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FACTORIZATION AND TRANSVERSE MOMENTUM FOR TWO-HADRON PRODUCTION IN INCLUSIVE e^+e^- ANNIHILATION

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We study factorization of processes involving two fragmentation functions in the case of very small transverse momenta. We consider two-hadron production in inclusive e^+e^- annihilation and demonstrate a new simple and illustrative method of factorization for such processes including leading-order α_S corrections.

Изучается факторизация в процессе с двумя фрагментационными функциями для случая очень малых поперечных импульсов. Рассмотрено двухадронное рождение в инклюзивном процессе e^+e^--аннигиляции и продемонстрирован новый простой и иллюстративный метод факторизации, включающий лидирующие α_S-поправки.

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INTRODUCTION

The electron–positron annihilation process is one of the basic hard scattering processes where the composite structure of hadrons is investigated. Especially, the two-hadron production in inclusive e^+e^- annihilation (THPIA), i.e., the process in which two hadrons are detected in the final state, attracts a lot of attention due to the possibility for both the experimental and theoretical studies of several fragmentation functions, in particular, the chiral-odd Collins functions [1–5]. Since available experimental data [2–4] correspond to substantially different values of Q^2, taking into account the evolution of fragmentation (in particular, Collins) functions becomes an important theoretical task. In turn, the studies of evolution are intimately correlated to the investigation of QCD factorization. However, it is well known that the factorization theorems do not hold for an arbitrary kinematic regime. Indeed, if a transverse momentum of the produced hadron is of the same order of magnitude as the large photon virtuality, $p_T \sim Q$, the corresponding hadronic tensor can be factorized and expressed in terms of the integrated fragmentation functions (see, for instance, a detailed analysis of e^+e^- annihilation in [6]). Even if p_T is much smaller than the large photon virtuality, but on the other hand, is much bigger than the characteristic hadronic size, $\Lambda_{\rm QCD}$, factorization of the e^+e^- annihilation cross section can still be proven with methods similar to those used for semi-inclusive DIS or Drell–Yan processes [7–9]. However, the regime for which the transverse momentum of produced hadron is of the order of $\Lambda_{\rm QCD}$ faces a conceptual problem how to identify the hard subprocess. Therefore, factorization becomes vague and requires a special care [9].

Factorization in THPIA, starting from the seminal paper [10], was only studied in the collinear approximation (see also [11]). Besides, the procedure of integration over the transverse virtual photon momentum (see [12]) was restricted to Born approximation.

Since the transverse momentum is of crucial importance for the Collins function, we perform a detailed analysis of its role in THPIA. We use ideas of [13,14] where it was shown that integration over the transverse momentum of a produced $\mu^+\mu^-$ pair or a hadron in the Drell–Yan process provides an effective propagator of a highly virtual photon and generates the hard subprocess structure. This approach was recently generalized [15] to include the case of the (weighted) transverse momentum average of semi-inclusive deep-inelastic scattering (SIDIS) with transverse momentum dependent fragmentation functions.

In this paper, we develop these ideas further and present a new method of factorization which may be applied for any two-current process. In particular, we demonstrate the application of the method in the case of e^+e^- annihilation when two hadrons, belonging to different back-to-back jets, are produced.

To demonstrate the method we are proposing, we consider, at the first stage, the simplest case of the spin-independent $k_\perp$-integrated fragmentation functions. We analyze contributions up to leading-order α_S corrections to the hard part to obtain the evolutions of the corresponding fragmentation functions. Also, we briefly outline a way to extend our approach for the study of the Collins effects.

1. KINEMATICS

We consider the process $e^+(l_1) + e^-(l_2) \to H(P_1) + H(P_2) + X(P_X)$ where positron and electron carrying momenta l_1 and l_2, respectively, annihilate into a time-like photon with momentum $q = l_1 + l_2$ for which $q^2 = Q^2$ is large. This time-like photon creates then two outgoing hadrons with momenta $P_i\,(i = 1, 2)$ (these hadrons belong to two distinct jets) and an undetected bunch of hadrons with total momentum P_X. For such kind of a process, it is convenient to introduce two invariants

$$z_i = \frac{2P_i \cdot q}{Q^2} \tag{1}$$

which are analogous to the Bjorken variable. Note that the variables z_1 and z_2 are the energy fractions of the detected hadrons in the e^+e^- center-of-mass system. They obey the following constraint due to the energy conservation:

$$\frac{z_1 + z_2}{2} < 1. \tag{2}$$

Moreover, due to the momentum conservation there is a stronger constraint on z_1 and z_2:

$$z_1 < 1, \quad z_2 < 1. \tag{3}$$

Within this region, these variables can vary independently.

To perform the Sudakov decomposition, we choose two dimensionless light-cone basis vectors:

$$n^*_\mu = \left(\frac{1}{\sqrt{2}},\ \mathbf{0}_T,\ \frac{1}{\sqrt{2}}\right), \quad n_\mu = \left(\frac{1}{\sqrt{2}},\ \mathbf{0}_T,\ -\frac{1}{\sqrt{2}}\right), \quad n^* \cdot n = 1. \tag{4}$$

In this paper, we choose the kinematics such that the photon and one of the hadrons have purely longitudinal momenta, while the other hadron has both longitudinal and perpendicular momenta:

$$\begin{aligned} P_{2\mu} &= \frac{z_2 Q}{\sqrt{2}} n^*_\mu + \frac{M_2^2}{z_2 Q\sqrt{2}} n_\mu, \quad q_\mu = \frac{Q}{\sqrt{2}} n^*_\mu + \frac{Q}{\sqrt{2}} n_\mu, \\ P_{1\mu} &= \frac{z_1 Q}{\sqrt{2}} n_\mu + \frac{M_1^2 + \mathbf{P}_1^{\perp\,2}}{z_1 Q\sqrt{2}} n^*_\mu + P^\perp_{1\mu}. \end{aligned} \tag{5}$$

The leptonic momenta are

$$\begin{aligned} l_{1\mu} &= \frac{Q[1-\cos\theta_2]}{2\sqrt{2}} n^*_\mu + \frac{Q[1+\cos\theta_2]}{2\sqrt{2}} n_\mu + l^\perp_{1\mu}, \quad l^\perp_{1\mu} = \left(\frac{Q}{2}\,\sin\theta_2,\, 0\right); \\ l_{2\mu} &= \frac{Q[1+\cos\theta_2]}{2\sqrt{2}} n^*_\mu + \frac{Q[1-\cos\theta_2]}{2\sqrt{2}} n_\mu + l^\perp_{2\mu}, \quad l^\perp_{2\mu} = \left(-\frac{Q}{2}\,\sin\theta_2,\, 0\right), \end{aligned} \tag{6}$$

where θ_2 is the angle between $\mathbf{P}_2$ and $\mathbf{l}_1$. This frame is called the «$\perp$»-frame (or the perpendicular frame) [12,16]. Below, we will omit terms of order M^2/Q^2. Such a kinematics has advantage for analysis of the experimental situation, where the momentum of one of the produced hadrons is measured.

