PHOTON–GRAVITON AMPLITUDES FROM THE EFFECTIVE ACTION

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We report on the status of an ongoing effort to calculate the complete one-loop low-energy effective actions in the Einstein–Maxwell theory with a massive scalar or spinor loop, and to use them for obtaining the explicit form of the corresponding M-graviton/N-photon amplitudes. We present explicit results for the effective actions at the one-graviton four-photon level and for the amplitudes at the one-graviton two-photon level. As expected on general grounds, these amplitudes relate in a simple way to the corresponding four-photon amplitudes. We also derive the gravitational Ward identity for the 1PI one-graviton – N-photon amplitude.

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INTRODUCTION

In string theory, the prototypical example of relations between gravity and gauge theory amplitudes are the «KLT» relations discovered by Kawai et al. [1]. Schematically, they are of the form

$$(\text{gravity amplitude}) \sim (\text{gauge amplitude})^2$$

and follow naturally from the factorization of the graviton vertex operator into a product of two gauge boson vertex operators (see, e.g., [2]):

$$V^{\text{closed}} = V^{\text{open}}_{\text{left}} \bar{V}^{\text{open}}_{\text{right}}. \tag{1}$$

These string relations induce also relations in field theory. For example, at four and five point one has [2]

$$\begin{aligned}
M_4(1,2,3,4) &= -is_{12}A_4(1,2,3,4)A_4(1,2,4,3), \\
M_5(1,2,3,4,5) &= is_{12}s_{34}A_5(1,2,3,4,5)A_5(2,1,4,3,5)+ \\
&\quad + is_{13}s_{24}A_5(1,3,2,4,5)A_5(3,1,4,2,5).
\end{aligned} \tag{2}$$

Here the M_n are n-point tree-level graviton amplitudes, and the A_n are (colour-stripped) tree-level gauge theory amplitudes. The $s_{ij} = (k_i + k_j)^2$ are kinematical invariants.

Although the work of [1] was at the tree level, by unitarity those tree-level relations induce also identities at the loop level. By now, many relations between graviton and gauge amplitudes have been derived along these lines at the one-loop level and beyond (see [3] and refs. therein). Presently a key issue here is the possibility that the finiteness of $N = 4$ SYM theory may extend to $N = 8$ supergravity (see [3,4] and P. Vanhove's talk at this conference). Finiteness of a quantum field theory usually implies extensive cancellations between Feynman diagrams, and it is presently still not well-understood what are the precise extent and origin of such cancellations in the supergravity case.

In this respect, gravity amplitudes are more similar to QED amplitudes than to non-Abelian amplitudes, since colour factors greatly reduce the potential for cancellations between diagrams. In QED, there are many cases of surprising cancellations between diagrams. A famous case is the three-loop QED β-function coefficient, involving the sum of diagrams shown in Fig. 1. As discovered by Rosner in 1967 [5], individual diagrams give contributions to the β-function coefficient that involve $\zeta(3)$, however those terms cancel out, leaving a simple rational number for the sum of diagrams. Such cancellations are usually attributed to gauge invariance, since they generally appear inside gauge-invariant sets of graphs. Even for QED, little is still known about the influence of these cancellations on the large-order behaviour of the QED perturbation series [6,7]. For recent gravity-inspired studies of the structure of QED amplitudes, see [8,9].
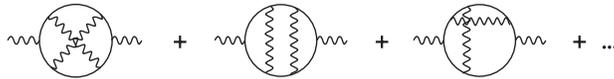

Fig. 1. Sum of diagrams for the three-loop QED photon propagator

Considering the enormous amount of work that has been done on the structural relationships between gauge and gravity amplitudes, it is surprising that relatively few results exist for mixed graviton–gluon or graviton–photon amplitudes [10–12]. In this talk, we report on the status of an ongoing effort to calculate the complete one-loop low-energy effective actions in the Einstein–Maxwell theory with a massive scalar or spinor loop, and to use them for obtaining the explicit form of the corresponding M-graviton/N-photon amplitudes [13–15]. The talk is organized as follows: in Sec. 1 we will shortly summarize what is presently known about the QED N-photon amplitudes. In Sec. 2 we summarize the results of [13,14] on the one-loop effective action in the Einstein–Maxwell theory, and also improve somewhat on the form of its one-graviton four-photon part

as compared to [14]. Section 3 is devoted to the graviton–photon–photon amplitude. Our conclusions are presented in the final section.

