ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2012. Т. 43. ВЫП. 5

NEW CHALLENGES IN UNIFIED THEORIES

*G. Zoupanos**

Theory Group, Physics Department, CERN, Geneva, Switzerland**

Among the research directions that we have presented during the Workshop SQS'2011, we have chosen to discuss here in some detail the derivation of the effective action in four dimensions of the ten-dimensional $N = 1$ heterotic supergravity coupled to $N = 1$ supersymmetric Yang–Mills resulting from the dimensional reduction over nearly Kähler manifolds.

PACS: 11.30.Pb; 12.10.Dm

INTRODUCTION

A large and sustained effort has been done in the recent years, aiming to achieve a unified description of all interactions. Out of this endeavor, two main directions have emerged as the most promising to attack the problem, namely, the superstring theories and noncommutative geometry. The two approaches, although at a different stage of development, have common unification targets and share similar hopes for exhibiting improved renormalization properties in the ultraviolet (UV) as compared to ordinary field theories. Moreover, the two frameworks came closer by the observation that a natural realization of noncommutativity of space appears in the string theory context of D-branes in the presence of a constant background antisymmetric field. However, despite the importance of having frameworks to discuss quantum gravity in a self-consistent way and possibly to construct there finite theories, it is very interesting to search for the minimal realistic framework in which finiteness can take place. In addition, the main goal expected from a unified description of interactions by the particle physics community is to understand the present-day large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. In other words, to achieve reduction of couplings at a more fundamental level. It is a thoroughly fascinating fact that many interesting ideas that have survived various theoretical and phenomenological tests, as well as the solution to the UV divergences problem, find a common ground in the framework of $N = 1$ Finite

*E-mail: zoupanos@cern.ch

**On leave from Physics Department, National Technical University of Athens, Zografou Campus, GR-15780 Zografou, Greece.

Unified Theories, which is one of the research directions that we have presented at the Workshop SQS'2011. From the theoretical side, they solve the problem of UV divergences in a minimal way. On the phenomenological side, since they are based on the principle of reduction of couplings (expressed via RGI relations among couplings and masses), they provide strict selection rules in choosing realistic models which lead to testable predictions [1].

Given the above considerations, our group is developing activities in the following research directions:

1. Dimensional Reduction of the heterotic string over nearly Kähler Manifolds [2, 4].

2. Higher dimensional Unification with Fuzzy Extra Dimensions [5–8].

3. Finite Unification [1].

Due to lack of space, we discuss in the present contribution only parts of the first research direction.

1. DIMENSIONAL REDUCTION OVER NEARLY KÄHLER MANIFOLDS

The main research objective in this direction is the full dimensional reduction of the Heterotic String using nonsymmetric coset spaces, which admit an $SU(3)$-structure, in the presence of background fluxes and gaugino condensates. CY manifolds were proposed as internal spaces for compactifications in view of the requirement that a four-dimensional $N = 1$ supersymmetry is preserved. Namely, they admit a nowhere-vanishing, globally defined spinor, which is covariantly constant with respect to the (torsionless) Levi-Civita connection. However, a wider class of manifolds exists for which the spinor is covariantly constant with respect to a connection with torsion. These are called manifolds with $SU(3)$-structure and clearly CY manifolds belong to this class too.

$SU(3)$-structure manifolds are characterized by a real 2-form J and a complex 3-form Ω, which may be defined as bilinear forms of the covariantly constant spinor, and they satisfy certain algebraic and differential relations. The structure forms J and Ω are not closed, and in particular their exterior derivatives define the five intrinsic torsion classes $W_i, i = 1, \ldots, 5$, which fully characterize the intrinsic torsion of the manifold and may be used in the classification of the different types of manifolds. It is worth noting that a complex manifold has vanishing torsion classes W_1 and W_2, while a Kähler manifold has in addition vanishing torsion classes W_3 and W_4. A CY manifold has all the torsion classes equal to zero, and only in this case the structure forms J and Ω are evidently closed.

In the present study, we focus on an interesting class of $SU(3)$-structure manifolds called nearly Kähler manifolds. In this case all the intrinsic torsion classes but W_1 are zero. The fact that the torsion class W_1 does not vanish means, according to the above, that the manifolds we deal with are not complex. The homogeneous nearly Kähler manifolds in six dimensions have been completely classified, and they are the coset spaces $G_2/SU(3)$, $Sp4/SU(2) \times U(1)$ and $SU(3)/U(1) \times U(1)$ and the group manifold $SU(2) \times SU(2)$.

