ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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ON THE MASSES OF ELEMENTARY PARTICLES

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We make an attempt to describe the spectrum of masses of elementary particles, as it comes out empirically in six distinct scales. We argue for some rather well-defined mass scales, like the electron mass; we elaborate on the assumption that there is a minimum mass associated to any electric charge. Another natural mass scale is $\Lambda = \Lambda_{\rm QCD}$ coming arbitrarily at quantizing a classically conformal $SU(3)_c$ theory. Indeed, some scales of masses will cover also masses of composite particles or mass differences. We extend some plausible arguments for other scales, as binding or self-energy effects of the microscopic forces, plus some speculative uses, here and there, of gravitation. We also consider briefly exotics like supersymmetry and extra dimensions in relation to the mass scale problem, including some mathematical arguments (e.g., triality), which might throw light on the three-generation problem. We also address briefly the issues of dark matter and dark energy. The paper is rather tentative and speculative and does not make many predictions, but it aims to explain some features of the particle spectrum.

В работе делается попытка описать спектр масс элементарных частиц эмпирически, исходя из шести различных шкал. Приводятся доводы в пользу некоторых наиболее хорошо определенных шкал масс, таких как масса электрона. Предлагается подход, основанный на предположении, что существует минимальная масса, связанная с любым электрическим зарядом. Другая естественная шкала масс — это $\Lambda = \Lambda_{\rm QCD}$, возникающая при квантовании классической конформной $SU(3)_c$-теории. Действительно, некоторые шкалы масс включают в себя также массы составных частиц или разницы масс. Приводятся аргументы в пользу других шкал, такие как эффекты связи или собственной энергии микроскопических сил плюс некоторые спекулятивные заключения из теории гравитации. Также в качестве возможного описания проблемы массы коротко рассматривается экзотика в виде суперсимметрии и дополнительных измерений, включая некоторые математические аргументы (например, тройственность), которые могут пролить свет на проблему трех поколений. Коротко обсуждаются проблемы темной материи и темной энергии. Статья представляет собой рассуждение на тему и не имеет предсказательного характера. Но все же ее основной целью является объяснение некоторых свойств спектра частиц.

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1. MOTIVATION

One of the most unsatisfactory features of our understanding of the microworld is the status of the spectrum of masses: The masses of elementary particles are not predicted at all, and in the Standard Model (SM) they are just

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given by arbitrarily variable couplings to the overall scalar Higgs boson, undiscovered so far; the coupling is just adjusted as to reproduce the experimental mass; and this, of course, is none an explanation!

For the admittedly large predictive power of the theory of SM one needs first enter by hand these masses and the coupling constants, as well as some information on the types of acting particles, like spin, charge, etc. Then many scattering processes, plenty of decay constants and some bound states can be accurately predicted by the theory: The three known microscopic forces can be described successfully by the respective gauge theories, and in the three cases many checks can be performed, and are fairly well borne out by the experiments; it is only when one asks questions about the mass spectrum or the range of the coupling «constants», that the answers are scarce, or in cases nonexistent at all; indeed, the total number of parameters to be fixed beforehand to compare experiments with theory is rather large, well beyond twenty [1]. Of course, low-energy calculations in strong interactions (Quantum Chromodynamics, QCD) are marred for our inability to perform nonperturbative calculations, but even there some successes (e.g., for many hadrons as bound states) have been achieved by lattice calculations, etc.

However, we notice that the particle mass spectrum is not completely chaotic, and some levels and groupings are clearly apparent phenomenologically. In the present essay we look at the problem of identifying these levels, and provide, when possible and sensible, a *rationale* for them. These groupings might include also masses for some composite particles, e.g., the pion mass or the neutron–proton mass differences will be considered in some of the mass scales we shall discuss.

One of our tenets will be the interpretation of the electron mass scale ≈ 0 (1 MeV), with a *minimum* mass supporting particles with electric charge. Another one will be the scale $\Lambda_{\rm QCD}$, separating the two regimes, confinement and asymptotic freedom, of QCD.

The coupling constants are also used as given, but some speculations based on running towards Grand Unification are also contemplated, as well as some appeals to extra dimensions and/or supersymmetry. For a recent alternative use of the Higgs scalar(s) in the SM see [2].

2. THE SCALES OF MASSES: GENERAL DISCUSSION

If we look at the experimental masses of particles around us, they clearly gather in some groups. Here we give just a broad introduction to the subject, with a specific discussion of each level later on.

We neatly observe *six mass scales* (see, e.g., Particle Data Group, PDG [3]):

1. *Massless* particles, $m = 0$. As far as we know, the following particles

$$\text{Photon}\,\gamma, \quad \text{Gluon}\,g, \quad \text{and} \quad \text{Graviton(?)}\,h \tag{1}$$

seem to be massless to a large precision (e.g., $m_\gamma < 1 \cdot 10^{-18}$ eV [3]). In theory, the gluon mass is zero; the graviton is yet to be found, but it is expected to be massless also.

2. *Neutrino mass scale*. The next level is the neutrino mass scale: although only square differences are measured so far for neutrinos, there is some consensus on two neutrino mass difference values and the corresponding mixing angles, the third mixing angle being rather small. The PDG quoted values for the masses are as follows:

$$|m_2^2 - m_1^2| \approx (9 \cdot 10^{-3})^2 \text{ eV}^2 \quad \text{and} \quad |m_3^2 - m_2^2| \approx (4 \cdot 10^{-2})^2 \text{ eV}^2. \tag{2}$$

Neutrinos are the lightest *leptons**, with presumably bare masses of the order of 10^{-2} eV.

3. *Electron mass scale*. At a value more than a million times higher, it does show up the electron mass scale, around the MeV: besides the electron e, we include in this level also the first-generation quarks u, d:

$$\text{electron,} \quad m_e = 0.512 \text{ MeV}; \quad \text{quarks:} \quad m_u \approx 2 \text{ MeV}, \quad m_d \approx 4 \text{ MeV}. \tag{3}$$

Of course, quark masses (*current* masses for u, d) are *deduced*, by a somewhat indirect way, from several experimental pieces of data; see, e.g., [4, 5].

4. The *muon and* $\Lambda_{\rm QCD}$ *scales*. The muon lepton μ was a fully unexpected surprise when discovered (1937); today the muon mass *level* is well populated, with the strange quark s, the composite pion π, the so-called QCD scale, $\Lambda_{\rm QCD}$, etc.; all these masses are around 100–250 MeV. The scale includes also the pion, although it is not elementary, and the strange quark s:

$$m_\mu = 106 \text{ MeV} \quad (m_\pi \approx 137 \text{ MeV}), \quad m_s \approx 104 \text{ MeV}. \tag{4}$$

Around $\Lambda = \Lambda_{\rm QCD} \approx 250$ MeV, the scale of QCD, the regime changes, roughly speaking, from asymptotic freedom ($q^2 \gg \Lambda^2$) to confinement ($q^2 \ll \Lambda^2$). Recall, in QCD, Λ is an arbitrary parameter to be fixed by experiments.

