ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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NUCLEAR THRESHOLD EFFECTS AND NEUTRON STRENGTH FUNCTIONS

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The present work is devoted to studying the spectroscopic aspects (in terms of neutron strength function) of the threshold effects of nonlight nuclei. Relations of the neutron strength functions with the anomalous effects observed at threshold of neutron analogue channel, in deuteron stripping reactions on $A \approx 90$ mass target nuclei, and in nucleon-mirror reactions with $A \approx 30$ nuclei, are established. It is proved that these threshold effects follow the mass dependence of the neutron strength functions. The connection between threshold effects and neutron strength functions proves that the threshold effects are related to spectroscopy of ancestral zero-energy neutron particle resonance. One can conclude that the threshold effects depend not only on penetration factors of opening neutron channel, as in cusp theory, but also on multichannel reaction dynamics as well as on spectroscopy of neutron threshold state. The present study is based on reduced scattering matrix, describing the effect of invisible threshold channel on open observed channels. The determining role of nuclear reaction dynamics, quasiresonant scattering, and spectroscopy of neutron threshold state in threshold effects is evinced.

Данный обзор посвящен исследованию методом нейтронных силовых функций спектроскопических аспектов пороговых эффектов нелёгких ядер. Установлены связи нейтронных силовых функций с аномальными эффектами, наблюдаемыми в пороговом нейтронном канале, в реакциях стриппинга дейтрона на ядрах мишеней с $A \approx 90$, а также в нуклонно-зеркальных реакциях с ядрами $A \approx 30$. Доказано, что эти пороговые эффекты следуют массовой зависимости нейтронных силовых функций. Соотношение между пороговыми эффектами и нейтронными силовыми функциями доказывает, что пороговые эффекты связаны со спектроскопией первичных резонансов частиц с нулевой энергией нейтрона. Можно сделать вывод о том, что пороговые эффекты зависят не только от факторов проникновения открывающегося нейтронного канала, как в «cusp theory», но и от динамики многоканальной реакции, а также от спектроскопии порогового состояния нейтрона. Данное исследование основано на сокращенной матрице рассеяния, описывающей влияние невидимого порогового канала на открытые наблюдаемые каналы. Подчеркивается определяющий вклад в пороговые эффекты динамики ядерной реакции, квазирезонансного рассеяния, а также спектроскопии порогового состояния нейтрона.

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INTRODUCTION

The basic law of nuclear reactions is conservation of the flux; if a new reaction channel opens, a redistribution of the flux in old open channels appears. The modification of an open-channel cross section, due to opening of a new one, is called threshold effect. The formulation of the problem of threshold effects, and its first solution, are due to Wigner and it is known as the cusp theory [1]. Decade later, the problem of the threshold cusp has been formally approached by Breit in the frame of R-matrix theory [2], and by Baz in the frame of S-matrix theory [3].

The strength function is an averaged spectroscopic factor, defined as overlap or mixing of a single-particle state with the actual states. The group of actual levels, carrying out a substantial fraction of the single-particle state, constitutes the giant resonance (giant model of Lane–Thomas–Wigner [4]). The strength function will display maxima whenever a single-particle state is present.

The problems of the nuclear threshold effects [1] and of the neutron strength function [4] were formulated in the early days of the low-energy nuclear physics; nevertheless, they both are still topical basic subjects of research in contemporary physics of nuclear reactions [5–8], although apparently, as nonrelated ones. The interrelation between threshold effects and spectroscopic factors has been pointed out in previous papers [9, 10] and recently approached for exotic nuclei [11, 12], turning to a topic of current interest. The present work, continuing previous ones [13–16], aims to study spectroscopic aspects of threshold effects with non-light nuclei. One establishes, theoretically and from analysis of experimental data, the relation between nuclear threshold effects and neutron strength functions.

This paper is devoted to nuclear threshold effects with nonlight nuclei. It is complementary to the review paper of Abramovich, Guzhovskii, and Lazarev [6], which was focussed on light nuclei.

1. THRESHOLD PHENOMENA: PHYSICAL APPROACH

The threshold effects observed in open channels depend on the amount of flux absorbed by the new opening (threshold) channel. If the threshold channel exhibits no (coulombian and centrifugal) barriers, (i. e., it is an s-wave neutron one), then absorption of the flux by the threshold channel is suddenly produced and it results in the cusp threshold effect. Because the flux in s-wave neutron channel is proportional to channel wave number, it results that the cusp has an infinite energy derivative at threshold energy; from here the cusp denomination. For higher partial waves, the centrifugal barrier inhibits flux transfer between threshold channel and open ones and, consequently, results in smaller threshold effect.

The Wigner–Breit–Baz threshold cusp should be a universal effect, appearing at the threshold of every new s-wave neutron channel. However, extensive experimental studies, along many decades [6], have shown that the nuclear threshold effects are rather rare and diverse. One can mention, with respect to diversity, evidences for p-wave threshold effects in isospin coupled channel reactions; the cusp theory does not predict a significant p-wave threshold effect.

The physical idea underlying this study is that the threshold effects are determined by the dynamics of the flux to and from threshold channel, which in its turn is controlled by the reaction mechanism. According to this representation, different groups of threshold effects should be related to different types of reaction mechanisms. For example, the threshold cusp is related to potential nonresonant scattering. The factor governing the flux transfer to threshold channel is then the penetration factor, resulting in genuine Wigner–Breit–Baz cusp.

The primary factor governing the leakage of the flux from resonance to neutron channel is neutron preformation in internal region, i. e., neutron reduced width [17]. The other factor governing the flux leakage into reaction channel is penetration factor through channel potential barrier (favouring s-wave neutron channel). The two factors, particle preformation in the resonance (particle reduced width) and its penetration factor of channel barrier, define the resonance «decay partial width» to (neutron) channel; the flux leakage from resonance to (neutron) reaction channel is determined by the corresponding resonance partial decay width. The resonance total width, i. e., the sum of the channel's partial widths, gives the flux leakage in all reaction channels. The threshold effect is directly related to flux absorbed by threshold channel; this means that the resonance total width has to be dominated by neutron threshold channel partial width, or the resonance's neutron reduced width is very large approaching its maximal value (Wigner unit of reduced width γ_W). According to this representation, significant threshold effects are related to a resonance, coincident with neutron threshold; the resonance has to decay preferentially in neutron threshold channel. This type of threshold effect is related to interchannel flux transfer via a compound nucleus reaction.

Another mechanism for interchannel flux transfer is that of multichannel couplings. The flux transfer between channels a and n, via channels coupling, is proportional to transition amplitude matrix element T_{an} ($S = 1 + 2iT$; S — scattering matrix, T — transition matrix). Of peculiar interest is the case of a single-channel resonance, π, in the threshold channel, n, reduced width $\gamma_{\pi n} \sim \gamma_W$ (for all other channels a, $\gamma_{\pi a} = 0$). The single-channel resonance induces, via direct channels couplings, in competive open channels, a coupled channel resonance or quasiresonance; it consists of a single-channel resonance preceded and/or followed by direct transitions to other reaction channels. A measure of the flux transfer between the open a and threshold n channels is then proportional both to channel coupling T_{an} and to the resonance's reduced width $\gamma_{\pi n}$, namely $T_{an}\gamma_{\pi n}$. The flux transfer in the reaction (a, b), via threshold channel n, is then $\alpha_{ab} \simeq T_{an}\gamma_{\pi n}^2\, T_{nb}$. The involved transfer reaction should be a direct single-step one (as described by DWBA); multistep couplings could obscure the threshold effect which, anyway, is expected to be small.

The two conditions — resonance's energy coincidence with neutron threshold and large threshold channel reduced width — define the «neutron threshold state».

It has a large threshold channel reduced width; due to the state large overlap on threshold channel, it is acting as amplifier of flux transfer to and from neutron threshold channel. The threshold state is a quasistationary state, coincident in energy with threshold, which has a large reduced width ($\simeq$ Wigner unit γ_W) for decay in the threshold channel. The reduced width is a measure of single-particle character of the level in interior region. The probability of finding a pair of threshold particles out of channel radius is proportional to the threshold channel reduced width; a very large reduced width will result into level «explosion» out of channel radius.

These schematic descriptions are valid only for an isolated resonance. Both evince the vital role, for producing threshold effects, of threshold single-channel resonance coupled either by compound nucleus or by direct channels couplings to open observed channels.

The sharp resonances are observed in reactions on light nuclei at low energies; the resonance's spectroscopic parameters are reduced widths which do measure resonance's overlap to reaction channels. For medium and heavy nuclei one observes no longer sharp resonances but rather smooth cross sections; the resonant levels become very closely spaced and their widths are larger than their separations. It is therefore necessary to define a corresponding statistical spectroscopic quantity, by averaging over many levels. This is the strength function, which is defined as total value of the reduced width per unit energy interval of (λ) resonances, $S_{\lambda n} = \overline{\gamma^2_{\lambda n}}\rho_\lambda$, where ρ_λ is the density of (λ) levels. The strength function is ratio of averaged width to the mean spacing D between adjacent levels, $\rho_\lambda = 1/D_\lambda$. The strength function is an averaged quantity like the nuclear-level density. Regions where it is appropriate to discuss levels densities, instead of single levels, are also regions where it is useful to think in terms of strength function instead of individual reduced widths.