A very interesting feature of the six-dimensional nonsymmetric coset spaces is that they have simple and well-known geometry. Indeed, the most general S-invariant metric can be easily determined, and the S-invariant p-forms are known explicitly. Indeed, as far as the most a general S-invariant metric is concerned, it is always diagonal and depends on the number of radii that each spaces admits. In particular, $G_2/SU(3)$ admits only one radius R_1, $Sp4/SU(2) \times U(1)$ admits two radii R_1, R_2, and $SU(3)/U(1) \times U(1)$ admits three radii R_1, R_2, R_3. Then the metric fluctuations can be parameterized by one, two and three scalar fields, respectively. Regarding the S-invariant p-forms on these manifolds, the common feature is that they do not admit S-invariant 1-forms. On the contrary, S-invariant 2-forms ω_i exist in all cases, and in particular there is one for $G_2/SU(3)$, two for $Sp4/SU(2) \times U(1)$ and three for $SU(3)/U(1) \times U(1)$. Moreover, all the three spaces admit two S-invariant 3-forms ρ_1 and ρ_2.

In two recent publications [2, 3], we have studied the dimensional reduction of the low-energy field theory limit of the Heterotic String on nearly Kähler manifolds. In particular, the bosonic sector of the ten-dimensional action was studied, which contains the Einstein–Hilbert action in ten dimensions, the dilaton, the NS–NS 3-form H and the gauge fields. Initially, all the fields of the theory were expressed in terms of their four-dimensional counterparts, and the ten-dimensional action was dimensionally reduced to four dimensions. Subsequently, a case-by-case analysis for all the nearly Kähler manifolds was performed by applying the general results in all the specific cases, thus obtaining the corresponding effective actions in four dimensions for each case. The resulting theories are $N = 1$ supersymmetric E_6 GUTs, and they also contain terms which could be interpreted as soft scalar masses and trilinear soft terms in four dimensions, in case the minimization of the full potential would lead to Minkowski vacuum. Their superpotential is determined in a straightforward way via the heterotic Gukov–Vafa–Witten formula, and their Kähler potential is determined using results of the special Kähler geometry. It would be interesting to study further the possibility to determine a Minkowski vacuum with stabilized moduli in this context. Several directions exist in order to explore this possibility, such as gaugino condensation and the KKLT scenario. Moreover, the inclusion of more general fluxes combined with the ones already considered could serve as another possibility to determine such vacua.

On the other hand, progress has been made recently concerning the dimensional reduction of the $N = 1$ supersymmetric E_8 gauge theory, resulting in the field theory limit of the heterotic string over the nearly Kähler manifold $SU(3)/U(1) \times U(1)$. More specifically, an extension of the Standard Model (SM) inspired by the $E_8 \times E_8$ heterotic string was derived [4] (see also the contribution of N. Irges and G. Zoupanos in the present proceedings). In order that a reasonable effective Lagrangian is presented, we neglected everything else other than the $N = 1$ ten-dimensional supersymmetric Yang–Mills sector associated with one of the gauge factors and certain couplings necessary for anomaly cancellation. A compactified space-time $M_4 \times B_0/Z_3$ was considered, where B_0 is the nearly Kähler coset manifold $S/R = SU(3)/(U(1) \times U(1))$ and Z_3 a freely acting discrete group on B_0. Then we reduced dimensionally the E_8 on this manifold and employed the Wilson flux mechanism, leading in four dimensions to a gauge theory with the spectrum of an $N = 1$ supersymmetric theory. We computed the effective four-dimensional Lagrangian and demonstrated that an extension of the SM is obtained. The gauge group contains, beyond that of the SM, two extra anomalous $U(1)$ factors. One of them is precisely the Baryon number (B) and the other a Peccei–Quinn (PQ) symmetry. They both break at a high scale, leaving their corresponding global quantum numbers conserved at low energies. Their anomalies are cancelled by a combination of Stueckelberg and Wess–Zumino terms which render the effective Lagrangian gauge-invariant [9]. After Electroweak (EW) symmetry breaking the low-energy spectrum is affected by the presence of the extra $U(1)$'s via heavy neutral gauge bosons, the so-called Z' gauge bosons, just as in D-brane models [10]. The mass scale of these Z' bosons is determined by the radii of the compact space and can be as low as a few TeV. The Z' associated with B is naturally leptophobic. Anomalous $U(1)$'s typically have an additional distinctive signature, the so-called axi-Higgs [9], which is an axion-like particle that survives at low energies. It originates from the anomaly-cancelling terms and should be considered as the low-energy remnant of the ten-dimensional Green–Schwarz (GS) mechanism. We consider the latter one of the most characteristic low-energy signatures of string theory.