5. The *nucleon mass scale*. Again, proton p and neutron n are not elementary, but the charm meson c is included, as well as the third charged lepton, τ, and the bottom quark b; all group around the GeV scale:

$$\begin{aligned} &c \text{ (charmed quark)}, \ m_c \approx 1.27 \text{ GeV}, \\ &b \text{ (bottom quark)}, \ m_b \approx 4.2 \text{ GeV}. \end{aligned} \tag{5}$$

$$\text{Tau lepton } \tau \text{ with } m_\tau = 1.8 \text{ GeV}. \tag{6}$$

*The term *lepton* has been introduced to include nonquark fermions by Rosenfeld L. in his book «Nuclear Forces». North-Holland, 1948.

$$\begin{aligned}&\text{(Proton } p\text{), as } m_p = 939 \text{ MeV;}\\&\text{(Neutron } n\text{), as } m_n - m_p \approx 1.2 \text{ MeV.}\end{aligned} \tag{7}$$

6. The *electroweak (broken) mass scale*. Finally, we have the electroweak mass scale, with the massive gauge bosons: $W^{\pm}$ and Z vector mesons, as carriers of the weak force, rank at the next level, with masses around 100 GeV; also $\langle H \rangle$, the expectation value of the (original neutral, scalar) Higgs field H, is in the same ballpark. The value of the original (1934) Fermi coupling constant G_F (with $G_F^{-1/2} \approx 292$ GeV) was of course also comparable. The last discovered quark (1995), the top t, is also placed in this level. Hopefully the new-to-be-discovered Higgs particle(s) would have a mass on the same range, so we have

$$m_{W^{\pm}} = 80 \text{ GeV}, \quad m_Z = 91 \text{ GeV}, \tag{8}$$

$$m_t = 173 \text{ GeV}, \quad m_H > 114 \text{ GeV}, \quad \langle H \rangle = 247 \text{ GeV}. \tag{9}$$

Some bounds on the Higgs mass are discussed in the recent paper [6].

With Supersymmetry (Susy) one needs more than one Higgs, but the minimum mass quoted is around the cited limits; see later.

Interactions. These are the clear-cut mass scales we see experimentally; they group ostensibly in the six above-mentioned scales. Now the question of *interactions* arises, as physically masses should come from forces, from interactions. There should therefore be relations between *masses* and *forces*. About the forces present in physics, we take the conventional view of the *four* interactions: Einstein's *general relativity* as a theory of (pseudo-)Riemannian space-time (with $-+++$ signature), with the geometric description of the gravitation force: geodesic motion for test particles in a given gravitational field, and curvature generated by matter as in Einstein equations of gravitation (1915). Of course, due to the weakness of gravitation on the ordinary microscopic scale, we can take as the spacetime manifold just Minkowski space, which is flat. Nevertheless, gravitation is an essential part of the whole of physics, so one would not be surprised if it also enters somehow into the microworld, at least as an ordering parameter.

And there are three *microscopic forces*, described as gauge theories, that is, mathematically as connections in some vector bundles, with the structure group being the composite (nonsimple) Lie group $G = SU(3)_c \times SU(2)_{\mathrm{wi}} \times U(1)_Y \equiv (3, 2, 1)$ (c for colour and wi, Y for weak isospin and hypercharge) and the associated principal and vector bundles. Naturally, the Quantum Theory requires renormalization; a very good source book is [7]. Of course, the group G by itself implies only the existence of the $8 + 3 + 1 = 12$ gauge vector bosons with «spin» or helicity: $s = 1 = |h|$, in the adjoint representation of the gauge group G, and physically massless *if* there is no spontaneous symmetry breaking (but see again [2]), which seems to be the case for colour $SU(3)_c$ and for electromagnetism, $U(1)_{\mathrm{em}}$: the latter is a subgroup of the $SU(2)_{\mathrm{wi}} \times U(1)_Y$

group, the precise «location» being measured by the Weinberg angle $\theta_W \approx 30°$. The matter contents are the fundamental (or vector) *representations* of the groups: quarks and leptons, but there are more possibilities; the spin of the matter particles is not predicted, but it is $s = 1/2$ overwhelmingly; we do not know why. The putative Higgs(es) would have spin zero.

It is perhaps interesting to quote here Witten's analysis [8] for the dimension of the natural *internal* spaces acted upon by the group (3,2,1) of the SM: it has to be 7-dimensional, so here there is an argument for a total of $4+7 = 11$-dimensional space-time (no longer flat), the same dimension to support maximal supergravity, to be considered briefly later, which also lives in 11 dimensions! [9, 10]. The group-theoretical favourite space is the homogeneous space $CP^2 \times CP^1 \times RP^1$, or $[SU(3)/U(2)] \times [SU(2)/U(1)](= S^2 = CP^1)\times [S^1](= RP^1)$.

However, in the modern theory [11], also in 11 dimensions, compactification might be very different, for example, with a G_2 holonomy [12]. Does electroweak breaking have something to do with 11-dimension space? With maximal supergravity?

Summing up, we see the particle spectrum spread out in six levels, roughly speaking, as (1): $m_\gamma = 0$; (2): $m_\nu \approx 10^{-2}$ eV; (3): $m_e \approx 1$ MeV; (4): $m_\mu \approx 100$ MeV; then (5): $m_c \approx 1$ GeV; and finally (6): $M_Z \approx 100$ GeV. The known four forces seem to be, at first sight at least, at a loss to explain these mass levels; although level (3) seems dominated by the e.m. forces, and (4) could be due to the arbitrary $\Lambda_{\rm QCD}$ and perhaps the (6) scale is due to (electro-)weak force breaking (?). Level 5 for nucleons is beginning to be understood from QCD lattice calculations.

With this information as input, we want to see now whether some rational explanation(s) can be advanced for these mass levels, and for the particles they encompass.

3. THE MASSLESS LEVEL

The *massless* property of the *photon* γ is true experimentally to an astonishing degree, $m_\gamma < 10^{-18}$ eV, so Coulomb forces fall off exactly with the $1/r^2$ law; also the photon seems to be exactly electrically neutral ($q_\gamma < 5 \cdot 10^{-30} e$ [3]). We understand this, as the photon is the carrier of the e.m. force, with $U(1)$ as gauge group, and the group being Abelian, the adjoint representation is the trivial one, so γ is chargeless, and as the $U(1)$ gauge group it is neither spontaneously nor explicitly broken, the γ remains massless.

