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ON-MASS-SHELL RENORMALIZABILITY OF THE MASSIVE YANG–MILLS THEORY

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Renormalizable theory of electroweak interactions without scalar particles can be constructed by modifying the Standard Model. One should remove all terms with the scalar field from the Lagrangian in the unitary gauge. The resulting electroweak theory without the Higgs particle is on-mass-shell renormalizable and unitary. Thus, the experimental nonobservation of the Higgs boson will not mean a problem for the concept of renormalizability in quantum field theory but will confirm the scalar free theory.

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The massive Yang–Mills theory [1] was long considered to be nonrenormalizable [2], see also [3, 4] and references therein. The only known way to get renormalizable and unitary theory with massive Yang–Mills bosons was due to the Higgs mechanism of spontaneous symmetry breaking [5]. This mechanism is used in the Standard $SU(2) \times U(1)$ Model of electroweak interactions [6] which is established to be renormalizable [7], see also [8] and references therein. In this way, one introduces in the model the famous scalar Higgs particle which one can still hope to see in experiments.

Quite recently it was found that the massive Yang–Mills theory is in fact on-mass-shell renormalizable [9]. Correspondingly, one can simplify the Standard $SU(2)\times U(1)$ Model of electroweak interactions by removing from the Lagrangian in the unitary gauge all terms containing the scalar field. The resulting electroweak theory without the scalar Higgs particle remains on-mass-shell renormalizable and unitary. Thus, at present there are two self-consistent renormalizable theories of electroweak interactions: one with the exotic scalar particle and one without. It is up to experiments to choose between them. The nondiscovery of the Higgs particle at the Large Hadron Collider will indicate the validity of the scalar-free electroweak theory.

In the present paper we elaborate some points connected with the statement of renormalizability of the theory without scalar particles. The paper is an extended version of the preprint [10].

We will work within perturbation theory. To regularize ultraviolet divergences, we will use for convenience dimensional regularization [11] with the space-time dimension $d = 4 - 2\epsilon$, ϵ being the regularization parameter.

As the simplified case of the real electrowek theory, let us consider the known model given by the initial $SU(2)$-invariant Lagrangian of interaction of vector bosons and scalar fields possessing the spontaneously broken symmetry

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\Phi)^+ D_\mu\Phi - \lambda(\Phi^+\Phi - v^2)^2 \tag{1}$$

with the doublet of scalar fields $\Phi(x)$ in the fundamental representation of the group.

Here, $D_\mu\Phi = \left(\partial_\mu - ig\frac{\tau^a}{2}W^a_\mu\right)\Phi$ is the covariant derivative, τ^a are the Pauli matrices, $\lambda > 0$, $v^2 > 0$.

To get the complete Lagrangian, one makes the shift of the scalar field

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} i\phi_1(x) + \phi_2(x) \\ \sqrt{2}v + \chi(x) - i\phi_3(x) \end{pmatrix},$$

fixes the gauge and adds ultraviolet counterterms.

Let us consider two gauges: the widely used R_ξ gauge [7, 12] with an arbitrary parameter ξ and the unitary gauge.

In the R_ξ gauge, one gets the theory described by the generating functional of Green functions

$$Z_{R_\xi}(J,K) = \frac{1}{N}\int dW\, d\phi\, d\chi\, d\bar{c}\, dc \exp\left(i\int dx(L_{R_\xi} + J^a_\mu W^a_\mu + K\chi)\right),$$

