{aligned} D_+(x,\beta) &= D(x,\beta) - D_-(x,\beta), \\ D_-(x,\beta) &= \frac{\beta^2}{3x}(1-x)^3, \\ G_+(x,\beta) &= \frac{\beta}{x} + \frac{\beta^2}{4x}[3 + 4\ln(1-x) - (1-x)(3+x)], \\ G_-(x,\beta) &= \beta\frac{(1-x)^2}{x} + \frac{\beta^2}{2x}\Bigg[(x^2-2x)\ln\frac{1}{x} + \\ &\qquad\qquad + 2(1-x)^2\ln(1-x) + x(1-x)\Bigg]. \end{aligned}$$

Using these results one can obtain the cross section for the case when both initial leptons have the same (positive) chirality and the (un)detected particles — the scattering leptons — move close to the beam axis in opposite directions:

$$\frac{d\sigma_{++}}{ds_1} = \frac{\beta^2}{s_1}\left[\phi_1\left(\frac{s_1}{s}\right)\sigma_{\gamma\gamma}^{+-}(s_1) + \phi_2\left(\frac{s_1}{s}\right)\sigma_{\gamma\gamma}^{++}(s_1)\right]. \quad (1.47)$$

For the case of different chiralities of the initial leptons we have

$$\frac{d\sigma_{+-}}{ds_1} = \frac{\beta^2}{s_1}\left[\phi_2\left(\frac{s_1}{s}\right)\sigma_{\gamma\gamma}^{+-}(s_1) + \phi_1\left(\frac{s_1}{s}\right)\sigma_{\gamma\gamma}^{++}(s_1)\right], \quad (1.48)$$

with

$$\begin{aligned} \phi_1(x) &= 2\left[\ln\frac{1}{x} - \frac{1}{2}(1-x)(3-x)\right], \\ \phi_2(x) &= (2+4x+x^2)\ln\frac{1}{x} - 3(1-x^2), \end{aligned}$$

and the superscripts in the cross sections in the right-hand side of the equations denote the chiralities of photons.

For the case when the (un)detected particles are the positrons, scattered on a small angle, and the photon moving in opposite direction we have

$$\begin{aligned} \frac{d\sigma_{++}}{ds_1} &= \frac{1}{s_1}\left[\psi_1\left(\frac{s_1}{s},\beta\right)\sigma_{e\gamma}^{++}(s_1) + \psi_2\left(\frac{s_1}{s},\beta\right)\sigma_{e\gamma}^{+-}(s_1)\right], \\ \frac{d\sigma_{+-}}{ds_1} &= \frac{1}{s_1}\left[\psi_2\left(\frac{s_1}{s},\beta\right)\sigma_{e\gamma}^{++}(s_1) + \psi_1\left(\frac{s_1}{s},\beta\right)\sigma_{e\gamma}^{+-}(s_1)\right], \end{aligned} \quad (1.49)$$

with

$$\psi_1(x,\beta) = \beta x^2 + \frac{3}{4}\beta^2(2x + x^2 + 4\ln(1-x)),$$

$$\psi_2(x,\beta) = \beta(1-x)^2 + \frac{3}{2}\beta^2\Bigg[(x^2-2x)\ln\frac{1}{x} + \\ + 2(1-x)^2\ln(1-x) + x(1-x)\Bigg]. \quad (1.50)$$

For the case when photons are moving along beams direction we have the case of annihilation

$$\begin{aligned} \frac{d\sigma_{++}}{ds_1} &= \frac{\beta^2}{3s_1}\left(1-\frac{s_1}{s}\right)^3 \frac{1}{|1-\Pi(s_1)|^2}[\sigma^{+-}_{e^+e^-}(s_1) + \sigma^{-+}_{e^+e^-}(s_1)], \\ \frac{d\sigma_{+-}}{ds_1} &= \frac{1}{s_1}\frac{1}{|1-\Pi(s_1)|^2}\Phi_{e\bar{e}}\left(\frac{s_1}{s},\beta\right)\sigma^{+-}_{e^+e^-}(s_1). \end{aligned} \quad (1.51)$$

Below we give the explicit expressions for cross sections of some electroweak processes which can be investigated at colliding electron–positron beams. For the process of two W bosons production with tagging two gammas moving along the beam directions we have

$$\sigma^{e\bar{e}\to(\gamma\gamma)W^+W^-}(s) = \int\limits_1^\rho dx\,\Phi_{e\bar{e}}\left(\frac{x}{\rho},\beta\right)\sigma^{e\bar{e}\to W^+W^-}(x), \quad (1.52)$$

with [16]

$$\sigma^{e\bar{e}\to W^+W^-}(x) = \sigma_{e,0}\frac{v}{2x}\left[\frac{1+4x+8x^2}{8x^2}l - \frac{5}{4} + \right. \\ \left. + \frac{1}{2(3x-1)}\left[\frac{8x+1}{8x^2}l - \frac{4x^2+20x+3}{12x}\right] + \frac{v^2(4x^2+20x+3)}{24(3x-1)^2}\right], \quad (1.53)$$

$$x = \frac{s_1}{4M_W^2}, \quad \rho = \frac{s}{4M_W^2}, \quad v^2 = 1-\frac{1}{x}, \quad l = \frac{1}{v}\ln\frac{1+v}{1-v},$$

with $\sigma_{e,0} = 51$ pb.

For the process of two W bosons production after tagging $e^\pm$ which move along the beam direction we have

$$\sigma^{e\bar{e}\to(e\bar{e})W^+W^-}(s) = 4\beta^2\int\limits_1^\rho \frac{dx}{x}\phi_{\gamma\gamma}\left(\frac{x}{\rho},\beta\right)\sigma^{\gamma\gamma\to W^+W^-}(x), \quad (1.54)$$

where the same notations are used and

$$\sigma^{\gamma\gamma\to W^+W^-}(x) = \sigma_{\gamma,0}v\left[1+\frac{3(1+x)}{16x^2} - \frac{3}{16x^2}\left(1-\frac{1}{2x}\right)l\right], \quad (1.55)$$

with $\sigma_{\gamma,0} = 86$ pb.

For the process of $W\nu$ production after tagging positron and photon which move along the beam direction we have

$$\sigma^{e\bar{e}\to(e^+\gamma)W^-\nu}(s) = 2\beta \int\limits_1^{s/M_W^2} \frac{dx}{x}\left[1+\left(1-\frac{xM_W^2}{s}\right)^2\right]\sigma^{e\gamma\to W\nu}(x),$$

$$\sigma^{e\gamma\to W\nu}(x) = \sigma_W\left[\frac{2x^2+x+1}{x^3}\ln\frac{1}{x} - \frac{(1-x)(4x^2+5x+7)}{4x^3}\right], \tag{1.56}$$

with $\sigma_W = 47$ pb.

For the process of two $Z\nu$ bosons production after tagging positron and photon which move along the beam direction we have

$$\sigma^{e\bar{e}\to(e^+\gamma)Ze}(s) = 2\beta \int\limits_1^{s/M_Z^2} \frac{dx}{x}\left[1+\left(1-\frac{xM_Z^2}{s}\right)^2\right]\sigma^{e\gamma\to Ze}(x),$$

$$\sigma^{e\gamma\to Ze}(x) = \sigma_Z\left[\frac{x^2-2x+2}{x^3}\left[24+\ln\frac{(1-x)^2}{x}\right] - \frac{(1-x)(x+7)}{2x^3}\right], \tag{1.57}$$

with $\sigma_Z = 6$ nb. These formulae as well as the Brodsky–Kinoshita–Terazawa formula have rather bad accuracy ($\sim 20\%$) but can be used for rough estimations. So for the year luminosity $\mathcal{L} \sim 10^4$ (pb)$^{-1}$ and $\sqrt{s} = 200$ GeV one can expect a number of W-meson pairs on the level 10^4 per year.

In photon–photon colliders there is a possibility of studying the interaction of two charged leptons of different kinds. Tagging two leptons moving along the photon beam axes we have

$$\sigma^{\gamma\gamma\to\bar{l}_1\bar{l}_2\Phi} = \beta_1\beta_2 \int\limits_{s_{th}}^{s} \frac{ds_1}{s_1}\eta\left(\frac{s_1}{s}\right)\sigma^{l_1 l_2\to\Phi}(s_1), \quad \beta_i = \frac{\alpha}{2\pi}\left(\ln\frac{s}{m_i^2}-1\right), \tag{1.58}$$

with (compare with $\phi_{\gamma\gamma}$)

$$\eta(z) = \int\limits_z^1 \frac{dx}{x}P_\gamma^e(x)P_\gamma^e\left(\frac{z}{x}\right) = (1+2z)^2\ln\frac{1}{z} - 2(1-z)(1+3z). \tag{1.59}$$

1.4. Calculation of the Radiative Corrections to the Cross Section for Electron–Nucleus Scattering by the Method of Structure Function. To describe the process of inelastic electron–proton scattering

$$e(p_1) + p(\mathcal{P}) \to e(p_2) + X, \tag{1.60}$$

we employ the standard variables accepted for DIS,

$$\begin{gathered} q = p_1 - p_2, \quad V = 2\mathcal{P}p_1, \quad x = -\frac{q^2}{2\mathcal{P}q}, \\ y = \frac{2\mathcal{P}q}{V}, \quad \tau = \frac{M^2}{V}, \quad Q^2 = Vxy, \end{gathered} \tag{1.61}$$

with m (M) — the mass of the electron (target).

The general formula for the inclusive cross section for electron scattering has the following form [17]:

$$\begin{aligned} \frac{\epsilon_2 d^3\sigma(p_1,p_2)}{d^3p_2} = \int^1 dz_1 \int^1 \frac{dz_2}{z_2^2} \sum_{A,A'} D_e^A(z_1,\beta_Q)\bar{D}_{A'}^e(z_2,\beta_Q)\times \\ \times \left.\frac{\tilde{\epsilon}_2 d^3\sigma_{AA'}^{\mathrm{hard}}}{d^3\tilde{p}_2}(z_1p_1,\tilde{p}_2)\right|_{\tilde{p}_2=\frac{p_2}{z_2}}, \end{aligned} \tag{1.62}$$

$$\beta_Q = \frac{\alpha}{2\pi}\left(\ln\frac{Q^2}{m^2} - 1\right),$$

where $D_e^A(z,\beta_Q)$ is the structure function giving the distribution with respect to the energy fraction z of a specific A parton with virtuality up to Q^2 in the electron; $\bar{D}_{A'}^e(z,\beta_Q)$ is the fragmentation function of a specific A parton with virtuality up to Q^2 in the electron with energy fraction z; and $\sigma_{AA'}^{\mathrm{hard}}$ is the hard cross section for scattering of a parton A with transformation into a parton A'. In QED, the role of the partons is played by electrons, positrons, and photons. The lower limits of the z_1 and z_2 integrals in (1.62) are determined from the kinematical conditions specific to the parton process.

In the LLA, where in the RC one sums only terms that for each power of α contain the factor $\ln(Q^2/m^2)$, in the formula (1.62) one must set $A = A' = e$, take the Born cross section in the electromagnetic interaction, for the cross section $\sigma_{AA'}^{\mathrm{hard}}$, and use the Gribov–Lipatov approximation $D_e^A(z,\beta_Q) = \bar{D}_A^e(z,\beta_Q)$. Going over to the variables x and y and denoting $D_e^e \equiv D$, from (1.62) we obtain

$$\frac{d\sigma(x,y)}{dx\,dy} = \int^1\int^1 \frac{dz_1\,dz_2 D(z_1,\beta_Q)D(z_2,\beta_Q)}{z_2^2 z_1}\frac{y\,d\sigma^{\mathrm{hard}}(\tilde{x},\tilde{y})}{\tilde{y}\,d\tilde{x}d\tilde{y}}, \tag{1.63}$$

where the tilde denotes the corresponding variables for the parton process:

$$\tilde{x} = \frac{z_1yx}{z_1z_2+y-1}, \quad \tilde{y} = \frac{z_1z_2+y-1}{z_1z_2}, \quad Q^2 = Vxy. \tag{1.64}$$

The structure function $D(z, \beta_Q)$ satisfies Lipatov's evolution equations. A closed form expression does not exist for them. However, since in practice the parameter β_Q is small (< 0.1), even in calculations with the accuracy 0.1% it is sufficient to retain only two terms of the expansion in this parameter. This allows one to use the obtained formulae in the whole region of variation of the variables x, y.

In the case of fully inclusive cross section, the function D takes the form:

$$D = D^{\gamma} + D^{\pm}.$$

In experiments which ignore events with production of an additional e^+e^- pair, one has $D = D^{\gamma}$ (see (1.24)). In the LLA, the hard cross section appearing in (1.63) coincides with the cross section calculated in the lowest order of PT in the electromagnetic interaction:

$$\left.\frac{d\sigma^{\rm hard}(x,y)}{dx\,dy}\right|_{\rm LLA} = \frac{d\sigma^{(0)}(x,y)}{dx\,dy}. \tag{1.65}$$

The cross section $d\sigma^{(0)}$ is expressed in terms of the structure functions $W_{1,2}(x, Q^2)$ in a well-known way [18]. Expressing, in addition, the parton variables $\tilde{x}$ and $\tilde{y}$ in terms of x and y, we obtain

$$\frac{d\sigma^{(0)}(\tilde{x},\tilde{y})}{\tilde{y}\,d\tilde{x}\,d\tilde{y}} = \frac{z_2}{z_1} 4\pi\alpha^2 \left(\frac{z_1}{z_2}Q^2\right) \left(Q^2 x y^2\right)^{-1} \left[\left(1 - y - \frac{M^2}{Q^2}x^2y^2\right) \times \right.$$
$$\left. \times \frac{Q^2}{2Mx} W_2\left(\frac{xz_1y}{z_1z_2+y-1}, \frac{z_1}{z_2}Q^2\right) + xy^2 M W_1\left(\frac{xz_1y}{z_1z_2+y-1}, \frac{z_1}{z_2}Q^2\right)\right].$$

Here $\alpha(Q^2) = \alpha/|1 - \Pi(q^2)|$, and $\Pi(q^2)$ is the photon polarization operator.

The lower limits of the z_1, z_2 integrations in (1.63) are obtained from the condition of existence of the parton cross section $d\sigma^{\rm hard}$. One can identify the following two contributions to the integral (1.63): the proper inelastic contribution beginning at $(\mathcal{P} + \tilde{q})^2 = M_{\rm th}^2$ (where $M_{\rm th}$ is the inelastic threshold $M_{\rm th} = M + m_\pi$), and the radiative tail of the elastic peak (ERT), for which $(\mathcal{P} + \tilde{q})^2 = M^2$. With the help of (1.61) and (1.63) we find that for the first contribution the region of integration over z_1 and z_2 is limited by the inequality

$$z_1z_2 + y - 1 - xyz_1 \geqslant z_1\delta, \quad \delta = \frac{M_{\rm th}^2 - M^2}{V}. \tag{1.66}$$

For ERT, the region of integration degenerates into a curve which is obtained from (1.66) by discarding the inequality and setting δ to zero. The contribution of the curve to the double integral is nonzero because W_1 and W_2 contain δ functions corresponding to the elastic process; thus, for scattering by a proton or neutron one has

$$\begin{aligned} MW_1(x,Q^2) &= \frac{1}{2}\delta(x-1)\left[F_1(q^2)+F_2(q^2)\right]^2, \\ \frac{1}{M}W_2(x,Q^2) &= \frac{2}{Q^2}\delta(x-1)\left[F_1^2(q^2)+\frac{Q^2}{4M^2}F_2^2(q^2)\right], \end{aligned} \tag{1.67}$$

where $F_{1,2}$ are the Pauli form factors.

It is possible to design an experiment in such a way as to select the events where the structure of the target is not changed. In this case the ERT is of interest in its own right.

Calculations beyond the leading logarithmic approximation were considered in [17].