The *gluons* g are the carriers of the (colour) strong force, whose gauge group is $SU(3)_{\rm colour}$, so there are eight$= 3^2 - 1$ of them; they have not been seen isolated, but known only indirectly; though all studies imply also that the

QCD gauge group $SU(3)$ is exact, so the gluon g must also be massless (but coloured). Now the continuation of the proven *asymptotic freedom* property of strong QCD forces (that is, the UV limit $q^2 \to \infty$ is trivial, it is a free theory; this «justifies» that the colour self-energy of gluons or quarks generates *no* mass for them!) will perhaps imply *infrared slavery* [13, 14], so confinement will hide the true masslessness property of the gluon [15]. Experimentally, a mass of a few MeV for gluons cannot be ruled out as today [3]. Contrary to photons, which are chargeless, the gluons carry colour (with the dim-8 adjoint representation of $SU(3)_c$, as said); so it must be anticipated that some consequences of the colourful gluons like gluonium «atoms», «glue» contribution to the mass of hadrons (see below) etc., will show up experimentally.

Speculations for the $SU(3)_c$ group as coming from the octonion numbers are also sometimes contemplated [16, 17]: $SU(3)$ is the stabilizer or «little group» of the octonion-algebra automorphism group G_2, acting on the S^6 sphere of unit imaginary octonions. Also, manifolds with G_2 holonomy, as said, are the favourite ones for compactifying from 11 to our mundane 4 dimensions [12]; in any case, it is just remarkable that the $SU(3)$ group appears at least *three times* in the phenomenology/theory, to wit: colour, flavour (i.e., the original $SU(3)$ of Gell-Mann and Ne'eman, 1961), as well as the holonomy group of the heterotic string compactification Calabi–Yau (CY) space.

The «graviton» h has never been found, and reasonable doubts exist (e.g., by F. J. Dyson [18, 19]) it never will; but we take the conventional view that the long-range decay of gravitation, i.e., the $1/r^2$ gravitational force law, will «translate» into the massless character for the putative graviton, too. The natural mass for any gauge boson is zero, unless the gauge group is broken; there seems to be no reason why the $U(1)_{\rm em}$ group should be broken, neither the very same Lorentz group L_0 should be spontaneously broken (*explicit* breakings of Lorentz invariance are also contemplated nowadays, but do not take stand in the issue; see, e.g., [20]). We all hope that at the end of the day the gravitation interaction should join the other microworld forces, but at the moment there is a clear-cut distinction; some ideas along a unification line-of-thought will be presented as we go along.

So the only gauge symmetry broken is the $SU(2)$ group of weak isospin (wi); more precisely, that part of $SU(2)_{\rm wi} \times U(1)_Y$ that leaves the mixed $U(1)$ group of e.m. as an exact symmetry, the mixing being determined by the Weinberg angle θ_W [21]. As a consequence, we have the three massive boson states: $W^{\pm}$ and the neutral Z. In our philosophy that *any electrically charged particle must have a mass*, we realize why $SU(2)_{\rm wi}$ cannot be exact: the Ws are charged. Of course, the arguments do not tell about the magnitude of the breaking, and experimentally the expectation value of the Higgs field, $\langle H \rangle$, on the 100–250 GeV range, is a factor of 10^5 of the minimal mass scale to support an electric charge: The reason why also the Z is so massive and why the mass is not the minimum e.m. mass, like the electron mass, is unclear at the moment; it should come probably from some argument intrinsic to the e.w. force breaking mechanism. The W must, of course, be massive, because it is charged, and it is, but here the charge does not determine the mass, as it seems to be the case for the electron and first-generation quarks. What about the Z mass? It is clear that the whole weak-isospin triplet $(W^{\pm}, Z)$ is broken symmetrically, so $m_W \approx m_Z$ is not unexpected.

So we believe we throw a little light on the necessity of the breaking of $SU(2)_{\mathrm{wi}}$, and in the exact nature of both the $U(1)_{\mathrm{em}}$ and the $SU(3)_c$ gauge groups. . .

4. THE NEUTRINO MASS SCALE

The story of neutrinos is worth recalling briefly in our context [22]: first hypothetized as neutral particles and with a tiny (if at all) mass by Pauli (unpublished) in 1930, they were instrumental in Fermi's successful «four-fermion» beta decay theory (1934) [23]; even Fermi already asked himself about the neutrino mass. When parity violation was discovered in 1957 (supposedly conjectured by Lee and Yang, 1956; decisive experiments started by C. S. Wu in January, 1957), the two-component neutrino theory of H. Weyl (1929) was resurrected to «justify» parity violation, in the models of Salam, Landau, and Lee–Yang (1957); neutrinos still entered massless in the «universal Fermi interaction, $V - A$», of Sudarshan (1955) and Feynman–Gell-Mann (1958). To recall that neither Fermi's original treatment nor the parity-violation refinement of Lee and Yang dealt with not renormalizable theories. . . by exactly the same argument that gravitation was not, namely the appropriate coupling constant has length dimensions; in fact, $[G_N] = [G_F] = (\mathrm{Length})^2$. Besides the specific $V - A$ form of the theory, the main advance of this post-war period was the extension of the original beta-decay theory to the whole world of weak interactions, including muon decay and capture, decay of strange particles, etc. B. Pontecorvo [24] seems to be about the first person to conceive unified weak interactions as the natural extension of nuclear beta decay, around 1947.

Two *different* neutrinos $(\nu_e \neq \nu_\mu)$ were first recognized/identified in 1962, but the issue of the neutrino masses did not arise experimentally until the turn of the century, with the «solar missing neutrino problem» (see, e.g., [25, 26]). After some troubles, neutrino(s) were adjudicated undoubtedly positive mass *differences* around the year 2001 (atmospheric neutrinos & Kamiokande experiments [27]); a third neutrino has also been identified nowadays. In fact, only *squares* of neutrino mass *differences* were measured, with the values quoted at the beginning, which contain large errors. For an update of the neutrino masses and mixing angles, see [28].

Massive neutrinos raise many questions; one is the following: in the late fifties, Weyl neutrinos were presented as a *rationale* for parity violation, as they were intrinsically left-handed (hence massless). Then, one might ask, what happens to the argument, that massless neutrinos being instrumental in «explaining» parity violation, once neutrinos have mass? For a short discussion of this, see, e.g., [29]. There is also some speculation about the neutrino mass differences as a generation effect [30], so perhaps the tau-type neutrino would have different mass scale that the other neutrinos. Other possibility is a self-energy effect coming from the weak force, see below.

