ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2012. Т. 43. ВЫП. 5

ASPECTS OF EFFECTIVE ACTION FOR SUPER CHERN–SIMONS-MATTER MODELS

I. L. Buchbinder [1,*], *N. G. Pletnev* [2,**]

[1]Department of Theoretical Physics, Tomsk State Pedagogical University, Tomsk, Russia

[2]Department of Theoretical Physics, Institute of Mathematics, Novosibirsk, Russia

We develop the superfield background method and study the effective action in the $\mathcal{N}=2$, $d3$ supersymmetric Chern–Simons-matter systems. The one-loop low-energy effective action for non-Abelian supersymmetric Chern–Simons theory is computed to order F^4 by use of $\mathcal{N}=2$ superfield heat kernel techniques.

PACS: 11.30.Pb

INTRODUCTION

During the last few years, quantum aspects of $d3$ supersymmetric theories at perturbative level have attracted considerable attention. This was inspired by the papers [1–4], where for an IR description of stacks of M2-branes, highly supersymmetric three-dimensional conformal field theories were proposed in the same sense as maximally supersymmetric Yang–Mills theory provides an effective description of stacks of D-branes. Such models are referred to as the Bagger–Lambert–Gustavsson (BLG) and Aharony–Bergman–Jafferis–Maldacena (ABJM) theories. ABJM models are defined as three-dimensional $\mathcal{N}=6$ superconformal $U(N)\times U(N)$ Chern–Simons-matter theory with level $(k,-k)$. It is conjectured to describe N M2-branes located at the fixed point of the C^4/Z_k orbifold in the static gauge. It is also argued that the ABJM model is dual to M-theory on $AdS_4\times S^7/Z_k$ at large N. For $SU(2)\times SU(2)$ gauge group, the $\mathcal{N}=6$ supersymmetry is enhanced to $\mathcal{N}=8$, and the ABJM model coincides with the BLG model. All these new superconformal field theories involve a nondynamical gauge field, described by a Chern–Simons-like term in the Lagrangian, which is coupled to matter fields, parameterizing the degrees of freedom transverse to the worldvolume of the M2-branes.

It has been known for a long time that the quantization of a membrane world-volume theory is very challenging, and one of the difficulties is the nonlocality

*E-mail: joseph@tspu.edu.ru
**E-mail: pletnev@math.nsc.ru

associated with the deformation of membrane without changing its volume. A quantum supermembrane theory faces a serious problem of quantum mechanical instability [5]. As a result, a single (quantum mechanical) supermembrane does not make sense and we get a multibody problem in its nature, which can be regarded as the origin of the continuous spectrum. Therefore, from the field theory side, the action of M2-brane should go away from the infrared fixed point to a nonperturbative Yang–Mills–Chern–Simons system.

We want to draw attention to the fact that an effect similar to the quantum effect potentially occurs in all super Chern–Simons-matter models and even in pure non-Abelian Chern–Simons theory and its supersymmetric versions. Then the BLG and ABJM Lagrangians and supersymmetry transformations presented in [1,2] can be thought as representing the leading order terms in Planck scale expansion of a (not yet determined) nonlinear M2-brane theory. This circumstance is analogous to the fact that the $\mathcal{N}=4, D4$ super Yang–Mills theory represents the leading order terms of the Born–Infeld action, which is believed to describe the dynamics of coincident D3-branes. Therefore, it would be interesting to determine the full theory, in which the leading order terms are the BLG or ABJM Lagrangians. This ambitious program is similar to non-Abelian supersymmetric extension of the Born–Infeld-type action in the $\mathcal{N}=4, D4$ super Yang–Mills quantum field theory (see as an example of just a few links [6], from a large list of references). The off-shell loop corrections in Chern–Simons-matter theory attract much attention since they generate nontrivial quantum dynamics for classically nondynamical gauge field (see, e.g., [7]). The natural way to study these corrections is given by effective action which can be treated as a method to derive the new, higher order in strength, gauge-invariant and supersymmetric functionals.

The aim of this paper is to construct the background field method for $\mathcal{N}=2$ super Chern–Simons theories, study the effective action in terms of unconstrained $\mathcal{N}=2, d3$ superfields and calculate the leading low-energy contributions to the effective action. Although the various classical and quantum aspects of $\mathcal{N}=2, d3$ supersymmetric theories were extensively studied (see, for example, [8, 9]), the superfield background field method, allowing one to develop manifestly gauge-invariant and $\mathcal{N}=2$ supersymmetric perturbation theory, has not been formulated up to now. It is just this problem that is being solved in the present paper. As the applications of background field method we show that in case of pure non-Abelian $\mathcal{N}=2$ super Chern–Simons theory, η-invariant vanishes, but off-shell contributions to the effective action have a nontrivial complicated structure.