1. PROPERTIES OF THE QED N-PHOTON AMPLITUDES

We shortly summarize what is known about the N-photon amplitudes in scalar and spinor QED (results given refer to the spinor case unless stated otherwise).

Although the one-loop four-photon amplitude was calculated by Karplus and Neumann already in 1950 [16], progress towards higher leg or multiloop photon amplitudes has been extremely slow. The one-loop six-photon amplitude (recall that by Furry's theorem there are no amplitudes with an odd number of photons) was obtained only quite recently [17], and only for the massless case. On-shell amplitudes for gauge bosons are nowadays generally given in the helicity eigenstate decomposition; using CP invariance, the six-photon amplitude then has four independent components, which can be chosen as $A(+ + + + ++)$, $A(+ + + + +-), A(+ + + + --), A(+ + + - --)$ (in the gluonic case there will be more independent components since the ordering of the legs matters).

Apart from these explicit low-order calculations, there are also a number of all-N results. First, for massless QED there is Mahlon's vanishing theorem [18], stating that $A_N(+ + + \ldots + +) = A_N(- + + \cdots + +) = 0$ for $N > 4$. Mahlon also obtained a closed formula for the first nonvanishing case of two negative helicities $A(- - + \ldots +)$ in terms of dilogarithms [19].

More recently, Badger et al. [8] have shown that the massless N-photon amplitudes for $N \geqslant 8$ fulfill the «no triangle» property; that is, after the usual reduction from tensor to scalar integrals they involve only box integrals but not triangle ones. This is analogous to the «no triangle» property of $\mathcal{N} = 8$ supergravity [20], which is important for the possible finiteness of that theory.

An explicit all-N calculation is possible for the low-energy limit of the massive photon amplitudes, where all photon energies are small compared to the electron mass, $\omega_i \ll m$. The information on the N-photon amplitudes in this limit is contained in the well-known Euler–Heisenberg [21] (for spinor QED) resp. Weisskopf [22] (for scalar QED) effective Lagrangians:

$$\mathcal{L}_{\mathrm{spin}}(F) = -\frac{1}{8\pi^2} \int_0^\infty \frac{dT}{T^3} \, \mathrm{e}^{-m^2 T} \left[\frac{(eaT)(ebT)}{\tanh(eaT)\tan(ebT)} - \frac{1}{3}(a^2 - b^2)T^2 - 1 \right],$$

$$\mathcal{L}_{\mathrm{scal}}(F) = \frac{1}{16\pi^2} \int_0^\infty \frac{dT}{T^3} \, \mathrm{e}^{-m^2 T} \left[\frac{(eaT)(ebT)}{\sinh(eaT)\sin(ebT)} + \frac{1}{6}(a^2 - b^2)T^2 - 1 \right].$$

(3)

Here T is the proper time of the loop scalar or spinor particle and a, b are defined by $a^2 - b^2 = B^2 - {}^2$, $ab = \mathbf{E} \cdot \mathbf{B}$. Extracting the on-shell amplitudes from the

effective action is a standard procedure in quantum field theory. In the helicity decomposition, one finds [23]

$$A_{\text{spin}}^{(\text{EH})}[\varepsilon_1^+; \ldots; \varepsilon_K^+; \varepsilon_{K+1}^-; \ldots; \varepsilon_N^-] = -\frac{m^4}{8\pi^2}\left(\frac{2ie}{m^2}\right)^N (N-3)! \times$$

$$\times \sum_{k=0}^{K} \sum_{l=0}^{N-K} (-1)^{N-K-l} \frac{B_{k+l} B_{N-k-l}}{k!l!(K-k)!(N-K-l)!} \chi_K^+ \chi_{N-K}^- \quad (4)$$

and a similar formula for the scalar QED case [23]. Here the B_k are Bernoulli numbers, and the variables $\chi_K^{\pm}$ are written, in spinor helicity notation (our spinor helicity conventions follow [24])

$$\chi_K^+ = \frac{(K/2)!}{2^{K/2}}\Big\{[12]^2[34]^2 \cdots [(K-1)K]^2 + \text{ all permutations}\Big\},$$

$$\chi_K^- = \frac{(K/2)!}{2^{K/2}}\Big\{\langle 12\rangle^2\langle 34\rangle^2 \cdots \langle (K-1)K\rangle^2 + \text{ all permutations}\Big\}. \quad (5)$$