The Standard Model (SM), conceived since 1970 and completed around 1975, still supposed massless neutrinos... But in fact, a slight enlargement of the SM will accommodate massive neutrinos without too much trouble.

Some actual questions about neutrinos are, for example:

1. What determines the small scale, $\approx 10^{-2}$–10^{-3} eV, for some neutrino masses? We have no clue, but we offer here the following *negative* argument: nature works with the axioms of a *totally compulsory (fascist) state*: all which is not forbidden is mandatory; there is nothing to impose zero mass for the neutrinos (as there is for the photons!), hence neutrinos have to have a mass! As they have no charges, the mass could be less than the electron mass (and it seems to be!). On the positive side, we expect that once gravitation forces will be accommodated with quantum mechanics (see later), a kind of gravitational and/or weak interaction self-energy of the neutrinos (they have weight, after all!) could generate a mass for them. That is, as neutrinos experience the (purely) weak force, a self-mass is not to be ruled out, of «similar» origin to that of the electron mass or first quark masses. For a clear-cut «gravitation neutrino» see [31]. However, «a priori» it is difficult to understand the ratios

$$\frac{m_\nu}{m_e} \approx 10^{-8}, \quad \frac{m_e}{m_Z} \approx 10^{-5}. \tag{10}$$

2. Are *the three* neutrinos massive? Are they more than three? At the moment only two mass differences do exist, but we believe (and predict, really) that the three neutrinos have *no* reason to be massless, hence the three of them *must be massive**... and as the reasons should be similar, the three of them should have masses in the same range, meV, for example (massive $\approx\geqslant 1$ eV interacting neutrinos are to be excluded by astrophysical reasons); but see [30]. Experimentally, direct measurements of neutrino masses are still out of question, but it might come up to be possible in the future (for example, after careful measurements of the end-spectrum of some nuclear beta decay processes like tritium decay, double beta decay (neutrinoless or neutrinoful), etc.). There are several experiments planned to resolve this issue.

*Some people claim that, as the three neutrinos oscillate, the three must be massive.

3. On the other hand, neutrino masses apparently do not experience the «generation effect» present in other leptons and in quarks: electron, muon or tauon have very different masses, and so have, e.g., the up u, the charmed c and the top t quarks, as well as d, s, and b. So there must be a generation effect, perhaps related to charges, which is not (?) present in neutrinos, and which we do not understand yet; but again, this is not all clear-cut.

4. How do neutrinos mix? The Cabbibo–Kobayashi–Maskawa (CKM) matrix for flavour mixing suggests a corresponding neutrino mixing matrix, which does exist, but at the moment is incompletely known. Although the third mixing angle should be rather small, if nonzero, as expected, it will allow for an extra $U(1)$ phase contribution to the CP violation, which is rather welcome, to explain the matter–antimatter asymmetry present in the actual Universe!

For speculations about the masses of the three neutrinos, see [32, 33].

Are there other hints for the existence of a neutrino scale, turned out in mass, to be so small? Yes, there are *cosmological* arguments: (i) The existence of a positive cosmological constant Λ, producing *accelerated* cosmic expansion on top of Hubble's constant-velocity flow, is out of question since about the year 2000, and its value translates into the meV scale, close in fact, to the neutrino mass scale [3]. And (ii) besides, the average density of energy in the Universe should also be in this range, as the cosmological constant amounts for about 0.7 of the mean density of the actual Cosmos*. As the evolution of the Universe is most likely consequence of gravitation, one sees another hint, perhaps, that the neutrino masses should be related to the gravitation-dominated actual evolution of the Universe as a whole**.

The neutrinos are still very mysterious. Are they Dirac or Majorana particles [34]? A particle of type ($m > 0$, $s = 1/2$) has four components, interpreting negative energies as antiparticle states; but the neutrinos active in beta decay are fundamentally chiral (that is, the beta-decay neutrino is left-handed, as if it were a massless fermion, and the antineutrino would be right-handed). What about the other two degrees of freedom? There is the famous *see-saw* mechanism of Gell-Mann and Ramond [35], which relates neutrino masses, electroweak breaking scale and the Grand Unified Theories (GUT) mass scale. Leaving for later speculations on the GUT scale, the bland argument in [35] is that the electroweak scale (around 100 GeV) is $\propto$ to the «square root» of the GUT scale times the actual neutrino scale, to wit:

$$\begin{gathered} M_Z^2/(m_\nu \times M_{\rm GUT}) \approx 1, \text{ e.g., with } M_Z = 90 \text{ GeV}, \\ m_\nu \approx 10^{-2} \text{ eV if } M_{\rm GUT} \approx 10^{16} \text{ GeV}. \end{gathered} \tag{11}$$

*The critical density of the Universe is [3] $\approx 1 \cdot 10^{-5} h^2$ (GeV/c^2) $\cdot$ cm^{-3}, and 70% is contributed by the repulsive Cosmological Constant.

**We thank A. Seguí (Zaragoza) for an illuminating discussion of this point.

It remains to be seen how compelling is this see-saw mechanism.

Another line of argument, with the same conclusion is perhaps more cogent: let us start with the cosmological constant value Λ (expressed as an energy); in the future it must be related (at least) with gravitation; now neutrinos undergo gravitation forces, so there is no big surprise (?) if both effects are in the same ballpark, let us say, the meV regime... For a recent study of the Cosmology at the meV-scale, see [36].

Neutrinos are very abundant in our Universe, and they are created continuously in the interior of burning stars: so it would not be a whole surprise if they fill a cosmological role, contributing to Λ (the cosmological constant), for example.

5. THE ELECTRON MASS SCALE

We quote first some data [3]:

$$\text{electron mass: } m_e \approx 0.511 \text{ MeV}$$
$$\text{(with precision } \pm 13 \text{ meV, better than } 10^{-7}), \tag{12}$$

$$\text{up quark } u,\ m_u \approx 2.4 \text{ MeV; down quark } d,\ m_d \approx 4.8 \text{ MeV.} \tag{13}$$

The first generation quarks, u and d, have large errors in their masses, about 50%. The d is heavier than the u in spite of the charge of the u being twice that of the d; the given masses are understood as *current* masses (as opposed to *constituent* masses, possible for higher mass quarks).

Our philosophy, to repeat, is this: there must be a *minimal* supporting mass for any electric charge, because the nontrivial UV behaviour of QED (in modern parlance, QED should be an inconsistent, «trivial» theory); in the conventional, renormalized theory, the electron *bare* mass is *infinite*, and everything is computable from the experimental mass, taken at face value. The empirical electron mass fixes an electron radius (as expressed already more than 100 years ago by Lorentz (and Poincaré)) by the formula $e^2/r \approx m_e c^2$: for $r \approx$ nuclear radius ($= 2.8 \cdot 10^{-15}$ m), the mass comes out to be $\approx 1/2$ MeV. See also [37].