1. $\mathcal{N}=2, d3$ SUPERFIELD MODELS AND BACKGROUND FIELD QUANTIZATION

We start with a brief description of the $\mathcal{N}=2, d3$ super Chern–Simons theory [10, 11]. The constrained geometry of $\mathcal{N}=2$ supergauge field is formulated in $R^{3|4}$ superspace with coordinates $z^M = \{x^m, \theta^\alpha, \bar\theta^\alpha\}$ in terms of the gauge covariant derivatives $\mathcal{D}_M \equiv \{\mathcal{D}_m, \mathcal{D}_\alpha, \bar{\mathcal{D}}_\alpha\} = D_M + i\Gamma^a_M T^a$, where D_M are the «flat» covariant derivatives and the gauge connection Γ_M takes the values in the Lie algebra of a compact gauge group. The vector multiplet in three dimensions is built from one real scalar ϕ, one complex spinor λ_α, one vector field A_m and one real auxiliary scalar D, all in the adjoint representation of the gauge group. The gauge covariant derivatives obey the superalgebra:

$$\begin{gathered}\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = -2i\mathcal{D}_{\alpha\beta} + 2i\varepsilon_{\alpha\beta}G, \quad \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 0,\\ [\mathcal{D}_\rho, \mathcal{D}_{\alpha\beta}] = \varepsilon_{\rho(\alpha}\bar W_{\beta)}, \quad [\bar{\mathcal{D}}_\rho, \mathcal{D}_{\alpha\beta}] = -\varepsilon_{\rho(\alpha}W_{\beta)},\\ [\mathcal{D}_{\alpha\beta}, \mathcal{D}_{\rho\sigma}] = -i\varepsilon_{(\alpha\rho}F_{\beta)\sigma} - i\varepsilon_{(\alpha\sigma}F_{\beta)\rho}.\end{gathered} \tag{1.1}$$

The superfield strengths are the linear superfield G and chiral W_α and antichiral $\bar W_\alpha$ superfields satisfy the Bianchi identities.

In $\mathcal{N} = 2, d3$ superspace, the gauge invariant Chern–Simons action reads [10]

$$S_{\rm CS} = \frac{ik}{4\pi}{\rm tr}\int_0^1 dt \int d^7z \bar D^\alpha\{{\rm e}^{-2V} D_\alpha\, {\rm e}^{2V}\}\, {\rm e}^{-2V}\partial_t\, {\rm e}^{2V}. \tag{1.2}$$

Here the extra parameter t satisfies the boundary conditions $V(t = 0) = 0$, $V(t = 1) \equiv V$. After rescaling the potential as $V_{\rm new} \equiv 2\sqrt{k/\pi}V$, we see that the coupling constant is $\sqrt{\pi/k}$.

The superfield Lagrangian for N_f matter chiral superfields Q^i coupled to non-Abelian $\mathcal{N} = 2$ vector multiplet has the form: $S_{\rm matter}[V, Q, \bar Q] = {\rm tr}\int d^7z \sum_{i=1}^{N_f} \bar Q^i\, {\rm e}^{q_i V} Q^i$, where the matter field $Q^i = \{f^i, \psi^i\}$, with global $U(N_f)$ flavor symmetry, is in an arbitrary representation R of the gauge group. Such an $\mathcal{N} = 2$ theory can be formulated for any gauge group G and chiral superfields in any representation, with arbitrary superpotential. The more extended supersymmetric theories can be formulated using some sets of $\mathcal{N} = 2$ superfields. It should be noted that the most elegant presentation of a large class of classically marginal models of Chern–Simons matter with manifestly realized $\mathcal{N} = 3$ off-shell supersymmetry is provided in the $\mathcal{N} = 3$ harmonic superspace [8].

Maximally supersymmetric theories in $2 + 1$ dimensions with $SO(8)$ R-symmetry were constructed in [1]. These theories have an interesting property that the closure of the supersymmetry requires particular combinations of the gauge group and the matter content, whereas there is no such restriction for $\mathcal{N} \leqslant 3$. The essential feature of these theories is that the matter fields $X^I = X^I_a T^a$, $I = 1, \ldots, 4$ take the values in a metrized version of the Lie 3-algebra $\mathcal{A}_n$: $[T^a, T^b, T^c] = f^{abc}_d T^d$, $h^{ab} = {\rm tr}\,(T^a, T^b)$, where the structure constants $f^{abcd} = f^{abc}_e h^{ed}$ are totally antisymmetric in upper indices and are subject to some basic identity. The gauge field takes values in the Lie algebra associated with the Lie 3-algebra $A_m = A_{m,ab}t^{ab}$, where the generators act in the fundamental representation as $(t^{ab})^c_d = f^{abc}_d$. When h^{ab} is positive definite, there is the only such $\mathcal{A}_4$ 3-Lie algebra (with $f^{abcd} \propto \varepsilon^{abcd}$, $h^{ab} = \delta^{ab}$) which satisfies all reasonable physical requirements. On the associated Lie algebra there exist two invariant tensors which have the required structure of a Killing form, namely $G^{ab,cd} = f^{abcd}$, $g^{ab,cd} = f^{abe}_f f^{cdf}_e$. Extending the BLG model to higher numbers of M2-branes by reducing the number of supersymmetries led to two generalizations of the notion of a 3-algebra: the generalized 3-Lie algebras and the Hermitian 3-algebras [3]. A classification of the possible $\mathcal{N} = 6$ theories of ABJM-type was presented in [3]. In all cases, the underlying 3-bracket is no longer required to be totally antisymmetric.