1.5. Radiative Tail in π_{e2} Decay and Some Comments on μ–e Universality. The result of the lowest-order perturbation theory calculations of the photon and positron spectra in radiative π_{e2} decay can be generalized to all orders of perturbation theory using the structure-function method [18]. An additional source of radiative corrections to the ratio of the positron and muon channels of pion decay, due to emission of virtual and real photons and pairs, is considered. It depends on details of the detection of the final particles and is large enough to be taken into account in theoretical estimates with a level of accuracy of 0.1% [19, 20].

As a first step in the calculation of the spectra of radiative pion decays, we reproduce the earlier-obtained results (see review [21]), treating the pion as a point-like particle. In these papers the positron energy spectrum in radiative pion decay was calculated:

$$\begin{aligned} \frac{d\Gamma}{\Gamma_0 dy} = \frac{\alpha}{2\pi}\Bigg[\frac{1+y^2}{1-y}(L-1) - 2(1-y) - \\ -(1-y)\ln(1-y) + \frac{1+y^2}{1-y}(2\ln y + 1)\Bigg], \end{aligned} \tag{1.68}$$

$$y_{\min} \leqslant y \leqslant 1 + \frac{m_e^2}{m_\pi^2},$$

where $y = 2\varepsilon_e/m_\pi$ is the positron energy fraction; ε_e is the positron energy (here and below we have in mind the rest frame of the pion), $L = \ln(m_\pi^2/m_e^2) = 11.2$ is the «large logarithm», and m_π, m_e are the masses of the pion and positron. The quantity

$$Sfurth\Gamma_0 = \frac{G^2|V_{ud}|^2}{8\pi} f_\pi^2 m_e^2 m_\pi \left(1 - \frac{m_e^2}{m_\pi^2}\right)^2 = 2.53 \cdot 10^{-14}\ \text{MeV} \tag{1.69}$$

is the total width of π_{e2} decay, calculated in the Born approximation.

We will now calculate the photon spectrum. Consider first the emission of a soft real photon. The corresponding contribution to the total width may be obtained by the standard integration of the differential widths:

$$\frac{d\Gamma^{\rm soft}}{\Gamma_0} = -\frac{\alpha}{4\pi^2}\int\frac{d^3k}{\omega}\left(\frac{P}{Pk}-\frac{p_e}{p_ek}\right)^2\Bigg|_{\omega\leqslant\Delta\varepsilon\ll m_\pi/2}, \tag{1.70}$$

where P, p_e, k are the four-momenta of the pion, positron, and photon, respectively; $P^2 = m_\pi^2$, $p_e^2 = m_e^2$, $\omega^2 = \mathbf{k}^2+\lambda^2$, and λ is the photon mass. The result has the form

$$\frac{\Gamma^{\rm soft}}{\Gamma_0} = \frac{\alpha}{\pi}\left[-b(\sigma)\ln\frac{2\Delta\varepsilon}{\lambda}+1-\frac{1+\sigma}{2(1-\sigma)}\ln\sigma - \right. \\ \left. -\frac{1+\sigma}{4(1-\sigma)}\ln^2\sigma-\frac{1+\sigma}{1-\sigma}{\rm Li}_2(1-\sigma)\right], \tag{1.71}$$

where

$$b(\sigma) = \frac{1+\sigma}{1-\sigma}\ln\sigma+2, \quad \sigma = \frac{m_e^2}{m_\pi^2}. \tag{1.72}$$

Consider now the hard-photon emission process regarding pion to be a point-like particle

$$\pi^+(P)\to e^+(p_e)+\nu_e(p_\nu)+\gamma(k). \tag{1.73}$$

The standard procedure of final-states summing of the squared modulus of its matrix element

$$M_{if} = \frac{GV_{ud}}{\sqrt{2}}e\bar{u}(p_e)\left[-\hat{\epsilon}-\frac{P\epsilon}{Pk}(\hat{P}-\hat{k})+ \right. \\ \left. +\frac{1}{2p_ek}\hat{\epsilon}(\hat{p}_e+\hat{k}+m)\hat{P}\right](1-\gamma_5)v(p_\nu), \tag{1.74}$$

and integration over the neutrino phase volume leads to the spectral distribution over the photon energy fraction $x = 2k^0/m_\pi$:

$$\frac{d\Gamma^{\rm hard}}{\Gamma_0 dx} = \frac{\alpha}{2\pi}\frac{x(1-x-\sigma)}{(1-\sigma)^2}\left[-\frac{4(1-\sigma)}{x^2}-\frac{1}{1-x}+ \right. \\ \left. +\frac{1}{x(1-x-\sigma)}\left(\frac{1}{x}(1+(1-x)^2)+2\sigma-2\frac{\sigma^2}{x}\right)\ln\frac{1-x}{\sigma}\right]. \tag{1.75}$$

Further integration of this spectrum gives the result:

$$\int\limits_{x_{\min}}^{1-\sigma} \frac{d\Gamma^{\text{hard}}}{\Gamma_0 dx} dx = \frac{\alpha}{2\pi} \left[-2b(\sigma) \ln \frac{1-\sigma}{x_{\min}} - 2\frac{1+\sigma}{1-\sigma} \text{Li}_2(1-\sigma) + \right.$$
$$\left. + \frac{3(1-2\sigma)}{2(1-\sigma)^2} \ln \sigma + \frac{19-25\sigma}{4(1-\sigma)} \right], \quad x_{\min} = \frac{2k^0_{\min}}{m_\pi}. \quad (1.76)$$

Putting $k^0_{\min} = \Delta\varepsilon$ in this formula and adding the soft photon contribution, we obtain (in agreement with Kinoshita's 1959 result [20]) the contribution to the width from inner bremsstrahlung of a point-like pion:

$$\frac{\Gamma_{\text{IB}}}{\Gamma_0} = \frac{\alpha}{\pi} \left\{ b(\sigma) \left[\ln \frac{\lambda}{m_\pi} - \ln(1-\sigma) - \frac{1}{4} \ln \sigma + \frac{3}{4} \right] - \right.$$
$$\left. -2\frac{1+\sigma}{1-\sigma} \text{Li}_2(1-\sigma) - \frac{\sigma(10-7\sigma)}{4(1-\sigma)^2} \ln \sigma + \frac{15-21\sigma}{8(1-\sigma)} \right\}. \quad (1.77)$$

Now let us return to the positron spectrum. The contributions containing the large logarithm L may be associated with the known kernel of the Altarelli–Parisi–Lipatov evolution equation (see (1.4)). Using the factorization theorem, we may generalize this spectrum to include the leading logarithmic terms in all orders of perturbation theory. This may be done in terms of nonsinglet structure functions $D(y,\eta)$. In the case of the photon spectrum, the function $D(1-x,\eta)$ appears. The nonsinglet part of it works here, $D = D^\gamma$, explicit expression for it is given above in (1.24). The expressions for the spectra are as follows:

$$\frac{d\Gamma}{\Gamma_0 dy} = D(y,\eta) \left[1 + \frac{\alpha}{\pi} K_e(y) \right],$$
$$K_e(y) = -(1-y)\left(1 + \frac{1}{2}\ln(1-y)\right) + \frac{1+y^2}{2(1-y)}(2\ln y + 1), \quad y = \frac{2\varepsilon_e}{m_\pi},$$
$$\frac{d\Gamma}{\Gamma_0 dx} = D(1-x,\eta) \left[1 + \frac{\alpha}{\pi} K_\gamma(x) \right], \quad (1.78)$$
$$K_\gamma(x) = -\frac{1-x}{x} + \frac{1+(1-x)^2}{2x} \ln(1-x), \quad x = \frac{2\omega}{m_\pi}, \quad \eta = \frac{\alpha}{2\pi}\left(\ln\frac{1}{\sigma} - 1\right).$$

Let continue our consideration of the RC to $\pi l2$ decay in the lowest order of perturbation theory. There are two kinds of sources of RC. The first one connected with emission of soft and hard real photons must be completed with the contribution of virtual photons emission. The result is [20]

$$\frac{\Gamma}{\Gamma_0} = 1+\rho, \quad \rho = \frac{\alpha}{\pi}\left[\frac{3}{2}\ln\frac{\Lambda}{m_\pi} + \frac{3}{4} + F(\sigma)\right],$$
$$F(\sigma) = -b(\sigma)\ln(1-\sigma) - \frac{\sigma(8-5\sigma)}{4(1-\sigma)^2}\ln\sigma + \frac{13-19\sigma}{8(1-\sigma)} - \frac{2(1+\sigma)}{1-\sigma}\mathrm{Li}_2(1-\sigma), \tag{1.79}$$

with Λ-ultraviolet cut-off parameter. Besides we see that the quantity η is finite at electron zero-mass limit.

Studying the spectral distributions one can verify that the factor $1+\frac{3\alpha}{2\pi}\ln\frac{\Lambda}{m_\pi}$ is absorbed by $(V_{ud}G)_0^2$ and turn them to the observable quantities $(V_{ud}G)^2$.

As for the ratio of the widths of $\pi e2$ and $\pi\mu 2$ channels, the dependence on the ultraviolet cut-off parameter disappears.

Another source of RC is the moving electron mass. The renormalization group equation for the running mass $m(q^2)$ has the form [22]:

$$\frac{dm(q^2)}{d\tilde{L}} = -\frac{3\alpha(\tilde{L})}{4\pi}m(q^2), \quad \alpha(q^2) = \frac{\alpha}{1-\dfrac{\alpha}{3\pi}\tilde{L}},$$
$$m(0) = m_e, \quad \tilde{L} = \ln\frac{q^2}{m_e^2}, \tag{1.80}$$

which solution is $m(q^2) = m_e\left(1-\frac{\alpha}{3\pi}\tilde{L}\right)^{9/4}$. So the bare electron mass implied at the beginning of our considerations must be replaced by the running one.

Working in the frames of the Standard Model the evolution of the coupling constant from the momentum scales of order ρ-meson mass up to Z-, W-bosons masses must be taken into account resulting in the factor [23]

$$S_{\mathrm{EW}}(m_\rho, m_Z) = 1.0235. \tag{1.81}$$

The total expression for the ratio

$$R_{\pi l2} = \frac{\Gamma(\pi\to e\nu) + \Gamma(\pi\to e\nu\gamma)}{\Gamma(\pi\to\mu\nu) + \Gamma(\pi\to\mu\nu\gamma)} \tag{1.82}$$

is

$$R_{\pi l2} = R_0 S_{\mathrm{EW}}(m_\rho, m_Z)\left(1-\frac{\alpha}{3\pi}L\right)^{9/2}\left[1+\frac{\alpha}{\pi}\left[\frac{13}{8}-\frac{\pi^2}{3}-F_\mu\right]+\delta_{\mathrm{str}}\right] =$$
$$= R_0[1+\delta_{\mathrm{point}}+\delta_{\mathrm{str}}], \quad \delta_{\mathrm{point}} = -0.01786, \tag{1.83}$$

with $F_\mu = F(m_\mu^2/m_\pi^2)$ (see Eq. (1.79)) and

$$R_0 = \frac{m_e^2}{m_\mu^2}\frac{(1-m_e^2/m_\pi^2)^2}{(1-m_\mu^2/m_\pi^2)^2} = 1.28347\cdot 10^{-4}. \tag{1.84}$$

The quantity $\delta_{\rm str}$ reflects the strong interactions manifestations. To estimate its value is the nowadays problem of experimental efforts as well it can be estimated in the frames of the strong interactions theoretical models [23–25].

1.6. QED Structure Function of Charged Pion. Multiple Production. The kernel of the evolution equation for spinless charged particle can be extracted from the contribution to the cross section of point-like pion pair in annihilation of electron–positron with emission of a hard photon by pions. In leading logarithmical approximation collinear kinematics is relevant, which leads to [26]:

$$\frac{d\sigma^{e\bar{e}\to\pi\bar{\pi}\gamma}}{d\nu} = \frac{\alpha^2}{3s}\frac{2\alpha}{\pi}\frac{1-\nu}{\nu}L, \tag{1.85}$$

where $\sqrt{s}$ is the total energy in the center-of-mass frame; $L = \ln s/m^2$ — «large» logarithm; m — pion mass; $\nu = 2\omega/\sqrt{s}$ — energy fraction of the photon.

In analogy with the case of fermions we can construct the kernel of evolution equation of scalar charged particle:

$$\begin{gathered} P_{(1)}^{\pi^+}(z) = P_{(1)}^{\pi^-}(z) = \left[\frac{2z}{1-z}\right]_+ = \lim_{\Delta\to 0}[P_\Delta^\pi\delta(1-z) + P_\theta^\pi(z)\theta(1-z-\Delta)], \\ P_\Delta^\pi = 2\ln\Delta + 2, \quad P_\theta^\pi(z) = \frac{2z}{1-z}. \end{gathered} \tag{1.86}$$

In the usual way the higher iterations of pion kernel can be constructed:

$$P_{(n)}(z) = \int\limits_z^1 \frac{dx}{x}P_{(n-1)}(x)P_{(1)}\left(\frac{z}{x}\right), \tag{1.87}$$

with the property

$$\int\limits_0^1 ds\, P_{(n)}(z) = 0, \quad n = 1,2,3,\ldots \tag{1.88}$$

The corresponding nonsinglet structure function $D^\pi(z,\beta_\pi)$, $\beta_\pi = \alpha L/2\pi$

$$D^\pi(z,\beta_\pi) = \delta(z-1) + \beta_\pi P_{(1)}^\pi(z) + \frac{1}{2!}\beta_\pi^2 P_{(2)}^\pi(z) + \ldots \tag{1.89}$$

can be written in the smoothed form

$$D^{\pi}(z,\beta) = D^{\pi}(z,L) = 2\beta(1-z)^{2\beta-1}(1+2\beta) - 2\beta + \mathcal{O}(\beta^2). \qquad (1.90)$$

The differential cross section of creation of n charged pion or lepton, at high energy collisions of particles a, b with the total center-of-mass frame (cms) energy $\sqrt{s}$, which takes into account RC in LLA due to electromagnetic interaction of particles in final state, neglecting the initial state interaction, can be put in the form:

$$\frac{d\sigma^{ab\to l_1,l_2,\ldots,l_n,X}}{dx_1\cdots dx_n} = \\ = \int\limits_{x_1}^{1} \frac{dy_1}{y_1} \cdots \int\limits_{x_n}^{1} \frac{dy_n}{y_n} D^{l_1}\left(\frac{x_1}{y_1},\beta_1\right)\cdots D^{l_n}\left(\frac{x_n}{y_n},\beta_n\right) \frac{d\sigma_B^{ab\to l_1,l_2,\ldots,l_n,X}}{dy_1\cdots dy_n}, \qquad (1.91)$$

with $\beta_j = \dfrac{\alpha}{2\pi}\ln\dfrac{s}{m_j^2}$, $d\sigma_B$ is the cross section in the Born approximation, x_i — energy fractions of the created particles.

For the case of decay of a heavy particle H with mass $M \gg m_j$, we obtain in the same approximation:

$$\frac{d\Gamma^{H\to l_1,l_2,\ldots,l_n,X}}{dx_1\cdots dx_n} = \\ = \int\limits_{x_1}^{1} \frac{dy_1}{y_1} \cdots \int\limits_{x_n}^{1} \frac{dy_n}{y_n} D^{l_1}\left(\frac{x_1}{y_1},b_1\right)\cdots D^{l_n}\left(\frac{x_n}{y_n},b_n\right) \frac{d\Gamma_B^{H\to l_1,l_2,\ldots,l_n,X}}{dy_1\cdots dy_n}, \qquad (1.92)$$

with $b_j = \dfrac{\alpha}{2\pi}\ln\dfrac{M_H^2}{m_j^2}$.

1.7. Radiative Corrections to DVCS Electron Tensor. An interesting information about the structure of the proton can be found in radiative deep-inelastic electron–proton scattering experiments (DIS), analyzing the interference between the amplitudes of the radiative electron block (Bethe–Heitler amplitude) and the amplitudes of the radiative proton block. In the literature one can find different suggestions for the determination of the relevant contributions to the differential cross section, concerning in particular lepton–hadron scattering [27].