These variables appear naturally in the low-energy limit. Since they require even numbers of positive and negative helicity polarizations, in this low-energy limit we find a «double Furry theorem»: Only those helicity components are non-zero where both the number of positive and negative helicity photons are even. It is easy to show that this even holds true to all loop orders. For the MHV («maximally helicity violating» = «all +» or «all −») case (4) and its scalar analogue imply that the scalar and spinor amplitudes differ only by the global factor of -2 for statistics and degrees of freedom:

$$A_{\text{spin}}^{(\text{EH})}[\varepsilon_1^+; \ldots; \varepsilon_N^+] = -2 A_{\text{scal}}^{(\text{EH})}[\varepsilon_1^+; \ldots; \varepsilon_N^+]. \quad (6)$$

This well-known relation is actually true also away from the low-energy limit and can be explained by the fact that the MHV amplitudes correspond to a self-dual background, in which the Dirac operator has a quantum-mechanical supersymmetry [25].

2. ONE-LOOP EFFECTIVE ACTION IN EINSTEIN–MAXWELL THEORY

The calculation of the one-loop effective action in the Einstein–Maxwell theory is usually done using heat kernel techniques. The first calculation of relevance in our present context of the on-shell photon–graviton amplitudes was performed by Drummond and Hathrell [26], who obtained the terms in the fermionic effec-

tive Lagrangian involving one curvature tensor and two field strength tensors:

$$\mathcal{L}_{\text{spin}}^{(\text{DH})} = \frac{1}{180(4\pi)^2 m^2} \times$$

$$\times \left(5RF_{\mu\nu}^2 - 26R_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha} + 2R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} + 24(\nabla^{\alpha}F_{\alpha\mu})^2 \right) \quad (7)$$

(we will often absorb the electric charge e into the field strength tensor F).

Recently, some of the present authors used the worldline formalism [27, 28] to obtain master formulas for the Scalar and Spinor QED low-energy effective actions that generalize the above QED Lagrangians to the level of the N-photon – one-graviton amplitudes in the Einstein–Maxwell theory [13]. Those master formulas involve two-parameter integrals of trigonometric power series in the field strength matrix; to extract from them the part relevant at the one-graviton N-photon level, one has to expand the integrand in powers of $F_{\mu\nu}$ up to F^n, after which the integrals are polynomial and thus can be done by computer. After this, the gauge and gravitational Bianchi identities can be used to greatly reduce the number of terms. At the one-graviton – two-photon level, one finds [13]

$$\mathcal{L}_{\text{scal}}^{h\gamma\gamma} = \frac{1}{360\,m^2(4\pi)^2}\Big[5(6\xi - 1)RF_{\mu\nu}^2 + 4R_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha} - 6R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} -$$

$$- 2(\nabla^{\alpha}F_{\alpha\mu})^2 - 8(\nabla_{\alpha}F_{\mu\nu})^2 - 12F_{\mu\nu}\Box F^{\mu\nu}\Big], \quad (8)$$

$$\mathcal{L}_{\text{spin}}^{h\gamma\gamma} = \frac{1}{180\,m^2(4\pi)^2}\Big[5RF_{\mu\nu}^2 - 4R_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha} - 9R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} +$$

$$+ 2(\nabla^{\alpha}F_{\alpha\mu})^2 - 7(\nabla_{\alpha}F_{\mu\nu})^2 - 18F_{\mu\nu}\Box F^{\mu\nu}\Big]. \quad (9)$$

The result for the spinor loop case differs from the Drummond–Hathrell Lagrangian (7) by a total derivative term [13]. The parameter ξ appearing in the scalar case represents a nonminimal coupling to gravity. At the next, $N = 4$ level (there are no amplitudes with an odd number of photons by an extension of Furry's theorem to the photon–graviton case) this procedure is already quite laborious. It was carried through in [14], but here we give the results in a slightly more compact form than was obtained there:

$$\mathcal{L}_{\text{spin}}^{h,4\gamma} = -\frac{1}{8\,\pi^2}\frac{1}{m^6}\Bigg[-\frac{1}{432}R(F_{\mu\nu})^4 + \frac{7}{1080}R\,\text{tr}[F^4] - \frac{1}{945}R_{\alpha\beta}(F^4)^{\alpha\beta} -$$

$$- \frac{1}{540}R_{\alpha\beta}(F^2)^{\alpha\beta}(F_{\gamma\delta})^2 + \frac{4}{135}R_{\alpha\mu\beta\nu}(F^3)^{\alpha\mu}F^{\beta\nu} + \frac{1}{108}R_{\alpha\mu\beta\nu}F^{\alpha\mu}F^{\beta\nu}(F_{\gamma\delta})^2 +$$