2. TOWARDS DIMENSIONAL REDUCTION OF THE HETEROTIC STRING

2.1. Nearly Kähler Manifolds. According to the discussion in Sec. 1, CY manifolds are extensively used in string compactifications because they preserve $\mathcal{N} = 1$ supersymmetry in four dimensions [11]. This feature stems from the existence of a nowhere-vanishing, globally defined spinor on these manifolds, which is covariantly constant with respect to the Levi-Civita connection. However, another possibility is to use a connection with torsion. The six-dimensional manifolds which admit a spinor which is covariantly constant with respect to the new connection are called manifolds with an $SU(3)$-structure. Their structure is completely specified by a real two-form J and a complex three-form Ω, which are both globally-defined and nonvanishing and they satisfy the compatibility conditions

$$J \wedge J \wedge J = \frac{3}{4} i\Omega \wedge \Omega^*, \quad J \wedge \Omega = 0. \tag{1}$$

In addition, the exterior derivatives of these structure forms define the components $\mathcal{W}_i, i = 1, \ldots, 5$, of the intrinsic torsion as

$$\begin{aligned} dJ &= \frac{3}{4} i(\mathcal{W}_1 \Omega^* - \mathcal{W}_1^* \Omega) + \mathcal{W}_4 \wedge J + \mathcal{W}_3, \\ d\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega. \end{aligned} \tag{2}$$

The intrinsic torsion classes $\mathcal{W}_i$ can be used to classify several types of manifolds [12].

Nearly Kähler manifolds are defined in the above context as the ones whose only nonvanishing torsion class is $\mathcal{W}_1$. The homogeneous nearly Kähler manifolds in six dimensions have been classified in [13], and they are the coset spaces $G_2/SU(3)$, $Sp_4/SU(2) \times U(1)_{\rm nonmax}$, $SU(3)/U(1) \times U(1)$ and the group manifold $SU(2) \times SU(2)$. The first three cases are well-known to be the only nonsymmetric coset spaces S/R in six dimensions which preserve the rank, namely $\operatorname{rank} S = \operatorname{rank} R$. They have been studied extensively in [14] in the reduction of ten-dimensional gauge theories to four dimensions. Therefore, it is interesting to study the dimensional reduction of the heterotic supergravity Yang–Mills theory over these spaces and determine the corresponding effective actions in four dimensions. In our examination we ignore the group manifold case since it cannot lead to chiral fermions in four dimensions.

In order to perform the dimensional reduction from ten to four dimensions, the forms on which several fields will be expanded have to be specified. A natural basis of expansion forms consists of the S-invariant forms of the manifolds [15]. Let us mention that all the nonsymmetric coset spaces do not admit S-invariant one-forms. However, S-invariant two-forms, which we shall denote by ω_i, exist in all cases, and in particular there is one for $G_2/SU(3)$, two for $Sp_4/(SU(2) \times U(1))_{\rm nonmax}$ and three for $SU(3)/U(1) \times U(1)$. Finally, all the three spaces admit two S-invariant three-forms, which we shall denote by ρ_1 and ρ_2. Forms of higher rank also exist, but they are not important in our context. The above forms are related to the structure forms J and Ω of the $SU(3)$-structure as

$$J = R_i^2 \omega_i, \;\; \Omega = V(\rho_2 + i\rho_1), \tag{3}$$

where R_i refers to the radii of the manifolds and V to their volume. In particular, $G_2/SU(3)$ admits only one radius R_1, $Sp_4/(SU(2) \times U(1))_{\rm nonmax}$ admits two radii R_1, R_2 and, finally, $SU(3)/U(1) \times U(1)$ admits three radii R_1, R_2, R_3. Therefore, the volume is R_1^3, $R_1^2 R_2$ and $R_1 R_2 R_3$, respectively, in each case.