In view of the large experimental programme which is underway or foreseen at present accelerators and of the precision of the data in electron–proton elastic and inelastic scattering, the necessity to achieve an adequate precision in the calculation of RC is a very actual problem.

The theoretical description at the lowest order is based on the work of Schwinger [28] and Mo and Tsai [29]. The last one contains an application to ep radiative scattering to experimental data. A further improvement was given in the known paper of Yennie, Frautschi, and Suura [30], where a simple formula was derived to describe the emission of virtual and real (soft) photons with energy lower than a value $\Delta\varepsilon$, of the order of the experimental resolution. Such photons cannot be detected in exclusive experiments. In inclusive or semi-inclusive experiments, the emission of hard, undetected photons should also be taken into account, as it escapes the detection.

The emission of an additional photon (virtual or real) is associated with a suppression factor of the order of $\alpha = 1/137$, the fine structure constant. It corresponds to a small correction to the cross section, which can be estimated to be 0.5%. However, a precise calculation of RC at higher order of PT is highly required in modern experiments at high energy. There are at least two reasons for this. Firstly, due to the emission of photons by light charged leptons, RC have an enhancement by a factor called «large logarithm»,

$$L = \ln \frac{Q^2}{m^2}, \tag{1.93}$$

where Q is the characteristic momentum or the energy parameter and considerably exceeds the lepton rest mass m. Therefore the effective expansion parameter becomes αL. Applying the general theorem about the factorization of soft and virtual photon contribution [30], one obtains this factor in the form:

$$W \sim \beta \exp\left[(2\beta - 1)\ln\frac{\Delta\varepsilon}{\varepsilon}\right] = 2\beta\left(\frac{\Delta\varepsilon}{\varepsilon}\right)^{2\beta-1}, \tag{1.94}$$

where $\Delta\varepsilon$ is the energy of the photon emitted by an electron of energy ε.

Secondly, a kinematical effect, called «returning» mechanism, due to hard photon emission from one of the initial charged particles may become important, in particular for processes where the cross section increases when the initial energy decreases.

Both mechanisms were studied in the lowest order of PT. Including higher orders brings, in general, large computing difficulties. However, mostly due to the study of QCD [7, 8] processes, a powerful method was developed based on scale invariance (or renormalization group). In this frame, the behavior of the amplitudes and of the cross section can be described in the limit of vanishing lepton mass in the leading $\sim (\alpha L)^n$ and next-to-leading $\sim \alpha(\alpha L)^n$ approximations. The application of this method to the calculation of RC provides an accuracy at the thousandth level.

The cross section including RC in LLA has the expression of the convolution of universal functions (lepton structure functions (LSF)) with a kinematically shifted cross section, calculated in the Born approximation. The NLA contributions are taken into account by a K factor. In this case, for two light leptons in the initial channel, one can write:

$$d\sigma(p_1, p_2, \ldots) = \int dx_1\, dx_2 D(x_1, \beta) D(x_2, \beta) d\sigma_B(x_1 p_1, x_2 p_2, \ldots) \left(1 + \frac{\alpha}{\pi} K\right),$$

and, for the case of a single light lepton in the initial state:

$$d\sigma(p_1, \ldots) = \int \frac{dx\, D(x, \beta)}{x} d\sigma_B(x p_1, \ldots) \left(1 + \frac{\alpha}{\pi} K\right).$$

The LSF $D(x, \beta)$ obeys the evolution equations of a twist-2 operator. For most QED applications it is sufficient to consider only the nonsinglet LSF, which has been derived in [10].

Unpolarized and polarized DVCS data are considered to provide useful information for the extraction of the properties of generalized parton distributions (GPD). When the accuracy of the experiment is better than 10%, the role of radiative corrections becomes important and a careful study of higher order contributions is mandatory. QED RC to virtual Compton scattering on proton ($ep \to ep\gamma$) were calculated in the lowest order in [31], where a detailed study of one-loop virtual corrections including first-order soft photon emission contribution was done. Higher order RC were included by exponentiation procedure, which is valid only for small $\Delta\varepsilon$.

One can write schematically the cross section for the DVCS process as the sum of three contributions:

$$d\sigma^{\rm tot}(e^- p \to e^- p\gamma) = d\sigma^{\rm BH} + d\sigma^{\rm DVCS} + d\sigma^{\rm odd}, \qquad (1.95)$$

where $d\sigma^{\rm BH}$ is the Bethe–Heitler cross section (Fig. 1, *a*, *b*), $d\sigma^{\rm DVCS}$ corresponds to the radiation of the photon from the proton (Fig. 1, *c*, *d*), and the last term corresponds to the interference between these two mechanisms.

Below in the different Subsections we define the kinematics and derive the formalism for the odd part of the cross section of the radiative $e^-\mu^+$ scattering with RC taking into account RC in the LLA, consider the contributions of three

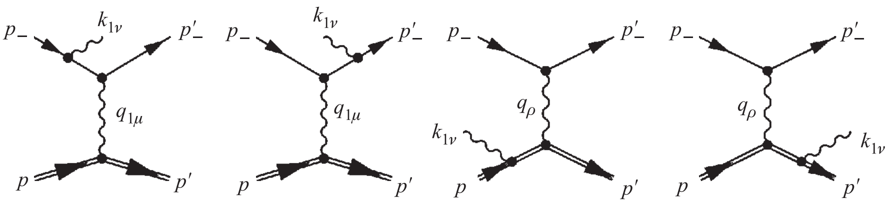

Fig. 1. The Born–Feynman diagrams for virtual Compton scattering

gauge-invariant classes of one-loop virtual corrections, soft and additional hard photon emissions in collinear kinematics. The relevant generalization for all orders in LLA in the form of electron SF is performed. Further we extended our calculation to ep scattering under realistic assumptions. We calculate the charge-even and charge-odd contributions to the cross section for the reactions $e^-p \to e^-p\gamma$ and $e^+p \to e^+p\gamma$ and the charge asymmetry, as well. The role of RC in LLA is discussed.

1.7.1. Electron-Muon Radiative Scattering in the Born Approximation. Let us consider the radiative $e^-\mu^+$ scattering

$$e^-(p_-) + \mu(p) \to e^-(p'_-) + \mu(p^{'}) + \gamma(k_1) \tag{1.96}$$

as a model for DVCS in electron–proton radiative scattering, considering the muon as a structureless proton. The contribution to the differential cross section of reaction (1.96), which corresponds to the so-called up-down interference of the amplitudes describing the radiation from the electron and the muon blocks, in the lowest order of PT, can be written as

$$\begin{gathered}(d\sigma)^{e\mu\gamma}_{\rm odd} = \frac{4(4\pi\alpha)^3}{stt_1} H_{\mu\nu\rho} E_0^{\mu\nu\rho} d\Gamma, \\ d\Gamma = \frac{d^3p'_-}{2\varepsilon'_-}\frac{d^3p'}{2\varepsilon'}\frac{d^3k}{2\omega}\frac{\delta^4(p_- + p - p'_- - p' - k_1)}{(2\pi)^5},\end{gathered} \tag{1.97}$$

p'_- and ε'_- (p_- and ε_-) are the momentum and the energy of the scattered electron (muon). The odd DVCS tensors for electron and muon are

$$\begin{gathered}E_0^{\mu\nu\rho}(p_-, k_1, p'_-) = \frac{1}{4}\mathrm{Tr}\, \hat{p}_-^{'} \left(\gamma^\nu \frac{\hat{p}'_- + \hat{k}_1}{\chi'_-}\gamma^\mu - \gamma^\mu \frac{\hat{p}_- - \hat{k}_1}{\chi_-}\gamma^\nu\right) \hat{p}_-\gamma^\rho, \\ H_{\mu\nu\rho} = \frac{1}{4}\mathrm{Tr}\,(\hat{p}' + M)\left(\gamma_\rho \frac{\hat{p} - \hat{k}_1 + M}{-\chi}\gamma_\nu + \gamma_\nu \frac{\hat{p}' + \hat{k}_1 + M}{\chi'}\gamma_\rho\right)(\hat{p} + M)\gamma_\mu.\end{gathered} \tag{1.98}$$

The on-mass shell conditions and kinematics invariants are defined:

$$\begin{gathered}p_-^2 = p_-^{'2} = m^2, \quad k_1^2 = 0, \quad p^2 = p'^2 = M^2, \\ \chi_- = 2k_1p_-, \quad \chi'_- = 2k_1p'_-, \quad \chi = 2k_1p, \quad \chi' = 2k_1p', \\ s = 2p_-p, \quad s_1 = 2p'_-p', \quad t = -Q^2 = -2p_-p'_-, \\ t_1 = q_1^2 = 2M^2 - 2pp', \quad u = -2p_-p', \quad u_1 = -2p'_-p, \\ s + s_1 + t + t_1 + u + u_1 = 0,\end{gathered} \tag{1.99}$$

where m and M are the electron and the muon (proton) masses. Throughout this Subsection we will suppose

$$s \sim s_1 \sim -t \sim -t_1 \sim -u \sim -u_1 \sim \chi_- \sim \chi'_- \sim \chi \sim \chi' \gg m^2, \tag{1.100}$$

and we will systematically omit terms of the order of m^2/s compared to those of order of unity. This kinematical region corresponds to large-angle final particle emission in the lab frame, where the calculation is performed.

In order to make the comparison with the experimental data, we chose the following set of four independent variables:

$$Q^2 = -t, \quad t_1, \quad x_{\mathrm{Bj}} = \frac{Q^2}{2pq}, \quad q = p_- - p'_- \quad \text{and} \quad \phi, \tag{1.101}$$

where ϕ is the azimuthal angle between the plane containing the three-momenta of the initial and the scattered electrons $(\mathbf{p}_-, \mathbf{p}'_-)$ and the hadronic plane, containing the momentum transfer to the electron, $\mathbf{q}$, and the scattered muon momentum $\mathbf{p}'$ [32].

The phase volume can be rewritten in terms of these variables (see details in [33]) as

$$d\Gamma = \frac{d\Phi_4}{2^8\pi^4 R}, \quad d\Phi_4 = \frac{1}{sx_{\mathrm{Bj}}} d\phi\, dQ^2\, dt_1\, dx_{\mathrm{Bj}}, \quad R = \left[1 + \frac{4M^2 x_{\mathrm{Bj}}^2}{Q^2}\right]^{1/2}. \tag{1.102}$$

The Born cross section (in the lowest order of PT) has the form:

$$(d\sigma)^{e\mu\gamma}_{\mathrm{odd}} = \frac{\alpha^3}{2\pi s t t_1 R} W\, d\Phi_4, \quad W = 2H_{\mu\nu\rho}E_0^{\mu\nu\rho}. \tag{1.103}$$

In the case of massless muon we recover the result from [2]:

$$W_{M=0} = (s^2 + s_1^2 + u^2 + u_1^2)\left[\frac{s}{\chi_-\chi} + \frac{s_1}{\chi'_-\chi'} + \frac{u}{\chi'\chi_-} + \frac{u_1}{\chi'_-\chi}\right]. \tag{1.104}$$

Below we consider the radiative corrections to this part of the differential cross section. We show that when the energy fraction of the scattered electron is not fixed, we obtain in LLA:

$$\frac{d\sigma^{e\mu\gamma}_{\mathrm{odd}}}{d\Phi_4} = \frac{\alpha^3}{2\pi s Q^2 t_1} \int\limits_{x_0(\phi)}^{1} \frac{dx}{x} D(x,\beta) \frac{W(x)}{[1-\Pi(xt)][1-\Pi(t_1)]} \Psi(x),$$
$$\Psi(x) = \frac{1}{R'I}\left[1 - \frac{sx_{\mathrm{Bj}}(1-x)}{Q^2}\right]^{-1}, \tag{1.105}$$
$$R' = \sqrt{1 + \frac{4M^2 x'^2_{\mathrm{Bj}}}{xQ}}, \quad I = \left|\frac{dc'_x}{dc'}\right|,$$

where $\Pi(Q^2)$ is the contribution to vacuum polarization from the light lepton (electron) and $W(x) = W(p_- \to p_- x)$, $D(x,\beta)$ is the nonsinglet SF of the electron D^γ (see Eq. (1.24)).

The physical requirements $\varepsilon'_- > 0$ and the on-mass shell condition for the real photon lead to the restrictions:

$$x > x_0(\phi), \quad 1 - \frac{Q^2}{s x_{\mathrm{Bj}}} > 0. \tag{1.106}$$

The helicity-dependent part of DVCS cross section on proton is

$$\frac{d^4\Sigma}{d\phi} = \frac{1}{2}\left(\frac{d\sigma^{\rightarrow}}{d\phi} - \frac{d\sigma^{\leftarrow}}{d\phi}\right) \tag{1.107}$$

and it is sensitive to the imaginary part of the DVCS amplitude. Let us calculate the proton Compton amplitude in the structureless approximation, and parameterize the nucleon structure by a general factor G.

The relevant part of the matrix element squared can be written as

$$\Delta|M^{\rightarrow}|^2 - \Delta|M^{\leftarrow}|^2 \sim \mathrm{Im}\,(G)[\mathbf{p}_- \times \mathbf{p}'_-]\mathbf{k}\,\mathcal{F}, \tag{1.108}$$

with

$$\mathcal{F} = (2t + 4m^2)\left(\frac{1}{\chi_1\chi_2} + \frac{1}{\chi_2\chi'_1}\right) + 2(s - u_1)\left(\frac{1}{\chi_1\chi_2} - \frac{1}{\chi_2\chi'_1}\right) - \\ - 2\chi_2\left(\frac{1}{\chi_1\chi'_2} + \frac{1}{\chi'_1\chi'_2}\right) + \frac{4(s - M^2)}{\chi_1\chi_2} + \frac{4(u_1 - M^2)}{\chi'_1\chi'_2}. \tag{1.109}$$

1.7.2. One-Loop Virtual Corrections. In LLA only FD, where a single photon is transferred between the muon and the electron blocks, contribute to cross section (see Fig. 2). In our considerations we omit FD with two virtual exchanged photons due to the cancellation of such contributions when one includes the amplitude corresponding to soft-photon emission between electron and muon blocks. The details of this «up-down cancellation», which holds in LLA, were

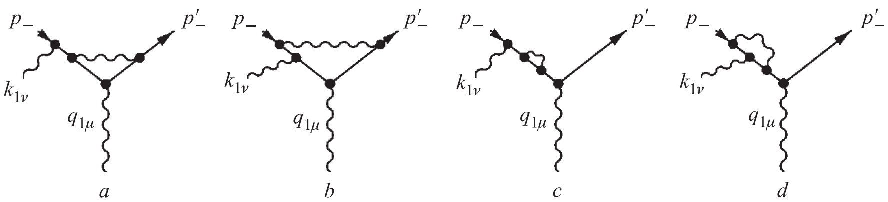

Fig. 2. Some one-loop FD for virtual Compton scattering

discussed in [34] and refs. therein. The corresponding contribution goes beyond the limits of accuracy of the present calculation.