$$\begin{aligned} L_{R_\xi} = &-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{m^2}{2}W^a_\mu W^a_\mu - mW^a_\mu\partial_\mu\phi^a + \frac{1}{2}\partial_\mu\phi^a\partial_\mu\phi^a + \frac{1}{2}\partial_\mu\chi\partial_\mu\chi - \\ &- \frac{M^2}{2}\chi^2 + \frac{g}{2}W^a_\mu(\phi^a\partial_\mu\chi - \chi\partial_\mu\phi^a + \epsilon^{abc}\phi^b\partial_\mu\phi^c) + \frac{mg}{2}\chi W^a_\mu W^a_\mu + \\ &+ \frac{g^2}{8}(\chi^2 + \phi^a\phi^a)W^2_\mu - \frac{gM^2}{4m}\chi(\chi^2 + \phi^a\phi^a) - \frac{g^2M^2}{32m^2}(\chi^2 + \phi^a\phi^a)^2 - \\ &- \frac{1}{2\xi}(\partial_\mu W^a_\mu + \xi m\phi^a)^2 + \partial_\mu\bar{c}^a(\partial_\mu c^a - g\epsilon^{abc}c^bW^c_\mu) - \xi m^2\bar{c}^ac^a - \\ &- \frac{g}{2}\xi m\chi\bar{c}^ac^a + \frac{g}{2}\xi m\epsilon^{abc}\bar{c}^ac^b\phi^c + \text{counterterms}. \end{aligned} \tag{2}$$

This theory describes three physical massive vector bosons with the mass $m = gv/\sqrt{2}$, and the physical Higgs field χ with the mass $M = 2\lambda v$. Here are also Goldstone ghosts ϕ^a and Faddeev–Popov ghosts c^a with masses ξm^2. The structure of the counterterms (consistent with gauge invariance and Slavnov–Taylor identities [13, 14] to ensure unitarity) is well known, see, e.g., [8].

This is the renormalizable gauge; i.e., Green functions are finite. The corresponding propagators in momentum space are

$$\begin{gathered}\langle T(W_\mu^a W_\nu^b)\rangle = -i\delta^{ab}\left(\frac{g_{\mu\nu}-k_\mu k_\nu/m^2}{k^2-m^2}+\frac{k_\mu k_\nu/m^2}{k^2-\xi m^2}\right),\\ \langle T(\phi^a\phi^b)\rangle = -i\delta^{ab}\frac{1}{k^2-\xi m^2},\\ \langle T(\bar{c}^a c^b)\rangle = -i\delta^{ab}\frac{1}{k^2-\xi m^2},\\ \langle T(\chi\chi)\rangle = -i\frac{1}{k^2-M^2}.\end{gathered} \tag{3}$$

In the unitary gauge defined by the gauge condition $\phi^a = 0$, one has the Lagrangian

$$\begin{aligned}L_U = -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + \frac{m^2}{2}W_\mu^a W_\mu^a + \frac{1}{2}\partial_\mu\chi\partial_\mu\chi - \frac{M^2}{2}\chi^2 + \\ + \frac{mg}{2}\chi W_\mu^a W_\mu^a + \frac{g^2}{8}\chi^2 W_\mu^a W_\mu^a - \frac{gM^2}{4m}\chi^3 - \frac{g^2M^2}{32m^2}\chi^4 + \text{counterterms}.\end{aligned} \tag{4}$$

The propagators in the unitary gauge are obtained from those of the R_ξ gauge in Eq. (3) by taking the limit $\xi \to \infty$. The theory in the unitary gauge is renormalizable only on mass shell; i.e., Green functions are divergent at $\epsilon \to 0$, but the S-matrix elements are finite. In this gauge, all unphysical particles (longitudinal quanta of vector fields and ghosts) are absent and unitarity of the theory is manifest.