In order to ensure that the two hadrons are in different jets, we introduce two different variables [10]:

$$\mathcal{Z} = \frac{2P_1\cdot q}{Q^2} \equiv z_1, \quad \mathcal{U} = \frac{P_1\cdot P_2}{P_1\cdot q}. \tag{7}$$

However, using Eq. (5), the difference between $\mathcal{U}$ and z_2 is of the order of $1/Q^2$:

$$\mathcal{U} = z_2\left[1 + \mathbf{P}_1^{\perp\,2}/(z_1^2\,Q^2)\right]^{-1}, \tag{8}$$

and can be neglected in the leading-order approximation which we are considering in this paper.

The perpendicular projection tensor is defined as usual:

$$g^\perp_{\mu\nu} = g_{\mu\nu} - \hat{q}_\mu\hat{q}_\nu + \hat{T}_\mu\hat{T}_\nu, \tag{9}$$

where the two normalized vectors $\hat{q}$ and $\hat{T}$ are constructed as

$$\hat{T}_\mu = \frac{T_\mu}{T}, \quad T_\mu = P_{2\,\mu} - \frac{P_2\cdot q}{Q^2} q_\mu, \quad \hat{q}_\mu = \frac{q_\mu}{Q}. \tag{10}$$

For simplicity, we consider the unpolarized case. The differential cross section of the corresponding e^+e^- annihilation is given by

$$\begin{aligned} d\sigma(e^+e^-) = \frac{1}{2\,Q^2}\frac{d^3\,\mathbf{P}_1}{(2\pi)^3\,2E_1}\,\frac{d^3\,\mathbf{P}_2}{(2\pi)^3\,2E_2}\times \\ \times\sum_X\int\frac{d^3\,\mathbf{P}_X}{(2\pi)^3 2E_X}\,(2\pi)^4\,\delta^{(4)}(q-P_1-P_2-P_X)\,\left|\mathcal{M}(e^+e^-)\right|^2. \end{aligned} \tag{11}$$

In terms of leptonic and hadronic tensors, we have

$$d\sigma(e^+e^-) = \frac{\alpha^2}{4Q^6}\,\frac{d^3\mathbf{P}_1}{E_1}\,\frac{d^3\mathbf{P}_2}{E_2}\,\mathcal{L}^{\mu\nu}\mathcal{W}_{\mu\nu}, \tag{12}$$

where the hadronic tensor $\mathcal{W}_{\mu\nu}$ is defined as

$$\mathcal{W}_{\mu\nu} = \sum_X \int \frac{d^3\,\mathbf{P}_X}{(2\pi)^3 2E_X}\,\delta^{(4)}(q - P_1 - P_2 - P_X)\times \\ \times\,\langle 0|J_\mu(0)|P_1, P_2, P_X\rangle\langle P_1, P_2, P_X|J_\nu(0)|0\rangle. \tag{13}$$

We can rewrite the part of the phase space corresponding to the detected hadron with momentum P_1 as

$$\frac{d^3\mathbf{P}_1}{(2\pi)^3\,2E_1} = \frac{dz_1}{(2\pi)^3\,2z_1}\,d^2\mathbf{P}_{1\perp}. \tag{14}$$

Because the leptonic tensor $\mathcal{L}_{\mu\nu}$ is independent of the hadronic momentum P_1, it is useful to introduce the averaged hadronic tensor, $\overline{\mathcal{W}}^{(\perp)}_{\mu\nu}$,

$$\overline{\mathcal{W}}^{(\perp)}_{\mu\nu} = \int d^2\mathbf{P}_{1\perp}\,\mathcal{W}^{(\perp)}_{\mu\nu}. \tag{15}$$

Finally, using (14) and (15), the cross section (12) can be presented as

$$E_2\,\frac{d\sigma(e^+e^-)}{d^3\mathbf{P}_2} = \frac{\alpha^2}{4Q^6}\,\frac{dz_1}{z_1}\,\mathcal{L}^{\mu\nu}\overline{\mathcal{W}}^{(\perp)}_{\mu\nu}. \tag{16}$$

Note that this averaging procedure produces the hard subprocess in the Born approximation.

2. FACTORIZATION PROCEDURE: BORN DIAGRAM

Let us now discuss factorization of hadronic tensors corresponding to the hadron production in e^+e^- annihilation at low p_T. In this case, as pointed out before, a serious conceptual problem is known to be associated with the difficulty to identify (or, in other words, to separate out) the hard subprocess. In [14], it was shown that in the case of Drell–Yan process a suitable integration over the transverse momentum provides effectively the separation of the hard subprocess. We extend this approach to the factorization of the e^+e^--annihilation hadronic tensor including leading-order α_S corrections. Furthermore, the reproduction of the well-known DGLAP evolution kernel for both the quark and anti-quark fragmentation functions can be considered as a proof of longitudinal factorization.

First, we consider a simple Born diagram depicted in Fig. 1, *a*. The corresponding hadronic tensor reads

$$\mathcal{W}_{\mu\nu} = \int d^4k_1\,d^4k_2\,\delta^{(4)}\,(k_1 + k_2 - q)\;\mathrm{tr}\left[\gamma_\nu\,\Theta(k_2)\,\gamma_\mu\,\bar{\Theta}(k_1)\right] + (1 \leftrightarrow 2), \tag{17}$$

