ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2013. Т. 44. ВЫП. 3

Q^2-EVOLUTION OF PARTON DENSITIES AT SMALL x VALUES. COMBINED H1&ZEUS F_2 DATA

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We use the Bessel-inspired behavior of the structure function F_2 at small x, obtained for a flat initial condition in the DGLAP evolution equations, with «frozen» and analytic modifications of the strong coupling constant to study precise combined H1&ZEUS data for the structure function F_2 published recently.

PACS: 12.38.Cy

INTRODUCTION

A reasonable agreement between HERA data [1–3] and the next-to-leading-order (NLO) approximation of perturbative Quantum Chromodynamics (QCD) has been observed for $Q^2 \geqslant 2$ GeV^2 (see reviews in [4] and references therein), which gives us a reason to believe that perturbative QCD is capable of describing the evolution of the structure function (SF) F_2 and its derivatives down to very low Q^2 values, where all the strong interactions are conventionally considered to be soft processes.

A standard way to study the x behavior of quarks and gluons is to compare the data with the numerical solution to the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [5] by fitting the parameters of x-profile of partons at some initial Q_0^2 and the QCD energy scale Λ [6, 7]. However, for the purpose of analyzing exclusively the small-x region, there is the alternative of doing a simpler analysis by using some of the existing analytical solutions to DGLAP equations in the small-x limit [8, 9].

The ZEUS and H1 collaborations have presented the new precise combined data [10] for the SF F_2. The aim of this short paper is to compare the combined H1&ZEUS data with the predictions obtained by using the so-called doubled asymptotic scaling (DAS) approach [9].

To improve the analysis at low Q^2 values, it is important to consider the well-known infrared modifications of the strong coupling constant. We will use the «frozen» and analytic versions (see [11] and references therein).

1. PARTON DISTRIBUTIONS AND THE STRUCTURE FUNCTION F_2

Here, for simplicity we consider only the leading-order (LO) approximation*. The structure function F_2 has the form

$$F_2(x,Q^2) = ef_q(x,Q^2), \quad f_a(x,Q^2) = f_a^+(x,Q^2) + f_a^-(x,Q^2) \quad (a=q,g), \tag{1}$$

where $e = (\sum_1^f e_i^2)/f$ is an average charge squared.

The small-x asymptotic expressions for parton densities $f_a^{\pm}$ look like

$$\begin{aligned} f_g^+(x,Q^2) &= \left(A_g + \frac{4}{9}A_q\right) I_0(\sigma)\,\mathrm{e}^{-\overline{d}_+ s} + O(\rho), \\ f_q^+(x,Q^2) &= \frac{f}{9}\frac{\rho I_1(\sigma)}{I_0(\sigma)} + O(\rho), \\ f_g^-(x,Q^2) &= -\frac{4}{9}A_q\,\mathrm{e}^{-d_- s} + O(x), \\ f_q^-(x,Q^2) &= A_q\,\mathrm{e}^{-d_-(1)s} + O(x), \end{aligned} \tag{2}$$

where I_ν $(\nu = 0,1)$ are the modified Bessel functions,

$$s = \ln\left(\frac{a_s(Q_0^2)}{a_s(Q^2)}\right), \quad \sigma = 2\sqrt{|\hat{d}_+|\, s \ln\left(\frac{1}{x}\right)}, \quad \rho = \frac{\sigma}{2\ln(1/x)}, \tag{3}$$

and

$$\hat{d}_+ = -\frac{12}{\beta_0}, \quad \overline{d}_+ = 1 + \frac{20f}{27\beta_0}, \quad d_- = \frac{16f}{27\beta_0} \tag{4}$$

denote singular and regular parts of the anomalous dimensions $d_+(n)$ and $d_-(n)$, respectively, in the limit $n \to 1$**. Here n is a variable in the Mellin space.

2. «FROZEN» AND ANALYTIC COUPLING CONSTANTS

In order to improve an agreement at low Q^2 values, the QCD coupling constant is modified in the infrared region. We consider two modifications that effectively increase the argument of the coupling constant at low Q^2 values (see [12]).

*The NLO results can be found in [9].

**We denote the singular and regular parts of a given quantity $k(n)$ in the limit $n \to 1$ by $\hat{k}/(n-1)$ and $\overline{k}$, respectively.