In order to get a compact form of the Feynman rules, it is convenient to use the capital Roman letters $A, B, \ldots$ to denote the indices in associated gauge Lie algebra [1, 3, 4]. In terms of the gauge algebra indices, the invariant form is given by $\langle X, Y\rangle = X^{ab}Y^{cd}f_{abcd} = X^A Y^b G_{AB}$. The structure constants $F_{ABC} = F^D_{AB}G_{DC}$, where $F^E_{AB} \equiv C^{ef}_{ab,cd} = 2f_{ab[c}{}^{[e}\delta^{f]}_{d]}$, are totally antisymmetric due to *ad*-invariance of $\langle\cdots\rangle$. Moreover, it is convenient to use the multi-indices ai combining flavor and 3-algebra indices for $Q^{ai} = Q^I$. For example, we have for vertices $\langle \bar{Q}_i, VQ^i\rangle = Q^I V^A (T_A)_I{}^J \bar{Q}_J$.

By construction, all these models have at least $\mathcal{N} = 2$ supersymmetry. Higher supersymmetry depends on the underlying 3-algebra and the choices of the superpotential. Therefore formally, the structure of the effective action in the sector of gauge fields (without violating the gauge symmetry) should have a universal form. The difference of effective actions of one model from another is stipulated by the choice of explicit 3-algebra representations and relations between various Casimir invariants for such Lie 3-algebras.

We quantize the $\mathcal{N} = 2$ super Chern–Simons theory in the quantum-chiral but background vector representation. As a first step, we split the initial superfields $V, Q, \bar{Q}$ into background $V, Q, \bar{Q}$ and quantum $v, q, \bar{q}$ parts by the rule $e^V \to e^V e^v$, $Q \to Q + q$. Our aim now is to construct an effective action as a gauge-invariant and $\mathcal{N} = 2$ supersymmetric functional of the background superfield V. The presence of the parameter t in (1.2) is very essential, and the direct integration in (1.2) can be explicitly done only in the Abelian case. However, the first (that noted in [10]) variation of (1.2) and second-order expansion in powers of quantum field v contain no t integration (modulo a total spinor derivative):

$$S \sim \int_0^1 dt\,\partial_t(v\bar{D}^\alpha\Gamma_\alpha) + \frac{1}{2}\int_0^1 dt\,\partial_t(v\bar{D}^\alpha\mathcal{D}_\alpha v) + \mathcal{O}(v^3). \tag{1.3}$$

It is well known that the linear in v term in (1.3) should be dropped when considering the effective action. The quadratic part S_2 of quantum action given in (1.3) depends on V via the dependence of $\mathcal{D}_M$ on background superfield. Each term in the action (1.3) is manifestly invariant with respect to the background gauge transformations.