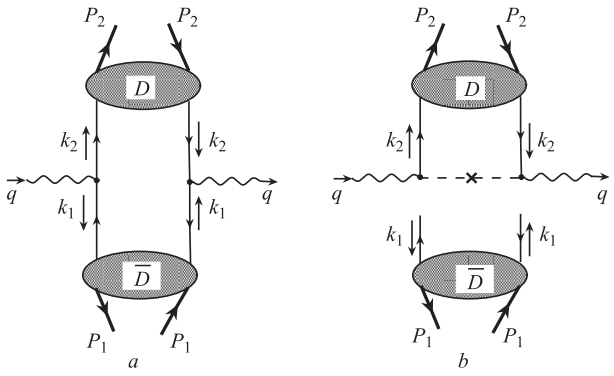

Fig. 1. Non-factorized (a) and factorized (b) Born diagrams

where the four-dimension δ function, representing the momentum conservation at the quark-photon vertex, will be treated as the «hard» part. The non-perturbative quark and anti-quark correlation functions $\Theta(k_2)$ and $\bar{\Theta}(k_1)$ are given by

$$\begin{aligned}
\Theta_{\underline{\alpha}\,\underline{\beta}}(k_2) &= \int \frac{d^4\,\xi}{(2\pi)^4}\,\mathrm{e}^{ik_2\cdot\xi}\,\langle 0|\psi_{\underline{\alpha}}(\xi)|P_2,P_{X_2}\rangle\langle P_2,P_{X_2}|\bar{\psi}_{\underline{\beta}}(0)|0\rangle,\\
\bar{\Theta}_{\underline{\alpha}\,\underline{\beta}}(k_1) &= -\int \frac{d^4\,\eta}{(2\pi)^4}\,\mathrm{e}^{-ik_1\cdot\eta}\,\langle 0|\bar{\psi}_{\underline{\beta}}(0)|P_1,P_{X_1}\rangle\langle P_1,P_{X_1}|\psi_{\underline{\alpha}}(\eta)|0\rangle,
\end{aligned} \tag{18}$$

where the underlined indices are the Dirac indices. The summation and integration over the intermediate undetected states are implied. Note that owing to the delta function in (17) the hadronic tensor cannot be expressed through the factorized $k_\perp$-integrated fragmentation functions. To express the corresponding correlation functions through the fragmentation functions, we suggest a factorization scheme inspired by the Ellis–Furmanski–Petronzio (EFP) approach [17]. We first consider the formal identical transformation:

$$\begin{aligned}
d^4k_1 \to d^4k_1\,dz_1'\,\delta\left(\frac{P_1^-}{k_1^-}-z_1'\right) &= d^4k_1\,\frac{dz_1'}{(z_1')^2}\,\delta\left(k_1\cdot\tilde{n}^*-\frac{1}{z_1'}\right),\\
d^4k_2 \to d^4k_2\,dz_2'\,\delta\left(\frac{P_2^+}{k_2^+}-z_2'\right) &= d^4k_2\,\frac{dz_2'}{(z_2')^2}\,\delta\left(k_2\cdot\tilde{n}-\frac{1}{z_2'}\right),
\end{aligned} \tag{19}$$

where the two vectors

$$\tilde{n}_\mu^* = \frac{n_\mu^*}{P_1\cdot n^*}, \quad \tilde{n}_\mu = \frac{n_\mu}{P_2\cdot n} \tag{20}$$

have been introduced. Variables z_i' may be interpreted as partonic fractions of the corresponding momenta of produced hadrons.

As mentioned above, we treat the four-dimensional δ function as the hard part of the corresponding tensor. This assumption will be justified below. Our analysis is limited by the study of the leading twist contributions. It means that we keep only the first terms of the

expansion:

$$\delta^{(4)}\left(k_1+k_2-q\right)=\delta^{(4)}\left(\frac{P_{1\,\mu}}{z_1'}+\frac{P_{2\,\mu}}{z_2'}-q\right)+O(k_\perp)\simeq$$
$$\simeq\delta\left(\frac{P_2^+}{z_2'}-q^+\right)\,\delta\left(\frac{P_1^-}{z_1'}-q^-\right)\,\delta^{(2)}\left(\frac{\mathbf{P}_1^\perp}{z_1'}\right)\,. \quad (21)$$

The hadronic tensor is now rewritten as

$$\mathcal{W}_{\mu\nu}^{(\perp)}=\int\frac{dz_1'}{(z_1')^2}\times$$
$$\times\int\frac{dz_2'}{(z_2')^2}\delta\left(\frac{P_2^+}{z_2'}-q^+\right)\,\delta\left(\frac{P_1^-}{z_1'}-q^-\right)\,\delta^{(2)}\left(\frac{\mathbf{P}_1^\perp}{z_1'}\right)\,\mathrm{tr}\left[\gamma_\nu\,\Theta(z_2')\,\gamma_\mu\bar{\Theta}(z_1')\right], \quad (22)$$

where

$$\Theta(z_2')\stackrel{\mathrm{def}}{=}\int d^4k_2\,\delta\left(k_2\cdot\tilde{n}-\frac{1}{z_2'}\right)\Theta(k_2), \quad (23)$$

$$\bar{\Theta}(z_1')\stackrel{\mathrm{def}}{=}\int d^4k_1\,\delta\left(k_1\cdot\tilde{n}^*-\frac{1}{z_1'}\right)\bar{\Theta}(k_1). \quad (24)$$

Since we study spin-independent fragmentation functions, we first have to project the correlation functions (23) and (24) onto the corresponding Lorentz vector structures. Starting from the quark correlator function, we write

$$\Theta(z_2')\Longrightarrow\frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\Theta(z_2')\right]\gamma_\alpha. \quad (25)$$

Using (18) and the integral representation of the δ function in (23), the vector projection of the quark correlation function (see (25)) can be represented in terms of the spin-independent fragmentation function as

$$\frac{1}{4}\int\frac{d\lambda_2}{2\pi}\,\mathrm{e}^{i\lambda_2/z_2'}\int d\xi^+\,d\xi^-d^2\boldsymbol{\xi}_\perp\,\delta(\lambda\tilde{n}^--\xi^-)\,\delta(\xi^+)\,\delta^{(2)}(\boldsymbol{\xi}_\perp)\times$$
$$\times\,\mathrm{tr}\left[\gamma_\alpha\,\langle0|\psi(\xi^+,\xi^-,\boldsymbol{\xi}_\perp)|P_2,P_{X_2}\rangle\langle P_2,P_{X_2}|\bar{\psi}(0)|0\rangle\right]\gamma_\alpha=\frac{D(z_2')}{z_2'}\,\hat{P}_2. \quad (26)$$