The strength function is a measure of the mean strength of reduced widths of actual compound nucleus resonances. This spectroscopic quantity is also defined as the overlap of single-particle state and the actual states, giving how much the single-particle state is mixed with actual states of the nucleus. It is expected that the strength function will display maxima whenever a single-particle state is present. The (broad) giant resonances correspond to each of the single-particle states of the compound system when the residual interaction was neglected. The (neutron) single-particle reduced width $\gamma^2_{\mathrm{sp}(n)}$ is shared among the complicated levels (λ) of the compound nucleus in such a way that $\Sigma_\lambda \gamma^2_{\lambda n} = \gamma^2_{\mathrm{sp}(n)}$. The enhancement of the cross section or of the strength function, resembling to a large width resonance, reveals existence of single-particle states in nucleon scattering on nuclei. The giant resonances are (neutron) single-particle resonances which are split, by residual interactions, into complicated compound nucleus states. They are not more described by single-particle reduced widths but rather by the statistical neutron strength function. Both the reduced width of an isolated resonance and the strength function of a giant resonance control the flux dynamics in threshold channel which, in turn, is determining the magnitude of the threshold effects.

For threshold effects with nonlight nuclei, with a high density of levels, one has to average over energy. One obtains, then, that the strength of the threshold effect $\overline{\alpha_{ab}}$ is proportional to (single-particle) «neutron strength function», $\overline{\gamma_{\pi n}^2}$. For this work, devoted to threshold effects with nonlight nuclei, it is of peculiar interest the neutron single-particle resonances, coincident with threshold, and their (spectroscopic) strength functions.

The nuclear threshold effect is dependent, via neutron strength function, on ancestral neutron threshold state. This dependence proves that the origin of the threshold effect is neutron single-particle state, which is acting as an amplifier for the flux transfer to and from neutron threshold channel. According to present result, the flux transfer, responsible for threshold effect, is governed by spectroscopic factor of the neutron single-particle state, energy-coincident with threshold, as well as by channels couplings. The scattering matrix threshold term, displaying neutron threshold state, has to be described as zero-energy neutron single-particle resonance; the threshold energy dependence is given by the logarithmic derivative of the neutron channel.

The previous discussion emphasises the role of the neutron single-particle threshold state in producing threshold effects. The threshold effect, originating in a neutron threshold state is proportional to neutron strength function. The magnitude of threshold effect does depend not only on reaction mechanism but also on spectroscopical amplitude of ancestral quasistationary threshold state. A threshold state does act as an amplifier for flux transfer to threshold channel thus enhancing the threshold effect.

2. THRESHOLD EFFECTS: THEORETICAL APPROACH

Formally, the problem of the threshold effects can be viewed as a scattering problem in the truncated space of open (observed, retained) channels; one has to take into account the coupling of open channels on the threshold (invisible, eliminated) channel. The usual approach to multichannel scattering problems in truncated space of channels is either reduced R-(K-)matrix [18], or effective Hamiltonian (projector method) [6, 19]. Since the scattering matrix is primary object of the scattering theory, the concept of «reduced» or «effective» operator should be extended to the S-matrix.

Consider the multichannel system of N open (retained) channels, decoupled from the threshold (unobserved, eliminated) channel n. The «bare» independent open channels are described by the unitary scattering matrix $S_N^0 = ||S_{ab}^0||$. By coupling the threshold channel $n = N + 1$, to N open ones, *via* S_{na}-matrix elements, one obtains the reduced scattering matrix $S_N = ||S_{ab}||$ for the retained channels; it includes both bare S_N^0 scattering matrix and the effect ΔS of eliminated channel

$$S_{ab} = S_{ab}^0 + \Delta S_{ab} = S_{ab}^0 + S_{an}(1 + S_{nn})^{-1} S_{nb}.$$

This formula, valid only above threshold, is obtained via parameterizations of scattering matrix S in terms of K-matrix or of the collision matrix U in terms of R-matrix, provided natural boundary conditions are used [10].

The formal merits of the reduced scattering/collision matrix are: it is valid both near and far away from threshold, it is valid both for potential and resonant scattering and it is valid even for threshold channel with barrier, as, $e.g.$, a p-wave one. The physical merit of the method is that it does establish a relation between the threshold effects, ΔS, and the reaction mechanism in the threshold channel, via S_{nn}-matrix element. Different reaction mechanisms will result in different types of threshold anomalies.

The threshold cusp is related to the nonresonant potential scattering. In zero-energy limit of potential scattering, $S_{nn} \to 1$, the reduced S-matrix results in the cusp formula, $\Delta S_{ab} = 1/2 S_{an} S_{nb}$ [20]. The flux transfer involved in a cusp effect is essentially determined by the penetration factors of the threshold channel S-matrix elements.

A compound nucleus resonance (π), located in neutron threshold vicinity, $|E_\pi - E_n| < \Gamma_\pi$, and decaying preferentially in the neutron threshold channel, $\Gamma_\pi \sim \Gamma_{\pi n}$, induces a non-negligible threshold effect for s-wave only (Γ_π and $\Gamma_{\pi n}$ — resonance total and partial widths). The flux transfer to and from the neutron threshold channel is, in this case, controlled not only by the penetration factors, as in cusp theory, but also by the spectroscopic neutron reduced width; the reduced width is primary factor governing the flux leakage from the compound nucleus to the channels [17]. However the threshold compound nucleus resonance cannot account for a p-wave threshold effect.

A p-wave threshold effect does require (1) resonant energy dependence of the threshold channel related S-matrix elements, S_{nn}, S_{an}, and S_{nb}, and (2) direct interaction in open channels, S_{ab}^0 — monotone energy dependence. Otherwise, the effective term of the reduced S-matrix, ΔS, goes to zero in threshold range. In the following we will approach the p-wave threshold effect problem in different formal ways, all converging to the same physical conclusion: a non-negligible p-wave threshold effect involves (1) a neutron threshold single-particle resonance and (2) its direct interaction coupling to open channels [10].

The two formal conditions could be physically realized in terms of the final-state interaction [21]. The final fragments, neutron and corresponding residual nucleus, have an interaction producing a resonance at zero energy; the Jost function has, then, a zero in the complex k-wave plane, just below the real axis, near origin. The S-matrix elements, having in denominator the Jost function, are strongly enhanced; this is final- (or initial) state interaction. It refers in our case to the S-matrix elements S_{nn}, S_{an} related to the n-channel only. On the other hand, it is required that the potential responsible for transition between channels should be assumed perturbative (direct interaction transitions). The forces producing reaction are responsible for direct interaction transitions and for interaction in the (threshold) channel where the final-state interaction does produce a resonance. The final-state interaction is effective mainly at low energies, where one-channel resonances are produced by centrifugal barrier effects [21].

The reduced S-matrix can be explicit in case of coexistence of both the direct («background» β) and resonant (ρ) scatterings [22]. In case of the single-channel resonance π in eliminated channel n ($\gamma_{\pi n} \neq 0$; all other $\gamma_{\pi a} = 0$), the scattering matrix, for open retained channels, becomes

$$S_{ab} = S_{ab}^{\beta} - 2i \frac{T_{an}^{\beta} \gamma_{\pi n}^2 T_{nb}^{\beta}}{E_{\pi} - E + \mathrm{Re}\, T_{nn}^{\beta} \gamma_{\pi n}^2 - i(1 - \Sigma_l |T_{ln}^{\beta}|^2) \gamma_{\pi n}^2}$$

with the transition matrix for direct scattering defined by $S^{\beta} = 1 + 2iT^{\beta}$ and index l running over all channels, either open (a) or threshold (n). If single-channel resonance is a neutron threshold state, the strength of the threshold effect induced in open channels, $\alpha_{ab} \sim T_{an}^{\beta}\ \gamma_{\pi n}^2\ T_{nb}^{\beta}$, is proportional both to the single-channel resonance reduced width and to the channel coupling strengths.

The reduced scattering/collision matrix should be extended below threshold, too. It should be constructed in such a way in order to display the single-particle states in threshold channel. For both reasons, one has to work with the collision matrix, a formalism dealing explicitly with the threshold channel logarithmic derivative. The collision matrix U (defined up to hard-sphere phase shifts) is parameterized in terms of the R-matrix, R (describing resonances in inner configuration space), and of the logarithmic derivative, L (describing reaction channels) [23].