We now proceed to the quantization of the theory in a manifest $\mathcal{N}=2$ supersymmetric form. To construct the effective action, we can use the Faddeev–Popov Ansatz. Within the framework of the background field method, we should fix only the quantum gauge transformations keeping the invariance under the background gauge transformations. It is convenient to choose the gauge-fixing functions in the form analogous to $\mathcal{N}=1, d4$ theories: $\bar{f}=\mathcal{D}^2 v$, $f=\bar{\mathcal{D}}^2 v$. These functions are covariantly (anti)chiral and transform under the quantum gauge transformations. Therefore, the ghost action is the same as in the four-dimensional $\mathcal{N}=1$ case: $S_{\rm FP}=\mathrm{tr}\int d^3x\, d^4\theta\,(b+\bar{b})L_{(1/2)v}[c+\bar{c}+\coth(L_{(1/2)v})(c-\bar{c})]$, where $c,\bar{c},b,\bar{b}$ covariantly chiral and antichiral superfields. The effective action for pure Chern–Simons theory is given by the following functional integral: $\mathrm{e}^{i\Gamma_{\rm CS}[V]}=\mathrm{e}^{iS_{\rm CS}[V]}\int\mathcal{D}v\,\mathcal{D}b\,\mathcal{D}c\,\delta[f-\bar{\mathcal{D}}^2 v]\,\delta[\bar{f}-\mathcal{D}^2 v]\,\mathrm{e}^{iS_2[V,v]+\mathcal{O}(v^3)+iS_{\rm FP}}$. Unlike in $\mathcal{N}=1, d4$ case, we average this expression with the following weight (see some details for $\mathcal{N}=2, d3$ theory in [12]): $1=\int\mathcal{D}f\mathcal{D}\bar{f}\mathcal{D}\varphi\mathcal{D}\bar{\varphi}\,\exp\left\{\frac{i}{2\alpha}\int d^5z f^2+\frac{i}{2\beta}\int d^5\bar{z}\bar{f}^2+i\int d^5z\varphi^2+i\int d^5\bar{z}\bar{\varphi}^2\right\}$, where α, β are the gauge-fixing parameters and the anticommuting third ghost superfield φ is background covariantly chiral. As a result, we see that the $\mathcal{N}=2$ super Chern–Simons theory is described within the background field approach by three ghosts. However, the opposite of $4d$ case, the Nielsen–Kallosh ghost, gives no rise to the effective action even at one-loop level.

Further we will study only one-loop effective action in gauge superfield sector. In this case, it is sufficient to consider, under the functional integral for $\Gamma_{\rm CS}[V]$, only the quadratic part of gauge-fixed action for quantum fields. Then one gets

$$S_2+S_{\rm gf}=\frac{1}{2}\mathrm{tr}\int d^7z\, v\frac{1}{4}\left(\mathcal{D}^\alpha\bar{\mathcal{D}}_\alpha+\bar{\mathcal{D}}^\alpha\mathcal{D}_\alpha+\frac{1}{\alpha}\mathcal{D}^\alpha\mathcal{D}_\alpha+\frac{1}{\beta}\bar{\mathcal{D}}^\alpha\bar{\mathcal{D}}_\alpha\right)v\equiv \equiv\frac{1}{2}\mathrm{tr}\int d^7z\, v\mathcal{H}_v v. \quad (1.4)$$

Now we should add the contribution of matter superfields. As a result, we get the following representation for the one-loop effective action in the gauge field sector:

$$\mathrm{e}^{i\Gamma^{(1)}[V]}=\mathrm{Det}^{-1/2}(\mathcal{H}_v)\,\mathrm{Det}(\mathcal{H}_{\rm FP})\,\mathrm{Det}^{-1/2}(\mathcal{H}_{\rm hyper}), \quad (1.5)$$

where $\mathcal{H}_{\rm FP}=\begin{pmatrix}0 & (1/16)\mathcal{D}^2\bar{\mathcal{D}}^2\\ -(1/16)\bar{\mathcal{D}}^2\mathcal{D}^2 & 0\end{pmatrix}\delta^{(7)}(z,z')$. The matter superfield contributions to the effective action differ from the contributions of ghosts only by the sign and choice of the representation of a gauge group.

2. ONE-LOOP EFFECTIVE ACTION

In this section, we investigate off-shell one-loop corrections to the action for $\mathcal{N}=2$ super Chern–Simons quantum field theory. It is well known that the one-loop effective action is given in terms of functional determinants of the differential operators in quadratic part of action for quantum fields. In the theory under consideration, all these operators are the generalized d'Alembertians acting on superfields. According to the previous section, there are three basic d'Alembertians which arise in the covariant supergraphs: (i) the vector d'Alembertian $\Box_{\rm v}$; (ii) the chiral d'Alembertian $\Box_+$; and (iii) the antichiral d'Alembertian $\Box_-$. The vector d'Alembertian is defined by

$$\Box_{\rm v}=\mathcal{H}_v^2=\frac{1}{16}\left[-\mathcal{D}\bar{\mathcal{D}}^2\mathcal{D}-\bar{\mathcal{D}}\mathcal{D}^2\bar{\mathcal{D}}+\frac{1}{\alpha\beta}\{\mathcal{D}^2,\bar{\mathcal{D}}^2\}-\right.$$
$$\left.-16G^2-\frac{8i}{\alpha}\bar{W}^\alpha\mathcal{D}_\alpha+\frac{8i}{\beta}W^\alpha\bar{\mathcal{D}}_\alpha\right]=\Box_{\rm cov}+\left(-1+\frac{1}{\alpha\beta}\right)\frac{1}{16}\{\mathcal{D}^2,\bar{\mathcal{D}}^2\}+$$
$$+\frac{i}{2}\left(W^\alpha-\frac{1}{\alpha}\bar{W}^\alpha\right)\mathcal{D}_\alpha-\frac{i}{2}\left(\bar{W}^\alpha-\frac{1}{\beta}W^\alpha\right)\bar{\mathcal{D}}_\alpha. \quad (2.1)$$