To show equivalence of S-matrix elements in two gauges, one uses the functional integral technique. Let us repeat it for the case of the Landau gauge $\xi = 0$ (for simplicity), which corresponds in fact to the Lorentz gauge $\partial_\mu W_\mu^a = 0$ (the L gauge). The generating functional of Green functions in the L gauge is

$$\begin{aligned}Z_L(J,K) = \frac{1}{N}\int dW\,d\phi\,d\chi\,\exp\left(i\int dx(L_R + J_\mu^a W_\mu^a + K\chi)\right)\times \\ \times\,\Delta_L(W)\,\delta(\partial_\mu W_\mu),\end{aligned} \tag{5}$$

where $\Delta_L(W)$ is the Faddeev–Popov determinant [15] and L_R is obtained from L_{R_ξ} by omitting terms depending on ξ and c^a (and by corresponding modification of counterterms). The Lagrangian L_R is invariant under the following gauge transformations:

$$\begin{gathered} W_\mu^a \to \left(W_\mu^\omega\right)^a = W_\mu^a + \partial_\mu \omega^a + \tilde{g} f^{abc} W_\mu^b \omega^c + O(\omega^2), \\ \phi^a \to (\phi^\omega)^a = \phi^a - \tilde{m}\omega^a - \frac{\tilde{g}}{2} f^{abc}\phi^b\omega^c - \frac{\tilde{g}}{2}\chi\omega^a + O(\omega^2), \\ \chi \to \chi^\omega = \chi - \frac{\tilde{g}}{2}\phi^a\omega^a + O(\omega^2), \end{gathered} \tag{6}$$

where

$$\tilde{g} = \frac{z_1}{z_2} g, \quad \tilde{m} = \frac{z_1}{z_2} m,$$

and z_1, z_2 are renormalization constants of the triple W vertex and the W field.

One inserts in the functional integral the unity

$$\Delta_U(\chi) \int dw\, \delta(\phi^\omega) = 1. \tag{7}$$

Making the known change of variables

$$W_\mu \to W_\mu^{\omega^{-1}}, \quad \phi \to \phi^{\omega^{-1}}, \quad \chi \to \chi^{\omega^{-1}}, \quad \omega^{-1} \to \omega$$

and integrating over ω, one obtains

$$Z_L(J,K) = \frac{1}{N} \int dW\, d\phi\, d\chi \times \\ \times \exp\left(i \int dx (L_U + J_\mu W_\mu^{\tilde{\omega}} + K\chi^{\tilde{\omega}})\right) \Delta_U(\chi)\, \delta(\phi), \tag{8}$$

where $\tilde{\omega}$ is defined from the equation

$$\partial_\mu (W_\mu^{\tilde{\omega}})^a = \partial_\mu (W_\mu^a + \partial_\mu \tilde{\omega}^a + \tilde{g}(f^{abc} W_\mu^b \tilde{\omega}^c)) + O(\tilde{\omega}^2) = 0. \tag{9}$$

The Lagrangian L_U is given in Eq. (4).

The functional $\Delta_U(\chi)$ can be presented on the surface $\phi^a = 0$ as

$$\Delta_U(\chi) = \det \left| \tilde{m} + \frac{\tilde{g}}{2}\chi(x) \right|^3 = \mathrm{const} \cdot \exp\left(\int \delta^d(0) \ln \left(1 + \frac{g}{2m}\chi(x)\right)^3 dx \right).$$

In dimensional regularization this functional is just a constant and can be absorbed in the normalization factor N, although this simplification is not essential for the following derivation.

One obtains

$$Z_L(J,K) = \frac{1}{N} \int dW\, d\chi \exp\left(i \int dx\, (L_U + J_\mu W_\mu^{\tilde{\omega}} + K\chi^{\tilde{\omega}})\right), \tag{10}$$

where $\chi^{\tilde{\omega}}$ is taken on the surface $\phi^a = 0$.

The expression (10) differs from the generating functional of Green functions in the unitary gauge

$$Z_U(J,K) = \frac{1}{N}\int dW\, d\chi \exp\left(i\int dx\,(L_U + J_\mu W_\mu + K\chi)\right) \qquad (11)$$

only by source terms. It is known that this difference is not essential for S-matrix elements, see, e.g., [8]. Thus, the physical equivalence of the L gauge and the U gauge is proved.

From Eq. (10) one sees that the counterterms of L_U are given by the counterterms of L_R at $\phi^a(x) = 0$.