2.2. Dimensional Reduction. The bosonic sector of the Lagrangian of the heterotic supergravity coupled to supersymmetric Yang–Mills [16], which is the low-energy limit of the heterotic superstring theory, can be written as

$$\hat{e}^{-1}\mathcal{L}_b = -\frac{1}{2\hat{\kappa}^2}\bigg(\hat{R}\hat{*}\mathbf{1} + \frac{1}{2}\,\mathrm{e}^{-\hat{\phi}}\hat{H}_{(3)}\wedge\hat{*}\hat{H}_{(3)} + \frac{1}{2}d\hat{\phi}\wedge\hat{*}d\hat{\phi} + \\ + \frac{\alpha'}{2}\,\mathrm{e}^{-\hat{\phi}/2}\,\mathrm{Tr}\,(\hat{F}_{(2)}\wedge\hat{*}\hat{F}_{(2)})\bigg). \quad (4)$$

The above Lagrangian, written in the Einstein frame, contains the ten-dimensional Einstein–Hilbert action, the kinetic term of the ten-dimensional dilaton $\hat{\phi}$, the kinetic term for the gauge fields $\hat{A}$ and the corresponding one for the three-form $\hat{H}$. The hats denote ten-dimensional fields, while $\hat{\kappa}$ is the gravitational coupling constant in ten dimensions with dimensions $[\text{length}]^4$; $\hat{e}$ is the determinant of the metric, while $\hat{*}$ is the Hodge star operator in ten dimensions.

In order to dimensionally reduce the above Lagrangian to four dimensions, we perform the following Ansätze for the fields appearing in (4). The metric Ansatz is

$$d\hat{s}^2 = \mathrm{e}^{2\alpha\varphi(x)}\eta_{mn}\,\mathrm{e}^m\,\mathrm{e}^n + \mathrm{e}^{2\beta\varphi(x)}\gamma_{ab}(x)\,\mathrm{e}^a\,\mathrm{e}^b, \quad (5)$$

where $\mathrm{e}^{2\alpha\varphi(x)}\eta_{mn}$ is the four-dimensional metric and $\mathrm{e}^{2\beta\varphi(x)}\gamma_{ab}(x)$ is the internal metric, while e^m are the one-forms of the orthonormal basis in four dimensions and e^a are the left-invariant one-forms on the coset space. The exponentials rescale the metric components in order to obtain an action without any prefactor for the four-dimensional Einstein–Hilbert part. In order to achieve this, we have to choose the values of α and β to be $-\sqrt{3}/4$ and $\sqrt{3}/12$, respectively. Moreover, the dilaton is trivially reduced by $\hat{\phi}(x,y) = \phi(x)$, since it is already a scalar in ten dimensions.

The three-form $\hat{H}$ is given in general by

$$\hat{H} = \hat{d}\hat{B} - \frac{\alpha'}{2}\hat{\omega}_{\mathrm{YM}}, \quad (6)$$

where $\hat{B}$ is the Abelian two-form potential, which we expand in the S-invariant forms of the internal space as

$$\hat{B} = B(x) + b^i(x)\omega_i(y), \quad (7)$$

and $\hat{\omega}_{YM}$ is the Yang–Mills–Chern–Simons form. In (6) a term involving the Lorentz–Chern–Simons form has to be included too. However, it is not needed in the minimal supergravity Lagrangian and therefore we shall not consider it here.

Finally, in order to reduce the gauge sector, we employ the Coset Space Dimensional Reduction (CSDR) scheme [14]. The original CSDR of a multidimensional gauge field $\hat{A}$ on a coset S/R is described by a generalized invariance condition

$$\mathcal{L}_{X^I}\hat{A} = DW_I = dW_I + [\hat{A}, W_I], \tag{8}$$

where W_I is a parameter of a gauge transformation associated with the Killing vector X_I of S/R and $\mathcal{L}_{X^I}$ denotes the Lie derivative with respect to X^I. The Ansatz for the higher dimensional gauge field that solves the invariance condition (8) is

$$\hat{A}^{\tilde{I}} = A^{\tilde{I}} + \phi_A^{\tilde{I}}\,\mathrm{e}^A, \tag{9}$$