In the calculation, only FD drawn in Fig. 2 can be considered. The corresponding part of the total matrix element is denoted as M^γ. The total contribution to the DVCS tensor can be restored from the interference of these amplitudes (Fig. 2) with the Born one (Fig. 1, *d* or 1, *c*):

$$E^{\rm virt}_{\mu\nu\rho} = [1 - P(p_- \leftrightarrow -p'_-)] M^\gamma_{\mu\nu} (M_\rho)^\star. \tag{1.110}$$

The matrix element describing the electron self-energy (see Fig. 2, *c*, *d*) and the vertex function of the real photon emission by the initial electron have the form [6]:

$$\frac{\alpha}{2\pi} \overline{u}(p'_-)\gamma_\mu \left[A_1 \left(\hat{e} - \hat{k_1} \frac{ep_-}{k_1 p_-} \right) + A_2 \hat{k_1} \hat{e} \right] u(p_-). \tag{1.111}$$

The contribution of the structure A_1 disappears in the limit $m \to 0$, whereas A_2 survives, providing the following contribution to the DVCS tensor:

$$E^{\rm virt_1}_{\mu\nu\rho} = \frac{\alpha}{\pi} \frac{1}{\chi_-} \left(\ln \frac{\chi_-}{m^2} - \frac{1}{2} \right) {\rm Sp}\ \hat{p}'_- \gamma_\mu \hat{k_1} \gamma_\nu \hat{p}_- \gamma_\rho. \tag{1.112}$$

The contributions of the virtual photon emission vertex of type FD (Fig. 2, *a*) as well as of the box-type (Fig. 2, *b*) have the form:

$$E^{\rm virt_2}_{\mu\nu\rho} = \frac{\alpha}{4\pi} \int \frac{d^4k}{i\pi^2} \left\{ \frac{S_1}{-\chi_-} + \frac{S_2}{(p_- - k)^2 - m^2} \right\} \times \\ \times \frac{1}{(k^2 - \lambda^2)[(p'_- - k)^2 - m^2][(p_- - k_1 - k)^2 - m^2]}, \tag{1.113}$$

where

$$S_1 = \frac{1}{4} {\rm Sp}\, \hat{p}'_- \gamma_\lambda (\hat{p}'_- - \hat{k}) \gamma_\mu (\hat{p}'_- - \hat{k}_1 - \hat{k}) \gamma_\lambda (\hat{p}_- - \hat{k}_1) \gamma_\nu \hat{p}_- \gamma_\rho, \\ S_2 = \frac{1}{4} {\rm Sp}\, \hat{p}'_- \gamma_\lambda (\hat{p}'_- - \hat{k}) \gamma_\mu (\hat{p}'_- - \hat{k}_1 - \hat{k}) \gamma_\nu (\hat{p}_- - \hat{k}) \gamma_\lambda \hat{p}_- \gamma_\rho. \tag{1.114}$$

Their calculation requires scalar, vector, and tensor (up to rank three) integrals with three and four denominators, which are listed in [35].

Both $E^{\rm virt_1}_{\mu\nu\rho}$ and $E^{\rm virt_2}_{\mu\nu\rho}$ do not satisfy gauge invariance. Only the right-hand side of the expression (1.110) restores the property of gauge invariance.

After applying (1.110), the sum of the vertex contributions excluding FD with Dirac form factor (see Fig. 3) are:

$$E^{\rm virt}_{\mu\nu\rho} = E^{\rm virt_1}_{\mu\nu\rho} + E^{\rm virt_2}_{\mu\nu\rho} = E^0_{\mu\nu\rho} \frac{\alpha}{\pi} \left[-\frac{1}{4} L^2 + \frac{1}{2} \ln \frac{m^2}{\lambda^2} (1 - L) + \frac{3}{4} L + \mathcal{O}(\infty) \right]. \tag{1.115}$$

In this expression it was assumed that all terms proportional to $k_{1\nu}$ give a vanishing contribution, due to the Lorentz condition $e(k_1)k_1 = 0$.

1.7.3. Soft Photon Emission and Dirac Form Factor Contributions. Finally let us consider the vertex-type corrections to the electron scattering vertex without real photon emission (see Fig. 3, *a*) and the contribution of additional soft photon emission with energy not exceeding $\Delta\varepsilon$.

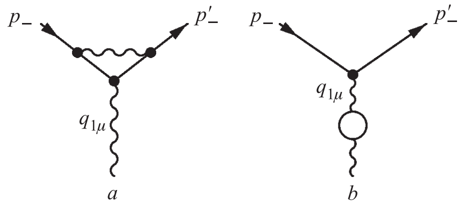

Fig. 3. Dirac and vacuum polarization contribution for one-loop FD

Both contributions are proportional to the Born DVCS terms:

$$E^{\mathrm{soft}+D}_{\mu\nu\rho} = E^0_{\mu\nu\rho}\left(\frac{\alpha}{\pi}\Gamma_1(q^2) + \delta_{\mathrm{soft}}\right),$$
$$\delta_{\mathrm{soft}} = -\frac{4\pi\alpha}{(2\pi)^3}\int \frac{d^3k_2}{2\omega_2}\left(\frac{p_-}{p_-k_2} - \frac{p'_-}{p'_-k_2}\right)^2\bigg|_{\omega_2 \ll \Delta\varepsilon}, \tag{1.116}$$

where

$$\frac{\alpha}{\pi}\Gamma_1(q^2) = \frac{\alpha}{\pi}\left[\ln\frac{m}{\lambda}(1-L) - \frac{1}{4}L^2 + \frac{3}{4}L + \frac{\pi^2}{12} - 1\right],$$
$$\delta_{\mathrm{soft}} = \frac{\alpha}{\pi}\left[(L-1)\ln\frac{(\Delta\varepsilon)^2 m^2}{\lambda^2\varepsilon_-\varepsilon'_-} + \frac{1}{2}L^2 - \frac{1}{2}\ln^2\frac{\varepsilon'_-}{\varepsilon_-} - \frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta}{2}\right)\right], \tag{1.117}$$

where ε_- is the energy of the incident electron and θ is electron scattering angle.

Combining all contributions containing large logarithms, we arrive to the lowest order expansion of the right-hand side, which does not contain the auxiliary parameter λ. Omitting the terms of order of unity we obtain

$$E^{\mathrm{summed}}_{\mu\nu\rho} = E^{\mathrm{virt}}_{\mu\nu\rho} + E^{\mathrm{soft}+D}_{\mu\nu\rho} = E^0_{\mu\nu\rho}\frac{\alpha}{\pi}\left[\ln\frac{(\Delta\varepsilon)^2}{\varepsilon_-\varepsilon'_-} + \frac{3}{2}\right](L-1). \tag{1.118}$$

1.7.4. Additional Hard-Photon Emission Contribution. The contributions arising from the emission of an additional hard photon with energy $\omega_2 > \Delta\varepsilon$ can be written in the form of two terms. The first one, corresponding to collinear kinematics, contains a large logarithm of type L and can be calculated with the help of the quasi-real electron method [4]. It has a form:

$$\frac{\alpha}{2\pi}\int\limits_{x_0(\phi)}^{1-\Delta_1} dx[P(x)(L_1-1) + 1 - x]E^0_{\mu\nu\rho}(p_-x, p'_-, k_1) \tag{1.119}$$

for the case of photon emission close to the initial electron, and

$$\frac{\alpha}{2\pi} \int\limits_{y(1+\Delta_2)}^{1} \frac{dz}{z} \left[P\left(\frac{y}{z}\right)(L_2 - 1) + 1 - \frac{y}{z} \right] E^0_{\mu\nu\rho}\left(p_-, \frac{z}{y} p'_-, k_1\right) \tag{1.120}$$

for the case of photon emission close to the scattered electron with

$$\Delta_1 = \frac{\Delta\varepsilon}{\varepsilon_-}, \quad \Delta_2 = \frac{\Delta\varepsilon}{\varepsilon'_-}, \quad P(z) = \frac{1+z^2}{1-z}, \tag{1.121}$$

with

$$L_1 = \ln \frac{\varepsilon_-^2 \theta_0^2}{m^2}, \quad L_2 = \ln \frac{\varepsilon_-'^2 \theta_0^2}{m^2}. \tag{1.122}$$

This contribution arises when the photons are emitted in a narrow cone, within an angle $\theta_0 \ll 1$, along the directions of the initial and the scattered electrons.

The contribution from noncollinear kinematics $\theta > \theta_0$ cancels the θ_0 dependence and does not contain large logarithms. Omitting nonleading terms, we can write $L_1 = L_2 = L$.

By summing up all contributions, we can put the cross section of the radiative production in the form:

$$E_{\mu\nu\rho}(p_-, p'_-, k_1) = \int\limits_0^1 dx\, D(x,\beta) \int\limits_y^1 \frac{dz}{z} D\left(\frac{y}{z}, \beta\right) \frac{E^0_{\mu\nu\rho}\left(xp_-, \frac{z}{y} p'_-, k_1\right)}{q^2(x,z) q_1^2} \times \\ \times \frac{1}{[1 - \Pi(q^2(x,z))][1 - \Pi(q_1^2)]}. \tag{1.123}$$

Here $1/[1 - \Pi(q^2(x,z)]$, $q^2(x,z) = q^2 xz/y$ is the polarization vacuum factor; $D(x,\beta)$ is defined at (1.9).

This expression is in agreement with the result previously obtained for the whole differential cross section in [36] where the RC to the muon block were also taken into account. Performing the integration on the scattered electron energy fraction y and using the normalization property of the LSF (1.11), we recover the expression (1.105).

The differential cross section for reaction (1.96) in LLA can therefore be expressed in terms of the shifted Born cross section as [36]:

$$d\sigma^{e^\pm \mu \to e^\pm \mu \gamma}(p_\pm, \ldots) = \int \frac{dx D(x,\beta)}{[1 - \Pi(xt)][1 - \Pi(t_1)]} d\sigma_B^{e^\pm \mu \to e^\pm \mu \gamma}(xp_\pm, \ldots), \tag{1.124}$$

with the following expression:

$$d\sigma_B^{e^\pm\mu\to e^\pm\mu\gamma}(p_\pm,\ldots) = \frac{2^7\pi^3\alpha^3}{stt_1}\left(s^2+s_1^2+u^2+u_1^2\right)\times$$
$$\times\left[-\frac{t_1}{\chi_-\chi'_-} - \frac{t}{\chi\chi'} \mp \left(\frac{u}{\chi_-\chi'} + \frac{u_1}{\chi'_-\chi} + \frac{s}{\chi_-\chi} + \frac{s_1}{\chi'_-\chi'}\right)\right] d\Gamma \quad (1.125)$$

for the nonshifted cross section. The explicit expression for the shifted cross section is derived in a straightforward way, by replacement of the shifted kinematics.

1.7.5. Numerical Calculation. Application to ep DVCS. Let us consider the case of unpolarized electron and unpolarized proton target and give an estimation of the RC to the cross section calculated in the Born approximation. We consider, in particular, the calculation for reaction (1.96) as a model for $e^\pm+p\to e^\pm+p+\gamma$, replacing the muon mass by the proton one.

The four-fold differential cross section, $d^4\sigma(\phi)$, has been calculated according to Eqs. (1.124), (1.125) for kinematical conditions as in [47]. The results for electron (plot *a*) and positron (plot *b*) scattering are shown in Fig. 4 before (solid line) and after (dashed line) applying radiative corrections. One can see that at $\phi=\pi$ the cross section for electrons (positrons) has a minimum (maximum) and that RC induce a ϕ-dependent relative correction.

The calculated relative effect may be applied to the experimental data. In [47], RC were calculated for $e^-+p\to e^-+p+\gamma$ following [31] and applied to the

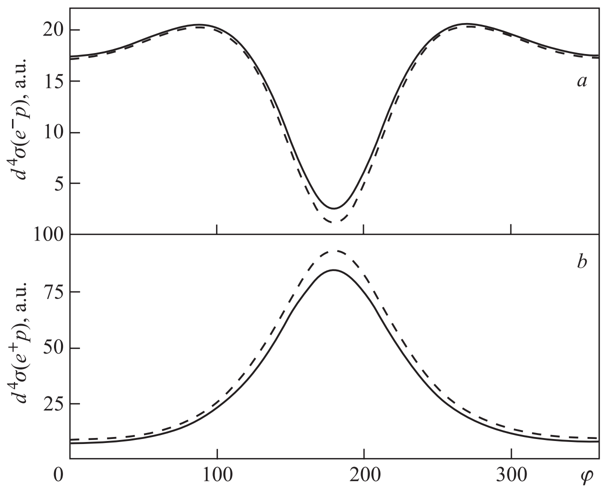

Fig. 4. *a*) Azimuthal distributions for $e^-p\to e^-p\gamma$ (i.e., $e^-\mu\to e^-\mu\gamma$ with M_μ =1 GeV) for the kinematics corresponding to [47]: $Q^2=2.3$ GeV^2, $-t_1=0.28$ GeV^2, $x_{\rm Bj}=0.36$ (solid line). The result after applying radiative correction is also shown (dashed line). *b*) The same for positron scattering

data with the help of a Monte Carlo simulation. This procedure resulted in a correction of the yield by a factor $F = 0.91 \pm 0.02$ which is constant with respect to ϕ, convoluted with $\Delta\varepsilon$-dependent corrections, which were included in a Monte Carlo simulation together with acceptance corrections. The overall effect was to increase the experimental yield of about 20%, roughly constant with ϕ.

In case of e^-p, LLA radiative corrections induce, on the one side, a lowering of the cross section, with respect to the calculated Born cross section, and on the other side, a change of the ϕ dependence. This strong ϕ dependence is an effect of hard photon emission. In an exclusive measurements, where the four momenta of all the particles involved are precisely determined, the importance of this effect could be quantitatively determined.

Let now consider the charge asymmetry:

$$A_{\mathrm{ch}} = \frac{d\sigma^{e^-\mu\to e^-\mu\gamma} - d\sigma^{e^+\mu\to e^+\mu\gamma}}{d\sigma^{e^-\mu\to e^-\mu\gamma} + d\sigma^{e^+\mu\to e^+\mu\gamma}}. \tag{1.126}$$

We can consider the calculation of A_{ch} as a model for radiative ep scattering (after replacing the muon mass with the proton mass). In the Born and LLA approximation A_{ch} is shown in Fig. 5, *a*, and the relative difference in Fig. 5, *b*.

The charge asymmetry is large, and may exceed 0.5 for in-plane kinematics. Radiative corrections are of the order of 5% with a smooth ϕ dependence. This quantity is especially interesting as it is in principle measurable at electron–positron rings with fixed target.

The helicity-dependent cross section Eq. (1.107) and the radiative corrections, calculated in LLA as a function of ϕ, is shown in Fig. 6. As expected, we obtain

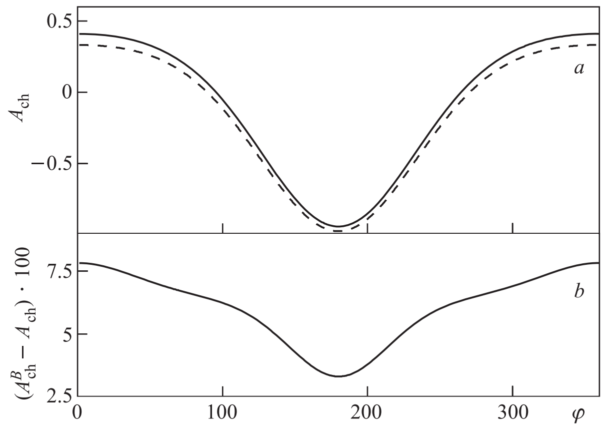

Fig. 5. *a*) Azimuthal angle dependence of the charge asymmetry (Eq. (1.125)) in the Born (solid line) and LLA (dashed line) approximation. *b*) The relative value in per cent. The same kinematics as in Fig. 4

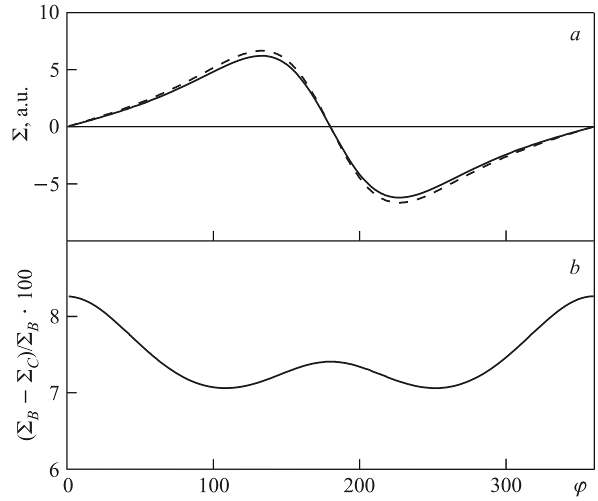

Fig. 6. *a*) Azimuthal dependence of the helicity asymmetry: Born calculation (solid line), radiative corrected (dashed line). *b*) Relative value of the corrections in per cent (bottom). The same kinematics as in Fig. 4

an antisymmetric function, that can be expanded in harmonics by $\sin\phi,\ \sin 2\phi\ldots$, the coefficients of which have physical meaning of all order twist contributions.