Here, the fragmentation function $D(z_2')$ can also be written as

$$D(z_2')=\frac{z_2'}{4(2\pi)}\int d\xi^-\,\mathrm{e}^{iP_2^+\xi^-/z_2'}\,\mathrm{tr}\left[\gamma^+\,\langle0|\psi(0,\xi^-,\mathbf{0}_\perp)|P_2,P_{X_2}\rangle\langle P_2,P_{X_2}|\bar{\psi}(0)|0\rangle\right], \quad (27)$$

provided that the minus coordinate component ξ^- is equal to $\lambda_2\tilde{n}^-$. Note that this fragmentation function obeys the momentum conservation sum rules [6, 16, 18]:

$$\sum_{h,\,s}\int dz\,z\,D(z)=1. \quad (28)$$

In a similar manner, we project the anti-quark correlation function:

$$\frac{1}{4}\mathrm{tr}\left[\gamma_\alpha\bar{\Theta}(z_1')\right]\gamma_\alpha = \frac{1}{4}\int\frac{d\lambda_1}{2\pi}\mathrm{e}^{-i\lambda_1/z_1'}\int d\eta^+\,d\eta^-\,d^2\boldsymbol{\eta}_\perp\,\delta(\lambda\tilde{n}^{*+}-\eta^+)\,\delta(\eta^-)\,\delta^{(2)}(\boldsymbol{\eta}_\perp)\times$$
$$\times\,\mathrm{tr}\left[\gamma_\alpha\,\langle 0|\bar{\psi}(0)|P_1,P_{X_1}\rangle\langle P_1,P_{X_1}|\psi(\eta^+,\eta^-,\boldsymbol{\eta}_\perp)|0\rangle\right]\gamma_\alpha = \frac{\bar{D}(z_1')}{z_1'}\,\hat{P}_1. \quad (29)$$

Taking into account Eqs. (25)–(29), we bring the hadronic tensor in the following form:

$$\mathcal{W}_{\mu\nu}^{(\perp)} = \int\frac{dz_1'}{(z_1')^3}\,\bar{D}(z_1')\int\frac{dz_2'}{(z_2')^3}\,D(z_2')\times$$
$$\times\,\delta\left(\frac{P_1^-}{z_1'}-q^-\right)\delta\left(\frac{P_2^+}{z_2'}-q^+\right)\delta^{(2)}\left(\frac{\mathbf{P}_1^\perp}{z_1'}\right)\,\mathrm{tr}\left[\gamma_\nu\hat{P}_2\gamma_\mu\hat{P}_1\right]. \quad (30)$$

Now, inserting P_2^+ and P_1^- defined via the kinematical variables z_2 and z_1 (see (5)) and calculating the trace in (30), we get

$$\begin{aligned}\mathcal{W}_{\mu\nu}^{(\perp)} &= -4g_{\mu\nu}^\perp\,\delta^{(2)}(\mathbf{P}_1^\perp)\left[z_1\int dz_1'\,\bar{D}(z_1')\delta(z_1-z_1')\right]\left[z_2\int\frac{dz_2'}{(z_2')^2}\,D(z_2')\delta(z_2-z_2')\right],\\ &= -4g_{\mu\nu}^\perp\,\delta^{(2)}(\mathbf{P}_1^\perp)\,\left[z_1\bar{D}(z_1)\right]\,\left[\frac{D(z_2)}{z_2}\right].\end{aligned} \quad (31)$$

From (31), one can see that, though the hadronic tensor $\mathcal{W}_{\mu\nu}^{(\perp)}$ has a formally factorized form, it does not have much sense because of the two-dimensional δ function. To eliminate it, we have to integrate over the perpendicular momentum or, in other words, to go over to the averaged hadronic tensor (15),

$$\overline{\mathcal{W}}_{\mu\nu}^{(\perp)} = -4g_{\mu\nu}^\perp\int d^2\mathbf{P}_{1\,\perp}\,\delta^{(2)}(\mathbf{P}_1^\perp)\,\left[z_1\bar{D}(z_1)\right]\,\left[\frac{D(z_2)}{z_2}\right] =$$
$$= -4g_{\mu\nu}^\perp\,\left[z_1\bar{D}(z_1)\right]\,\left[\frac{D(z_2)}{z_2}\right]. \quad (32)$$

On the other hand, as we will discuss below, integration over $d^2\,\mathbf{P}_{1\,\perp}$ will generate the effective diagram depicted in Fig. 1, *b*. Indeed, after integration, one can represent the one-dimensional δ function $\delta(z_2-z_2')$ as the imaginary part of the effective propagator:

$$\delta(z_2-z_2') \Longrightarrow \Im\mathrm{m}\frac{1}{z_2-z_2'} \Longrightarrow \Im\mathrm{m}\frac{1}{[q-P_2/z_2']^2}. \quad (33)$$

The latter will reduce the diagram in Fig. 1, *a* to the diagram plotted in Fig. 1, *b*. Notice that the appearance of this hard effective propagator justifies a posteriori the suggested generalization of the EFP factorization scheme. Namely, the four-dimensional δ function should be treated as the hard part, and the entering parton momenta should be replaced, at the leading twist level, by their longitudinal parts.

Let us now discuss the above-mentioned procedure from the viewpoint of the approach of averaging given by

$$d^2\mathbf{P}_{1\perp} = 2z_1 \iint d^4P_1 \delta(P_1^2)\,\delta\left(\frac{2P_1\cdot q}{Q^2} - z_1\right). \tag{34}$$

Indeed, integrating over d^4P_1 with the four-dimensional δ functions (see (30)), one can observe that the one-dimensional δ function which is responsible for the mass-shell condition goes to the δ function that can be understood as the imaginary part of some effective «propagator» with large photon virtuality, i.e.,

$$\delta(P_1^2) \Longrightarrow \delta([q - P_2/z_2']^2) \sim \Im\mathrm{m}\frac{1}{[q - P_2/z_2']^2}. \tag{35}$$

Diagrammatically, it corresponds to the case when the Born diagram transforms to the diagram plotted in Fig. 1, *b*. The dashed line implies the effective propagator or the factorization link. So, the averaged hadronic tensor can now be written in terms of the factorization link,

$$\overline{\mathcal{W}}_{\mu\nu}^{(\perp)} = \int d^2\mathbf{P}_{1\perp}\mathcal{W}_{\mu\nu}^{(\perp)} = \frac{4g_{\mu\nu}^{\perp}}{\pi}\,\left[z_1\bar{D}(z_1)\right]\left[z_2\int\frac{dz_2'}{(z_2')^2}\,D(z_2')\Im\mathrm{m}\frac{1}{z_2 - z_2'}\right]. \tag{36}$$