It is clear that the most convenient gauge choice is $\alpha=\beta=1$. The covariantly chiral d'Alembertian is defined by

$$\Box_+=\Box_{\rm cov}+iW^\alpha\mathcal{D}_\alpha+\frac{i}{2}(\mathcal{D}^\alpha W_\alpha)+G^2,$$
$$\Box_+\Phi=\frac{1}{16}\bar{\mathcal{D}}^2\mathcal{D}^2\Phi,\quad \bar{\mathcal{D}}_\alpha\Phi=0. \quad (2.2)$$

The operator $\mathcal{H}_v$ has the «first order in power ∂». Therefore, we must worry about the phase of the functional determinant. Following the pioneering work [7], we define the phase of the path integral by means of the superfield eta-invariant as $\eta_{\mathcal{H}}(s)=(1/2)\lim\limits_{s\to0}\sum\limits_i\operatorname{sign}\lambda_i|\lambda_i|^{-s}=\operatorname{Tr}\left(\mathcal{H}(\mathcal{H})^{-(s+1)/2}\right)$. Then $\dfrac{1}{\sqrt{\operatorname{Det}[\mathcal{H}]}}=\dfrac{1}{\sqrt{\operatorname{Det}|[\mathcal{H}]|}}\,\mathrm{e}^{i(\pi/4)\eta_{\mathcal{H}}(0)}$. In case of non-supersymmetric Chern–Simons theories, the phase was discussed in [7]. Our aim is to compute the $\eta_{\mathcal{H}}(0)$ in the theory under consideration. To do that, one uses the identity $\eta_{\mathcal{H}}(s)=\dfrac{1}{\Gamma((s+1)/2)}\int\limits_0^\infty dt\,t^{(s-1)/2}\operatorname{Tr}\mathcal{H}\,\mathrm{e}^{-t\mathcal{H}^2}$ and then puts $s=0$. For evaluating the integral, we replace the background field V by the field gV, with g being a real parameter. As a result, one gets the operator $\mathcal{H}(g)$, such that $\mathcal{H}(1)=\mathcal{H}$ and $\mathcal{H}(0)$ is background-field-independent. Differentiating the above equation, one obtains

$\delta_g \eta_{\mathcal{H}(g)}(s) = \dfrac{1}{\Gamma((s+1)/2)} \int\limits_0^\infty dt\, t^{(s-1)/2} 2t^{1/2} \dfrac{d}{dt} \operatorname{Tr}\{t^{1/2}\delta_g \mathcal{H}(g)\, \mathrm{e}^{-t\mathcal{H}^2(g)}\}$. Now we see that $\delta_g \eta_{\mathcal{H}(g)}$ is regular at $s=0$, and its value is given by a local invariant

$$\lim_{s\to 0} \delta_g \eta_{\mathcal{H}(g)}(s) = -\frac{2}{\sqrt{\pi}} \operatorname{Tr}\,(\sqrt{t}\delta_g \mathcal{H}(g)\, \mathrm{e}^{-t\mathcal{H}^2(g)})\Big|_{(t=0)}. \tag{2.3}$$

It is easy to see that the nonzero contribution in $\delta_g \eta_{\mathcal{H}(g)}(0)$ can result only from zero and first terms of power series expansion of (2.3) in t. Then, it is obvious that to obtain a nonzero result, we must put exactly four spinor derivatives on all Grassmann δ-functions. However, the operator $\delta_g \mathcal{H}(y)$, where $\delta_g \mathcal{D}_\alpha = [\mathcal{D}_\alpha, \mathrm{e}^{-gV}\delta_g\, \mathrm{e}^{gV}]$, $\delta_g \bar{\mathcal{D}}_\alpha = 0$, has a combination of spinor derivatives $\bar{\mathcal{D}}_\alpha$, $\mathcal{D}_\alpha$ of first-degree and the operator $\mathcal{H}^2(g)$ also has first-order spinor derivatives. Therefore, both the first terms vanish. It is known that the background field dependent $\eta_{\mathcal{H}}(0)$ gives rise to a finite shift of coupling constant in non-Abelian Chern–Simons action. Our result means that such a shift is absent in the theory under consideration.