The reduced collision matrix U_N refers to the retained (N) channels, but by taking into account the effect of the eliminated (n)-channel. The collision matrix describing the N «bare» retained channels, uncoupled to eliminated (n)-channel, is U_N^0. The dynamical term of reduced collision matrix, U_N, is the submatrix $(L^{-1} - R)_N^{-1}$ while that of «bare» collision matrix of retained open channels, U_N^0, is the matrix $(L_N^{-1} - R_N)^{-1}$. The two collision matrices are related by an effective term, ΔU_N, describing coupling between open retained (N) and closed eliminated channels (n),

$$U_N = U_N^0 + \Delta U_N.$$

The effective term ΔU_N of reduced collision matrix, valid both below and above n-threshold, is [24],

$$\Delta U_N = M(U_N^0) R_{Nn} (L_n^{-1} - \mathcal{R}_{nn})^{-1} R_{nN} M^T(U_N^0).$$

The matrix M is a function only on «bare» collision matrix U_N^0. The coupling of the threshold channel n to open ones is represented by nondiagonal R-matrix elements R_{nN}. The terms dependent on threshold channel are the logarithmic derivative, L_n, and the reduced R-matrix element, $\mathcal{R}_{nn}$,

$$\mathcal{R}_{nn} = R_{nn} - R_{nN}(R_N - L_N^{-1})^{-1} R_{Nn}.$$

Above, ($>$), and below threshold, ($<$), one has to insert the corresponding logarithmic derivatives $L_n^>$ or $L_n^<$, respectively. The (n)-channel threshold effects, on retained channels (N), are expressed by the product $R_{Nn}(L_n^{-1} - \mathcal{R}_{nn})^{-1} R_{nN}$, resembling to the additional term of $\mathcal{R}_N$ reduced R-matrix. However, there is a difference, namely, the «bare» R-matrix element R_{nn} of eliminated n-channel is here replaced by its effective counterpart $\mathcal{R}_{nn}$; the reduced $\mathcal{R}_{nn}$-matrix element does include also rescattering effects from complementary open channels.

Remark, the last equations contain basic formulae of the cusp theory, both above and below n-threshold. For neutron s-wave scattering, the logarithmic derivatives are $L_n^> = i\rho$ and $L_n^< = -\rho$ ($\rho = k_n a$; k_n — channel wave number, a — channel radius). It follows $\Delta U_N^< = \Delta U_N^> ((L_n^>)^{-1} - \mathcal{R}_{nn})/((L_n^<)^{-1} - \mathcal{R}_{nn})$ which in zero energy limit, ($\rho \to 0$), reduces to the cusp theory result, $\Delta U_N^< = i\Delta U_N^>$ [20].

Below threshold, a pole in the $U_N^<$ collision matrix elements could be obtained from condition $\mathcal{R}_{nn}^{-1} = L_n^< = S_n^{(-)}$ ($S_n^{(-)}$ — shift function). In noncoupling limit, $\mathcal{R}_{nn}$ reduces to single-channel R-matrix element R_{nn}. Or this is just the bound state condition of the R-matrix theory [23]; a bound state appears at that energy at which the internal (R_{nn}^{-1}) and external $S_n^{(-)}$ logarithmic derivatives do match. This result is an R-matrix proof that the single-particle state from a closed channel does induce resonance in competing open channels of the multichannel system.

For positive energy eliminated channels the corresponding states should be quasistationary ones. A pole in $U_N^>$ is now obtained by a condition which is analogous to the bound state one, $\mathcal{R}_{nn}^{-1} = L_n^>$; the logarithmic derivative $L_n^>$ is corresponding, at positive energy, to the shift function $S_n^{(-)}$ defined for negative energy. Such a condition determines a quasistationary state, $e.g.$, [25]; the outgoing wave at infinity corresponds to the quasistationary state decay, resulting in eigenenergies which are complex (energy and width of quasistationary state). The quasistationary state is defined, according to R-matrix theory, by condition $|1 - RL| = 0$ (see [23, p. 297]). A quasistationary state originating in an eliminated channel induces a quasiresonant structure in other open competing channels.

The resonance condition, $1 - L_n \mathcal{R}_{nn} = 0$, can be approached in different ways. The logarithmic derivative could have resonant form, e. g., as that proposed by Baz and collaborators [26] in a generalized variant of cusp theory. If the energy dependence of logarithmic derivative L_n is considered a parametric one, then the root of resonance equation becomes energy-dependent; it is Kapur–

Peierls approach to resonance [23]. Another approach is in terms of complex energy pole H_π which is the root of implicit equation $1 - L_n\mathcal{R}_{nn} = 0$. The boundary conditions are those of out waves at the state energy $H_\pi = E_\pi - i\Gamma_\pi$ (not at prescribed energy E). The resonance condition can be related to R-matrix parameters, by means of the level matrix A [23]

$$(1 - \mathcal{R}L)^{-1} = 1 + L\gamma_\pi^2 A_{\pi\pi}, \quad A_{\pi\pi}^{-1} = E_\pi - E - i\Gamma_\pi - L\gamma_\pi^2,$$

$$\mathcal{R}(1 - \mathcal{R}L)^{-1} = \gamma_\pi^2 A_{\pi\pi}.$$

Let us consider the single-particle energy to be in a region with many actual nuclear states. For this case of R-matrix fluctuant elements, one can follow the assumptions and procedures developed in nuclear physics. There one studies the fragmentation of (bound or quasistationary) single-particle state amongst the actual states of nucleus, by energy averaging over last ones [27]. By using energy averaging procedures one has to avoid the threshold branch point; one can consider only energy averaging intervals which could be very near threshold but avoiding its overlap [28, p. 146]. Another physical assumption used, is that R-matrix elements are factorizable $R_{Nn}R_{nN} \sim R_{NN}R_{nn}$ [27]. One proves [28] that the energy averaging is equivalent to replacement of the real energy E by a complex quantity $\mathcal{E}$, $\overline{R_{nn}(E)} = R_{nn}(\mathcal{E})$; further this is related to reduced R-matrix element, $R_{nn}(\mathcal{E}) = \mathcal{R}_{nn}(E)$, provided the decay widths are much larger than level spacings [23]. Within these assumptions, one obtains the result according to the n-channel related term in averaged collision matrix

$$\overline{\Delta U_N} \sim \mathcal{R}_{nn}(E)\frac{1}{1 - L_n\mathcal{R}_{nn}(E)}$$

which is just dynamical term of the n-channel collision matrix element which, in its turn, is proportional to neutron strength function $\sim \overline{\gamma_n^2}/D$.

The averaged effective collision matrix term, E_π — resonance energy (Γ_π — decay width including the spreading one),

$$\overline{\Delta U_{ab}} = \frac{\overline{\alpha_{ab}}}{E_\pi - E - L_n\overline{\gamma_{\pi n}^2} - i\overline{\Gamma_\pi}}$$

results into threshold effect strength, $\overline{\alpha_{ab}}$, proportional to neutron strength function

$$\overline{\alpha_{ab}} = \Gamma_{an}\frac{\overline{\gamma_{\pi n}^2}}{D}\Gamma_{nb}$$

with Γ_{an} and Γ_{nb} as coupling strengths of the threshold channel n to open ones.

The threshold effect has a peculiar property due to energy dependence of n-channel logarithmic derivative $L_n = S_n + iP_n$ (S_n — shift factor, P_n —

penetration factor). In Thomas approximation [23], for the logarithmic derivative, the resonance parameters (energy E_π, total width Γ_π) are renormalized in terms of R-matrix compression factor $\beta_{\pi n}$ [31],

$$\beta_{\pi n} = \frac{1}{1 + \gamma_{\pi n}^2 (dS_n/dE)_{E=E_\pi}}, \quad E_\pi \to \beta_{\pi n} E_\pi; \quad \Gamma_\pi \to \beta_{\pi n} \Gamma_\pi.$$

It is proved, at least in neutron channel case, that derivative of shift factor, dS_n/dE, is non-negative and this results in $\beta_{\pi n} = 1/[1 + \gamma_{\pi n}^2 (dS_n/dE)_{E=E_\pi}] \leqslant \leqslant 1$. The compression factor is significantly smaller than unity provided the reduced width is large and shift factor is nonconstant. A large reduced width, of order of Wigner unit, is vital in obtaining small value of compression factor. The shift factor is constant faraway from threshold; near threshold there is monotone increasing with energy. Accordingly, the compression factor is essential nonunity only near neutron channel threshold and for large neutron reduced width. The distorsion of the resonance's shape can be viewed as compression of the energy scale in the threshold range. The compression factor [31] results into a shift to the threshold of the resonance's position as well as into a width compression. For $\beta \to 0$, the resonance is shifted just to zero (threshold) energy. A large reduced width is essential in obtaining small values of β.

The threshold effect is an interference of the background scattering B, and threshold $\Delta\overline{U}$, collision matrix terms,

$$\Delta\sigma_{ab} \simeq \mathrm{Re}\,(B_{ab}\overline{\Delta U_{ab}^*}).$$

The background scattering/reaction is described in terms of effective interactions (optical model, DWBA). These two background reaction models are not subject of threshold effects; they generate collision matrix elements which are continuous across thresholds. The threshold term of collision matrix is suitably described, in R-matrix theory, by a (neutron) single-particle resonance; the threshold energy dependence is that of the neutron-channel logarithmic derivative. All involved parameters (neutron single-particle, optical potential, DWBA ones) have to be congruent with global reaction data and they are not subject of modification or fit. The only parameter specific to threshold effect (in addition to neutron-channel logarithmic derivative) is the strength of the anomalous threshold term. According to present philosophy the strength of the threshold effect is proportional to neutron strength function. If true, one proves that the ancestral origine of the threshold effect is neutron single-particle resonance, which is acting as an amplifier for the flux transfer to and from neutron threshold channel. In the cusp theory, the flux transfer responsible for threshold effect, is controlled only by the penetration factor of the neutron threshold channel. In the present approach, the flux transfer is governed also by spectroscopic factor of the neutron single-particle state, energy-coincident with threshold. Another factor entering into play is flux transfer *via* channels couplings; it could be represented by DWBA matrix elements if one-step transitions are dominant. Accordingly the strength of the threshold effect is proportional to the product of neutron spectroscopic factor and of (observed open channel, invisible threshold channel) DWBA matrix elements.