The radiative corrections to the helicity-dependent cross section are of the order of several per cent, with a small modulation in ϕ.

Let us summarize this Subsection. We calculated radiative corrections to VCS in the high-energy limit. The emission of hard photon in collinear kinematics is also included. The sum of all contributions (including soft photon emission) does not depend neither on the fictitious photon mass λ or on the soft photon energy $\Delta\varepsilon$, and it is consistent with the renormalization group prediction.

We applied the calculation, which is rigorous for the μ case, to proton scattering, after correcting for the mass. The proton structure can be taken into account in terms of electromagnetic form factors, which are function of t_1 and are not influenced by the conversion procedure to the shifted kinematics. However, let us note that taking into account nucleon form factors may violate the current conservation condition [48]. A self-consistent procedure requires an *ad hoc* modification of the nucleon propagator. This can be done including the excited states of the nucleon, such as the Δ resonance [49, 50]. It appears that elastic and inelastic processes partly compensate the effects of the strong interaction. Based on arguments of analyticity and unitarity [1, 50], one can expect that, taking into account the complete set of inelastic states in the intermediate state of the virtual Compton amplitude, an almost complete cancellation takes place, up to the contribution of structureless proton. This is the reason for which the approximation of structureless proton can be considered realistic. Moreover, if one builds the relevant ratios, such as $A_{\rm ch}$, the effect of form factors is essentially canceled.

The effect of hard photon emission is considerable, and the «returning mechanism» which is essentially expressed in the form of convolution of the shifted Born cross section with the electron LSF, may become important. At our knowledge, such mechanism was not considered in the previous literature for the reaction under consideration here.

Comparing with the scheme adopted to correct the experimental data (i.e., taking into account the first order RC, partly calculated with the help of a Monte Carlo and partly applying a constant factor to the final results), the present approach suggests a ϕ-dependent correction, mostly due to hard photon emission. The importance of this effect could be tested in a truly exclusive experiment and it may affect the extraction of the physical information from the Fourier analysis of the ϕ dependence of the relevant observables.

1.8. Deep Inelastic Scattering (DIS) in the Limit $y \to 1$. There is a believe that the kinematic region $1 - y \ll 1$ in DIS experiments cannot be described correctly due to huge RC which exceed the lowest order by more than 100% [44]

$$y = \frac{2p_1q}{2p_1P}, \quad q = p_1 - p_2, \tag{1.127}$$

here p_1, p_2 are 4-momenta of initial and final electrons; P is 4-momentum of proton.

This fact is the reason why the experimental results at $y > 0.8$ region as a rule are excluded from data processing. We argue here [51] that for the correct description of high y region RC, all orders of PT must be taken into account.

For the solution of this task the renormalization group approach is modified in such a way to include Sudakov-type suppression form factor. We will consider here the experimental setup with no emission of hard photons along an initial lepton.

For this aim let us consider two lowest order RC of PT. The emission of additional soft pions and soft pairs of the same order of energy as the one ε_2 of a scattered lepton do not exceeding $\Delta\varepsilon \ll \varepsilon$ becomes relevant:

$$\Delta\varepsilon \sim \varepsilon_2 = \varepsilon(1-y) \ll \varepsilon_1 = \varepsilon. \tag{1.128}$$

The cross section with RC can be put in the form:

$$\frac{d\sigma}{d\sigma_B} = 1 + \delta, \quad \delta = \frac{\alpha}{\pi}\Delta^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \Delta^{(2)} + \ldots, \tag{1.129}$$

while the lowest order RC are

$$\Delta^{(1)} = (l_t - 1)\left(\ln\frac{\Delta\varepsilon}{\varepsilon_1} + \ln\frac{\Delta\varepsilon}{\varepsilon_2}\right) + \frac{3}{2}l_t - \frac{1}{2}\ln^2(1-y) - \\ - \frac{\pi^2}{6} - 2 + \mathrm{Li}_2\left(\frac{1+c}{2}\right), \quad (1.130)$$

with

$$-t = 2\varepsilon^2(1-y)(1-c) \gg m_e^2, \quad l_t = \ln\left(\frac{-t}{m_e^2}\right), \quad c = \cos\theta, \qquad (1.131)$$

where $\theta = \widehat{\mathbf{p}_1\mathbf{p}_2}$ and ε_2 are the scattering angle and the energy of the scattered lepton in the laboratory frame. The mentioned above reasons allow us to put:

$$\ln\frac{\Delta\varepsilon}{\varepsilon_1} + \ln\frac{\Delta\varepsilon}{\varepsilon_2} = \ln(1-y). \qquad (1.132)$$

As a result, we have some deviation from the well-known Δ-part of evolution equation kernel (1.4). In our approach θ-part does not work, $\Delta = 1 - y$ (see the term containing l_t in (1.130)):

$$P_\Delta^{(1)} = 2\ln\Delta + \frac{3}{2} \to \left(2\ln(1-y) + \frac{3}{2}\right) - \ln(1-y). \qquad (1.133)$$

At the second order of PT the emission of two soft photons and soft pair (with total energy not exceeding $\Delta\varepsilon$) as well as a single-photon emission with 1-loop RC and, finally the 2-loop virtual corrections must be taken into account: $\Delta^{(2)} = \delta_{\gamma\gamma} + \delta_{\rm sp}$. We will not consider here the contribution from emission of real and virtual pairs. It can be taken into account by replacing the coupling constant by the moving one.

Contributions to RC from virtual and real photons emission have the form:

$$\delta_{\gamma\gamma} = \frac{1}{2}(\Delta^{(1)})^2 - \frac{\pi^2}{3}(l_t - 1)^2 + \frac{3}{2}l_t\left(2 + \frac{\pi^2}{6} - \mathrm{Li}_2\left(\frac{1+c}{2}\right)\right) + \mathcal{O}(1). \qquad (1.134)$$

This result agrees with RG predictions [10] at $y = 0$ and, in addition, contains the terms of type $\ln^2(1-y)$, $l_t\ln(1-y)$, which become relevant in the limit $y \to 1$.

Let us discuss this points more closely. We suppose that there is no hard photon emission by the initial lepton which can provide the «returning to resonance» mechanism. Really this mechanism for the case $\varepsilon_2/\varepsilon = 1 - y \ll 1$ will correspond to very small transversal momentum squared $Q_1^2 \sim \epsilon^2(1-y)^2 \ll Q^2$.

Let us now average DIS cross section over a small interval $\tilde{Q}^2 \sim Q^2$ introducing an additional integration in the right part of formula (1.63):

$$\int d\tilde{Q}^2 \delta\left(\left(\frac{z_1 x Q^2}{z_2}\right) - \tilde{Q}^2\right), \quad x = 1 - \left(\frac{\Delta\epsilon}{\epsilon}\right). \tag{1.135}$$

Small variations of transfer momentum arise from RC emission of soft real and virtual partons (photons and leptons). Taking into account that the hard cross section in this region is flat, we obtain for the ratio of DIS cross sections with and without RC:

$$\begin{gathered}\frac{d\sigma}{d\sigma_B} = F(x, \beta_t) = \int\int D(z_1, \beta_t) D(z_2, \beta_t) dz_1\, dz_2\, \theta(x z_1 - z_2), \\ \beta_t = \frac{\alpha}{2\pi}(l_t - 1).\end{gathered} \tag{1.136}$$

Using the differential evolution equations for nonsinglet structure functions $D(x, \beta_t)$

$$\frac{\partial D}{\partial l_t} = \frac{\alpha(l_t)}{2\pi} \int\limits_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) D(y, l_t), \quad D(y, 0) = \delta(1 - y),$$

one can obtain a differential equation for $F(x, \beta_t)$

$$\frac{\partial F}{\partial l_t} = \frac{\alpha(l_t)}{\pi} \int\limits_x^1 dz\, P\left(\frac{x}{z}\right) F(z, \beta_t), \quad F(x, 0) = 1. \tag{1.137}$$

This equation was solved in [10]:

$$F(x, \beta_t) = \left(\ln\frac{1}{x}\right)^{2\chi} \frac{\exp[\chi(3/2 - 2C_E)]}{\Gamma(1 + 2\chi)}, \quad \chi = -3\ln\left(1 - \frac{\alpha}{3\pi} l_t\right). \tag{1.138}$$

Terms containing $\ln(1 - y)$ are not taken into account in the evolution procedure. We argue here that there is a reason to take them into account as a general factor which can be obtained from the known factor of Yienie, Frautchi, and Suura [30], with replacement of the logarithm of the ratio of the photon to the lepton mass by $\ln\Delta, \Delta = \Delta\varepsilon/\varepsilon$ in accordance with the Bloch–Nordsick theorem.

Replacing $\ln(1/x) = 1 - y$ we obtain for DIS cross section:

$$\left.\frac{d\sigma}{d\sigma_B}\right|_{y \to 1} = R\left(1 + \frac{\alpha}{\pi} K\right),$$

$$R = \frac{1}{(1-\Pi(Q^2))^2}\frac{(1-y)^{2\chi}}{\Gamma(1+2\chi)}\exp\Big((3/2-2C_E)\chi- \\ -\frac{\alpha}{2\pi}(\ln^2(1-y)+2l_t\ln(1-y))\Big), \quad (1.139)$$

$$|K|\sim 1, \quad \chi = -3\ln\left(1-\frac{\alpha}{3\pi}l_t\right) = \frac{\alpha}{\pi}l_t + \frac{\alpha^2}{6\pi^2}l_t^2+\ldots,$$

where $C_E = 0.577$ is the Euler constant and $d\sigma_B$ is the DIS cross section in the Born approximation. One can be convinced that Eq. (1.139) agrees with the results of the lowest order calculation (1.133), (1.134) up to nonleading terms, which are parameterized in the form of K factor. Equation (1.139) provides us for $|K|\sim 1$ with the accuracy on the level of 1%. The behavior of quantity $R(x,y)$ for different values of the Bjorken parameter x at HERMES kinematic conditions is illustrated in Fig. 7.

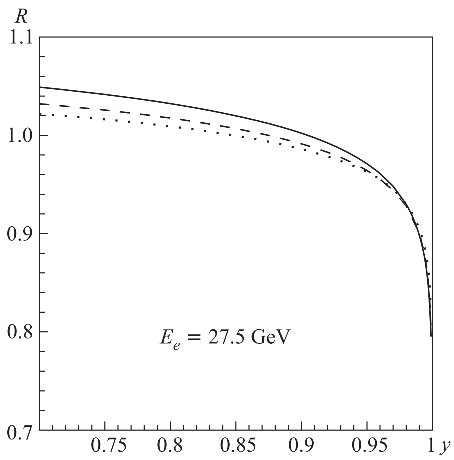

Fig. 7. Ratio $R = d\sigma/d\sigma_B$ for $x = 0.1$ (solid line), $x = 0.01$ (dashed line) and $x = 0.001$ (dotted line) versus y for $0.7 \leqslant y \leqslant 0.999$ at HERMES kinematic condition

1.9. 2γ and 3γ Annihilation of High-Energy e^+e^- Beams. In this Subsection we study the process of Compton scattering, namely the annihilation of electron and positron in 2γ (3γ), in the high-energy region, which is under consideration here:

$$\begin{aligned} &e^-(p_-)+e^+(p_+)\to\gamma(k_1)+\gamma(k_2), \quad s=(p_-+p_+)^2\gg m_e^2=m^2 \\ &e^-(p_-)+e^+(p_+)\to\gamma(k_1)+\gamma(k_2)+\gamma(k_3) \end{aligned} \quad (1.140)$$

including RC. This process was first considered in the well-known papers by L. Brown and R. Feynman, and H. Harris and L. Brown in the early 1950s and then revised in 1973 by H. Berends and R. Gastmans [37–39].

Nowadays these processes are used at colliders, as normalization processes which provide an independent way to measure the luminosity of beams. The large-angle emission of final photons provides a clean signal for an independent method for measuring the luminosity. The precise knowledge of this process must also be taken into account when estimating the background in channels with neutral meson production.

The differential cross section of two-gamma production in the Born approximation has the form

$$\frac{d\sigma_B}{dO_1} = \frac{\alpha^2}{sv}\left[\frac{1+v^2c^2}{1-v^2c^2} + 2v^2(1-v^2)\frac{1-c^2}{(1-v^2c^2)^2}\right], \tag{1.141}$$

with $v = \sqrt{1-(4m^2/s)}$, $c = \cos\theta$ and θ being the polar angle between the initial electron and photon (with momentum k_1 in the center-of-mass reference frame, which is implied below). In the high-energy limit for large-angle photon emission we can put $v = 1$ in (1.141) and obtain

$$\frac{d\sigma_B}{dO_1} = \frac{\alpha^2(1+c^2)}{s(1-c^2)}.$$

The conservation law, on-mass shell conditions and kinematic invariants are defined for the $e\bar{e} \to \gamma\gamma$ process as

$$\begin{gathered} p_+ + p_- = k_1 + k_2, \quad p_+^2 = p_-^2 = m^2, \quad k_1^2 = k_2^2 = 0, \\ \chi_{1,2} = 2p_-k_i, \quad \chi_1 + \chi_2 = s. \end{gathered} \tag{1.142}$$

The corresponding «total» cross section estimated for the region $\chi_1 \sim \chi_2 \sim s$ is of the order of $\pi\alpha^2/s$, which is rather large compared to the processes involving weak and strong interactions. Their knowledge with RC of higher order is urgent since these processes of QED nature provide large background in the studies of interactions of different nature than QED.

The beam calibration based on annihilation processes has an essential advantage compared to the methods based on Bhabha scattering and annihilation into a pair of charged particles $e\bar{e} \to \mu\bar{\mu}, \tau\bar{\tau}$. Indeed, the process of 2γ annihilation at large angles is of the same order of magnitude as Bhabha scattering, but does not encounter the problems related to final state interactions, taking into account the vacuum polarization.

The theorem on factorization [8, 11] of hard and soft momenta in the cross section of exclusive processes permits one to include RC in the leading logarithmic approximation

$$\frac{\alpha}{\pi} \ll 1, \quad \frac{\alpha}{\pi}L \sim 1, \quad L = \log\frac{s}{m^2}, \tag{1.143}$$

in all orders of perturbation theory in terms of structure functions of electron and positron $D(x,\beta)$. The differential cross section of annihilation in two quanta can be written as

$$d\sigma(p_-,p_+) = \int\limits_0^1 dx\, D(x,\beta) \int\limits_0^1 dy\, D(y,\beta) d\sigma_B(xp_-, yp_+)\left(1 + \frac{\alpha}{\pi}K_2\right), \tag{1.144}$$

with the «shifted» Born cross section of the form:

$$d\sigma_B(xp_-, yp_+) = \frac{2\alpha^2}{sxy} \frac{x^2(1-c)^2 + y^2(1+c)^2}{[x(1-c)+y(1+c)]^2} dO_1. \tag{1.145}$$

The main attention should be devoted to calculation of K factor $K = 1 + (\alpha/\pi)K_2$; its knowledge permits one to increase the accuracy of theoretical description up to 10^{-3} level.

The list of all relevant cross sections can be find in Subsec. 1.2.