Why the u quark (charge $+2/3$) is lighter than the d (charge $-1/3$)? We do not know, we only remark that both masses are bigger than the electron mass... but not much bigger. Perhaps some subtle unknown QCD argument would explain this mismatch in the future... but in any case it is satisfactory for us to see that the first-generation quarks u, d have masses just compatible with being electrically charged. In QCD the first-generation quarks u, d are given masses as the global chiral symmetry $SU(2)_L \times SU(2)_R$ is broken, *both* spontaneously (witness the low mass pion) and explicitly (witness quark masses).

Are there other (e.m.?) mass differences of the order of the electron mass? Plenty, starting by n–p mass differences (and also the positive/neutral pion π (139.6 MeV for π^+ vs. 135 MeV for π^0). But also, the validity of isospin invariance (Heisenberg, 1932) is rather good in nuclei, which guarantees that, e.g., tritium and He^3 have very close masses, which gives 31 years for the long lifetime of tritium, 18 keV for the reaction energy, and the best case for limiting β-decay neutrino mass. We are pretty sure experimentally that, when a mass difference is assignable to e.m. differences, or to isospin violation in the old language, then these differences are in the MeV range; this holds equally in elementary hadrons like the Σ triplet of hyperons as well as in ordinary nuclei.

One feature, for example, that comes close to be explained, is that the neutron–proton mass difference should be positive, as n is, in quark content, (udd), p is (uud), and the down quark is more massive than the up one [38].

6. THE MUON AND $\Lambda_{\rm QCD}$ SCALES

We first recall Rabi's dictum [39] «who ordered that?» in reference to the very existence of the muon, discovered, as said, in 1937. For W. Heisenberg, the muon was the biggest mystery of elementary particles [40]; still today, the only «reason» we see for the existence of (three) generations of leptons is from the anthropic point of view: it is CP violation (experimentally unavoidable: this is why we do exist*!), which requires (Kobayashi–Maskawa) at least three quark generations: so the true answer to Rabi's old question of why muons exist is this [41]: it was YOU yourself who ordered them, as your very existence depends on the presence of at least three generations, muons being part of the second, to explain overabundance of matter vs. antimatter!

Unfortunately, it turns out that the measured amount of the CP-violation strength (in neutral K decay, for example) is not enough to explain in quantitative terms the abundance of matter vs. antimatter in our observable Universe, but it is on the right track. We expect that the possible CP violation in the *neutrino* mass matrix (see above) should help...

As the muon mass is in the same batch as the pion mass (100 MeV vs. 137 MeV), one should look perhaps for a *common mechanism* generating their masses. For the pion π there is such a mechanism; it goes with the name of «chiral symmetry breaking», an emergent phenomenon of strong forces, not totally understood as today. This global (i.e., nongauge) chiral symmetry (i.e.,

*With three (or more) generations the CKM mixing quark matrix allows for an extra $U(1)$ angle, conducive of CP violation; Kobayashi and Maskawa, 1973.

$SU(2)_L \times SU(2)_R$) is not shared by the vacuum, and the corresponding Goldstone boson is an hypothetical massless pion, which becomes massive by some explicit breaking… giving a mass to the π much less than the average hadron masses. We amplify these remarks below.

It is remarkable as it is unexplained (but see later for a similar relation involving the bottom quark b and the τ lepton) that the strange quark s and the muon μ (and also the pion π) are in the same ballpark. Also it is remarkable that an «e.m.» correction to either pion or muon masses, that is, an α-order correction to the masses (where $\alpha = e^2/\hbar c \approx 1/137$ is the fine structure constant) gives one back the electron mass scale! [32]. For a recent report relating muon mass with many other masses, see the essay [42].

QCD is a gauge theory of quarks and gluons, with $SU(3)_c$ as the gauge group. It has been proposed since 1972 (Gell-Mann and Fritzsch) as the true theory of strong interactions; in this theory, there is a limit in which one couples massless quarks (the first generation, u and d; it is a worse approximation, but still viable, with three quarks, adding the strange quark s) to the gluon field; massless helicity $\pm 1/2$ particles can couple the two helicities differently, as in the weak interaction. Now QCD in this limit admits however a global (i.e., nongauge) $SU(2)_L \times SU(2)_R$ internal invariance group. But this symmetry is *spontaneously broken* to $SU(2)$ diagonal for some obscure mechanism (which we shall not try to select: fermion condensates, anomaly cancellation, etc., have been proposed as solutions). But of course, there is then the attendant Goldstone mechanism, as there are directions from the vacuum which require no effort to move on: the Nambu–Goldstone (NG) bosons are massless. When this idea was proposed in the early 1960s [43] it was generally rejected, because if something was certain in the hadron spectrum was the absence of massless particles. On the other hand, we have had the pion π since 1947, and by mid-1960 it was clearly the lowest mass hadron, by far: the pion is very light on the hadron scale, it is pseudoscalar, and carries isospin 1, all consistent with the way the chiral group is broken, so it may be the NG boson. Could it be, asked Weinberg [44] and others, that the massive but very light pion π sould be a reminder of that spontaneously broken chiral symmetry, which became explicitly broken by some nonchiral invariant term, giving a little mass to the pion, which then would become a «pseudogoldstone» particle? One possible explicit chiral breaking term is the quark mass, in its turn unavoidable in our framework that gives a mass to any charged particle, and the quarks are charged! In other words, chiral symmetry is broken both spontaneously as well as explicitly, but we understand the second process (as unavoidable) better! Recall also the pion is an isospin triplet, with two charged components $\pi^{\pm}$, which in our philosophy cannot be massless!

The next main question here is this: Is there any theoretical reason for *that* value for the pion mass? Will it still be the same (pion mass hundred times the minimal quark mass) for a QCD without quarks? Is it related to the «mass gap» in QCD, one of the Clay Mathematics Institute problems [45]? In all QCD treatments the chiral breaking mass scale is put by hand; the idea is that the flavour group $SU(2)_L \times SU(2)_R$ breaks spontaneously to $SU(2)_I \equiv SU(2)$ diagonal; as said, the consequential massless boson (Nambu–Goldstone) is the pion; explicit breaking should account for the u, d quark masses, and also for the very pion mass, much bigger. Lattice calculations with QCD account for many hadron masses, once the input is given, namely: the light QCD scale, around 200 MeV, and also the first generation quarks masses, around a few MeV [46].