The spuriously asymmetric form of (32) or (36) with respect to the interchanging of z_1 and z_2 emerges because of the integration over the momentum P_1 of one of the produced hadrons. Restoring the flavour dependence omitted above and by inserting (32) into (16), we get

$$\frac{d\sigma(e^+e^-)}{dz_1dz_2d\cos\theta_2} = \frac{3\pi\alpha^2}{2Q^2}\,(1+\cos^2\theta_2)\sum_{a,\bar{a}} e_a^2\bar{D}^a(z_1)\,D^a(z_2), \tag{37}$$

where the only remaining asymmetry is reflected in the polar angle of the detected hadron. After integration over θ_2, the result becomes completely symmetric:

$$\int d\cos\theta_2\,\frac{d\sigma(e^+e^-)}{dz_1dz_2d\cos\theta_2} = \frac{4\pi\alpha^2}{Q^4}\,\sum_{a,\bar{a}} e_a^2\bar{D}^a(z_1)\,D^a(z_2). \tag{38}$$

Concluding this section, we would like to stress that the factorization of the Born diagram can be implemented in a similar way as in the case of Drell–Yan process [12]. In this case, we transform the two-dimensional integration $d^2\mathbf{P}_{1\perp}$ in the phase space (see (14)) to the two-dimensional integration over the photon transverse momentum $d^2\mathbf{q}_T$ using the well-known relation: $d^2\mathbf{P}_{1\perp} = z_1^2d^2\mathbf{q}_T$ [12]. However, the leptonic tensor $\mathcal{L}^{\mu\nu}$ depends not only on Q^2 but also on $\cos\theta_2$. Therefore, factorization with the transformation to the integration over $d^2\mathbf{q}_T$ does not allow for calculating the angular dependence of the corresponding cross section.

3. FACTORIZATION PROCEDURE: LEADING-ORDER α_S CORRECTIONS

Now we proceed with the analysis of leading α_S corrections which are associated with the diagrams that involve gluon emission. It is known that large logarithms appearing in this case can be absorbed into the corresponding evolved fragmentation functions. We will pay a

special attention to the terms with mass singularities which are extracted from the diagrams with the emission of real gluons. The corresponding Feynman diagrams with α_S corrections are depicted in Fig. 2. The domain of integration over the loop momentum in each diagram is in the region where the considered parton is collinear to either P_1 or P_2 momentum directions. For the sake of definiteness, we assume that the hadron with momentum P_2 belongs to the quark jet, while the hadron with the momentum P_1 to the anti-quark jet. The opposite situation is trivially obtained by the interchange of the labels.

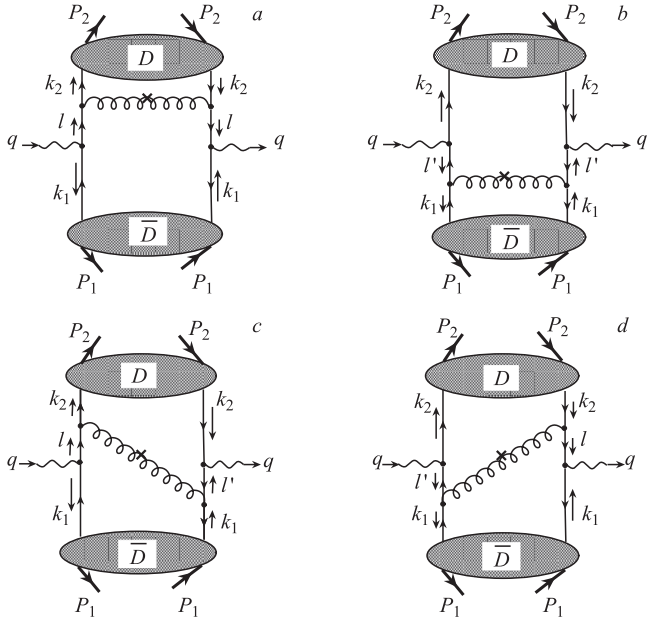

Fig. 2. Leading-order α_S diagrams

3.1. Evolution of the Quark Fragmentation Function. To begin with, let us consider evolution of the quark fragmentation function $D(z)$. At the Born level, the quark which decays into the detected hadron with the momentum P_2 and a bunch of undetected hadrons can emit a real gluon before and after interaction with the virtual photon. The diagram depicted in Fig. 2, a corresponds to the interaction of quark with the virtual photon before the emission of the real gluon. Choosing Feynman gauge for the gluon fields, we write the hadronic tensor corresponding to this diagram in the form resembling the Born diagram tensor:

$$\mathcal{W}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2}, a) = g^2 C_F \int d^4k_1\, d^4p\, \delta^{(4)}\left(k_1 + p - q\right)\, \mathrm{tr}\left[\gamma_\nu \bar{\Theta}(k_1)\, \gamma_\mu \Omega(p)\right]. \qquad (39)$$

However, instead of the quark correlator $\Theta(k_2)$, we have here the modified correlator $\Omega(p)$ defined as

$$\Omega(p) = \frac{\hat{p}}{p^2}\, \gamma_\alpha \int \frac{d^4k_2}{(2\pi)^3}\, \delta([p-k_2]^2)\, \Theta(k_2) \gamma_\alpha\, \frac{\hat{p}}{p^2}. \qquad (40)$$

In principle, factorization of (40) can be implemented in a similar way. However, in difference with the Born diagram, the modified tensor (40) in (39) is not completely soft because of the p-dependence.

To factorize (39), let us first express the parton momenta in terms of the corresponding fractions of the hadron momenta by means of the integral representation of unity. The definitions of the fractions for parton momenta k_1 and k_2 are the same as in (19), whereas for the loop momentum p we write

$$d^4p \to d^4p\,\frac{d\,y'}{(y')^2}\,\delta\left(p\cdot\tilde{n}-\frac{1}{y'}\right). \tag{41}$$

The pure soft part of (40) is associated with the quark correlator function $\Theta(k_2)$. At the same time, the quark propagators and the δ function emerging from the imaginary part of the gluon propagator have to be associated with the hard part of (40). Therefore, after expanding the δ function in (40) about the momentum k_2 around the direction defined by the hadron momentum P_2, we get for the correlator

$$\Omega(p)=\frac{\hat{p}}{p^2}\,\gamma_\alpha\int\frac{dz_2'}{(z_2')^2}\,\delta\left(p^2-\frac{2p\cdot P_2}{z_2'}\right)\,\Theta(z_2')\gamma_\alpha\,\frac{\hat{p}}{p^2}. \tag{42}$$