1.9.1. Virtual, Soft Real, and Hard Collinear Photon Emission Contribution. Using the known results of calculation of virtual corrections [37–39], we obtain

$$\frac{d\sigma_{\rm virt}}{d\sigma_B} = \delta_V = \frac{\alpha}{\pi}\left[-\frac{1}{2}L^2 - (L-1)\ln\frac{m^2}{\lambda^2} + \frac{3}{2}(L-1) + K_V\right]. \tag{1.146}$$

The emission of an additional soft photon which escapes detection has an energy which does not exceed some small quantity. The contribution for $\Delta E \ll E = \sqrt{s}/2$ is

$$\frac{d\sigma_{\rm soft}}{d\sigma_B} = \delta_S = -\frac{\alpha}{4\pi^2}\int \frac{d^3k}{\omega}\left(\frac{p_-}{p_-k} - \frac{p_+}{p_+k}\right)^2\Bigg|_{\omega<\Delta E} =$$
$$= \frac{\alpha}{\pi}\left[-\frac{1}{2}L^2 + L + 2(L-1)\ln\left(\frac{2\Delta E}{\lambda}\right) + K_S\right], \tag{1.147}$$

with $K_S = -\pi^2/3$. The total sum of virtual and real soft-photon emission has the form:

$$\frac{d\sigma_{\rm virt}}{d\sigma_B} + \frac{d\sigma_{\rm soft}}{d\sigma_B} = \frac{\alpha}{\pi}\left[(L-1)\left[2\ln\frac{\Delta E}{E} + \frac{3}{2}\right] + K_{\rm SV}\right], \tag{1.148}$$

with

$$K_{\rm SV} = \frac{\pi^2}{3} + \frac{1}{4(1+c^2)}\left[(5-6c+c^2)\ln\frac{1+c}{2} + (5+2c+c^2)\ln^2\frac{1+c}{2} + \right.$$
$$\left. + (5+6c+c^2)\ln\frac{1-c}{2} + (5-2c+c^2)\ln^2\frac{1-c}{2}\right]. \tag{1.149}$$

In Fig. 8 we show the dependence of $K_{\rm SV}(c)$. The contribution from emission of a hard photon in collinear kinematics can be obtained using the method of quasi-real electrons [4, 5]. For this purpose we introduce a numerically small angle θ_0 and consider the emission of an additional hard photon inside the cones $\theta < \theta_0$ along the electron and positron direction of motion. The chiral amplitude method cannot be applied to this kinematical region since chirality is not a good

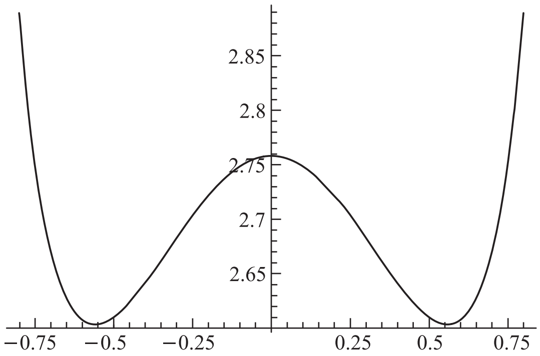

Fig. 8. Angular dependence of $K_{\rm SV}(c)$ (see (1.149))

quantum number for collinear kinematics. We distinguish emission along the initial electron $\theta < \theta_0$, where θ is the angle between 3-momenta of electron (we choose this direction as the z axis), and photon with energy ω

$$d\sigma^{\rm coll}_{e^-}(p_-,p_+) = \frac{\alpha}{2\pi}\int\limits_0^{1-\frac{\Delta E}{E}} dx\left[\frac{1+x^2}{1-x}(L_\theta - 1) + 1 - x\right] d\sigma_B(xp_-,p_+),$$
$$x = 1 - \frac{\omega}{E}. \tag{1.150}$$

The emission along the positron, $\pi - \theta < \theta_0$:

$$d\sigma^{\rm coll}_{e^+}(p_-,p_+) = \frac{\alpha}{2\pi}\int\limits_0^{1-\frac{\Delta E}{E}} dx\left[\frac{1+x^2}{1-x}(L_\theta - 1) + 1 - x\right] d\sigma_B(p_-,xp_+), \tag{1.151}$$

with $L_\theta = L + \ln(\theta_0/2)^2, \theta_0 \ll 1$ and the expressions for the shifted cross sections are given in (1.145).

1.9.2. Hard-Photon Emission Correction. Form of $K^{\rm hard}$. The contribution of the kinematics when all three hard photons are emitted in the so-called non-collinear kinematics (all three photons are emitted outside cones $\theta < \theta_0, \quad \pi - \theta \ll \theta_0$) can be obtained using the chiral amplitude method [2, 3]

$$d\sigma^{3\gamma}(p_+,p_-) = \frac{16\alpha^3}{3\pi^2 s} R\, d\Phi, \tag{1.152}$$

with*

$$R = \frac{\nu_3^2(1+c_3^2)}{\nu_1^2\nu_2^2(1-c_1^2)(1-c_2^2)} + \frac{\nu_2^2(1+c_2^2)}{\nu_1^2\nu_3^2(1-c_1^2)(1-c_3^2)} + \frac{\nu_1^2(1+c_1^2)}{\nu_3^2\nu_2^2(1-c_3^2)(1-c_2^2)} \tag{1.153}$$

and

$$d\Phi = \frac{1}{s}\frac{d^3q_1}{2\omega_1}\frac{d^3q_2}{2\omega_2}\frac{d^3q_3}{2\omega_3}\delta^4(p_- + p_+ - k_1 - k_2 - k_3) = \\ = \frac{(1-\nu_1)\nu_1 d\nu_1}{16(2-\nu_1(1-c_{13}))^2} dO_1\, dO_3, \quad c_{13} = 1 - \frac{2(1-\nu_2)}{\nu_1\nu_3}, \tag{1.154}$$

c_1, c_2, c_3 are the cosines of the photon emission angles to the initial electron 3-momentum and $\nu_i = \omega_i/E$ are the fractions of energy of final photons.

It can be seen that the dependence on the auxiliary parameters $(\Delta E/E)$ and θ_0 will be canceled in the expression of $K^{\rm hard}$ defined as

$$\frac{\alpha}{\pi} d\sigma_B(p_-, p_+) K^{\rm hard} = \int d\sigma^{3\gamma}(p_+, p_-)\Theta d\Phi + \\ + \frac{\alpha}{2\pi} \int\limits_0^{1-\frac{\Delta E}{E}} \frac{dx}{1-x}\left[(1+x^2)\ln\frac{\theta_0^2}{4} + (1-x)^2\right][d\sigma^{2\gamma}(xp_-, p_+) + d\sigma^{2\gamma}(p_-, xp_+)]. \tag{1.155}$$

Here the symbol Θ indicates the limits on the manifold of integration variable (additional hard photon) $d\Phi$. Such limitations are: all three energy fractions should be larger than $\Delta E/E$ (hardness condition). Moreover, conservation laws restrict $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$. In particular,

$$c_1\nu_1 + c_2\nu_2 + c_3\nu_3 = 0, \quad \nu_1 + \nu_2 + \nu_3 = 2, \quad \frac{\Delta E}{E} < \nu_i < 1. \tag{1.156}$$

Terms containing «large logarithm» L can be written in the form

$$\beta\left[\int\limits_0^1 dx P^{(1)}(x)\, d\sigma_B^{2\gamma}(xp_-, p_+) + \int\limits_0^1 dx P^{(1)}(x)\, d\sigma_B^{2\gamma}(p_-, xp_+)\right]. \tag{1.157}$$

The parameter x can be interpreted as the energy fraction of electron considered as a parton in the initial electron. In such a way, one can obtain the general form of cross section in the form of cross section of Drell–Yan process given above (see (1.144)). The total value of K_2 is $K_2 = K_{\rm VS} + K^{\rm hard}$.

*In paper [41] in the right-hand sides of 4.3 and 4.6, the factors $1/3!$ and $1/2$ are missing.

1.9.3. 3γ Annihilation Channel. For completeness, we also give the cross section of 3γ annihilation. In [42], it was shown that, in the leading logarithmical approximation, it has the form (1.144) with replacement $d\sigma_B$ by $d\sigma^{3\gamma}$ (see (1.155)). The value of K factor ($K = K_3$) is a complicated function of kinematical invariants. When calculating RC to the process $e\bar{e} \to 3\gamma$ one has to consider two kinds of 1-loop FD. One of them describes the interaction of the initial particles $e\bar{e}$. The relevant integrals including pentagon are presented in Subsec. 2.1. Another type contains the LBL block of conversion of virtual photon to 3 real ones.

A realistic estimation can be obtained by using the averaging procedure: replacing the ratio of amplitudes of nonleading contributions by an amplitude of leading contributions expressed in terms of cross sections. The cross section of 3γ annihilation through the light-by-light mechanism turns out to be dominant among nonleading terms. This contribution does not contain large logarithms L.

This cross section was calculated in [43]

$$\sigma_{lbl}^{e^+e^-\to 3\gamma}(s) = \frac{\alpha^5}{18\pi^2 s}N,$$
$$N = 200\xi_5 - 8\pi^2\xi_3 + \frac{7}{15}\pi^4 - 128\xi_3 + \frac{41}{3}\pi^2 - 124 \approx 15. \quad (1.158)$$

In [40], the so-called «total» cross section of the process $e^+e^- \to 3\gamma$ in the lowest order of PT was obtained. The detection energy threshold of final photons $2\omega_i/E = \nu_i > \eta$. Moreover it was implied $\psi_0, (\cos\psi_0 = z \sim 1)$ is the minimal angle between the plane of the photons momentum and beam axis was fixed. It has the form:

$$\sigma_{\rm tot}(z) = \frac{2\alpha^3}{s}[\phi_1(z)\ln\eta + \phi_2(z)]. \quad (1.159)$$

The explicit form of the functions $\phi_1(z), \phi_2(z)$ are:

$$\phi_1(z) = -2\ln(1-z^2) - \frac{3}{2}\ln^2(1-z^2) - 2\mathrm{Li}_2(z^2), \quad (1.160)$$

$$\phi_2(z) = -\frac{1}{2}\ln^2(1-z^2) + \frac{1}{6}\pi^2\left(\frac{2z^2}{1-z^2} - \ln(1-z^2)\right) + \frac{1}{6}\ln^3(1-z^2) -$$
$$- \left(\frac{2}{1-z^2} + 2\ln\frac{z^2}{1-z^2}\right)\mathrm{Li}_2(z^2) + \frac{3}{2}\int\limits_0^{z^2}\frac{dx}{x}\ln^2(1-x) -$$
$$- 2\int\limits_0^{z^2}\frac{dx}{x}\ln x\ln(1-x). \quad (1.161)$$

Keeping in mind the smooth behavior of the nonleading contributions in the kinematic region of large-angle photon emission, we estimate the K_3 using the «total cross section« approximation:

$$1+\frac{\alpha}{\pi}K_3 \approx 1+2\sqrt{\frac{\sigma_{lbl}}{\sigma_{\rm tot}}}, \quad K_3=\frac{1}{3}\sqrt{\frac{N}{\phi_1(z)\ln\eta+\phi_2(z)}}. \qquad (1.162)$$

Numerical estimation for $z=0.3$ and $\eta=0.05$ leads to $1+(\alpha/\pi)K_3 \approx 1.01$.

1.10. Initial State Radiative Corrections to the Annihilation of Electron and Positron to Hadrons Process. Cancellation of θ_0 Dependence. Born matrix element of annihilation of electron and positron to hadrons have the form:

$$M_B=\frac{4\pi\alpha}{s}J_Q^\mu(q)J_{H\mu}(q), \quad s=(p_+ + p_-)^2=q^2, \qquad (1.163)$$

with p_+, p_- — the 4-momenta of positron and electron, and the information on the hadronic state is suppressed. The conserved QED current has the form:

$$J_Q^\mu=\bar{v}(p_+)\gamma^\mu u(p_-), \quad J_Q^\mu q_\mu=0.$$

Using the conservation of the hadronic current the relation can be written as

$$\int d\Gamma_H J_{H\mu}J_{H\nu}^* = \frac{1}{3}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\int d\Gamma_H \sum_\lambda |J_{H\lambda}|^2, \qquad (1.164)$$

where the integration over the whole phase volume Γ_H of hadrons is implied. For the cross section in the Born approximation we obtain

$$d\sigma_B(s)=\frac{8\pi^2\alpha^2}{3s^2}\int d\Gamma_H(-|J_{H\mu}|^2). \qquad (1.165)$$

We note that both currents are the space-like 4-vectors so $-|J_{H\mu}|^2>0$.

Taking into account the radiative corrections due to emission of virtual and real soft photons by the initial leptons results as

$$\frac{\sigma^{\rm SV}}{\sigma_B}=1+\frac{2\alpha}{\pi}\left[(L-1)\left(\ln\frac{\Delta\epsilon}{\epsilon}+\frac{3}{4}\right)+\frac{1}{2}K\right], \quad K=\frac{1}{3}\pi^2-\frac{1}{2}, \qquad (1.166)$$

where $\Delta\epsilon \ll \epsilon$ is the maximal energy of the soft photon in the cms frame, $s=4\epsilon^2$ and $L=\ln s/m^2$.

Let us consider now the emission of a hard-photon process. We will suppose that the energy of photon exceeds $\Delta\epsilon$. It is convenient to use the «quasi-real electrons» method [4, 5]. It takes into account the emission of real hard photon in a narrow cone along the directions of electron and positron:

$$\sigma_<(s) = \frac{2\alpha}{\pi}\int\limits_{\Delta}^{1}\frac{dx}{x}\left[\left(1-x+\frac{x^2}{2}\right)\left(L-1+\ln\frac{\theta_0^2}{4}\right)+\frac{x^2}{2}\right]\sigma_B(s(1-x)),$$
$$\Delta = \frac{\Delta\epsilon}{\epsilon}, \tag{1.167}$$

with θ — the emission angle of photon measured from the initial electron direction of motion. The matrix element of photon emission at large angles $\theta > \theta_0, \pi-\theta > \theta_0$ is

$$M^\gamma = \frac{(4\pi\alpha)^{3/2}}{q_1^2}J^\gamma_{Q,\mu}J^\mu_H, \tag{1.168}$$

with $q_1 = q-k$ and

$$J^\gamma_{Q,\mu} = -\bar v(p_+)\left[\gamma_\mu\frac{\hat p_- - \hat k}{\chi_-}\hat e(k)+\hat e(k)\frac{-\hat p_+ + \hat k}{\chi_+}\right]u(p_-), \tag{1.169}$$

with $\chi_\pm = 2kp_\pm = (s/2)x(1\pm c), c = \cos\theta$ and polarization 4-vector of the photon is $e(k)$. Summation on the spin states of the module squared of QED current leads to

$$\sum|J^\gamma_{Q,\mu}|^2 = -8\left[\frac{\chi_+}{\chi_-}+\frac{\chi_-}{\chi_+}\frac{2ss_1}{\chi_+\chi_-}\right], \tag{1.170}$$

with $s_1 = q_1^2 = s(1-x), x = \omega/\epsilon$. Further integration on θ is straightforward with the result:

$$\sigma_> = \frac{2\alpha}{\pi}\int\limits_{\Delta}^{1}\frac{dx}{x}\sigma(s(1-x))\left[\left(1-x+\frac{x^2}{2}\right)\ln\frac{4}{\theta_0^2}-\frac{x^2}{2}\right]. \tag{1.171}$$

It can be seen that the auxiliary parameter θ_0 drops out from the sum $\sigma^\gamma = \sigma_< + \sigma_>$. The result of the total correction is

$$\sigma(s) = \int\limits_0^1 dx\int\limits_0^1 dy D^{\rm NS}(x,\beta)D^{\rm NS}(y,\beta)\sigma_B(sxy)\left(1+\frac{\alpha}{\pi}K\right). \tag{1.172}$$

1.11. Radiative Corrections to Muon Decay in Leading and Next-to-Leading Approximation. Electron Spectrum. The lowest order RC to the muon weak decay width were calculated about fifty years ago [45]. The result for the electron spectrum in muon decay including RC was obtained in the form

$$\frac{dW^{(1)}(x)}{dx} = \frac{dW_B(x)}{dx}\left[1+\frac{\alpha}{2\pi}h(x)\right], \quad x = \frac{E_e}{E_{\max}} \approx \frac{E_e}{M},$$
$$h(x) = A(x) + LB(x), \quad L = \ln\frac{M^2}{m^2}, \tag{1.173}$$

with the spectrum in the Born approximation

$$\frac{dW_B(x)}{dx} = 2W_B x^2(3-2x), \quad W_B = \frac{G^2M^5}{192\pi^3}. \tag{1.174}$$