The same scale is present also in the s, the strange quark mass, the third quark to be discovered (strange particles discovered in the late forties in cosmic rays (Rochester and Butler) interpreted as the need for the third, strange quark around 1962, with the Gell-Mann «$SU(3)$ flavour symmetry»); this symmetry is rather badly broken, so it is much poorer than isospin. We know today, since the old arguments of Glashow et al. [47] that quarks and leptons should accommodate in the same generations, lest we confront too much neutral currents with change of flavours. In particular, the fourth quark, the charmed one c, was predicted once the $SU(3)_{\text{flavor}}$ group became accepted, even approximately. Y. Ne'eman was one of the first [48] to try to relate the strange quark s with the μ lepton, unsuccessfully we must say. There is also an additional anomaly cancellation condition, first put forward in [49].

What is the reason for this intermediate scale? Granted we do not really fully understand any scale, this level, 100–250 MeV, is perhaps the most mysterious of all (that is, one can associate, e.g., the electron, proton or Z scale to self-energy or binding effects of the e.m., strong or broken weak force). So it comes as a partial relief when we notice that QCD exhibits a range of phenomena around the so-called Λ_{QCD}, close to 200 MeV. In particular, QCD is a classically conformal (scale-free) theory, where the phenomenon of *dimension transmutation* takes place: the dimensionless coupling constant α_{QCD} is «traded» for a renormalization energy scale, that we can identify with Λ_{QCD}. What is the relation with the s quark, or the μ lepton, or for that matter with the very QCD theory?

We insisted on the electron mass coming from QED self-energy; this clearly does not apply to the muon: instead, as Barut, Fritzsch and others have noticed (see, e.g., [50]), if the muon scale is a «natural» one, the electron mass is seen as an electromagnetic α-order correction: it is a very good adjustment to set

$$\frac{m_e}{m_\mu} \approx \left(\frac{2}{3}\alpha\right) \approx \frac{1}{206}. \tag{14}$$

If the above explanation stands, we shall never be able to deduce *masses*, only mass *ratios*. Some scale, e.g., Λ_{QCD}, should be taken for granted.

7. THE NUCLEON MASS LEVEL

The bound states of the strong force are to-day called *hadrons*, name due to L. B. Okun*. They come in two classes: mesons, made out of quark–antiquark pairs $\bar{q}q$, and baryons, made out of three quarks qqq (or $\bar{q}\bar{q}\bar{q}$); only $SU(3)_c$ singlets are allowed, because the confining character of the non-Abelian gauge force at the IR limit: colourful states do not appear then as free states. What about the binding energy due to this colour force? Although we are not much concerned here with reporting masses of particles composites of bound quarks, we can add some considerations. There are conceptually at least *three* different scales of colour binding energy:

1. In the broken chiral limit, the pion mass sets the minimal scale for colour binding, around 150 MeV. In that scale one can put, not only the isotriplet of pions, but the whole octet 0^- as pseudogoldstone bosons of broken $SU(3)_L \times SU(3)_R$ flavor, generated by the three lightest quarks, u, d, and s. In fact, in the eightfold-way (for $SU(3)_{\rm diag}$) the octet seen from the I isospin-$SU(2)$ subgroup splits in pions π ($I = 1$, three states), kaons K ($I = 1/2$, four states) and the singlet η ($I = 0$), all in the $\leqslant 1/2$ GeV range, consistent with: first generation $\bar{q}q$ mesons with NG mass reduction, the π: mass < 150 MeV; the four K mesons, mass < 500 MeV (already the s quark, entering the K meson bound states, contributes ≈ 100 MeV; also, the $SU(3)_{\rm flavor}$ is much more badly broken than $SU(2)_{\rm isospin}$). Finally, we have the singlet of the eta (η) particle, with mass 548 MeV: comparable to the kaon mass, as the strange quark s enters twice. Still there is a ninth p-scalar meson, η', with a mass 958 MeV: it is not protected neither by the NG mechanism nor by being strangeless: the mass turns out to be bigger, but sill < 1 GeV.

2. Quark–antiquark bound states, $\bar{q}q$, but outside the NG limit; for example, the spin-1 nonet $(\rho, \omega, K^*, \phi)$: all masses beyond 1/2 GeV, and less than 1 GeV, except the $\phi(1020)$: centrifugal spin 1, plus strange content plus absence of NG «explains» the masses, at least qualitatively. Then there are other meson multiplets, as recurrences, higher spins, etc.

3. Baryons as protons and neutrons are made of *three* quarks; the binding energy turns out to be bigger, and indeed much bigger than the constituents masses, a situation totally different of the atoms: in the H-atom, the binding energy is 13.6 eV, whereas the rest mass of $e + p$ is of the order of the GeV. But most of the nucleon mass is «binding energy», and, in spite of some success with lattice calculations, QCD is still far away to compare with the successes of the atomic binding energy calculations... [46]. Wilcek [51] is one of those

*Lev Borisovich Okun introduced the term «hadrons» in 1962 to mean strongly interacting paticles; today the term just means composite of quarks; see, e.g., *Migdal V. et al.* // Usp. Fiz. Nauk. 1988. V. 158. P. 540–542.

who rightly pointed out that it is *not* true that the «mass» of the Universe comes mainly from the Higgs, the «God's particle», but from the binding energy of the QCD force... as hidden in the nucleons.

But the nucleon mass is no doubt very clearly a new scale, shared also by the charm (c) and the bottom (b) quarks. Why is it that the nucleon mass is propagated to these two quarks? A total mystery, it seems to us... But related (may be) to the same problem of the s-quark mass, «propagated» from the chiral symmetry breaking, and perhaps again connected with the lepton–quark symmetry generation-wise that we mentioned.

Notice also the *oblique symmetry* in the second and third generations: $\{e, \nu_e\}$ go with $\{u, d\}$ as the first generation. Then $\{\mu, \nu_\mu\}$ go with $\{c, s\}$ as the second: only the strange quark s appears with $\Lambda_{\rm QCD}$-type mass. And then $\{\tau, \nu_\tau\}$ go with $\{t, b\}$ for the third generation, but only τ lepton matches with b quark. On top of all this, the c quarks lie in the same ballpark as the τ lepton and the b quark, whereas the top quark t goes to the next mass level, the $W^\pm$–Z level. Indeed, the «relation» between s quark and μ lepton repeats itself with the bottom quark b and the τ lepton, as a renormalization group effect. This *oblique symmetry* is very intriguing.