One-loop effective action, generated by vector multiplet, is given by the expression

$$\Gamma_v^{(1)}[V] = \frac{i}{4} \operatorname{Tr}\, \ln \mathcal{H}_v^2 = \frac{i}{4} \operatorname{Tr} \int\limits_0^\infty \frac{dt}{t}\, \mathrm{e}^{-m^2 t}\, \mathrm{e}^{-t\mathcal{H}_v^2}, \tag{2.4}$$

where m is an infrared regulator. We will calculate the asymptotic expansion of the heat kernel in the integrand that takes the form of an expansion in the powers of covariant derivatives. Structure of such an expansion is defined by superfield De Witt coefficient. At the component level, the nontrivial De Witt coefficients, a_n for $n \geqslant 4$, contain in bosonic sector the field strength terms of the form F^n. The first nontrivial coefficient, a_4, is well-known in $d4$ [6]. In $d3$ we also have an analogous box diagram with factors $(i/2)\mathcal{W}^\alpha(\mathcal{D}+\bar{\mathcal{D}})_\alpha$ at each vertex, and to get nonzero result, one should keep terms with two D's and two $\bar{D}$'s. Besides, we should treat the gauge strength as matrix in the adjoint representation $W^\alpha_{ac} \equiv f_{abc} W^{\alpha_b}$. Then we get for a four-points contribution to the effective action:

$$\Gamma_v^{(1)} = -\frac{1}{256\pi m^5} \int d^7 z g(a_1,a_2,a_3,a_4)(\mathcal{W}^\alpha(a_1)\,\mathcal{W}_\alpha(a_2)\,\mathcal{W}^\beta(a_3)\,\mathcal{W}_\beta(a_4) - \\ - \frac{1}{2}\mathcal{W}^\alpha(a_1)\,\mathcal{W}^\beta(a_2)\,\mathcal{W}_\alpha(a_3)\,\mathcal{W}_\beta(a_4)), \tag{2.5}$$

where in $\mathcal{N}=2, d3$ case $\mathcal{W}^\alpha \equiv (W-\bar{W})^\alpha$. Here we have used for colour structures the notation $g(a_1,a_2,\ldots,a_n) = f_{b_1 a_1 b_2} f_{b_2 a_2 b_3} \ldots f_{b_n a_n b_1}$, where f_{abc} are the structure constants for a gauge Lie-algebra or F_{ABC} for a gauge 3-algebra.

Note that these terms do not have the Abelian analogue. They simply vanish in the Abelian case.

Contribution to one-loop effective action from Faddeev–Popov ghosts is defined by the expression $\mathrm{Tr}\ \ln \mathcal{H}_{\mathrm{FP}} = \mathrm{Tr}_-\ \ln \Box_- + \mathrm{Tr}_+\ \ln \Box_+$. That allows one to write the ghost contribution to the effective action $\Gamma^{(1)}_{gh}$ in the form of an integral over a proper time $i\Gamma^{(1)}_{\mathrm{gh}} = \int\limits_0^\infty \frac{dt}{t}\,\mathrm{e}^{-tm^2}(K_+(t) + K_-(t))$. In this expression, m^2 is the infrared cutoff, and $K_+(t)$ and $K_-(t)$ are the functional traces of the chiral and antichiral heat kernels, respectively. It is well-known that $K_+(t) = K_-(t)$. Therefore, we discuss only the computation of the chiral kernel. One of the procedures in computations of the heat kernel is to make use of the Fourier integral representation of delta function: $\delta^{(7)}(z - z')\mathbf{1} = \int \frac{d^3p}{(2\pi)^3}\,\mathrm{e}^{i/2\rho^{\alpha\beta}p_{\alpha\beta}}\zeta^2\bar{\zeta}^2 I(z, z')$, where $\rho^m = (x - x')^m - i\zeta\gamma^m\bar{\theta}' + i\theta'\gamma^m\bar{\zeta}$, $\zeta = \theta - \theta'$, $\bar{\zeta} = \bar{\theta} - \bar{\theta}'$. Here $I(z, z')$ is the parallel displacement operator along the geodesic line connecting the points z' and z. The heat kernel $K_+(z, t)$ has an asymptotic Schwinger–De Witt expansion, which is written as $K_+(z, t) = \frac{i}{(4\pi t)^{3/2}} \sum\limits_{n=0}^{\infty} t^n a_n(z)$, $a_0 = a_1 = 0$. The $a_n(z)$ are the De Witt coefficients, which at the component level contain bosonic field strength terms of the form F^n. From dimensional considerations and the requirement of gauge invariance, we can expect that the first nontrivial coefficient a_2 in the non-Abelian case is $a_2 \sim \mathrm{tr}_{\mathcal{R}} \int d^5 z W^2 \sim \mathrm{tr}_{\mathcal{R}} \int d^7 z G^2$. One can show that the a_n with $n \geqslant 2$ are obtained in form of $\bar{\mathcal{D}}^2$ acting on superfield strengths and their covariant derivatives, and hence all terms in $K_+(z, z'|t)$ can be written as the gauge-invariant superfunctionals on full superspace. By differentiating the kernel $K_+(z, z'|t)$ with respect to t, one observes that $\frac{dK_+(t)}{dt} = \mathrm{tr}_{\mathcal{R}} \int d^7 z \frac{1}{4}\mathcal{D}^2\,\mathrm{e}^{-t\Box_+}\delta_+(z, z')|_{z=z'}$. It is convenient to introduce a new set of coefficients by writing $\mathcal{D}^2\,\mathrm{e}^{-t\Box_+}\delta_+(z, z')|_{z=z'} = \frac{1}{(4\pi t)^{3/2}} \sum t^n c_n(z)$, as an asymptotic series. Here $a_n(z) = \frac{1}{n - 3/2}\Big(-\frac{1}{4}\bar{\mathcal{D}}^2\Big)c_{n-1}(z)$. The effective action can then be written as