3. THRESHOLD EFFECTS WITH NONLIGHT NUCLEI

The purpose of this work is to establish relation between the neutron strength functions and the anomal effects observed with nonlight nuclei, at the threshold of the neutron analogue channel. We have in mind the threshold effects with nonlight nuclei: deuteron stripping threshold effect with $A \approx 80$–110 mass nuclei and isotopic threshold effect in proton–neutron mirror reactions with $A \approx 30$ mass nuclei. By analyzing the existing experimental data on these two groups of threshold effects, one proves that the magnitude of the threshold effect is proportional to neutron strength function, in their dependence on mass number. This connection between threshold effects and neutron strength function proves that the threshold effects are directly related to spectroscopy of ancestral zero-energy neutron single-particle resonance. The threshold effects depend not only on penetration factors of opening neutron channel, as in cusp theory, but also on channels couplings and on spectroscopic amplitude of neutron threshold state.

On Analysis of Threshold Effects. The magnitude of the threshold effect is proportional both to direct channel coupling strengths $(T^{\beta}_{an}T^{\beta}_{nb})$ and to the single-channel neutron reduced width $(\gamma^2_{\pi n})$. As mentioned before, the single-particle state is spread out, by residual interactions, over the actual (compound nucleus) levels. By averaging over actual levels one obtains the result that the effect is proportional to neutron strength function, $S_{\pi n} \sim \overline{\gamma^2_{\pi n}}$. (The only fluctuant quantities, related to threshold effect, are neutron reduced width and total resonance width; the other terms $(T^{\beta}, L_n = S_n + iP_n)$ are monotone energy-dependent and are not involded in averaging.)

In order to extract from the data the relation threshold effect — neutron strength function, one has to take into account the energy dependences on input (a) and exit (b) channels, i. e., those of the $T^{\beta}_{an}T^{\beta}_{nb}$ factors. Let us consider the case of the exit proton channel $(b = p)$, coupled by isospin interaction to neutron threshold channel; the proton and neutron channels are isospin analogue ones. The exit proton energy is fixed by its coulomb relation to the threshold energy of neutron analogue channel; $Q(p, n)$ for analogue channels has nearly the same value for nuclei of the same izomultiplet in a given mass area or with the same zero-energy neutron state. Therefore the term T^{β}_{np} could be considered nearly the same for the group of nuclei displaying the same zero-energy neutron single-particle state. For a threshold effect in proton elastic scattering one obtains that the product $T^{\beta}_{pn}T^{\beta}_{np}$ is nearly identical for all izomultiplet nuclei within mass area displaying the same zero-energy neutron single-particle state. The experimental threshold effect's magnitude α_{pp} is, up to a nearly constant factor, just the neutron strength function, $\alpha_{pp} = \text{const}\, S_{\pi n}$.

For transfer reactions populating the same proton and neutron isospin analogue channels (e.g., (d,p) and $(d,\bar{n})$ ones), it could happen that the Q-values ($Q(d,p)$) change significantly for different target nuclei of the same mass area. Consequently one has to «correct» the primary experimental data, for the input channel energy dependence. One can overcome this situation by remarking that the transfer analogue proton and neutron reactions have the same kinematical structure; their energy dependence on input channel appears only in DWBA radial integrals, via input (deuteron) channel distorted wave function. Consequently, one can consider that the input channel energy dependence of threshold experimental data is the same as that of the background cross section, $\alpha_{dp} \sim T_{dp}^{\beta}\alpha'$, with α' nearly independent of the input channel energy. The relation threshold effect — neutron strength function is now shifted into relation $\alpha' = \text{const}\, S_{\pi n}$.

These procedures have been used in establishing computational relations threshold effect — neutron strength function, both for isotopic threshold effect and deuteron-stripping threshold effect. The deuteron-stripping threshold effect requires a more sophisticated theoretical and computational analysis related to DWBA background. Also it is more rich in physical aspects and conclusions as compared to isotopic threshold effect; the last one is nevertheless more simple and transparent.

Deuteron-Stripping Threshold Effect. A threshold effect in direct reactions was evinced in deuteron stripping on medium-mass target nuclei, both in cross-section [29] and polarization [30] experiments. The main experimental cross-section characteristics of this threshold anomaly, as sistematized by Lane [31], are: (1) the anomaly does not appear for the lowest neutron threshold but rather it is related to opening of $(d,\bar{n})$ neutron analogue channel, (2) it manifests mainly as a dip (reversed resonant peak) in excitation functions; the dip half-width is typically 0.7 MeV, (3) the magnitude of the threshold dip is dependent on the mass of target nucleus. The first experimental conclusion [29] is an experimental evidence for isospin coupling of exit proton and analogue neutron channels. The isospin coupling of analogue proton and neutron channels is the basis for a coupled channel Born approximation model [32]. The cusp theory cannot account for deuteron-stripping threshold effect [33–35].

Related to the deuteron-stripping threshold effect, Lane has proposed a phenomenological model [31] based on zero-energy p-wave neutron single-particle resonance, specific to $A \approx 90$ mass nuclei; by isospin coupling it is reflected as a resonant structure in S-matrix element of analogue proton channel. The near-threshold p-wave neutron single-particle resonance ($l = 1$, $j = 3/2$, $1/2$; $E_j \sim 0$; $L_1 = S_1 + iP_1$ — p-wave neutron channel logarithmic derivative; b — boundary condition at channel radius a; γ_n^2 — neutron reduced width; Γ — resonance total width), is reflected, by isospin coupling as a resonant term in the proton channel S-matrix elements:

$$S_{dp} = B_{dp} + \Sigma_{j=3/2,1/2} \frac{\alpha_j(\hbar^2/ma^2)}{E_j - E - (S_1 + iP_1 - b)\gamma_n^2 - i\Gamma}.$$

The background S-matrix elements, B_{dp}, are generated by DWBA, $(\hbar^2/ma^2)$ is Wigner unit of the reduced width; the coupling constants α_j, related to isospin coupling strengths, are free parameters of this model. Although being formal equivalent to a single level formula, it is specific for the threshold effects due to the strong energy dependence of the neutron channel logarithmic derivative L_n near zero energy. This energy dependence results into a distorsion of the resonance shape, esp. for s and p waves.

For a compression factor which can reproduce the anomaly's width ~ 0.7 MeV, a reduced width γ_n^2 exceeding several Wigner units is necessary. (Such a large value of the reduced width can be obtained both from the shell model and optical model calculations or from an empirical formula, relating the width's increase to the nucleus surface's diffuseness, e.g., [17].)

According to Lane model, the strength of the threshold effect is dependent only on isospin coupling strength of proton-to-neutron analogue channel. If this assumption is taken literally, the threshold anomaly strength should be nearly the same for all nuclei in $A \approx 80$–100 mass area. If compared to Lane formula, the strength of the threshold effect (in effective collision matrix term) is given, in addition to channels couplings strengths, by the neutron strength function.

The mass dependence of threshold anomaly strength is established by analyzing the corresponding experimental cross-section data, for different target nuclei: ^{80}Se [36], ^{86}Kr [37], ^{88}Sr [38], ^{90}Zr [29,39,40], ^{92}Zr [41], ^{92}Mo [41], ^{94}Zr [41], ^{94}Mo [41], and ^{106}Cd [39]. The analysis took into account all experimental data on threshold effects in $A \sim 80$–110 mass-region, both cross section and analyzing power.

An empirical procedure for direct extraction from experimental data of anomaly's magnitude was devised for analyzing deuteron-stripping threshold effects observed with target nuclei $80 \leqslant A \leqslant 106$. The threshold effect strength is evaluated from maximal deviations of the cross section with respect to median (values) at half-widths points of the anomaly dip. The magnitude of the anomaly is normalized with respect to (d,p) background cross section including corresponding spectroscopic factor. This procedure results into a global parameter for the anomaly's magnitude with no explicit reference to different deuteron channels contributing to the same p-wave proton channel.

The «empirical» anomaly strengths and their errors are represented by triangles, while the recent experimental data on $3p$ neutron strength function for the investigated nuclei, are displayed by filled circles (Fig. 1). The values of the

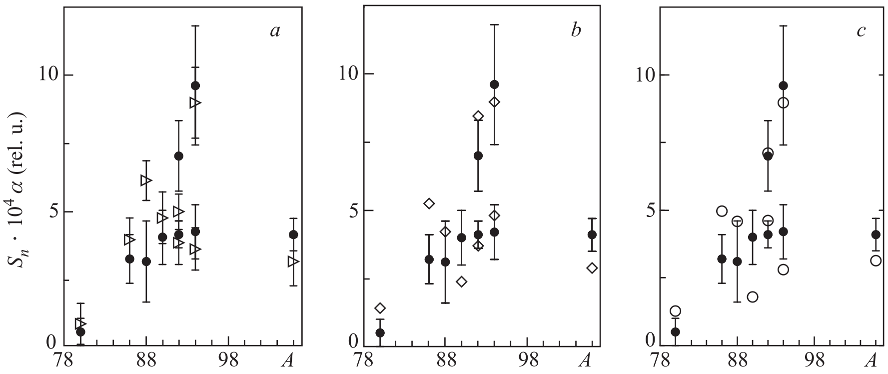

Fig. 1. The mass dependence of the: experimental $3p$-wave neutron strength function (•), the «empirical» (▷) and the «computational» (◇ and ○) strengths of threshold anomaly

anomaly strengths are given in relative units, as they have been obtained from the empirical analysis.