Nuclear binding energies, as opposed to *quark* binding energies, are small, if one considers nucleons as composed of three quarks; for example, the deuteron ($\mathrm{H}^2 = p - n$) binding energy is 2.2 MeV, out of 2 GeV rest mass. This is simple, if understood as a small, «molecular» effect. Molecules, in fact, have a binding energy much smaller than the H-atom binding, say centi-eV against eV. For nuclei, which should be justified soon, from QCD we hope, and it is expected that lattice calculations of complex nuclei should account for that nuclear binding energy [46]. For physicists of the old generation it came as a surprise when it turned out that the most of the «nuclear binding energy» is a sort of molecular or Van der Waals residual force...

So strong forces, as described by QCD, result in two mass scales, say $\Lambda_{\rm QCD} \approx 200$ MeV and $m_N \approx 1$ GeV, represented, e.g., by the pions π and the nucleons N; and these two scales propagate to bare quarks and leptons, as we pointed out. As stressed above, it seems that only $\Lambda_{\rm QCD}$ is primitive, and we should be eventually able to compute nucleon mass *ratios* from, say, lattice QCD calculations.

8. THE BROKEN ELECTROWEAK SCALE

The 100 GeV scale, our next level, is rather well populated: we have here the vector particles $W^\pm$ and Z, the top quark t, as elementary particles, and also the Higgs vacuum expectation $\langle H \rangle$, plus hopefully the Higgs particle itself, and of course the (old) Fermi coupling constant, $G_F \approx 298$ GeV, traded today by this expectation value $\langle H \rangle$. One then asks, what is the geometry of the spontaneous electroweak breaking? How is the vacuum manifold?

The same problem as before also arises: granted that for some reason the e.w. break scale mass is in the 100 GeV range, why it does attach to the top mass (and to the Higgs mass)? We have very bluntly seen the *generation problem*: each of the three generations defines a mass scale for quarks (1 MeV the first; 100 MeV (s) — 1 GeV (c) the second; and 1 GeV (b) — 100 GeV (t) the third), with quarks lying on that range: u and d for the smallest, s for the second, c and b for the third, t for the top. Perhaps the most expected result is a simple relation between the Higgs mass and the Higgs vacuum expectation value, but even this cannot be checked until the Higgs is discovered. Summing up, we have a generation effect, as well as an *oblique effect*, and the Higgs participates, as perhaps a kind of the fourth generation...

Again we have no clue as the e.w. scale; the bare dimensionless e. w. coupling constant is of the order of the e.m.'s α, but the weak interactions are «weak» because they are broken, and the breaking scale is much higher than both atomic and nuclear masses. With respect to the breaking itself, it is clear that it must be generated: the gauge fields $W^{\pm}$ are electrically charged. Since the begining of β-decay theories (Fermi, 1934) it was very clear that the weak currents were charged.

Among the speculations for the e.w. scale one can contemplate, for example: Supersymmetry, Grand Unification or compactification from Higher Dimensions... We shall say something more on this problem later in this review.

9. TWO MORE (THEORETICAL) SCALES: GUT AND SUSY

There are no particles found, supposedly elementary, with masses much beyond 100 GeV, although there are candidates; e.g., supersymmetric partners, very massive see-saw neutrinos, etc. Empirically we also have the nasty problem of the *dark matter*, constituting about 25% of the mass of the Universe.

However, the three *running coupling constants*, respectively for QCD, α_{QCD} and for e. w. forces α_{em} and α_{w}, by renormalization group calculations, starting with Georgi, Quinn, and Weinberg (1974) [52] seem to (roughly) coincide at an enormous scale, $\approx 10^{15}$ GeV. This important calculation points out at least to two items: 1) Grand Unification Theories, GUT: if the three interactions are equivalent at the energy scale of 10^{15} GeV, one should view the different values we observe «at rest» for the coupling constants as consequence of the different speed of running of the three coupling constants, which is well understood from renormalization group arguments. 2) By the way, the matching of the three couplings is much improved with Supersymmetry, which also extends about an order of magnitude the coincident energy (10^{15} to $2 \cdot 10^{16}$ GeV; as comparison, Planck's mass scale is $1.22 \cdot 10^{19}$ GeV); the couplings seem to unify at the value $\alpha_{\mathrm{GUT}} \approx 1/25$. For a modern treatment of gravitational corrections to the running coupling constants, see [53].

The first GUT group historically was $SU(5)$, found by Georgi and Glashow [13]. The unifying group has to have complex representations (to account for parity violation, so fermions and antifermions fill up complex *conjugate* representations ($=$ *irreps*)), and there are not so many possible groups: only $SU(n)$, $n \geqslant 3$, $SO(4n+2)$ with the spin *irreps*, and E_6 are the candidates among simple Lie groups; curiously, for a Lie group to have complex representations, one needs the centre of the group to have more than a single involutive element [54], and indeed a natural hierarchy of GUT groups is $SU(5)$ inside Spin(10) inside E_6 with centres Z_5, Z_4, Z_3.

But the matter is not yet mature... It is a bit surprising and uneasy for us to learn that electric and weak forces were successfully unified back in 1967 (Weinberg), but in the remaining 40+ years we have been unable to progress any further. Hints of GUT unification are still lacking, like the much-awaited proton decay.

There is also the famous (already quoted) *see-saw* mechanism of Ramond et al. [35]: the neutrino mass times the GUT scale is about the square of the Z mass ($m_\nu \times \Lambda_{\mathrm{GUT}} \approx m_Z^2$, or $10^{22} \approx 10^2$ eV^2)... At least they are related. So we have two or three mostly theoretical arguments for the existence of the 7th scale, around $10^{15}-10^{16}$ GeV. The appeal to gravitation is unavoidable, as the Planck mass scale is not far up (see the next), but at least this has a merit: by the mentioned see-saw mechanism, the (very small) neutrino mass scale matches with the cosmological constant Λ (in corresponding units), and it relates also to the (very large) GUT mass; this «smells» again of gravitational connotations, not yet understood.

Supersymmetry (*SUSY*) enters the game now: with the MSSM, i.e., the minimal SUSY extension of the SM model, the matching of the three coupling constants improves, as said, but to a larger scale: $2 \cdot 10^{16}$ GeV, ten times higher. It is one of the main reasons why people welcome SUSY; other reasons are: (ii) the hierarchy problem: the Higgs should acquire an enormous renormalization mass, unless it has a fermion superpartner; the Higgs mass is expected to be less than 200 GeV in any reasonable theory, see [6]; (iii) SUSY partners are candidates for dark matter, e.g., the «neutralinos»: the dark matter problem arises in astrophysics, as, e.g., the rotation curves of galaxies require much more mass than the one we «see»; the dark matter problem, together with the dark energy issue (which is about the repulsive acceleration of the Universe expansion) are perhaps the two more pressing problems today in cosmology and astrophysics. The conventional wisdom is to find Weakly Interacting Massive Particles (WIMP) contributing to the cold component of dark matter, and prevented from decaying by a certain «R» symmetry, forbidding transitions between genuine SUSY particles and normal ones.