Here M is the muon mass; m is the electron mass; L is the so-called «large logarithm» ($L \approx 12$). The result of the lowest order RC is presented in the expression $h(x)$, or in the functions $A(x)$ and $B(x)$ [9]

$$A(x) = 4\mathrm{Li}_2(x) - \frac{2\pi^2}{3} - 4 + 2\left[3\ln(1-x) - 2\ln x + 1\right]\ln x - 2\frac{1+x}{x}\ln(1-x) +$$
$$+ \frac{(1-x)(5+17x-16x^2)}{3x^2(3-2x)}\ln x + \frac{(1-x)(-22x+34x^2)}{3x^2(3-2x)}, \tag{1.175}$$
$$B(x) = 3 + 4\ln\frac{1-x}{x} + \frac{(1-x)(5+17x-34x^2)}{3x^2(3-2x)}.$$

One must remark that the result of the calculations does not suffer from the ultraviolet and the infrared divergences. Besides, it satisfies Kinoshita–Lee–Nauenberg (KLN) theorem [46] about the cancellation of mass singularities, namely the total width is finite in the limit of zero electron mass

$$\int dx, \frac{dW_B(x)}{dx}B(x) = 0, \tag{1.176}$$

besides,

$$\int dx\, x^2(3-2x)h(x) = \frac{25}{8} - \frac{\pi^2}{2}. \tag{1.177}$$

The mechanism of the realization of KLN theorem can be understand from the positions of parton interpretation of QED. Really, one can be convinced in the validity of the relation [52]

$$\frac{1}{2}x^2(3-2x)h(x) = (L-1)\int\limits_x^1 \frac{dy}{y}y^2(3-2y)P\left(\frac{x}{y}\right) + K(x),$$
$$K(x) = \frac{1}{2}x^2(3-2x)(A(x)+B(x)), \tag{1.178}$$

where $P(z)$ is the kernel (1.4) of the evolution equation of twist two operators. Using the property $\int\limits_0^1 dx\, P(x) = 0$, one can validate Eq. (1.176):

$$W_B\int\limits_0^1 dx\, x^2(3-2x)B(x) = \int\limits_0^1 dx\int\limits_x^1 \frac{dy}{y}\frac{dW_B(y)}{dy}P\left(\frac{x}{y}\right) = \\ = \int\limits_0^1 dy\frac{dW_B(y)}{dy}\int\limits_0^y \frac{dx}{y}P\left(\frac{x}{y}\right) = 0.$$

Considering the process in the Born approximation as a «hard» process and applying Collins factorization theorem about the contributions of the short and long distances, one can generalize the lowest order result to include all terms of the sort $(\alpha L/\pi)^n$ (LLA) as well as the terms of the sort $\alpha(\alpha L/\pi)^n$ (NLO) in the form:

$$\frac{dW(x)}{dx} = \int\limits_x^1 \frac{dy}{y}\frac{dW_B(y)}{dy}D\left(\frac{x}{y},\eta\right)\left(1+\frac{\alpha}{\pi}K(y)\right), \quad \eta = \frac{\alpha}{2\pi}(L-1), \tag{1.179}$$

and $D(z,\eta)$ is given in (1.10),

$$K(y) = y^2(3-2y)\left[2\mathrm{Li}_2(y) - \frac{\pi^2}{3} - \frac{1}{2} + [3\ln(1-y) - 2\ln y + 1]\ln y - \right. \\ \left. - \frac{1+y}{y}\ln(1-y) + 2\ln\frac{1-y}{y}\right] + \\ + \frac{1}{6}(1-y)\left[(5+17y-16y^2)\ln y + 5(1-y)\right]. \tag{1.180}$$

The total width as well will not contain «large logarithms» due to the property of $D(x,\eta)$ (see Subsec. 1.11). With the K factor in the lowest order of PT we obtain (compare with (1.177))

$$W = W_B\left[1 - \frac{\alpha}{2\pi}\left[\pi^2 - \frac{25}{4}\right]\right].$$

In the paper [53] the correction of order α^2 was calculated.

One can find useful relation

$$\int\limits_x^1 \frac{dy}{y^2} D\left(\frac{x}{y},\eta\right)\Psi(y) = \int\limits_x^1 \frac{dy}{y^2} D\left(\frac{x}{y},\eta\right)[\Psi(y)-\Psi(x)] + \\ + \frac{1}{x}\Psi(x)\int\limits_x^1 dz\, D(z,\eta) \quad (1.181)$$

with

$$\Psi(y) = y^3(3-2y)\left(1+\frac{\alpha}{\pi}K(y)\right).$$

For the comparison we give in Fig. 9 the numerical values of the quantity $\frac{96\pi^3}{G^2M^5}\left(\frac{dW^{(1)}(x)}{dx} - \frac{dW_B(x)}{dx}\right)\frac{\pi}{\alpha} = \frac{1}{2}x^2(3-2x)h(x)$ — the dashed curve (RC in the lowest order of PT, see (1.173)), and the quantity $\frac{96\pi^3}{G^2M^5}\left(\frac{dW(x)}{dx} - \frac{dW_B(x)}{dx}\right)\frac{\pi}{\alpha}$ — the solid curve, calculated in LLA and NLO approximations at all orders of PT (see (1.179)). One can see that the spectrum contrary to the result of the lowest order of PT is well defined in the whole region of x including $x \to 0$ and $x \to 1$. A similar result was obtained in [54]. Explicit expressions for the NLO terms of the two-loop level spectra as well as the creation of e^+e^- pairs in μ decay was considered in [55].

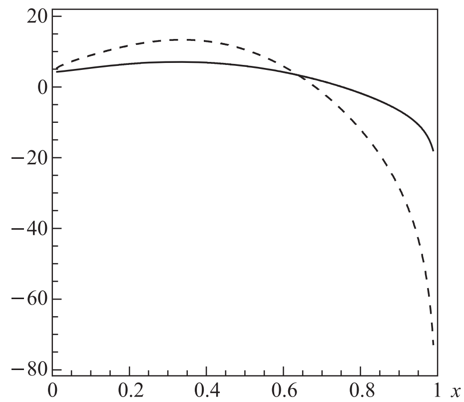

Fig. 9. The deviation of the electron spectrum in muon decay from the spectrum in the Born approximation: for the lowest order (dashed line), for all orders (solid line) of perturbation theory

1.12. Processes $e\bar{e} \to e\bar{e}$ and $e\bar{e} \to \mu\bar{\mu}$ in the Region of the Narrow Resonances. In the energy range of initial particles close to the mass of narrow resonance, the differential cross sections of the simplest QED processes are modified. The relevant amplitude besides the usual Born level form contains the additional terms taking into account the presence of resonance state in the annihilation channel.

For the case of elastic electron–positron scattering we obtain for the differential cross section in the lowest order approximation:

$$\frac{d\sigma_B^{e\bar{e}\to e\bar{e}}(\Delta)}{dO_e} = \frac{\alpha^2}{M^2}\left[\frac{1+c_1^4}{2s_1^4} + \frac{1}{2}(c_1^4+s_1^4)\left|1+\frac{(12\pi)^2 B_e^2\gamma^2}{\Delta+i\gamma}\right|^2 - \right.$$
$$\left. -\frac{c_1^4}{s_1^2}\mathrm{Re}\left[1+\frac{(12\pi)^2 B_e^2\gamma^2}{\Delta+i\gamma}\right]\right]. \quad (1.182)$$

For the case of the process of muon pair production we have

$$\frac{d\sigma_B^{e\bar{e}\to \mu\bar{\mu}}(\Delta)}{dO_e} = \frac{\alpha^2}{2M^2}\left[\frac{1}{2}(c_1^4+s_1^4)\left|1+\frac{(12\pi)^2 B_e B_\mu\gamma^2}{\Delta+i\gamma}\right|^2\right], \quad (1.183)$$

where $B_{e,\mu} = \Gamma_{e,\mu}/\Gamma$ — the branching ratio of the relevant mode of resonance decay; γ is the total width of the resonance, $\gamma = \Gamma/M$, $c_1, s_1 = \cos(\theta/2), \sin(\theta/2)$; θ is the scattering angle (center of mass of the initial beam is implied) between the negative charged initial and the scattered particles, and finally $\Delta = (s-M^2)/M^2$, M is the resonance mass. We imply $\Delta \ll 1$.

For the case when the energies of final particles are not specified (we imply the measurement of only the scattering angle) and beside the experiment is charge-blind (charge-odd effects are neglected) only emission of virtual and soft real photons by the initial leptons is relevant. Keeping in mind the relation

$$\int\limits_x^1 \frac{dy}{y} D(y,\beta) D\left(\frac{x}{y},\beta\right) = D(x,2\beta) \quad (1.184)$$

and the approximate relation

$$D(x,2\beta) = 4\beta\left[(1-x)^{4\beta-1}(1+3\beta) - \frac{1}{2}(1+x)\right] + O(\beta^2), \quad (1.185)$$

we can write down the cross section in the form

$$\frac{d\sigma}{dO} = \int\limits_0^1 dx\, D(x,2\beta)\frac{d\sigma_B(\Delta-(1-x))}{dO}. \quad (1.186)$$

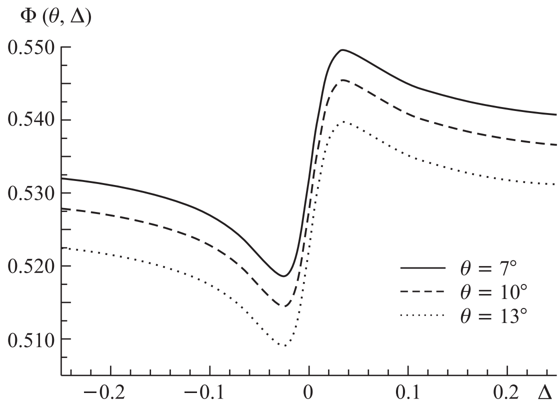

Fig. 10. The dependence of value $\Phi(\theta, \Delta) = \left(\frac{s}{\alpha^2}\right) \frac{d\sigma^{e\bar{e}\to\mu\bar{\mu}}}{d\Omega}$ as a function of $\Delta = \frac{s - M_Z^2}{s} = 2\left(\frac{2E - M_Z}{M_Z}\right)$ in the range $|2E - M_Z| < 5\Gamma_Z$. The angle θ is fixed

The distributions on Δ for the quantity $(s/\alpha^2)d\sigma^{e\bar{e}\to\mu\bar{\mu}}/dO$ are presented for several values of the scattering angles for both processes considered above (see Fig. 10) for the energy range close to Z-boson mass.

2. TABLE OF INTEGRALS. ONE-LOOP FEYNMAN INTEGRALS

2.1. Integrals for the Process $e^+e^- \to 3\gamma$. Consider now the integrals which appear in calculation of one-loop corrections to the process

$$e^+(p_+) + e^-(p_-) \to \gamma(k_1) + \gamma(k_2) + \gamma(k_3), \tag{2.1}$$

with the kinematics defined as

$$\begin{gathered} s = (p_- + p_+)^2, \quad \chi_i = 2p_-k_i, \quad \chi_i' = 2p_+k_i', \\ m_{ij} = 2k_ik_j, \quad p_\pm^2 = m^2, \quad k_i^2 = 0, \quad i = 1, 2, 3. \end{gathered} \tag{2.2}$$

One-loop Feynman integrals contain the denominators:

$$\begin{gathered} (0) = k^2 - \lambda^2, \quad (1) = (p_- - k)^2 - m^2, \quad (2) = (-p_+ - k)^2 - m^2, \\ (3) = (p_- - k_1 - k)^2 - m^2, \quad (4) = (-p_+ + k_3 - k)^2 - m^2. \end{gathered} \tag{2.3}$$

To calculate the 5-denominator Feynman integral (pentagon) we use the method developed in paper [56].

The answer reads as

$$w^2 J_{01234} = J_{1234}N_{1234} + J_{0234}N_{0234} + J_{0134}N_{0134} + \\ + J_{0123}N_{0123} + J_{0124}N_{0124}, \quad (2.4)$$

with

$$N_{1234} = 2\Delta_4 - w\sum_1^4 v_i, \quad N_{0234} = wv_1, \quad N_{0134} = wv_2, \\ N_{0124} = wv_3, \quad N_{0123} = wv_4, \quad w = \sum_1^4 r_i w_i, \quad \Delta_4 = e^{p_1p_2p_3p_4}e_{p_1p_2p_3p_4} \quad (2.5)$$

and we use the notation $e^{\mu p_1 p_2 p_3} = e^{\mu\beta\gamma\delta}p_{1\beta}p_{2\gamma}p_{3\delta}$, $e^{0123} = +1$, $e^{\alpha\beta\gamma\delta}$ — antisymmetric Levi–Civita tensor. For our case we have:

$$r_1 = r_2 = 0, \quad r_3 = -\chi_1, \quad r_4 = -\chi_3', \\ p_1 = -p_-, \quad p_2 = p_+, \quad p_3 = -p_- + k_1, \quad p_4 = p_+ - k_3 \\ v_1^\mu = e^{\mu p_+ k_2 k_3}, \quad v_2^\mu = e^{\mu p_- k_1 k_2}, \quad v_3^\mu = e^{\mu p_- p_+ k_3}, \quad v_4^\mu = e^{\mu p_- p_+ k_2}, \\ w^\mu = -\chi_1 v_3^\mu - \chi_3' v_4^\mu, \quad \sum_i v_i = e^{\mu q k_3 k_1}, q = p_+ + p_-. \quad (2.6)$$

A straightforward (but tedious) calculation leads to

$$w^2 = -\frac{1}{4}s\chi_1\chi_3' m_{12}m_{23}, \\ N_{1234} = \frac{1}{8}\{(sm_{13})^2 - sm_{13}[\chi_3(s-m_{23}) + \\ + \chi_1'(s - sm_{12})] + (\chi_3\chi_1' - \chi_1\chi_3')(s-m_{12})(s-m_{23})\}, \\ N_{0234} = \frac{1}{8}\chi_3'[s\chi_2' m_{13} - s\chi_1 m_{23} - s\chi_3' m_{12} + (\chi_3'\chi_2 - \chi_2'\chi_3)(s-m_{23})], \\ N_{0124} = \frac{1}{8}\chi_3'[2\chi_1\chi_3 + \chi_1\chi_3' + \chi_3\chi_1' - sm_{13}], \\ N_{0134} = \frac{1}{8}\chi_1[s\chi_2 m_{13} - s\chi_1 m_{23} - s\chi_3' m_{12} + (\chi_1\chi_2' - \chi_2\chi_1')(s-m_{12})], \\ N_{0123} = \frac{1}{8}\chi_1[2\chi_1'\chi_3' + \chi_1\chi_3' + \chi_3\chi_1' - sm_{13}]. \quad (2.7)$$

Real part of relevant integrals with four denominators

$$\mathrm{Re}\, J_{0123} = \frac{1}{s\chi_1}\left[8\xi_2 - L_sL_\lambda - L_s^2 + 2L_s \ln\frac{m_{23}}{\chi_1}\right],$$

$$
\begin{aligned}
\operatorname{Re} J_{0124} &= \operatorname{Re} J_{0123}\left(m_{23} \to m_{12}, \chi_1 \to \chi_3'\right), \\
\operatorname{Re} J_{0234} &= \frac{1}{m_{23}\chi_3'}\left[6\xi_2 - 2\mathrm{Li}_2\left(1 - \frac{\chi_3'}{\chi_1}\right) - \ln^2\left(\frac{m_{23}\chi_3'}{m^2\chi_1}\right)\right], \\
\operatorname{Re} J_{0134} &= \operatorname{Re} J_{0234}\left(m_{23} \to m_{12}, \chi_3 \to \chi_1'\right), \\
\operatorname{Re} J_{1234} &= \frac{1}{m_{12}m_{23}}\left[-\xi_2 + \ln^2\left(\frac{m_{12}m_{23}}{m^2 s}\right)\right].
\end{aligned} \tag{2.8}
$$