From this tensor, which is the same as in the Born diagram, we single out the spin-independent quark fragmentation function (see Eqs. (25) and (26)). Inserting the tensor (42) into the hadronic tensor (39), we get

$$\begin{aligned}\mathcal{W}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2},a)=\frac{g^2C_F}{4(2\pi)^3}\int\frac{dz_1'}{(z_1')^2}\int\frac{dy'}{(y')^2}\,\delta\left(\frac{P_2^+}{y'}-q^+\right)\delta\left(\frac{P_1^-}{z_1'}-q^-\right)\times\\ \times\,\delta^{(2)}\left(\frac{\mathbf{P}_{1\perp}}{z_1'}\right)\int\frac{dz_2'}{(z_2')^2}\int dp^+\int d^2\,\mathbf{p}_\perp\,\delta\left(\frac{1}{y'}-\frac{p^+}{P_2^+}\right)\,\frac{p^+-P_2^+/z_2'}{p_\perp^4(P_2^+/z_2')^2}\times\\ \times\,\mathrm{tr}\left[\gamma_\nu\bar{\Theta}(z_1')\,\gamma_\mu\hat{p}\,\gamma_\alpha\Theta(z_2')\,\gamma_\alpha\,\hat{p}\right]\Big|_{p^-\sim 1/Q},\end{aligned} \tag{43}$$

where we decomposed again the four-dimensional δ function around the corresponding hadron directions,

$$\delta^{(4)}\,(k_1+p-q)\Longrightarrow\delta\left(\frac{P_2^+}{y'}-q^+\right)\delta\left(\frac{P_1^-}{z_1'}-q^-\right)\delta^{(2)}\left(\frac{\mathbf{P}_{1\perp}}{z_1'}\right), \tag{44}$$

and calculated the integral over dp^- with the δ function coming from the imaginary part of the gluon propagator which fixes the minus component of the loop momentum:

$$\delta([p-k_2]^2)\Longrightarrow\frac{1}{2[p^+-P_2^+/z_2']}\delta\left(p^-+\frac{p_\perp^2}{2[p^+-P_2^+/z_2']}\right). \tag{45}$$

We remind that p^+ and P_2^+ are large vectors. Therefore, as one can see from (45), the minus component is thus suppressed as $1/Q$ and can be discarded in (43).

In (43), we still have integration over the plus and perpendicular components of the loop momentum. Just like in Fig. 2, a, the quark with momentum p emits a real gluon and transforms into the quark with momentum k_2. This means that

$$p^+ = \xi k_2^+, \quad dp^+ = \frac{P_2^+}{z_2'}\, d\xi. \tag{46}$$

Using (46) and calculating the corresponding integral and the trace in (39), we get the following contribution of diagram 2, a to the hadronic tensor:

$$\mathcal{W}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2}, a) = -2 g_{\mu\nu}^{\perp} \frac{\alpha_S}{\pi^2}\, C_F \delta^{(2)}\left(\mathbf{P}_{1\perp}\right) \left[z_1 \bar{D}(z_1)\right] \times \\ \times \left[\int \frac{dz_2'}{(z_2')^2}\, D(z_2') \int d\xi\, \delta\left(\xi - \frac{z_2'}{z_2}\right) (1-\xi) \int \frac{d^2\mathbf{p}_\perp}{\mathbf{p}_\perp^2}\right]. \tag{47}$$

As in the case of the Born diagram, one can show that the δ function $\delta(\xi - z_2'/z_2)$ in (47) should be associated with the imaginary part of the hard effective propagator, see Fig. 3, a. Note that similar arguments are valid for other diagrams needed for the study of the quark fragmentation function evolution.

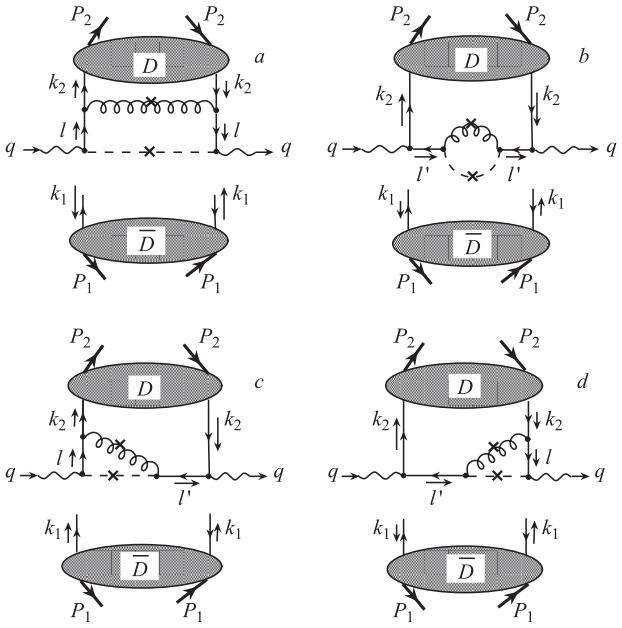

Fig. 3. Factorized leading-order α_S diagrams

Integration over the two-component loop momentum $\mathbf{p}_T$ should be implemented with the lower limits defined by the infrared cut-off λ^2 and the upper limit of Q^2. Therefore, the factor $\ln(Q^2/\lambda^2)$ which appears after this integration reflects the collinear singularity.

Turning back to the averaged hadronic tensor, we thus derive

$$\overline{\mathcal{W}}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2},a) = \int d^2\mathbf{P}_{1\,\perp}\,\mathcal{W}_{\mu\nu}^{(\perp)} = \\ = -2g_{\mu\nu}^{\perp}\,\frac{\alpha_S}{\pi}\,C_F \ln\left(\frac{Q^2}{\lambda^2}\right)\,\left[z_1\bar{D}(z_1)\right]\left[\int \frac{dz_2'}{(z_2')^2}\,D(z_2')\left(1-\frac{z_2'}{z_2}\right)\right]. \quad (48)$$

The tensor (48) has a completely factorized form and corresponds to Fig. 3, a which plays a role of a ladder diagram.