$$\Gamma^{(1)}_{\mathrm{gh}} = -\frac{1}{2\pi^{3/2}} \sum_{n=2}^{\infty} \frac{\Gamma(n - 3/2)}{(2n - 3)m^{2n-3}} \int d^7 z\, \mathrm{tr}_{\mathcal{R}}\, c_{n-1}. \tag{2.6}$$

Here $\mathcal{R}$ means adjoint representation. Matter contribution has the same form as (2.6), with $\mathcal{R}$ being a corresponding representation.

Our next goal is to discuss the computations of the superfield coefficients c_1, c_2 and c_3. In some respects, this procedure is similar to that used in [13] for constructing a gauge-invariant derivative expansion of the effective action in the Yang–Mills theory. Using the identities $\int d^2\eta\,\eta^2 = -4$, we present a ζ^2 as $\zeta^2 = \int d^2\eta\, \mathrm{e}^{\eta^\alpha \zeta_\alpha}$. Then $(d/dt)K_+(z,z',t)$ in the point coincidence limit becomes

$$K^\alpha_\alpha = \int \frac{d^3p}{(2\pi)^3} \frac{1}{4} d^2\eta X^\alpha X_\alpha\, \mathrm{e}^{-t\Delta} \cdot 1. \tag{2.7}$$

The operator $\Delta = (1/2)X^{\alpha\beta}X_{\alpha\beta} + iX^\alpha W_\alpha + G^2$ is defined by $X_m = \mathcal{D}_m + ip_m$, $X^\alpha = \mathcal{D}^\alpha + \eta^\alpha - p_{\alpha\beta}\bar{\zeta}^\beta$. Expanding the exponential in powers of proper time around $\mathrm{e}^{p^2 t}$ and integrating over p, we obtain the desired expansion, collecting together the coefficients at each degree of proper time t. Due to gauge invariance, these coefficients are actually expressed in terms of commutators of covariant derivatives.

Zeroth-order term does not depend on background fields. In the next terms of the expansion, we must take into account that $X^3_\alpha = 0$ and the integrals over odd powers of p vanish. Therefore, in the first order of expansion of (2.7), we have $-t(\Box + G^2)$, with a factor $i/(4\pi t)^{3/2}$, which is common in the expansion. In the next order of expansion after integration over p, we have exactly $+t\Box$ that cancels gauge-noninvariant contribution, and then one gets $c_1 = G^2$, as mentioned above*. As a result, in the given order of the expansion of the heat kernel, we obtain the super Yang–Mills action as a leading low-energy contribution to effective action. The IR cutoff parameter plays a role of the dimensional coupling constant:

$$\Gamma^{(1)}_{\mathrm{gh}} = -\frac{1}{4\pi}\, \mathrm{tr}_{Adj} \int d^5z \frac{1}{m} W^\alpha W_\alpha. \tag{2.8}$$

Discuss some consequences of (2.8). First, we see that the leading low-energy quantum correction to action is Yang–Mills and stipulated only by ghosts and matter, vector multiplet does not give rise. Second, since a contribution of matter to effective action has, up to a sign, the same form as ghost contribution, one can conclude that for appropriate matter in adjoint representation a total contribution of ghost and matter to effective action vanishes. Third, one applies the above consideration to the BLG model formulated in terms of $\mathcal{N} = 2, d3$ superfields. In this case, the ghost and matter superfields take the values in a real 3-algebra. The induced Yang–Mills action contains a factor $F^D_{AC}F^C_{BD} - 2(T_A)^J_I(T_B)^I_J = -2G_{AB}$ [3]. Therefore, the leading low-energy correction to action is $\Gamma_{\mathrm{YM}} = \dfrac{1}{2\pi} G_{AB} \int d^5z \dfrac{1}{m} W^{A\alpha} W^B{}_\alpha$.

*Such a cancellation of gauge-noninvariant terms should take place in any order of heat kernel expansion.