$$+ \frac{7}{270}(F^3)^{\mu\nu}\Box F_{\mu\nu} + \frac{1}{108}F^{\mu\nu}\Box F_{\mu\nu}(F_{\gamma\delta})^2 + \frac{1}{270}F_{\mu\nu;\alpha\beta}(F^2)^{\alpha\beta}F^{\mu\nu} -$$

$$- \frac{1}{540}(F_{\alpha\beta;\gamma})^2 (F_{\mu\nu})^2 - \frac{1}{945} F_{\mu\nu;\alpha} F^{\mu\nu}{}_{;\beta}(F^2)^{\alpha\beta} - \frac{11}{945} F_{\alpha\beta;\gamma} F_\mu{}^{\beta;\gamma}(F^2)^{\alpha\mu} -$$
$$- \frac{2}{189} F_{\alpha\beta;\gamma} F_{\mu\nu;}{}^{\gamma} F^{\alpha\mu} F^{\beta\nu} - \frac{2}{189} F_{\alpha\beta;\gamma} F_\mu{}^{\alpha}{}_{;\delta} F^{\beta\mu} F^{\gamma\delta} \Bigg], \quad (10)$$

$$\mathcal{L}_{\rm scal}^{h,4\gamma} = \frac{1}{16\,\pi^2} \frac{1}{m^6} \Bigg[-\frac{1}{144}\left(\bar\xi + \frac{1}{12}\right) R(F_{\mu\nu})^4 - \frac{1}{180}\left(\bar\xi + \frac{1}{12}\right) R\,{\rm tr}[F^4] -$$
$$- \frac{1}{945} R_{\alpha\beta}(F^4)^{\alpha\beta} + \frac{1}{1080} R_{\alpha\beta}(F^2)^{\alpha\beta}(F_{\gamma\delta})^2 - \frac{1}{270} R_{\alpha\mu\beta\nu}(F^3)^{\alpha\mu} F^{\beta\nu} +$$
$$+ \frac{1}{432} R_{\alpha\mu\beta\nu} F^{\alpha\mu} F^{\beta\nu}(F_{\gamma\delta})^2 - \frac{1}{540}(F^3)^{\mu\nu} \Box F_{\mu\nu} -$$
$$+ \frac{1}{432} F^{\mu\nu} \Box F_{\mu\nu}(F_{\gamma\delta})^2 - \frac{1}{540} F_{\mu\nu;\alpha\beta}(F^2)^{\alpha\beta} F^{\mu\nu} +$$
$$+ \frac{1}{1080}(F_{\alpha\beta;\gamma})^2 (F_{\mu\nu})^2 - \frac{1}{945} F_{\mu\nu;\alpha} F^{\mu\nu}{}_{;\beta}(F^2)^{\alpha\beta} -$$
$$- \frac{1}{1890} F_{\alpha\beta;\gamma} F_\mu{}^{\beta;\gamma}(F^2)^{\alpha\mu} + \frac{1}{1890} F_{\alpha\beta;\gamma} F_{\mu\nu;}{}^{\gamma} F^{\alpha\mu} F^{\beta\nu} +$$
$$+ \frac{1}{1890} F_{\alpha\beta;\gamma} F_\mu{}^{\alpha}{}_{;\delta} F^{\beta\mu} F^{\gamma\delta} \Bigg] \quad (11)$$

$(\bar\xi = \xi - 1/4)$. This improvement over the formulas given in [14] is due to the following consequence of the Bianchi identities, that had been overlooked in the list of identities used there:

$$R_{\alpha\mu\beta\nu}(F^2)^{\alpha\beta}(F^2)^{\mu\nu} = -\frac{1}{2} F_{\mu\nu;\alpha\beta}(F^2)^{\alpha\beta} F^{\mu\nu} - \frac{1}{2} R_{\alpha\mu\beta\nu}(F^3)^{\alpha\mu} F^{\beta\nu} -$$
$$- F_{\mu\nu;\alpha\beta}(F^2)^{\alpha\nu} F^{\beta\mu}. \quad (12)$$

3. THE GRAVITON–PHOTON–PHOTON AMPLITUDE

We proceed to the simplest amplitude case, the graviton–photon–photon amplitude shown in Fig. 2.