Diagram 2, b does not contribute to the terms containing the mass singularity. Indeed, the hadronic tensor corresponding to such a diagram is given by

$$\mathcal{W}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2},b) = g^2 C_F \int d^4k_1\,d^4p\,\delta^{(4)}\,(k_1+p-q)\;\mathrm{tr}\left[\bar{\Theta}(k_1)\,\Omega_{\mu\nu}(k_1,p)\right], \quad (49)$$

where

$$\Omega_{\mu\nu}(k_1,p) = \gamma_\alpha \int \frac{d^4k_2}{(2\pi)^3}\,\delta([p-k_2]^2)\,\frac{\hat{p}+\hat{k}_1-\hat{k}_2}{(p+k_1-k_2)^2}\,\gamma_\mu\Theta(k_2)\,\gamma_\nu\,\frac{\hat{p}+\hat{k}_1-\hat{k}_2}{(p+k_1-k_2)^2}\,\gamma_\alpha. \quad (50)$$

In the same manner as before, one can see that, after integration over $d\,p^-$ with the δ function originating from the gluon propagator, the denominator of (49) does not contain the term with $\mathbf{p}_\perp^2$ that produces a mass singularity. As a result, this contribution can be discarded. It is necessary to note that diagram 2, b effectively corresponds to diagram 3, b with the self-energy insertion into the quark propagator.

Two diagrams presented in Fig. 2, c and d contribute to the hadronic tensor in the following way:

$$\mathcal{W}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2},c,\,d) = 2g^2 C_F \int d^4k_1\,d^4p\,\delta^{(4)}\,(k_1+p-q)\;\mathrm{tr}\left[\gamma_\nu\bar{\Theta}(k_1)\,\Omega_\mu(k_1,p)\right], \quad (51)$$

where

$$\Omega_\mu(k_1,p) = \gamma_\alpha \int \frac{d^4k_2}{(2\pi)^3}\,\delta([p-k_2]^2)\,\frac{\hat{p}+\hat{k}_1-\hat{k}_2}{(p+k_1-k_2)^2}\,\gamma_\mu\Theta(k_2)\,\gamma_\alpha\,\frac{\hat{p}}{p^2}. \quad (52)$$

In (51), the denominator contains the necessary power of $\mathbf{p}_\perp^2$ and we have to keep only the zeroth order of $\mathbf{p}_\perp^2$ in the trace. Then, following the scheme outlined earlier for other diagrams, we derive the following expressions for the hadronic tensor:

$$\mathcal{W}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2},c,\,d) = \\ = -2g_{\mu\nu}^{\perp}\,\frac{\alpha_S}{\pi^2}\,C_F\delta^{(2)}\,(\mathbf{P}_{1\perp})\,\left[z_1\bar{D}(z_1)\right]\,\left[\int \frac{dz_2'}{(z_2')^2}\,D(z_2')\,\frac{2\,z_2'/z_2}{1-z_2'/z_2}\int \frac{d^2\,\mathbf{p}_\perp}{\mathbf{p}_\perp^2}\right]; \quad (53)$$

and for the averaged hadronic tensor:

$$\overline{\mathcal{W}}_{\mu\nu}^{(\perp),\,q}(\text{Fig. 2},c,\,d) = \\ = -2g_{\mu\nu}^{\perp}\,\frac{\alpha_S}{\pi}\,C_F \ln\left(\frac{Q^2}{\lambda^2}\right)\,\left[z_1\bar{D}(z_1)\right]\,\left[\int \frac{dz_2'}{(z_2')^2}\,D(z_2')\,\frac{2\,z_2'/z_2}{1-z_2'/z_2}\right]. \quad (54)$$

Thus, we have all the ingredients for the derivation of the factorized hadronic tensor including the α_S corrections and mass singularities. Summing of Eqs. (47) and (53) together with the contributions related to the virtual gluon emission gives

$$\overline{\mathcal{W}}_{\mu\nu}^{(\perp),\,q} = -2g_{\mu\nu}^{\perp}\,\frac{\alpha_S}{\pi}\,C_F\ln\left(\frac{Q^2}{\lambda^2}\right)\left[z_1\bar{D}(z_1)\right]\,\left[\int\frac{dz_2'}{(z_2')^2}\,D(z_2')\,\left(\frac{1+(z_2'/z_2)^2}{1-z_2'/z_2}\right)_+\right]. \quad (55)$$

The factorization scale μ_F can be introduced by the standard decomposition: $\ln(Q^2/\lambda^2) = \ln(Q^2/\mu_F^2) + \ln(\mu_F^2/\lambda^2)$, where the first term should be combined with the hard part of the corresponding hadronic tensor, whereas the second one with the soft part. If we choose $\mu_F^2 = Q^2$, the sum of (55) with the contribution of the Born diagram (32) leads to the following substitution for the quark fragmentation function:

$$D(z_2) \Longrightarrow D(z_2) + \frac{\alpha_S}{2\pi}\,C_F\ln\left(\frac{Q^2}{\lambda^2}\right)\int\limits_{z_2}^{1}\frac{dy_2}{y_2}\,D\left(\frac{z_2}{y_2}\right)\left(\frac{1+y_2^2}{1-y_2}\right)_+. \quad (56)$$

As a result, the quark fragmentation function acquires Q^2-dependence and satisfies the DGLAP evolution equation:

$$\frac{dD(z_2)}{d\ln Q^2} = \int\limits_{z_2}^{1}\frac{dy_2}{y_2}\,D\left(\frac{z_2}{y_2}\right)V_{qq}(y_2), \quad V_{qq}(y) = \frac{\alpha_S}{2\pi}\,C_F\,\left(\frac{1+y^2}{1-y}\right)_+. \quad (57)$$

3.2. Evolution of the Anti-quark Fragmentation Function. We will now consider the anti-quark fragmentation functions. The anti-quark sector in the fragmentation can be studied similarly to the quark fragmentation function case. However, there are some minor differences in comparison to the quark fragmentation function evolution. First of all, diagram $2,b$ will now play a role of the ladder diagram. Let us write down the hadronic tensor corresponding to this diagram:

$$\mathcal{W}_{\mu\nu}^{(\perp),\,\bar{q}}(\text{Fig. 2},b) = g^2C_F\int d^4k_2\,d^4m\,\delta^{(4)}\,(k_2+m-q)\;\mathrm{tr}\left[\gamma_\mu\Theta(k_2)\,\gamma_\nu\bar{\Omega}(m)\right], \quad (58)$$

where

$$\bar{\Omega}(m) = \frac{\hat{m}}{m^2}\,\gamma_\alpha\int\frac{d^4k_1}{(2\pi)^3}\,\delta([m-k_1]^2)\,\bar{\Theta}(k_1)\gamma_\alpha\,\frac{\hat{m}}{m^2}. \quad (59)$$