The procedure is not a very accurate one because it neglects the deuteron channel partitions compatible with proton p-wave channel, the spin-orbit coupling in exit proton channel and corrective factors as energy dependence of α parameters. In spite of its limitations, it exhibits the relation between the threshold effect's magnitude and neutron strength function. To avoid such inaccuracies, a numerical analysis was performed by using a standard DWBA method for the background's description and alternative subroutines for S-matrix threshold term. The purpose of numerical analysis is quantitative extraction, from experimental data, of threshold effect strengths, α, and, subsequently, of their spectroscopic component.

In the computational analysis of the threshold effect, the α-parameter is the subject of some general physical constraints. The parameter α is dependent, in addition to channels coupling strengths, on the deuteron input channel data (angular momenta $l_d j_d$) compatible with those $(l_p j_p)$ of p-wave single-particle resonance. If the resonance would be shifted far away from neutron threshold, the resonant-like term should behave as proton p-wave background. In that limit one deduces that the α-parameter's dependence on deuteron energy should be similar to that of the background terms of the scattering matrix which describe transitions to the p-wave proton channel; in other words, α-dependence on deuteron and proton energies is contained in (d, p) DWBA radial integrals corresponding to $l_p = 1$ proton partial wave. Formally this can be deduced from the relationship existing between the total angular momentum (specific to a resonance) and transferred angular momentum (specific to a direct process) parameterizations of the transition amplitude for a direct reaction [42].

The transition amplitude describing direct and anomalous reaction mechanisms in (d,p) reactions consists of a DWBA (and a resonant threshold) term; these terms are constructed, respectively, in transferred angular momentum coupling scheme [42] and in the total angular momentum coupling scheme [23], and they were added up in the code DWUCK [43]. The DWBA description of the background is realized in terms of deuteron [44], and proton [45], optical global (averaged) parameters and by a suitable choice of spectroscopic factors. The anomal threshold term is approached [31] in terms of R-matrix description [23] for $3p$-wave neutron single-particle resonance.

A physical constraint in analysis is the condition to obtain the same spectroscopic component of threshold effect, from all possible transitions (corresponding to $\alpha_{l_d j_d; l_p j_p}$ parameters) populating p-wave proton channel from different (l_d, j_d) deuteron channels; apart kinematical and penetration factors, the different transition strengths $\alpha_{l_d j_d; l_p j_p}$ should provide the same spectroscopic quantity, i. e., neutron strength function.

According to «computational» analysis, the magnitude of the p-wave threshold effect is proportional to the neutron strength function, in their mass dependence, Fig. 1. This analysis does evince the spectroscopic properties of the threshold effects; the strength of the threshold effect (in open channel) is proportional to the spectroscopic strength of the neutron threshold state (from opening channel). It is a proof that the threshold effects depend not only on the kinematical parameters but also on the spectroscopic factor of the ancestral neutron state in opening channel.

A «global» fit was performed for all the investigated cases, by describing the threshold effect strengths through a Lorentzian distribution function *versus* the nuclei masses. A similar distribution has been used in [8] to estimate the spin-orbit splitting of the $3p$ neutron strength function. The «global» fit does search for the best estimates nor of the each anomaly strength but rather for the parameters of the Lorentzian function (the width and position of the maximum). The threshold strengths Lorentzian function does fit for the existent investigated cases of the deuteron-stripping threshold effect, Fig. 2; the neutron strength function Lorentzians in [8] describe corresponding experimental data on the spin-orbit splitting.

Deuteron-Stripping Threshold Effect with A$\sim$110 Mass Nuclei. For $A \approx 90$ mass region, the nuclear shell just comes to enclose at $A = 90$, i. e., a $2d_{5/2}$ pure neutron single-particle state (spectroscopic factor ≈ 1) is encountered for ^{91}Zr ground state. The single-particle character of the residual states in stripping reactions becomes weaker while moving away $A \approx 90$. For most nuclei from $A \approx$ ≈ 110 mass region, the $3s_{1/2}$ subshell will be populated by a s-wave transferred neutron, but the corresponding spectroscopic factors of such states are, as a rule, small. If the background reaction spectroscopic coefficient is small, other competing processes, e.g., multistep ones, come to play a more important role in

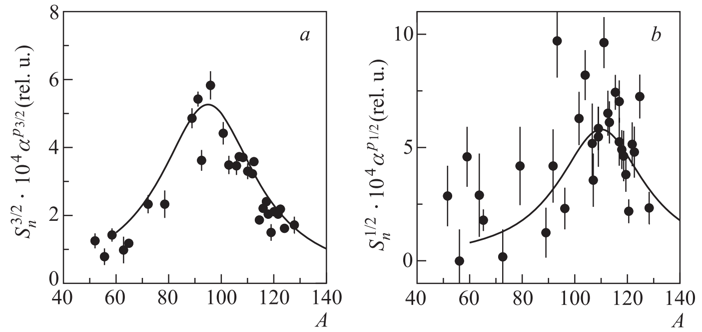

Fig. 2. The mass dependence of the experimental $3p$-wave neutron strength function (•) and of the α's anomaly strength derived within a Lorentzian distribution (solid curve) for the $p_{3/2}$ and $p_{1/2}$ spin orbit components

the reaction process [46]. The threshold effect, $\Delta\sigma$, superposed on background, will be less discernible if the multistep contributions to (d,p) cross section are important. Furthermore, these can mask the anomaly in the case of an energy fluctuant behaviour. It is expected, on basis of the above arguments, that one gets less experimental evidences of $3p$ threshold anomaly in $A \approx 110$ mass region, where the $3p_{1/2}$ neutron strength function lies its largest values.

A numerical experiment was devised in order to analyze the contribution of each of the $\alpha^{p_{3/2}}$ and $\alpha^{p_{1/2}}$ threshold effect strengths to the reaction cross section. The most simple partitions for the angular momenta in the deuteron channel, related to the proton p-wave in the exit channel, are obtained for a $s_{1/2}$-wave transferred neutron on a zero-spin target. This choice is compatible with a lot of deuteron-stripping reactions data in the mass range $110 \leqslant A \leqslant 130$.

The same amplitude and phase for $\alpha^{p_{3/2}}_{(l_d,j_d)}$ and $\alpha^{p_{1/2}}_{(l_d,j_d)}$ coefficients corresponding to $p_{3/2}$- and $p_{1/2}$-wave proton channel do result in different magnitudes of threshold effect. One has to notice the large amplitudes of the threshold dip for the ($l_d = 1$, $j_d = 2$) deuteron angular momentum corresponding to $p_{3/2}$-wave and its occurrence in both considered terms. For the $p_{1/2}$-wave case, at least one threshold contribution vanishes due to the cancellation of a Clebsch–Gordan coefficients product. Such differences are explained by a deeper introspection of DWBA cross-section terms. The different strengths of the threshold dip obtained from the above angular momenta partitions in deuteron channel result also from the product of the DWBA «kinematical complex» [46],

$$(2l_p+1)\sqrt{(2s_d+1)(2j+1)(2j_p+1)(2l_d+1)}\times \\ \times \langle l_p l 00 | l_d 0\rangle X(l_p s_p j_p, lsj, l_d s_d j_d)$$

multiplying the radial integrals. The quantum numbers l, s, j represent the orbital momentum, spin, and total spin of the neutron transferred particle. The «kinematical complex» could be related to the kinematical factors entering the relationship between the collision matrix in total angular momentum coupling scheme and the radial integrals from DWBA approach [42].

Similar numerical results have been obtained for $d_{5/2}$ transferred neutron, specific for $A \approx 90$ mass region. The deuteron channel with ($l_d = 3$, $j_d = 4$) angular momentum which is related only to $p_{3/2}$-wave in proton channel has a dominant contribution to threshold dip. The ($l_d = 1$, $j_d = 2$) channel analyzed above plays a similar role for the $s_{1/2}$-wave transfered angular momentum. If the $\alpha^{p_{3/2}}$ strength is set to zero, i. e., the $3p_{3/2}$ contribution of neutron strength function is canceled, the threshold effect in the cross section becomes very weak, despite the presence of a large $3p_{1/2}$ neutron strength function component. As a consequence of Yule–Haeberli rule [47], the analyzing power shape is reversed to a resonant form, contrary to the experimental behaviour. The destructive interference between $p_{3/2}$ and $p_{1/2}$ terms results in a small threshold effect when both have been taken into account. A $d_{3/2}$ transferred angular momentum is also possible for $A \geqslant 120$ target nuclei. However, neither $p_{3/2}$- nor $p_{1/2}$-α's strengths can reflect a strong threshold effect in the excitation functions as does the $p_{3/2}$ one for the $d_{5/2}$- or $s_{1/2}$-transfer deuteron-stripping reactions.

The inhibition of deuteron-stripping threshold anomaly related to $p_{1/2}$ neutron single-particle state is explained both in terms of the direct interaction (DI) process and also on the properties of the $3p_{1/2}$ neutron threshold resonance.

Three factors have been found to be related with (DI) process: (a) the Q-value dependence on (d,p) deuteron-stripping reaction, (b) the quantum kinematics involved in the DI transition amplitude and, (c) the spectroscopic factors of the residual nuclear state.