One can also prove directly at the level of Feynman diagrams that S-matrix elements coincide in the R_ξ gauge and the U gauge (it is well known that the path-integral formalism correctly reproduces in the compact form the direct diagrammatic approach). To this end, one can just take the limit $\xi \to \infty$ in diagrams of S-matrix elements in the R_ξ gauge since S-matrix elements do not depend on the gauge parameter ξ. Then one gets the presentation of S-matrix elements in terms of diagrams in the unitary gauge. The nice property of dimensional regularization is that the limit $\xi \to \infty$ can be taken before or after integrations: results coincide (the limit should be taken before removing regularization).

To consider renormalization for our purpose it is convenient to use the Bogoliubov–Parasiuk–Hepp subtraction scheme [16]. As is well known, in this scheme a counterterm of, e.g., a primitively divergent Feynman diagram is the truncated Taylor expansion of the diagram itself at some fixed values of external momenta. Hence, counterterms of mass-dependent diagrams are also mass-dependent. Needless to say that subtractions should respect Slavnov–Taylor identities.

Let us consider S-matrix elements in the R_ξ gauge without external Higgs bosons (i.e., with external W bosons only in this simplified model).

We will analyze the dependence of diagrams on the Higgs mass M by using the expansion in large M (after renormalization but before removing regularization). The algorithm for the large-mass expansion of Feynman diagrams is given, e.g., in [17] (where it is quite reasonably checked in calculations of the 4-loop diagrams for the Z-boson decay into hadrons). It can be rigorously derived, e.g., with the technique of [18].

We separate all diagrams into physical ones, which are not nullified in the limit $\xi \to \infty$, and unphysical ones, which are nullified. In this limit the propagator of the W boson reduces to the known unitary form

$$\lim_{\xi\to\infty}\langle T(W^a_\mu W^b_\nu)\rangle = -i\delta^{ab}\lim_{\xi\to\infty}\left(\frac{g_{\mu\nu}-k_\mu k_\nu/m^2}{k^2-m^2}+\frac{k_\mu k_\nu/m^2}{k^2-\xi m^2}\right) = \\ = -i\delta^{ab}\frac{g_{\mu\nu}-k_\mu k_\nu/m^2}{k^2-m^2}. \quad (12)$$

The propagators of the Goldstone bosons ϕ^a and ghosts c^a vanish in this limit and correspondingly all diagrams which contain these propagators are also nullified. The limit is quite subtle for diagrams containing ghosts loops coupled to the Higgs boson since the corresponding coupling constant itself contains the parameter ξ in the Lagrangian, see Eq. (2). For example, in the one-loop case one gets

$$\lim_{\xi\to\infty}\xi^2\int d^dp\frac{1}{(p^2-\xi m^2)((p+q)^2-\xi m^2)} = \int d^dp = 0, \quad (13)$$

where zero is obtained due to the famous property of dimensional regularization to nullify scaleless integrals. As we already stressed, the limit $\xi\to\infty$ commutes with integrations in Feynman integrals within dimensional regularization.

Thus, in our notations the physical diagrams are the diagrams that do not contain Goldstone bosons propagators or ghosts propagators and the unphysical diagrams are the diagrams that contain such propagators.

Within the large-M expansion, the physical diagrams with χ propagators contain either terms with integer negative powers of M^2

$$\frac{1}{M^{2n}}, \quad n = 1,2,3,\ldots$$

or terms with noninteger powers of M^2 (noninteger powers contain ϵ)

$$\frac{1}{M^{2(k+l\epsilon)}}, \quad k \text{ — integer}, \quad l \text{ — positive integer}.$$

This is because each vertex with the large factor M^2 has three or four attached χ propagators due to the structure of the Higgs boson self-coupling.