where $\tilde{I}$ is a gauge index and A an S-index, which can be split into indices i, a running in the group R and the coset, respectively. The same Ansatz is used for the reduction of the $\hat{\omega}_{\mathrm{YM}}$. It is important to mention that in the CSDR scheme the gauge group G in ten dimensions is broken down to the centralizer $C_G(R)$ of the group R in G in four dimensions. In the present cases, the initial $E_8 \times E_8$ gauge group is broken down to $E_6 \times E_8$ and therefore the resulting theories are $\mathcal{N} = 1$ supersymmetric E_6 GUTs. Moreover, in the CSDR scheme the soft supersymmetry breaking sector of the four-dimensional $\mathcal{N} = 1$ theories is obtained [17].

The effective action in four dimensions is obtained by substituting the above expressions in the original ten-dimensional action. This procedure results in the Lagrangian

$$\begin{aligned}\mathcal{L}_b = -\frac{1}{2\kappa^2}R*1 - \frac{1}{2}\,\mathrm{Re}\,(f)F^I\wedge *F^I + \frac{1}{2}\,\mathrm{Im}\,(f)F^I\wedge F^I - \\ -\frac{1}{\kappa^2}G_{i\bar{j}}d\Phi^i\wedge *d\bar{\Phi}^{\bar{j}} - V(\Phi,\overline{\Phi}).\end{aligned} \tag{10}$$

In (10) κ is the gravitational coupling in four dimensions, related to the ten-dimensional one by $\kappa^2 = \hat{\kappa}^2/V$; f is the gauge kinetic function, and $G_{i\bar{j}}$ is the Kähler metric. The potential has the form

$$V(\Phi,\bar{\Phi}) = \frac{1}{\kappa^4}\,\mathrm{e}^{\kappa^2 K}\left(K^{i\bar{j}}\frac{DW}{D\Phi^i}\frac{D\overline{W}}{D\bar{\Phi}^{\bar{j}}} - 3\kappa^2 W\overline{W}\right) + D - \text{terms}, \tag{11}$$

where the derivatives involved are the Kähler covariant derivatives, W is the superpotential, and by Φ^i we collectively denote the chiral supermultiplets. The superpotential W can be determined by the Gukov–Vafa–Witten formula [18, 19], which has the form

$$W = \frac{1}{4}\int_{S/R}\Omega\wedge(\hat{H} + idJ), \tag{12}$$

while the Kähler potential can be determined as the sum of two terms $K = K_S + K_T$, which are given by the expressions

$$K_S = -\ln(S + S^*), \quad K_T = -\ln\left(\frac{1}{6}\int\limits_{S/R} J \wedge J \wedge J\right), \tag{13}$$

where S is the dilaton superfield. A third contribution to the Kähler potential due to the complex structure moduli is not present here since the $SU(3)$-structures we consider are real.

Applying (12) to the cases of $G_2/SU(3)$, $Sp_4/(SU(2) \times U(1))_{\rm nonmax}$ and $SU(3)/U(1) \times U(1)$, respectively, the corresponding superpotentials take the form

$$W_1 = 3T_1 - \sqrt{2}\alpha' d_{ijk} B^i B^j B^k, \tag{14}$$

$$W_2 = 2T_1 + T_2 - \sqrt{2}\alpha' d_{ijk} B^i B^j \Gamma^k, \tag{15}$$

$$W_3 = T_1 + T_2 + T_3 - \sqrt{2}\alpha' d_{ijk} A^i B^j \Gamma^k, \tag{16}$$

where T_i are the superfields of the geometric moduli, A^i, B^i, Γ^i are the vector superfields, and d_{ijk} is the symmetric tensor of E_6. Also, applying (13), the corresponding Kähler potentials are

$$K_1 = -\ln(S + S^*) - 3\ln(T_1 + T_1^* - 2\alpha' B_i B^i), \tag{17}$$

$$K_2 = -\ln(S + S^*) - 2\ln(T_1 + T_1^* - 2\alpha' B_i B^i) - \\ - \ln(T_2 + T_2^* - 2\alpha' \Gamma_i \Gamma^i), \tag{18}$$

$$K_3 = -\ln(S + S^*) - \ln(T_1 + T_1^* - 2\alpha' A_i A^i) - \ln(T_2 + T_2^* - 2\alpha' B_i B^i) - \\ - \ln(T_3 + T_3^* - 2\alpha' \Gamma_i \Gamma^i). \tag{19}$$

Finally, in all the three cases the gauge kinetic function turns out to be $f(S) = S$.