Getting its low-energy limit from the three-point Lagrangians (8), (9) (or equivalently from (7) in the spinor case) is straightforward. In the helicity basis, and using the standard factorization of the graviton polarization tensor in terms of vector polarizations, $\varepsilon_{0\mu\nu}^{\pm\pm}(k_0) = \varepsilon_\mu^\pm(k_0)\varepsilon_\nu^\pm k_0)$, one finds that only the «all +»

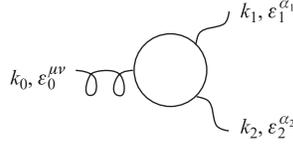Fig. 2. Graviton–photon–photon diagram

and «all $-$» components are nonzero:

$$A_{\text{spin}}^{(++;++)} = \frac{\kappa\,e^2}{90(4\pi)^2 m^2}\, [01]^2\, [02]^2,$$

$$A_{\text{spin}}^{(--;--)} = \frac{\kappa\,e^2}{90(4\pi)^2 m^2}\, \langle 01 \rangle^2\, \langle 02 \rangle^2. \tag{13}$$

Here the first upper index pair refers to the graviton polarization, and κ is the gravitational coupling constant. Moreover, those components fulfill the MHV relation (6),

$$A_{\text{spin}}^{(++;++)} = (-2)\, A_{\text{scal}}^{(++;++)},$$

$$A_{\text{spin}}^{(--;--)} = (-2)\, A_{\text{scal}}^{(--;--)}. \tag{14}$$

Also, these graviton–photon–photon amplitudes relate to the (low-energy) four-photon amplitudes in the following way: From (4), (5) the only nonvanishing components of those are

$$A^{++++}[k_1, k_2, k_3, k_4] \sim [12]^2[34]^2 + [13]^2[24]^2 + [14]^2[23]^2,$$

$$A^{++--}[k_1, k_2, k_3, k_4] \sim [12]^2\langle 34 \rangle^2,$$

$$A^{----}[k_1, k_2, k_3, k_4] \sim \langle 12 \rangle^2\langle 34 \rangle^2 + \langle 13 \rangle^2\langle 24 \rangle^2 + \langle 14 \rangle^2\langle 23 \rangle^2. \tag{15}$$

Replacing $k_1 \to k_0, k_2 \to k_0$ in the four-photon amplitudes, the middle one of these three components becomes zero, and the remaining ones become proportional to the corresponding components of (13),

$$A^{++++}[k_0, k_0, k_3, k_4] \sim 2[03]^2[04]^2 \ \sim A^{++;++}[k_0, k_3, k_4],$$

$$A^{----}[k_0, k_0, k_3, k_4] \sim 2\langle 03 \rangle^2\langle 04 \rangle^2 \sim A^{--;--}[k_0, k_3, k_4]. \tag{16}$$

Thus effectively two photons have coalesced to form a graviton, clearly a result in the spirit of the KLT relations.

At the next level of one graviton and four photons, the conversion of the effective action into amplitudes becomes already extremely laborious. Moreover,

here there are already one-particle reducible contributions to the amplitudes, with the graviton attached to a photon, and those are essential to arrive at a well-defined helicity decomposition. This is because the 1PI amplitudes are transversal in the photon indices, but not in the graviton ones; rather, one has the inhomogeneous Ward identity [15]

$$2k_{0\mu}A^{\mu\nu,\alpha_1\ldots\alpha_N}[k_0,\ldots,k_N] =$$
$$= -\sum_{i=1}^{N} A^{\mu\alpha_1\ldots\widehat{\alpha_i}\ldots\alpha_N}[k_0 + k_i, k_1,\ldots,\widehat{k_i},\ldots,k_N](\delta_\mu^{\alpha_i}k_i^\nu - \eta^{\alpha_i\nu}k_{i\mu}) \quad (17)$$

(where a «hat» means omission) which connects the one-graviton–N-photon amplitudes to the N-photon amplitudes.

CONCLUSIONS

We have presented here the first results of a systematic study of the mixed one-loop photon–graviton amplitudes with a scalar or spinor loop in the low-energy limit. At the one-graviton–two-photon level, we find a KLT-like factorization of the graviton into two photons. If this type of factorization persists for higher points, it would imply that, in the low-energy limit, the full information on the M-graviton–N-photon amplitudes is contained in the $N + 2M$-photon amplitudes. However, the three-point result may not be representative due to the absence of one-particle reducible contributions. The situation will be clearer after the completion of the one-graviton–four-photon calculation, which is presently in progress.

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