The 3-denominator integrals are

$$
\begin{aligned}
J_{024} &= \frac{1}{\chi_3'}\left[-2\xi_2 + \frac{1}{2}\ln^2\left(\frac{\chi_3'}{m^2}\right)\right], \\
\operatorname{Re} J_{012} &= \frac{1}{2s}\left[2\ln\frac{s}{m^2}\ln\frac{m^2}{\lambda^2} + \ln\frac{s}{m^2} - 8\xi_2\right], \\
J_{013} &= -\frac{1}{\chi_1}\left[-4\xi_2 + \frac{1}{2}\ln^2\left(\frac{\chi_1}{m^2}\right)\right], \\
\operatorname{Re} J_{014} &= \frac{1}{s-\chi_3}\left[-5\xi_2 + 2\mathrm{Li}_2\left(\frac{-m_{13}}{\chi_3'}\right) + \frac{1}{2}\ln^2\left(\frac{m_{12}}{\chi_3'}\right) + \right. \\
&\qquad \left. + \ln\frac{m_{12}}{s}\ln\frac{(s-\chi_3)^2}{\chi_3 m^2}\right], \\
J_{034} &= \frac{1}{2\left(\chi_3' - \chi_1\right)}\left[\ln^2\left(\frac{\chi_1}{m^2}\right) - \ln^2\left(\frac{\chi_3'}{m^2}\right)\right], \\
\operatorname{Re} J_{023} &= \frac{1}{s-\chi_1'}\left[-5\xi_2 + 2\mathrm{Li}_2\left(\frac{-m_{23}}{\chi_1}\right) + \frac{1}{2}\ln^2\left(\frac{m_{13}}{\chi_1}\right) + \right. \\
&\qquad \left. + \ln\frac{m_{23}}{s}\ln\frac{(s-\chi_1')^2}{\chi_1 m^2}\right], \\
\operatorname{Re} J_{134} &= \frac{1}{m_{12}}\left[-3\xi_2 + \frac{1}{2}\ln^2\left(\frac{m_{12}}{m^2}\right)\right], \\
J_{124} &= \frac{-1}{s-m_{23}}\left\{-\xi_2 + \mathrm{Li}_2\left(\frac{s-m_{12}}{m_{23}-m_{12}}\right) + \mathrm{Li}_2\left(\frac{m_{23}}{s}\right) - \right. \\
&\qquad \left. - \mathrm{Li}_2\left(\frac{m_{23}(s-m_{12})}{s(m_{23}-m_{12})}\right)\right\}, \\
\operatorname{Re} J_{234} &= \frac{1}{m_{23}}\left[-3\xi_2 + \frac{1}{2}\ln^2\left(\frac{m_{23}}{m^2}\right)\right],
\end{aligned} \tag{2.9}
$$

$$J_{123} = \frac{-1}{s - m_{23}} \left\{ \ln \frac{m_{23}}{s} \ln \frac{m_{23}}{m^2} + \mathrm{Li}_2 \left(1 - \frac{m_{23}}{s}\right) \right\} - \\ - \frac{1}{m_{23}} \mathrm{Li}_2 \left(1 - \frac{s}{m_{23}}\right)$$

and, besides,

$$J_{013} = J_{023} = \frac{1}{\chi_1} \left[-2\xi_2 - \frac{1}{2} l_s^2\right], \quad \mathrm{Re}\, J_{012} = \frac{1}{s} \left[2 l_s \ln \frac{m^2}{\lambda^2} + \frac{1}{2} l_s^2 - 4\xi_2\right],$$
$$\mathrm{Re}\, J_{123} = \frac{1}{s} \left[-3\xi_2 + \frac{1}{2} l_s^2\right], \quad J_{0123} = \frac{1}{s\chi_1} \left[3\xi_2 - l_s \ln \frac{m^2}{\lambda^2} - 2 l_s l_1\right],$$
$$l_s = \ln \frac{s}{m^2}, \quad l_1 = \ln \frac{\chi_1}{m^2}. \tag{2.10}$$

Algebraic system permits one to obtain vector and tensor integrals with three and four denominators.

Acknowledgements. Authors are grateful to many our collaborators who generously permit us to use the results obtained in common papers. One of us (EAK) is grateful to Lev Lipatov, Viktor Fadin and Valery Serbo for many years collaboration and numerous detailed and valuable discussions. We are also grateful to S. Bakmaev for help and encouragement. This work was supported by grants MK-1607.2008.2, INTAS-05-1000008-8328.

We present our apologies to authors working in the same direction, papers of who unfortunately avoid our attention.

REFERENCES

1. *Baier V. N. et al.* Inelastic Processes in Quantum Electrodynamics at High Energies // Phys. Rep. 1981. V. 78. P. 293.
2. *Berends F. A. et al.* Multiple Bremsstrahlung in Gauge Theories at High Energies. 2. Single Bremsstrahlung // Nucl. Phys. B. 1982. V. 206. P. 61.
3. *Berends F. A. et al.* Single Bremsstrahlung Processes in Gauge Theories // Phys. Lett. B. 1981. V. 103. P. 124.
4. *Baier V. N., Fadin V. S., Khoze V. A.* Quasi-Real Electron Method in High-Energy Quantum Electrodynamics // Nucl. Phys. B. 1973. V. 65. P. 381.
5. *Kessler P.* Sur une Methode Simplifiee de Calcul pour les Processus Relativist en Electrodynamique Quantique // Nuovo Cim. 1960. V. 17. P. 809.
6. *Kuraev E. A., Merenkov N. P., Fadin V. S.* The Compton Effect Tensor with Heavy Photon // Yad. Fiz. 1987. V. 45. P. 782 (in Russian);
Kuraev E. A., Merenkov N. P., Fadin V. S. Compton Tensor with Heavy Photon. INP Preprint 86-39. Novosibirsk, 1986.

7. *Gribov V. N., Lipatov L. N.* e^+e^- Pair Annihilation and Deep Inelastic ep Scattering in Perturbation Theory // Sov. J. Nucl. Phys. 1972. V. 15. P. 75; Yad. Fiz. 1972. V. 15. P. 1218;
Dokshitzer Y. L. Calculation of the Structure Functions for Deep Inelastic Scattering and e^+e^- Annihilation by Perturbation Theory in Quantum Chromodynamics // Sov. Phys. JETP. 1977. V. 46. P. 641 (in Russian); ZhETF. 1977. V. 73. P. 1216.
8. *Altarelli G., Parisi G.* Asymptotic Freedom in Parton Language // Nucl. Phys. B. 1977. V. 126. P. 298.
9. *Landau L. D., Lifschitz E. M.* Production of Electrons and Positrons by a Collision of Two Particles // Phys. Z. Sowjetunion. 1934. V. 6. P. 244.
10. *Kuraev E. A., Fadin V. S.* On Radiative Corrections to e^+e^- Single-Photon Annihilation at High Energy // Sov. J. Nucl. Phys. 1985. V. 41. P. 466; Yad. Fiz. 1985. V. 41. P. 733.
11. *Lipatov L. N.* The Parton Model and Perturbation Theory // Sov. J. Nucl. Phys. 1975. V. 20. P. 94; Yad. Fiz. 1974. V. 20. P. 181.
12. *Jadach S., Skrzypek M.* Exact Solutions of the QCD Evolution Equations Using Monte Carlo Method // Acta Phys. Polon. B. 2004. V. 35. P. 745.
13. *Tsai Y. S.* Radiative Corrections to e^+e^- Reactions to All Orders in α Using the Renormalization Group. Preprint SLAC-PUB-3129. 1983.
14. *Brodsky S. J., Kinoshita T., Terazawa H.* Dominant Colliding Beam Cross Sections at High Energies // Phys. Rev. Lett. 1970. V. 25. P. 972.
15. *Ivkin A. V. et al.* Parton's Picture of the Electroweak Processes in High-Energy e^+e^- Collisions // Sov. J. Nucl. Phys. 1991. V. 54. P. 1031; Yad. Fiz. 1991. V. 54. P. 1682.
16. *Ginzburg I. F. et al.* The $W^\pm$ Boson Production on the Colliding e^+e^-, γe, and $\gamma\gamma$ Beams // Nucl. Phys. B. 1983. V. 228. P. 285. Erratum // Nucl. Phys. B. 1984. V. 243. P. 550.
17. *Kuraev E. A., Merenkov N. P., Fadin V. S.* Calculation of the Radiative Corrections to the Cross Section for Electron Nucleus Scattering by the Method of Structure Functions // Sov. J. Nucl. Phys. 1988. V. 47. P. 1009; Yad. Fiz. 1988. V. 47. P. 1593.
18. *Kuraev E. A.* Radiative Tail in $\pi(e2)$ Decay and Some Remarks on $\mu - e$ Universality // JETP Lett. 1997. V. 65. P. 127.
19. *Berman S. M.* Radiative Corrections to Pion Beta Decay // Phys. Rev. Lett. 1958. V. 1. P. 468.
20. *Kinoshita T.* Radiative Corrections to $\pi - e$ Decay // Phys. Rev. Lett. 1959. V. 2. P. 477.
21. *Bryman D. A.* $\pi \to e$ Neutrino Decay: Window on the Generation Puzzle // Comments Nucl. Part. Phys. 1993. V. 21. P. 101.
22. *Landau L. D., Abrikosov A., Halatnikov L.* On the Quantum Theory of Fields // Nuovo Cim. Suppl. 1956. V. 3. P. 80.
23. *Marciano W. J., Sirlin A.* Radiative Corrections to π (lepton 2) Decays // Phys. Rev. Lett. 1993. V. 71. P. 3629.

24. *Terentev M. V.* Branching Ratio $\gamma(\pi \to e\nu)/\gamma(\pi \to \mu\nu)$ with Account of Electromagnetic Corrections and Contribution from Strong Interactions // Yad. Fiz. 1973. V. 18. P. 870.

25. *Cirigliano V., Rosell I.* Two-Loop Effective Theory Analysis of $\pi(K) \to e\bar{n}u/e(\gamma)$ Branching Ratios // Phys. Rev. Lett. 2007. V. 99. P. 231801.

26. *Kuraev E. A., Bystritsky Yu. M.* Emission in the Final State in Processes with the Production of Charged Pions and Light Leptons // JETP Lett. 2006. V. 83. P. 439; Pisma ZhETF. 2006. V. 83. P. 510.

27. *Belitsky A. V., Mueller D., Kirchner A.* Theory of Deeply Virtual Compton Scattering on the Nucleon // Nucl. Phys. B. 2002. V. 629. P. 323.

28. *Schwinger J. S.* Quantum Electrodynamics. III. The Electromagnetic Properties of the Electron: Radiative Corrections to Scattering // Phys. Rev. 1949. V. 76. P. 790.

29. *Mo L. W., Tsai Y. S.* Radiative Corrections to Elastic and Inelastic ep and μp Scattering // Rev. Mod. Phys. 1969. V. 41. P. 205.

30. *Yennie D. R., Frautschi S. C., Suura H.* The Infrared Divergence Phenomena and High-Energy Processes // Ann. Phys. 1961. V. 13. P. 379.

31. *Pasquini B. et al.* Dispersion Relation Formalism for Virtual Compton Scattering and the Generalized Polarizabilities of the Nucleon // Phys. Rev. C. 2000. V. 62. P. 052201.

32. *Bacchetta A. et al.* Single-Spin Asymmetries: The Trento Conventions // Phys. Rev. D. 2004. V. 70. P. 117504.

33. *Bytev V. V., Kuraev E. A., Tomasi-Gustafsson E.* Radiative Corrections to DVCS Electron Tensor. hep-ph/0710.0830.

34. *Kuraev E. A. et al.* Targed Normal Spin Asymmetry and Charge Asymmetry for $e\mu$ Elastic Scattering and the Crossed Processes // Phys. Rev. D. 2006. V. 74. P. 013003.

35. *Arbuzov A. B. et al.* Table of Integrals: Asymptotical Expressions for Non-Collinear Kinematics. hep-ph/0703048.

36. *Arbuzov A. B., Bytev V. V., Kuraev E. A.* Radiative Muon Pair Production in High-Energy Electron–Positron Annihilation Process // JETP Lett. 2004. V. 79. P. 593; Pisma ZhETF. 2004. V. 79. P. 729.

37. *Brown L. M., Feynman R. P.* Radiative Corrections to Compton Scattering // Phys. Rev. 1952. V. 85. P. 231.

38. *Harris I., Brown L. M.* Radiative Corrections to Pair Annihilation // Phys. Rev. 1957. V. 105. P. 1656.

39. *Berends F. A., Gastmans R.* Hard Photon Corrections for $e^+e^- \to \gamma\gamma$ // Nucl. Phys. B. 1973. V. 61. P. 414.

40. *Eidelman S. I., Kuraev E. A., Panin V. S.* Processes $e^+e^- \to e^+e^-\gamma, \mu^+\mu^-\gamma, \gamma\gamma\gamma$ with Emission of Final Particles at Large Angles // Nucl. Phys. B. 1979. V. 148. P. 245; *Geshkenbein B. V., Teren'tev M. V.* // Yad. Fiz. 1968. V. 8. P. 550.

41. *Arbuzov A. B. et al.* Radiative Corrections for Pion and Kaon Production at e^+e^- Colliders of Energies Below 2 GeV // JHEP. 1997. V. 9710. P. 006.

42. *Kuraev E. A., Silagadze Z. K.* Radiative Corrections to the Three Quantum Annihilation Cross Section in High-Energy Electron–Positron Collisions // Phys. At. Nucl. 1995. V. 58. P. 1741 (in Russian); Yad. Fiz. 1995. V. 58. P. 1843.

43. *Baier V., Kuraev E. A., Fadin V. V.* On Gluon Jet Formation in e^+e^- Collisions // Yad. Fiz. 1980. V. 31. P. 700.

44. *Bardin D.* Private communications.

45. *Berman S. M.* Radiative Corrections to Muon and Neutron Decay // Phys. Rev. 1958. V. 112. P. 267.

46. *Kinoshita T.* Mass Singularities of Feynman Amplitudes // J. Math. Phys. 1962. V. 3. P. 650;
Lee T. D., Nauenberg M. Degenerate Systems and Mass Singularities // Phys. Rev. B. 1964. V. 133. P. 1549.

47. *Munoz Camacho T. D. et al. (Jefferson Lab Hall A Collab.).* Scaling Tests of the Cross Section for Deeply Virtual Compton Scattering // Phys. Rev. Lett. 2006. V. 97. P. 262002.

48. *Kroll N. Ad Hoc* Modification of Quantum Electrodynamics // Nuovo Cim. A. 1966. V. 45. P. 65.

49. *Kondratyuk S. et al.* Delta Resonance Contribution to Two-Photon Exchange in Electron–Proton Scattering // Phys. Rev. Lett. 2005. V. 95. P. 172503.

50. *Bystritskiy Yu.M., Kuraev E. A., Tomasi-Gustafsson E.* Structure Function Method Applied to Polarized and Unpolarized Electron–Proton Scattering: A Solution of the $G_E(P)/G_M(P)$ Discrepancy // Phys. Rev. C. 2007. V. 75. P. 015207.

51. *Kuraev E., Galynsky M., Ilichev A.* Two Comments on the Structure Function Approximation for Deep-Inelastic Scattering // JETP Lett. 2003. V. 77. P. 256; Pisma ZhETF. 2003. V. 77. P. 305.

52. *Bartos E., Kuraev E.A., Secansky M.* Radiative Corrections to Muon Decay in Leading and Next-to-Leading Approximation for Electron Spectrum. hep-ph/0811.4242.

53. *van Ritbergen T., Stuart R. G.* Complete 2-Loop Quantum Electrodynamic Contributions to the Muon Lifetime in the Fermi Model // Phys. Rev. Lett. 1999. V. 82. P. 488.

54. *Arbuzov A., Czarnecki A., Gaponenko A.* Muon Decay Spectrum: Leading Logarithmic Approximation // Phys. Rev. D. 2002. V. 65. P. 113006.

55. *Arbuzov A., Melnikov K.* $O(\alpha^2 \ln(m(\mu)/m(e)))$ Corrections to Electron Energy Spectrum in Muon Decay // Ibid. V. 66. P. 093003.

56. *van Neerven W. L., Vermaseren J. A. M.* Large Loop Integrals // Phys. Lett. B. 1984. V. 137. P. 241.