Using the second and third terms in expansion in proper time under the integral (2.7), one finds the coefficient c_2 in the form

$$c_2 = \left\{ \frac{1}{2}G^4 + \frac{1}{12}[\mathcal{D}^m, \mathcal{D}^n][\mathcal{D}_m, \mathcal{D}_n] + \frac{1}{6}[\mathcal{D}^m, [\mathcal{D}_m, G^2]] - \frac{1}{2}[\mathcal{D}^\alpha, G^2]iW_\alpha + \right. \\ \left. + \frac{1}{6}[\mathcal{D}^m, [\mathcal{D}_m, \mathcal{D}^\alpha]]iW_\alpha + \frac{1}{3}[\mathcal{D}^m, \mathcal{D}^\alpha][\mathcal{D}_m, iW_\alpha] \right\}. \quad (2.9)$$

In principle, all commutators can be expressed in terms of strengths and their covariant derivatives. As a result, one gets in bosonic sector the terms of the form $\sim f^3, (\mathcal{D}f)^2$, where f_{mn} is bosonic strength. The next c_3 coefficient in expansion (2.6) has a complicated and cumbersome enough structure. Its explicit form is given in [14]. In leading bosonic component sector, this coefficient gives us the terms like $(f_{mn})^4$ and the products of some power of f_{mn} and some powers of covariant derivatives $\mathcal{D}_m f_{nl}$ with total dimension 8.

To conclude, we have developed the background method for constructing the gauge-invariant effective action in non-Abelian $\mathcal{N} = 2, d = 3$ supersymmetric Chern–Simons theory coupled to matter, and studied a structure of the one-loop effective action. These results open the possibilities for studying the quantum dynamics in various extended supersymmetric $d = 3$ models.

Acknowledgements. The authors thank the Organizing Committee of the SQS'2011 conference for warm welcome and partial support. The work was partially supported by RFBR grant, project No. 12-02-00121, and by a grant for LRSS, project No. 224.2012.2. Also, I. L. B. acknowledges the support from RFBR grants No. 11-02-90445. N. G. P. acknowledges the support from RFBR grant, project No. 11-02-00242.

REFERENCES

1. *Bagger J., Lambert N.* // Phys. Rev. D. 2007. V. 75. P. 045020; 2008. V. 77. P. 065008; *Gustavsson A.* // Nucl. Phys. B. 2009. V. 811. P. 66.
2. *Aharony O. et al.* // JHEP. 2008. V. 0810. P. 091.
3. *Bagger J., Lambert N.* // Phys. Rev. D. 2009. V. 79. P. 025002; *Cherkis S., Saemann C.* // Phys. Rev. D. 2008. V. 78. P. 066019; *Schnabl M., Tachikawa Y.* // JHEP. 2010. V. 1009. P. 103.
4. *Gustavsson A.* // Nucl. Phys. B. 2009. V. 807. P. 315; *Akerblom N., Saemann C., Wolf M.* // Nucl. Phys. B. 2010. V. 826. P. 456.
5. *de Wit B., Hoppe J., Nicolai H.* // Nucl. Phys. B. 1988. V. 305. P. 545; *de Wit B., Lusher M., Nicolai H.* // Nucl. Phys. B. 1989. V. 320. P. 135; *Taylor W.* // Rev. Mod. Phys. 2001. V. 73. P. 419.

6. *Chepelev I., Tseytlin A. A.* // Nucl. Phys. B. 1998. V. 515. P. 73;
Refolli A. et al. // Nucl. Phys. B. 2001. V. 613. P. 64.
7. *Witten E.* // Commun. Math. Phys. 1989. V. 121. P. 351;
Birmingham D. et al. // Phys. Rep. 1991. V. 209. P. 129;
Dilkes F. A. et al. // Intern. J. Mod. Phys. A. 1999. V. 14. P. 463.
8. *Buchbinder I. L. et al.* // JHEP. 2009. V. 0903. P. 096; V. 0910. P. 075.
9. *Buchbinder I. L., Pletnev N. G., Samsonov I. B.* // JHEP. 2010. V. 1004. P. 124; 2011. V. 1101. P. 121.
10. *Ivanov E. A.* // Phys. Lett. B. 1991. V. 268. P. 203.
11. *Zupnik B. M., Pak D. G.* // Teor. Mat. Fiz. 1988. V. 77. P. 97.
12. *Bianchi M. S., Penati S., Siani M.* // JHEP. 2010. V. 1005. P. 106.
13. *Diakonov D., Petrov V. Yu., Yung A. V.* // Phys. Lett. B. 1983. V. 130. P. 385.
14. *Buchbinder I. L., Pletnev N. G.* // JHEP. 2011. V. 1111. P. 085.