In the first case, which is more phenomenological, we introduce freezing of the coupling constant by changing its argument $Q^2 \to Q^2 + M_\rho^2$, where M_ρ is the ρ-meson mass (see [11] and discussions therein). Thus, in the formulae of Sec. 1 we have to carry out the following replacement:

$$a_s(Q^2) \to a_{\rm fr}(Q^2) \equiv a_s(Q^2 + M_\rho^2). \qquad (5)$$

The second possibility follows the Shirkov–Solovtsov idea [13] concerning the analyticity of the coupling constant that leads to additional power dependence of the latter. Then, in the formulae of the previous section the coupling constant $a_s(Q^2)$ should be replaced as follows:

$$a_{\rm an}^{\rm LO}(Q^2) = a_s(Q^2) - \frac{1}{\beta_0}\frac{\Lambda_{\rm LO}^2}{Q^2 - \Lambda_{\rm LO}^2} \qquad (6)$$

in the LO approximation and

$$a_{\rm an}(Q^2) = a_s(Q^2) - \frac{1}{2\beta_0}\frac{\Lambda^2}{Q^2 - \Lambda^2} + \ldots \qquad (7)$$

in the NLO approximation. Here the the symbol «...» stands for the terms that provide negligible contributions when $Q^2 \geqslant 1$ GeV [13].

Note here that the perturbative coupling constant $a_s(Q^2)$ is different in the LO and NLO approximations. Indeed, from the renormalization group equation we can obtain the following equations for the coupling constant:

$$\frac{1}{a_s^{\rm LO}(Q^2)} = \beta_0 \ln\left(\frac{Q^2}{\Lambda_{\rm LO}^2}\right) \qquad (8)$$

in the LO approximation and

$$\frac{1}{a_s(Q^2)} + \frac{\beta_1}{\beta_0}\ln\left[\frac{\beta_0^2 a_s(Q^2)}{\beta_0 + \beta_1 a_s(Q^2)}\right] = \beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right) \qquad (9)$$

in the NLO approximation. Usually, at the NLO level, $\overline{\rm MS}$ scheme is used; therefore, below we apply $\Lambda = \Lambda_{\overline{\rm MS}}$.

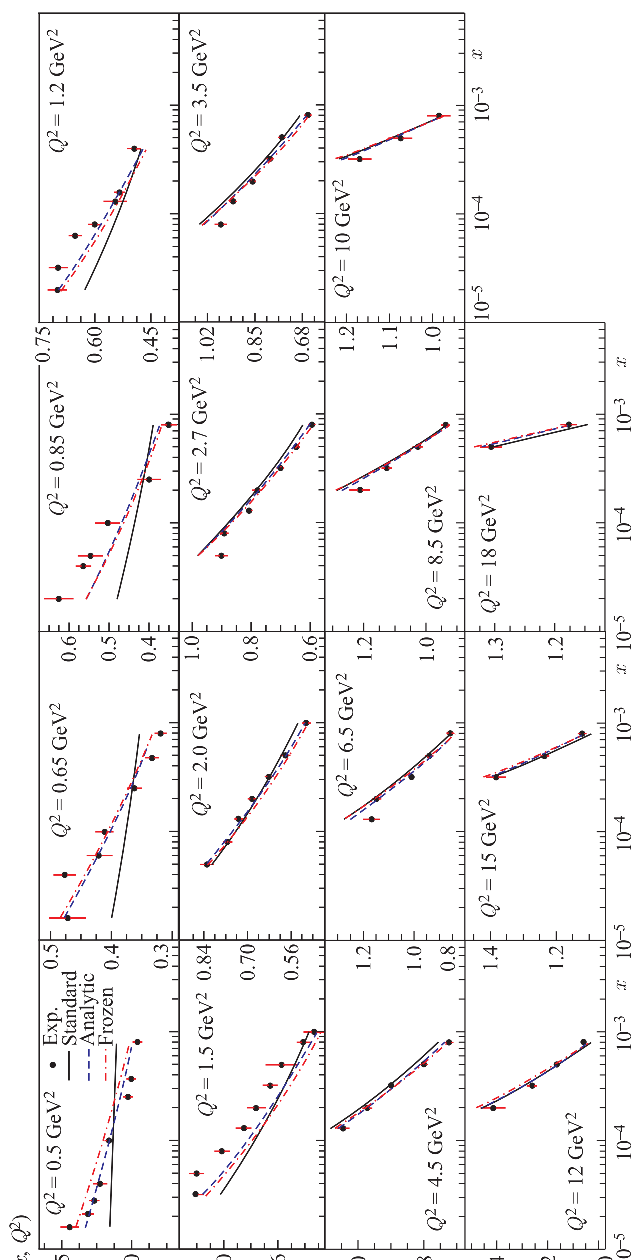
3. COMPARISON WITH EXPERIMENTAL DATA

By using the results of the previous section we have analyzed H1&ZEUS data for F_2 [10]. In order to keep the analysis as simple as possible, we fix $f = 4$ and $\alpha_s(M_Z^2) = 0.1168$ (i.e., $\Lambda^{(4)} = 284$ MeV) in agreement with more recent ZEUS results given in [1].