In contrast, unphysical diagrams can have polynomial in M terms due to the four-ϕ vertex with the large factor M^2. But they are ξ-dependent (they are nullified in the limit $\xi\to\infty$) and this polynomial terms cancel in S-matrix elements. This guarantees in particular the existence of the limit $M^2\to\infty$. The limit is quite delicate and exists only before the removing regularization. After removing regularization, terms of the type $M^{2n}/M^{2(l\epsilon)}$ produce terms of the type $M^{2n}(\ln(M^2))^k$, k, l, n are positive integers, which diverge in this limit. Before removing regularization we have the finite limit

$$\lim_{M\to\infty}\frac{M^{2n}}{M^{2(l\epsilon)}} = 0, \quad (14)$$

where zero is provided by the already mentioned property of dimensional regularization to nullify scaleless integrals.

We would like to stress once again that the limit $M \to \infty$ exists only in d dimensions. In 4 dimensions it does not exist, which is known as the nondecoupling of the Higgs boson.

In the renormalizable R_ξ gauge, one can present ultraviolet renormalization in a standard form of the Bogoliubov–Parasiuk R operation for individual diagrams. This ensures that after renormalization the M-dependent terms are finite at $\epsilon \to 0$ separately from M-independent terms. Thus, if one removes all M-dependent terms, one is left with a finite expression.

On the Lagrangian level it means in the unitary gauge that one removes from L_U all terms containing the field χ and also all M-dependent terms in the counterterms. The resulting theory is on-mass-shell finite. This is the massive Yang–Mills theory

$$\begin{gathered} Z(J) = \frac{1}{N} \int dW \exp\left(i \int dx (L_{\mathrm{YM}} + J^a_\mu W^a_\mu) \right), \\ L_{\mathrm{YM}} = -\frac{1}{4} z_2 \left(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \frac{z_1}{z_2} g f^{abc} W^b_\mu W^c_\nu \right)^2 + z_m m^2 W^a_\mu W^a_\mu . \end{gathered} \tag{15}$$

After renormalizability is established, one can fix renormalization constants z_1, z_2, and z_m within the theory (15) (without referring to the R_ξ gauge) by proper normalization conditions.

Thus, the Higgs mechanism can be considered as an efficient mathematical tool to observe on-mass-shell renormalizability of the massive Yang–Mills theory, which is far from to be obvious directly.

It is known that the Higgs theories of vector mesons possess the so-called tree level unitarity, see, e.g., [3, 19, 20] and references therein. Tree level cross sections of such theories grow at high energies slowly enough and do not exceed the so-called unitary limit imposed by the unitarity condition. The reversed statement is also proved: from the condition of tree level unitarity it follows that a theory of vector mesons should be a Higgs theory [19]. But one can see that tree level unitarity is not the necessary condition for renormalizability. Tree level unitarity is violated in the massive Yang–Mills theory. It indicates that higher-order contributions become relevant at high energies and one looses the perturbative control over the theory. This is due to the presence in external states of S-matrix elements of longitudinally polarized W bosons whose wave functions grow with energy like E/m, where E is typical energy of the process. But in the physical $SU(2) \times U(1)$ theory with inclusion of fermions the massive gauge bosons are highly unstable particles which do not appear as external states of S-matrix elements in complete calculations and correspondingly tree level unitarity is present (with or without the Higgs boson).

The above derivation of on-mass-shell renormalizability is also applicable to other gauge groups. It can be straightforwardly applied to the Standard $SU(2) \times U(1)$ Model of electroweak interactions. The presence of the $U(1)$ gauge boson and of fermions does not change the derivation. Again, one can remove from the Lagrangian in the unitary gauge all terms containing the scalar field. The fermion masses remain unchanged under this operation. The resulting electroweak theory without the Higgs particle is on-mass-shell renormalizable and unitary. Thus, the experimental nonobservation of the Higgs boson at the LHC will not mean a problem for the concept of renormalizability in quantum field theory but will confirm the scalar-free theory.

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