The empirical relation between Q-value of (d,p) reaction and the deuteron threshold energy have been verified on the basis of experimental evidences for the $A \sim 110$ mass region. The deuteron energy corresponding to neutron analogue channel is far away the stripping cross-section peak for many of the candidate (d,p) reactions; the threshold effect dip being more visible if superposed just on peak of background excitation function.

The quantum kinematics is determined by the transferred angular momenta and consequently, on the nuclear shell configuration of the residual and target nuclei of the stripping reaction. Both for the $d_{5/2}$ and $s_{1/2}$ transfer, the $j_p = 3/2$ proton channel is favoured in displaying threshold effect, due to the interplay of Racah coefficients entering the transition amplitude element. Also the proton p-wave radial integrals are slight larger for $j_p = 3/2$ than for $j_p = 1/2$ angular momenta.

Small spectroscopic factors of the involved residual states could reflect the incidence of multistep processes in the reaction mechanism. Their interference with one-step direct process will result in significant changes of magnitude of cross section that may mask small structures as threshold effects ones, superposed on the background excitation functions.

The predictions for the threshold effect cross sections in deuteron-stripping reactions, beyond $A \approx 107$ mass region, were approached by using DWUCK computer code [43], with additional routines for describing the resonant anomal interaction [48].

In order to evaluate the strength of the deuteron threshold effect, DWBA background, corresponding spectroscopic factors, and neutron threshold energies have been determined for all «candidate» stripping reactions. We have used, both for protons and deuterons, the averaged optical model parameters from [45, 49, 50]. In case of no experimental evidence of the neutron isobar analogue channel corresponding to the p-wave proton one, the Q-value of analogue neutron threshold channel was estimated using the empirical method for the coulomb displacement from [51]. The α-parameters, according to our purpose, had to obey the mass dependence from Fig. 2.

The criterion applied to select from a large number of «candidate» targets was the isotopic abundance. We have identified, on the basis on this criterion, more than twenty (d, p) reactions on nuclear targets starting with ^{107}Ag up to ^{130}Te target nuclei. All these nuclei do exhibit the $1/2$ spin-orbit component of $3p$ neutron strength function of significant magnitude.

To get a global picture for the entire investigated mass region as well as for a comparison with threshold effect reported for $A \sim 90$ mass region, it is represented within the same figure most of the predicted anomal cross sections (multiplied by corresponding scale factors) together with the largest experimental threshold effects measured for $^{88}\mathrm{Sr}(d,p)^{89}\mathrm{Sr}$ [38], and $^{94}\mathrm{Zr}(d,p)^{95}\mathrm{Zr}$ [41] stripping reactions, see Fig. 3. The experimental neutron strength function for ^{89}Sr and ^{95}Zr residual target nuclei did reproduce better the threshold effect if multiplying α-parameters by the corresponding spectroscopic factors.

The predicted threshold effect strengths, Δ', have been also calculated and represented, as scaled values with respect to the strongest threshold effect from $^{88}\mathrm{Sr}(d,p)^{89}\mathrm{Sr}$, versus mass number in Fig. 4. The stripping reactions under study were the following ones:

$$
\begin{gathered}
{}^{107}\mathrm{Ag}(1/2^-)(d,p)^{108}\mathrm{Ag}(1^+), \quad {}^{108}\mathrm{Pd}(0^+)(d,p)^{109}\mathrm{Pd}(5/2^+), \\
{}^{109}\mathrm{Ag}(1/2^-)(d,p)^{110}\mathrm{Ag}(1^+), \quad {}^{110}\mathrm{Pd}(0^+)(d,p)^{111}\mathrm{Pd}(5/2^+), \\
{}^{110}\mathrm{Cd}(0^+)(d,p)^{111}\mathrm{Cd}(1/2^+), \quad {}^{111}\mathrm{Cd}(1/2^+)(d,p)^{112}\mathrm{Cd}(0^+), \\
{}^{112}\mathrm{Cd}(0^+)(d,p)^{113}\mathrm{Cd}(1/2^+), \quad {}^{113}\mathrm{Cd}(1/2^+)(d,p)^{114}\mathrm{Cd}(0^+), \\
{}^{114}\mathrm{Cd}(0^+)(d,p)^{115}\mathrm{Cd}(1/2^+), \quad {}^{115}\mathrm{I}(1/2^+)(d,p)^{116}\mathrm{I}(1^+), \\
{}^{116}\mathrm{Sn}(0^+)(d,p)^{117}\mathrm{Sn}(1/2^+), \quad {}^{118}\mathrm{Sn}(0^+)(d,p)^{119}\mathrm{Sn}(1/2^+),
\end{gathered}
$$

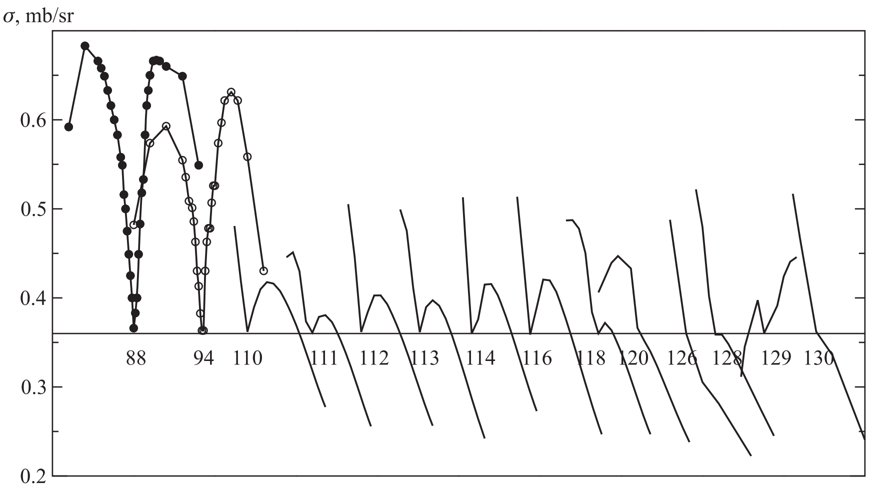

Fig. 3. Experimental differential cross sections for ^{88}Sr$(d,p)^{89}$Sr (•) and ^{94}Zr$(d,p)^{95}$Zr (○) stripping reactions at $\theta = 160^\circ$ scattering angle. Predicted values (solid line) for $100 \leqslant A \leqslant 130$ target nuclei have been scaled within the experimental ones. The displayed mass numbers, taken in an ascending order, label the following target nuclei: ^{88}Sr, ^{94}Zr, ^{110}Cd, ^{111}Cd, ^{112}Cd, ^{113}Cd, ^{114}Cd, ^{116}Sn, ^{118}Sn, ^{120}Sn, ^{126}Te, ^{128}Te, ^{129}Xe, and ^{130}Te

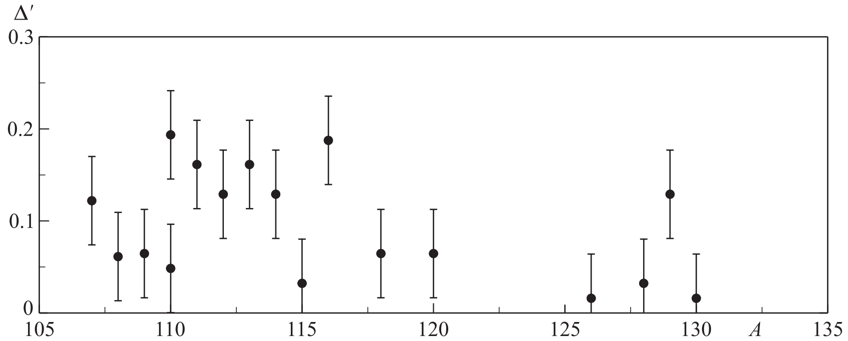

Fig. 4. The $\Delta' = (\sigma_{\max} - \sigma_{\min})/\sigma_{\min}$ predicted strengths of the deuteron anomaly for target mass nuclei with $A \geqslant 106$ determined within the numerical procedure. The values are normalized to the largest observed one from the $A \approx 90$ mass region, ^{88}Sr$(d,p)^{89}$Sr

$$^{120}\mathrm{Sn}(0^+)(d,p)^{121}\mathrm{Sn}(3/2^+), \quad ^{126}\mathrm{Te}(0^+)(d,p)^{127}\mathrm{Te}(3/2^+),$$
$$^{128}\mathrm{Te}(0^+)(d,p)^{129}\mathrm{Te}(3/2^+), \quad ^{129}\mathrm{Xe}(1/2^+)(d,p)^{130}\mathrm{Xe}(0^+),$$
$$^{130}\mathrm{Te}(0^+)(d,p)^{131}\mathrm{Te}(3/2^+).$$

One should remark that the predicted threshold effects with $A \sim 110$ target nuclei are much smaller (at least by a factor of 5) or even indiscernible if compared to those with $A \sim 90$ mass target nuclei.

The $3p$ neutron strength function contribution to deuteron-stripping threshold effect is modulated by a series of kinematical and dynamical factors as mentioned above. These factors do not contribute to evince better the $3p_{1/2}$-wave anomaly but rather they mask or even inhibit this threshold effect. However, one can consider some candidates for the threshold effects for $A \geqslant 110$ mass target nuclei (^{107}Ag, $^{110-114}$Cd, ^{116}Sn, ^{129}Xe) even they are by a factor of 5–10 smaller than those with $A \sim 90$ target nuclei.