But Supersymmetry raises more questions that it solves: SUSY, if it exists at all, must be broken, and this makes a new scale to enter: *the scale of SUSY breaking*. Below (Sec. 11) we elaborate more on Supersymmetry; at any rate, it might well signal the start of an eighth scale, perhaps on the TeV range!

10. THE PLANCK SCALE

Gravitation as a whole, as an interaction on its own, has been mainly left out intentionally, but now it is time to get it back. With $\hbar$, c, and G_N, we concoct units for everything, in particular the energy unit is $M_{\rm Pl}$, that is (with only $\hbar$, c, factors as units) 10^{19} GeV, not too far from the SUSY-GUT scale. What does this mean? We wish we knew! We should understand why the GUT scale is NOT much different from the Planck scale. Does this lead to a relation between gravity and the other forces? We believe so, in a mysterious way. However we want to emphasize one point.

There is no doubt that the naïve superposition of Quantum Mechanics and General Relativity is wrong: gravitational interactions are unavoidably not renormalizable, as $[G_N] \propto L^2$. As both theories have a clear domain of application, some modification is to be expected, soon or later. We bet our horses on non-commutative geometry (A. Connes [55]), modifying gravitation at microscopic scales, but it is only one of the several proposals (loop quantum gravitation is another: Ashtekar, 1986 [56]; not to speak of superstring theories [57]...). This has been the main reason why we did not consider gravitation as a theory on its own in this review, except for marginal comments, fixing perhaps a scale, and also influencing, may be, another two.

We also appeal to a recent paper by us [58] for the idea of changing the fundamental constants (in name, not in values). But the Planck mass stands as originally.

11. A NEW VIEW ON SUPERSYMMETRY

For the history of discovering Supesymmetry, see the book [59]: two of the principal papers of the Russian school were [60,61]. In 1971, also P. Ramond [62] introduced fermions in an incipient String Theory. Since 1974 (Wess and Zumino in [63]), it has been rightly considered as a natural extension of quantum field theory. Unavoidably, it was thought of as a mechanism to understand features of the real world, in absence of any clear experimental corroboration; for example, as mentioned, the mass of the Higgs gets unrenormalized to much higher scales if it has a SUSY partner (higgsino); also, the SUSY running of the couplings implies different Higgses for the upper vs. the lower quarks, and it goes some way to understand the mass differences between lower quarks and leptons for the second and third generation (s vs. μ, and b vs. τ). There are other blessings as well, which we omit. For the prospects of finding SUSY partners with the LHC machine, see [64].

When SUSY appeared, it was hard to swallow for the average physicist. We were used to consider fermions on the fundamental or vector representation of the gauge groups, whereas gauge vector (spin 1) bosons (*gaugeons*, one might be tempted to say) went with the adjoint representation; there is no more fundamental physical difference between particles and fields that the electron, as a fermion, obeying the exclusion principle, that accounts for all the chemistry, and the photons, with their cooperative states, and the «likeness» of photons to stay together (coherent states in the laser, etc.). But today perhaps we may start to understand better the issue, and the contradiction is not so poignant. Here is a very bold mathematical idea:

In precisely eight *space* dimensions (and *only* in those!) spinor and vector representations are isomorphic: the centre of the Spin(8) group is $V \equiv Z_2 \times Z_2$, and also the three representations: vector $\square$ or 8_V, and the two spinor *irreps* Δ_L and Δ_R, are *permutable* (isomorphic), as the symmetry group of that centre, $S_3 = \mathrm{Aut}\,(V)$, lifts to a *true* symmetry of the Spin(8) group: this is called Cartan's *triality* in mathematics, and it is very closely linked to the octonion division algebra; triality is very obvious from the Dynkin diagram for the $D_4 \approx O(8)$ group. On the other hand, the spin-statistics theorem is *not* valid in 8 space dimensions [65]; so one can contemplate a spinor(s)–vector *bona fide* symmetry (not supersymmetry!) which would descend to four dimensions, becoming the usual fermion–boson supersymmetry! The speculation that this is the origin of supersymmetry down to our mundane, three spatial dimensions, is a strong one, and we tentatively subscribe to it. On (possible) compactification, spinors become fermions, as we see them, with the attendant exclusion principle. Of course, the adjoint representation kills the centre (it is a faithful *irrep* of the $PO(8)$ group, $= \mathrm{Spin}(8)/V$), so if gauge groups appear in the process of compactification (as, e.g., removal of singularities: Achyara–Witten mechanism, [66]), conventional boson–fermion partners should appear.

CONCLUSIONS

The particles we believe nowadays considered elementary that one observes in nature group naturally in six well-defined scales, at least. The massless scale (1), the electron scale (3) and the nucleon scale (5), as present in two quarks (c, b) and a lepton (τ), are sort of understood: exact gauge carriers, support of minimal electric charge, regular binding energy from strong forces. One can perhaps anticipate some understanding of the necessity of Λ_{QCD}, as «dimension transmutation» of the scale-invariant QCD coupling by a mass (scale (4)). The electroweak gauge group has to be broken, as carriers are charged, and this points towards the Higgs scale (6). Only the neutrino scale (2) is not mentioned, and for it we also advanced some gravitation/cosmological arguments. But of course, all this is much more a research programme than a well established set of (unconnected?) hypotheses. In particular, we want to finish just to emphasize that the main problems remain as intractable as always: why are there three generations, with partial but also oblique symmetry?; neutrino masses seem to be insensible to generations (?), but the lower quarks (s, b) are seemingly related to the charged leptons (μ, τ), at least in the second and third generation, whereas the upper quarks signal the new scale: the charm quark c points towards the QCD binding energies, whereas the top quark (t) mass is in the regime of the e.w. breaking scale. Dark matter raises its ugly head pointing to another scale, with probably cosmological significance.

Some of the facts we have signalled have to be the way they are for *anthropic reasons*; we already alluded to three generations (at least) to support CP violation, and the enormous abundance of matter vs. antimatter; but there are other examples: neutrons heavier than protons are essential to form hydrogen, and after this, the remaining atoms and molecules. Related with this is the necessity of spin 1/2 fermions, to make structures via the exclusion principle. For a recent review of particle masses, with emphasis on neutrinos, see [67].

To end up, we would like to stress that the actual electroweak gauge symmetry *breaking* mechanism is rather ugly and *ad hoc*. At any rate, as we state at the very beginning, the masses obtained in the conventional SM by couplings to the Higgs are also very unsatisfactory as a matter of principle.

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