As can be seen from the figure and the table, the twist-two approximation is reasonable for $Q^2 \geqslant 4$ GeV^2. At lower Q^2 we observe that the fits in the cases with «frozen» and analytic strong coupling constants are very similar (see also [11, 14]) and describe the data in the low-Q^2 region significantly better than the standard fit. Nevertheless, for $Q^2 \leqslant 1.5$ GeV^2 there is still some disagreement with the data, which needs to be additionally studied. In particular, the Balitsky–Fadin–Kuraev–Lipatov (BFKL) resummation [15] may be important here [16]. It can be added in the generalized DAS approach according to the discussion in [17].

The results of LO and NLO fits to H1&ZEUS data [10], with various lower cuts on Q^2; in the fits the number of flavors f is fixed to 4

	A_g	A_q	Q_0^2, GeV^2	$\chi^2/n.d.f.$
		$Q^2 \geqslant 5$ GeV^2		
LO	0.623 ± 0.055	1.204 ± 0.093	0.437 ± 0.022	1.00
LO&an.	0.796 ± 0.059	1.103 ± 0.095	0.494 ± 0.024	0.85
LO&fr.	0.782 ± 0.058	1.110 ± 0.094	0.485 ± 0.024	0.82
NLO	-0.252 ± 0.041	1.335 ± 0.100	0.700 ± 0.044	1.05
NLO&an.	0.102 ± 0.046	1.029 ± 0.106	1.017 ± 0.060	0.74
NLO&fr.	-0.132 ± 0.043	1.219 ± 0.102	0.793 ± 0.049	0.86
		$Q^2 \geqslant 3.5$ GeV^2		
LO	0.542 ± 0.028	1.089 ± 0.055	0.369 ± 0.011	1.73
LO&an.	0.758 ± 0.031	0.962 ± 0.056	0.433 ± 0.013	1.32
LO&fr.	0.775 ± 0.031	0.950 ± 0.056	0.432 ± 0.013	1.23
NLO	-0.310 ± 0.021	1.246 ± 0.058	0.556 ± 0.023	1.82
NLO&an.	0.116 ± 0.024	0.867 ± 0.064	0.909 ± 0.330	1.04
NLO&fr.	-0.135 ± 0.022	1.067 ± 0.061	0.678 ± 0.026	1.27
		$Q^2 \geqslant 2.5$ GeV^2		
LO	0.526 ± 0.023	1.049 ± 0.045	0.352 ± 0.009	1.87
LO&an.	0.761 ± 0.025	0.919 ± 0.046	0.422 ± 0.010	1.38
LO&fr.	0.794 ± 0.025	0.900 ± 0.047	0.425 ± 0.010	1.30
NLO	-0.322 ± 0.017	1.212 ± 0.048	0.517 ± 0.018	2.00
NLO&an.	0.132 ± 0.020	0.825 ± 0.053	0.898 ± 0.026	1.09
NLO&fr.	-0.123 ± 0.018	1.016 ± 0.051	0.658 ± 0.021	1.31
		$Q^2 \geqslant 0.5$ GeV^2		
LO	0.366 ± 0.011	1.052 ± 0.016	0.295 ± 0.005	5.74
LO&an.	0.665 ± 0.012	0.804 ± 0.019	0.356 ± 0.006	3.13
LO&fr.	0.874 ± 0.012	0.575 ± 0.021	0.368 ± 0.006	2.96
NLO	-0.443 ± 0.008	1.260 ± 0.012	0.387 ± 0.010	6.62
NLO&an.	0.121 ± 0.008	0.656 ± 0.024	0.764 ± 0.015	1.84
NLO&fr.	-0.071 ± 0.007	0.712 ± 0.023	0.529 ± 0.011	2.79



x dependence of $F_2(x, Q^2)$ in bins of Q^2. The combined experimental data from H1 and ZEUS collaborations [10] are compared with the NLO fits for $Q^2 \geqslant 0.5$ GeV^2 implemented with the standard (solid lines), frozen (dash-dotted lines), and analytic (dashed lines) versions of the strong coupling constant

CONCLUSIONS

We have studied the Q^2-dependence of the structure function F_2 at small x values within the framework of perturbative QCD. Our twist-two results are well consistent with precise H1&ZEUS data [10] in the region of $Q^2 \geqslant 4$ GeV^2, where perturbative theory is thought to be applicable. The usage of «frozen» and analytic modifications of the strong coupling constant, $\alpha_{\rm fr}(Q^2)$ and $\alpha_{\rm an}(Q^2)$, is seen to improve an agreement with experiment at low Q^2 values, $Q^2 \leqslant 1.5$ GeV^2.

The work was supported by RFBR grant No. 11-02-1454-a.

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