Isotopic Threshold Effect. The isotopic threshold effect is neutron-threshold anomaly of proton reactions on mirror light-medium $A \sim 30$ mass nuclei; it is related to a p-wave neutron single-particle state coincident with threshold and in isospin coupling of proton and neutron charge-exchange channels.

The threshold effects with nonlight nuclei are related to quasiresonant scattering (coupled channel resonances) at neutron zero energy. A quasiresonant scattering process consists of (1) a single-particle resonance in the neutron channel, and (2) direct selective coupling of the neutron channel to the observed proton one. The first condition does select the mass region, while the second one does select the reaction channel. A single-particle resonance/state, located at zero energy, is a global property of a whole mass region; for example, the $2p$-wave neutron zero-energy single-particle resonance is specific for $A \sim 30$ mass nuclei, see, e.g., [52]. The neutron single-particle resonance is coupled selectively, by isospin interaction, only to a given (analogue proton) channel. The two coupled analogue channels are proton scattering and neutron charge-exchange reactions on mirror nuclei.

The possible proton induced reactions, satisfying the above two conditions, can be, for example, illustrated by the proton elastic scattering and (proton, neutron) charge exchange reaction on $^{27}_{13}\mathrm{Al}_{14}$ target nucleus. The two reaction channels $^{1}_{1}p_0 + {}^{27}_{13}\mathrm{Al}_{14}$ and $^{1}_{0}n_1 + {}^{27}_{14}\mathrm{Si}_{13}$ are coupled by isospin interaction $\mathbf{tT}$, because the two nuclei $^{27}_{13}\mathrm{Al}_{14}$ and $^{27}_{14}\mathrm{Si}_{13}$ are mirror nuclei (isotopic doublet, $T = 1/2$), e.g., [53]; also the proton $^{1}_{1}p_0$ and neutron $^{1}_{0}n_1$ are components of the nucleon (isodoublet, $t = 1/2$). The $2p$-wave neutron single-particle resonance, appearing at zero energy for $A \sim 30$ mass nuclei, does induce by isospin coupling a quasirezonant structure in the proton (analogue) channel; this is a threshold effect because of $2p$ state coincidence with opening of the neutron channel. The $2p$-wave neutron single-particle state at zero-energy is a global property of $A \sim$ ~ 30 mass nuclei; consequently, one can expect the same threshold effect with other nuclei in this mass region. The isotopic doublets ($T = 1/2$) or mirror nuclei in $A \sim 30$ mass region are, e.g., [53]: ($^{23}_{11}\mathrm{Na}_{12}$, $^{23}_{12}\mathrm{Mg}_{11}$), ($^{25}_{12}\mathrm{Mg}_{13}$, $^{25}_{13}\mathrm{Al}_{12}$), ($^{27}_{13}\mathrm{Al}_{14}$, $^{27}_{14}\mathrm{Si}_{13}$), ($^{29}_{14}\mathrm{Si}_{15}$, $^{29}_{15}\mathrm{P}_{14}$), ($^{31}_{15}\mathrm{P}_{16}$, $^{31}_{16}\mathrm{S}_{15}$), ($^{33}_{16}\mathrm{S}_{17}$, $^{33}_{17}\mathrm{Cl}_{16}$). The corresponding proton elastic scattering and neutron charge exchange channels are coupled by isospin interaction (analogue channels). The analogue channels for $A \sim 30$ mass mirror nuclei are, respectively; $p+^{23}_{11}\mathrm{Na}$ and $n+^{23}_{12}\mathrm{Mg}$; $p+^{25}_{12}\mathrm{Mg}$ and $n+^{25}_{13}\mathrm{Al}$; $p+^{27}_{13}\mathrm{Al}$ and $n+^{27}_{14}\mathrm{Si}$; $p+^{29}_{14}\mathrm{Si}$ and $n+^{29}_{15}\mathrm{P}$; $p+^{31}_{15}\mathrm{P}$ and $n+^{31}_{16}\mathrm{S}$; $p+^{33}_{16}\mathrm{S}$ and $n+^{33}_{17}\mathrm{Cl}$. The threshold effect, based on isospin coupling of proton and neutron channels, could manifest also in proton reactions on other isospin multiplets (as isobaric triplets, $T = 1$) from the same $A \sim 30$ mass area, e.g., in the reaction $p + ^{34}_{16}\mathrm{S}$ at $n + ^{34}_{17}\mathrm{Cl}$ neutron threshold. All above listed reactions are candidates for a threshold effect originating in $2p$-wave zero-energy neutron single-particle state, reflected by isospin interaction in proton elastic scattering channel.

A threshold effect is a small change in the background cross section of the observed channel; experimental observation of a threshold effect requires fulfillement of at least two conditions. Firstly, the number of partial waves involved in the observed scattering channel should be relatively small in order not to mask the threshold effect which is present only in one partial wave ($l = 1$ in this case). This condition is realized because the proton energy (or the number of effective partial waves in proton channel) necessary to open neutron mirror channel on $A \sim 30$ mass nuclei, are relatively small ones ($E_p \sim 6$ MeV; $l_p \leqslant 3$). A second condition for experimental observation of a threshold effect is the smooth behaviour of the background cross section on which the effect is superposed. Apparently this condition is not realized for proton elastic scattering on $A \sim 30$ mass target nuclei, because of small-width structures ($\sim$ 100–200 keV); part of them (esp. above neutron threshold) are Ericson fluctuations; part of them (below threshold) are compound nucleus structures, see, e.g., [54]. However the threshold effect, being related to a single-particle resonance, should have a rather broad structure ($\sim$ 500 keV) and, hence, it would manifest as an envelope (or an amplitude modulation) of the small-width structures [55].

The experimental excitation functions for proton elastic scattering on different $A \sim 30$ mass target nuclei, $^{23}\mathrm{Na}$ [56], $^{25}\mathrm{Mg}$ [57], $^{27}\mathrm{Al}$ [55–58], $^{29}\mathrm{Si}$ [59], $^{31}\mathrm{P}$ [56], $^{34}\mathrm{S}$ [56], do present small-width structures or fluctuations, as mentioned above; however a modulation of the fluctuating cross section does occur in threshold vicinity [55, 56]. In order to remove the small-width structures or fluctuations and to extract their envelope, three procedures were used: energy averaging, Fourier analysis, and an empirical analysis. These three procedures result in similar evidences for threshold effect [56].

The experimental data provide an evidence [60] of the neutron single-particle state origin of this threshold anomaly: the magnitude of the threshold effect is proportional to the neutron strength function. The empirical magnitude of the threshold effect $\Delta = (\sigma_{\max} - \sigma_{\min})/\sigma$ is the maximal deviation in the threshold domain $(\sigma_{\max} - \sigma_{\min})$ of the averaged proton cross-section data normalized to its corresponding energy integrated value σ. Another measure of the effect's magnitude, extracted from the analysis of the data, is the «computational» threshold effect, the strength α (heavy dots in Fig. 5). The threshold effect strength, Δ or α,

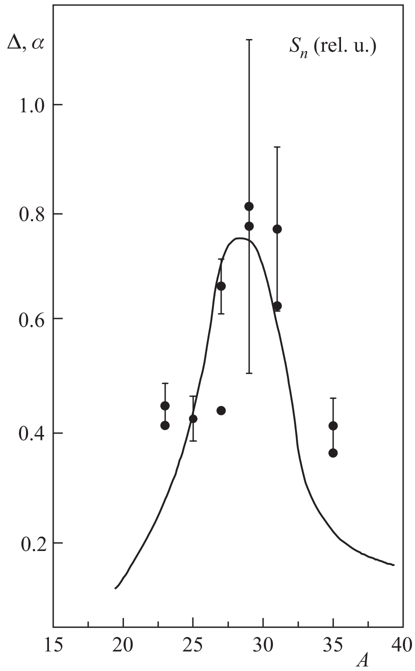

Fig. 5. Strengths of the isotopic threshold effect, Δ («empirical») and α («computational»), and the $2p$-wave neutron strength function, *versus* atomic mass number A

does follow the A mass dependence of the neutron strength function. This empirical result supports the philosophy of the work, according to that, the magnitude of the threshold effect is dependent on spectroscopic amplitude of the quasiresonant process, i. e., the neutron strength function. Apparently this result was the first experimental evidence that a coupled channel resonance depends not only on the channels-coupling strength but also on spectroscopical properties of ancestral single-particle state.

The isospin threshold effect was studied in a polarized transfer reaction [61]. The isospin coupling of proton and neutron channels could manifest also in direct transfer reactions as deuteron stripping. The stripping reaction $^{30}\mathrm{Si}(\mathbf{d},p)^{31}\mathrm{Si}$ was studied with polarized beam, by providing experimental evidence for threshold effect both in cross-section and analyzing power data. This experiment revealed also the possible coexistence of a threshold effect with an intermediate structure in the same transfer reaction; this is because angular momentum coupling schemes are not selective but rather nearly identical for both processes. The study [61, 62] of this transfer reaction does provide an alternative experimental evidence for this threshold effect. The study of this threshold effect has been extended [63] to other isospin-dependent reactions; it is based on the analysis of experimental data existing in literature. Evidence for this threshold effect was found in those reactions, too, esp. in $A \sim 30$ mass area and it was named «the isotopic threshold effect».