CONCLUSIONS

In this contribution we have presented the main steps of the derivation of the four-dimensional effective action which results from the heterotic supergravity coupled to supersymmetric Yang–Mills theory from ten dimensions to four, using homogeneous six-dimensional nearly Kähler manifolds as internal spaces.

Acknowledgements. It is a pleasure to thank the organizers for the warm hospitality. This work is supported by the NTUA programmes for basic research PEVE 2009 and 2010 and the European Union's ITN programme «UNILHC» PITN-GA-2009-237920.

REFERENCES

1. *Heinemeyer S., Mondragon M., Zoupanos G.* Finite Unification: Theory and Predictions // SIGMA. 2010. V. 6. P. 049.
2. *Chatzistavrakidis A., Zoupanos G.* Dimensional Reduction of the Heterotic String over Nearly Kähler Manifolds // JHEP. 2009. V. 0909. P. 077.
3. *Chatzistavrakidis A., Manousselis P., Zoupanos G.* Reducing the Heterotic Supergravity on Nearly Kähler Coset Spaces // Fortsch. Phys. 2009. V. 57. P. 527.
4. *Irges N., Zoupanos G.* Reduction of $N = 1, E_8$ SYM over $SU(3)/U(1) \times U(1) \times Z_3$ and Its Four-Dimensional Effective Action // Phys. Lett. B. 2011. V. 698. P. 146.
5. *Aschieri P. et al.* Dimensional Reduction over Fuzzy Coset Spaces // JHEP. 2004. V. 0404. P. 034.
6. *Chatzistavrakidis A., Zoupanos G.* Higher-Dimensional Unified Theories with Fuzzy Extra Dimensions // SIGMA. 2010. V. 6. P. 063.
7. *Chatzistavrakidis A., Steinacker H., Zoupanos G.* Orbifolds, Fuzzy Spheres and Chiral Fermions // JHEP. 2010. V. 1005. P. 100.
8. *Chatzistavrakidis A., Steinacker H., Zoupanos G.* Fuzzy Extra Dimensions and Particle Physics Models // PoS CNCFG. 2010. V. 2010. P. 014.
9. *Coriano C., Irges N., Kiritsis E.* On the Effective Theory of Low Scale Orientifold String Vacua // Nucl. Phys. B. 2006. V. 746. P. 77.
10. *Ghilencea D. M. et al.* TeV Scale Z' Bosons from D-Branes // JHEP. 2002. V. 0208. P. 016.
11. *Candelas P. et al.* Vacuum Configurations for Superstrings // Nucl. Phys. B. 1985. V. 258. P. 46.
12. *Grana M.* Flux Compactifications in String Theory: A Comprehensive Review // Phys. Rep. 2006. V. 423. P. 9.
13. *Butruille J.-B.* Homogeneous Nearly Kähler Manifolds. arXiv:math.DG/0612655.
14. *Kapetanakis D., Zoupanos G.* Coset Space Dimensional Reduction of Gauge Theories // Phys. Rep. 1992. V. 219. P. 1.
15. *Mueller-Hoissen F., Stuckl R.* Coset Spaces and Ten-Dimensional Unified Theories // Class. Quant. Grav. 1988. V. 5. P. 27.
16. *Bergshoeff E. et al.* Ten-Dimensional Maxwell–Einstein Supergravity, Its Currents, and the Issue of Its Auxiliary Fields // Nucl. Phys. B. 1982. V. 195. P. 97;
    Chapline G. F., Manton N. S. Unification of Yang–Mills Theory and Supergravity in Ten Dimensions // Phys. Lett. B. 1983. V. 120. P. 105.
17. *Manousselis P., Zoupanos G.* Dimensional Reduction over Coset Spaces and Supersymmetry Breaking // JHEP. 2002. V. 0203. P. 002.
18. *Gukov S., Vafa C., Witten E.* CFT's from Calabi–Yau Four-Folds // Nucl. Phys. B. 2000. V. 584. P. 69; Erratum // Nucl. Phys. B. 2001. V. 608. P. 477.
19. *Gukov S.* Solitons, Superpotentials and Calibrations // Nucl. Phys. B. 2000. V. 574. P. 169.