4. ON INTERMEDIATE STRUCTURE IN NUCLEAR THRESHOLD EFFECTS

The relation of threshold effects to neutron strength function can be approached either on mass dependence or on energy dependence.

The mass dependence (for a group of nuclei) of deuteron-stripping threshold effects follows, as proved, the mass dependence of neutron strength function. One has concluded that the threshold effect strength is proportional to spectroscopic amplitude of the neutron threshold state.

On the other side, one can study energy dependence of deuteron-stripping threshold effect (for a given nucleus). The microstructures of deuteron-stripping threshold effect for given residual nucleus can be put into correpondance with microstructure of its neutron strength function. This aspect can be addressed to intermediate structure in threshold effects.

Let us discuss the (micro)structures induced in threshold effects *via* neutron strength function. The structures in neutron strength function are related to structure of the nuclear state in compound system, formed by neutron coupled to residual nucleus state (from exit threshold channel). The system neutron + nonzero spin state can be coupled to various angular momenta. This way, the shell model configuration can be fragmented, through the residual interactions, into many components. If these components spread into each other by mixing into more complicated states, then there results a homogeneous Breit–Wigner shape for the neutron strength function. For a system neutron + zero spin state of residual nucleus, one can expect less fragments and the resulting components are not uniformly spread into each other; then there occur fluctuations in the Breit–Wigner line shape of the neutron strength function. G. E. Brown [64] has predicted fluctuations of the neutron strength function, depending on the nature of involved nuclear states: the intermediate structures are evinced as microstructures of the neutron strength function. The specific aspect of the intermediate structure in neutron strength functions and nuclear threshold effects is related to «threshold compression» (or nonlinearity) of energy scale. The apparent density of near-threshold microstructures (or fluctuations) could increase by an order of magnitude; only disparate microstructures could remain visible experimentally in excitation function of threshold effects.

Assuming this interpretation is correct, one could observe fluctuant threshold anomalies when the residual nucleus state in threshold channel has a simple relative structure (e.g., even–even nuclei). The nonfluctuating type of threshold anomalies, according to this view, should be related to complicated (nonzero spin) states of residual nucleus in threshold channel; see Lane systematics [31].

A similar interpretation was proposed by Lane [65] by taking into account the coupling 0^+–2^+ states in residual nucleus. Then one can observe a definite number of structure components in (neutron) threshold effect and this should be a global property for all even–even residual nuclei.

Analysis of experimental data for deuteron-stripping threshold effect does support the intermediate structure interpretation for fluctuations observed in the excitation function of this threshold effect [9, 66]. The deuteron-stripping threshold effect originates in $3p$-wave neutron threshold state whose microstructures are reflected in the $3p$-wave neutron strength function.

A similar analysis for isotopic threshold effect was not performed. In this case all threshold reaction channels have nonzero spins (mirror isodoublet nuclei) except only one case (a member of an isospin triplet). A Fourier analysis of these data, does display, in addition to gross single-particle component (~ 500 keV), small width compound nucleus structures and fluctuations (~ 100 keV).

Another approach to intermediate structure in threshold effects is in terms of «doorway states» of Feshbach [67, 68]. The nucleus continuum, explored by means of nuclear reactions, does exhibit a large spectrum of resonant-like structures: sharp narrow resonances, gross resonant structures, intermediate resonant structures. The narrow resonances are associated with compound nucleus, the gross resonant structures — to single-particle resonances, while the intermediate structure — to intermediate or doorway states. Intermediate structures, superposed on a continuum of statistical levels, are visible experimentally only if the «escape width» $\Gamma_\pi^\uparrow$ is larger than the «spreading width» $\Gamma_\pi^\downarrow$; otherwise it is spreaded in continuum of statistical levels. The spreading width of the intermediate state is proportional to probability for dissipating in compound nucleus states, while its escape width is proportional to probability of decay in incident reaction channels. The problem of the intermediate structure is to understand both the nature of the intermediate state and the mechanism which reduces its coupling to complicated compound nucleus states [67, 68]. The threshold state is highly excited state, embedded in a continuum of statistical levels. The threshold state has a small overlap to inner compound nucleus states because of its spatial extension, out of channel radius. The threshold state is decoupled from statistical levels by the «de-enhancement» factor β, resulting in a small spreading width $\Gamma_\pi^\downarrow$. In case of the threshold state, its «doorway» nature as well as the mechanism preventing its spreading in statistical continuum originate in its very large spatial extension (out of channel radius).

Later on, the concept of intermediate structure was included by Lane [27] in that of «line broadening». This approach does assume the existence of a «special state» which has a large overlap to one (or few) reaction channels, i. e., large escape width. By «residual interactions» the «special state» is mixed to «ordinary» or continuum states, resulting in «line-broadening» phenomenon. Lane considered that there are only few types of line broadening in nuclear physics. The threshold state could be an additional example of line broadening in nuclear physics. It has a large overlap only to threshold channel and it has a small overlap to inner compound nucleus states because of its spatial extension out of channel radius. The neutron threshold state is decoupled from statistical levels by the «de-enhancement» factor β, resulting in smaller spreading width $\Gamma_\pi^\downarrow$.

The problem of spectroscopic factors is, according to [27], subject of «line broadening». The ancestral single-particle state (whose spectroscopic amplitude is unity) is fragmented into actual nuclear states (with spectroscopic factors less than unity). There are two mechanisms of line broadening: residual interactions and change in boundary conditions. The last one is essential for near-threshold states. The threshold compression factor has a direct impact on neutron spectroscopic factor (reduced width).

CONCLUSIONS

In the present work, connection between threshold effects and neutron strength function proves that the threshold effects are directly related to spectroscopy of ancestral zero-energy neutron single-particle resonance.

Evidences for relation between nuclear threshold effects and neutron strength functions are obtained from deuteron-stripping threshold effect with $A \approx 80$–110 mass nuclei and from isotopic threshold effect in proton–neutron mirror reactions with $A \approx 30$ mass nuclei. The magnitude of the nuclear threshold effect is proportional to that of the neutron strength functions, in their dependence on mass.

The isotopic threshold effect is neutron-threshold anomaly of proton reactions on mirror light-medium nuclei; it originates in $2p$-wave neutron single-particle state coincident with threshold and in isospin coupling of proton and neutron charge-exchange channels. The magnitude of the effect is proportional to the $2p$-wave neutron strength function.

The deuteron-stripping threshold effects are determined both by reaction mechanism of (d, p) background reaction and by strength function of the neutron single-particle threshold state. The magnitude of the threshold effect in deuteron-stripping reactions on $A \approx 80$–110 mass target nuclei is proportional to the $3p$-wave neutron strength function spin-orbit components. Spectroscopic aspects of the threshold effect, with respect to the spin-orbit components, are discussed in relation to the neutron strength function.

The study of deuteron-stripping threshold effect provides relation to spin-orbit splitting of the $3p$-wave neutron strength function. For this purpose, the deuteron-stripping threshold effect strengths were evaluated according to mass dependence of the $3p$-wave neutron strength function spin-orbit components. Once the threshold effect parameters (depending on mass) are obtained, an inverse method was used, by considering the spin-orbit splitting of $3p$-wave neutron strength function, to reconstitute the cross-section threshold effects for other mass nuclei. The calculated threshold effect around $A \sim 110$ mass number is quite small in spite of the large values of $3p_{1/2}$ neutron strength function centered at $A \sim 120$. The diminution of the $3p_{1/2}$ deuteron-stripping threshold effect originates in factors related to quantum kinematics of corresponding transfer reactions and in incidence of multistep processes in the reaction dynamics.

For transfer reactions, the relation «threshold effect — neutron strength function» is convoluted by kinematical and dynamical factors; it is sensitive to single or multistep character of transfer reaction and is dependent on transfer reaction l- and Q-values. Such modulation factors could obscure the proportionality relation between threshold effect in transfer reaction and the neutron strength function. Specifically, in some cases, it resulted into inhibition of $3p_{1/2}$-wave spin-orbit component of neutron strength function in determining the deuteron-stripping threshold effect. The inhibiting factors, originating in reaction background, dependent on its multistep components, reflected in small spectroscopic factor, Q-value and transferred angular momentum.

The nuclear threshold effect is dependent, via neutron strength function, on ancestral neutron threshold state. This dependence proves that the origin of the threshold effect is zero-energy neutron single-particle state, which is acting as an amplifier for the flux transfer to and from neutron threshold channel. In the cusp theory, the flux transfer responsible for threshold effect is controlled only by the penetration factor of the neutron threshold channel. According to present result, the flux transfer is governed by spectroscopic factor of the neutron single-particle state, energy-coincident with threshold, as well as by channels couplings.

The neutron threshold state is highly excited state, embedded in a continuum of statistical levels. The threshold state has a small overlap to inner compound nucleus states because of its spatial extension, out of channel radius. This way the threshold state is decoupled from statistical levels resulting in a small spreading. The giant threshold state is an additional example of «line broadening» in nuclear physics. The neutron threshold state microstructures appear in neutron strength function, too, and could be reflected as fluctuations in excitation function of the threshold effect.

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