As in the case of the quark sector, we first introduce the definition of the corresponding parton fractions (see Eqs. (19) and (41)). Then, the corresponding δ functions have to be decomposed in the directions defined by the hadron momenta. We obtain

$$\delta^{(4)}\,(k_2+m-q) \Longrightarrow \delta\left(m^+ + \frac{P_2^+}{z_2'} - q^+\right)\,\delta\left(\frac{P_1^-}{y'} - q^-\right)\,\delta^{(2)}\left(\frac{\mathbf{P}_1^\perp}{y'}\right) \quad (60)$$

for the four-dimensional δ function which is responsible for the momentum conservation at the local vertex:

$$\delta([m-k_1]^2) \Longrightarrow \frac{1}{2[m^- - P_1^-/z_1']}\delta\left(m^+ + \frac{m_\perp^2}{2[m^- - P_1^-/z_1']}\right) \quad (61)$$

for the δ function in the tensor (59). Integration over dm^+ with δ function (61) fixes the plus component of the loop momentum to be a small variable. As a result, m^+ can be neglected in the corresponding expressions.

Omitting the details, since all stages of calculations are exactly the same as for the quark sector, below we write the expression for the averaged hadronic tensor corresponding to Fig. 2, b:

$$\overline{\mathcal{W}}^{(\perp),\,\bar{q}}_{\mu\nu}(\text{Fig. 2}, b) = \\ = -2g^{\perp}_{\mu\nu}\,\frac{\alpha_S}{\pi}\,C_F \ln\left(\frac{Q^2}{\lambda^2}\right)\left[\frac{D(z_2)}{z_2}\right]\left[z_1^2\int\frac{dz_1'}{(z_1')^2}\,\bar{D}(z_1')\left(1-\frac{z_1'}{z_1}\right)\right]. \quad (62)$$

The next non-zero contribution comes from the diagrams presented in Fig. 2, c and d. These diagrams give us the following expression:

$$\mathcal{W}^{(\perp),\,\bar{q}}_{\mu\nu}(\text{Fig. 2}, c,\, d) = 2g^2C_F\int d^4k_2\,d^4m\,\delta^{(4)}\,(k_2+m-q)\;\text{tr}\left[\Theta(k_2)\,\gamma_\nu\bar{\Omega}_\mu(k_2,m)\right], \quad (63)$$

where

$$\bar{\Omega}_\mu(k_2,m) = \frac{\hat{m}}{m^2}\,\gamma_\alpha\int\frac{d^4k_1}{(2\pi)^3}\,\delta([m-k_1]^2)\,\bar{\Theta}(k_1)\gamma_\mu\,\frac{\hat{m}-\hat{k}_1+\hat{k}_2}{(m-k_1+k_2)^2}\,\gamma_\alpha. \quad (64)$$

Calculating the tensor (63) in a way similar to the derivation of (51), we obtain the averaged hadronic tensor:

$$\overline{\mathcal{W}}^{(\perp),\,\bar{q}}_{\mu\nu}(\text{Fig. 2}, c,\, d) = \\ = -2g^{\perp}_{\mu\nu}\,\frac{\alpha_S}{\pi}\,C_F \ln\left(\frac{Q^2}{\lambda^2}\right)\left[\frac{D(z_2)}{z_2}\right]\left[z_1^2\int\frac{dz_1'}{(z_1')^2}\,\bar{D}(z_1')\frac{2z_1'/z_1}{1-z_1'/z_1}\right]. \quad (65)$$

Diagram $2, a$ in this case does not contribute to the evolution of the anti-quark fragmentation function due to the same arguments as we presented for the quark sector discussing diagram $2, b$.

Thus, combining all the diagrams and adding the contributions from the virtual gluon emissions, we get

$$\overline{\mathcal{W}}^{(\perp),\,\bar{q}}_{\mu\nu} = -2g^{\perp}_{\mu\nu}\,\frac{\alpha_S}{\pi}\,C_F \ln\left(\frac{Q^2}{\lambda^2}\right)\left[z_1^2\int\frac{dz_1'}{(z_1')^2}\,\bar{D}(z_1')\,\left(\frac{1+(z_1'/z_1)^2}{1-z_1'/z_1}\right)_+\right]\left[\frac{D(z_2)}{z_2}\right]. \quad (66)$$

Again, as for the quark case, the summation of the hadronic tensor (66) with the Born hadronic tensor modifies the anti-quark fragmentation function:

$$\bar{D}(z_1) \Longrightarrow \bar{D}(z_1) + \frac{\alpha_S}{2\pi}\,C_F \ln\left(\frac{Q^2}{\lambda^2}\right)\int\limits_{z_1}^{1}\frac{dy_1}{y_1}\,\bar{D}(z_1/y_1)\,\left(\frac{1+y_1^2}{1-y_1}\right)_+. \quad (67)$$

As a result of this, the anti-quark fragmentation function becomes Q^2-dependent and obeys the DGLAP evolution equation (see (57)). Besides, the introduction of the hard effective propagators as in the quark case leads to the factorized Feynman diagrams which are completely analogous to the diagrams in Fig. 3.

4. k_T-DEPENDENT FUNCTIONS: BORN APPROXIMATION

In the preceding sections, we focused on the case of the spin-independent integrated fragmentation functions which in a way was a test of our approach. Since we plan to extend our approach to k_T-unintegrated functions (in particular, Collins function) we discuss the Born approximation of e^+e^- annihilation, involving both the Collins fragmentation function and the spin-dependent fragmentation function. Namely, we consider the production of transverse polarized quark–antiquark pair associated with $\sigma_{\mu\nu}$ structures in the corresponding matrix elements (see below).

More exactly, the nonperturbative blob (see, for example, Fig. 1), related to the detected baryon, whose transverse polarization is correlated to the transverse polarization of the quark, can be described by the spin-dependent fragmentation function. At the same time, in the other blob, the transition of the transverse polarized antiquark with the intrinsic transverse momentum into the unpolarized hadron is described by the Collins fragmentation function. In the latter case, the mentioned transition is related to the azimuthal asymmetry distribution of hadrons. Note that the symmetric case, with two similar fragmentation functions, has been well studied from both the theoretical and experimental points of view [2,19–21]. The asymmetric situation requires the simultaneous measurement of the meson azimuthal asymmetry in one jet and baryon (say, Λ) polarization in another one. Such measurements may be performed at BELLE[1].

We use the coordinate (or the impact parameter) representation where the explicit definition of the transverse momentum is not necessary. In this case, the corresponding hadronic tensor takes the form

$$\Delta\mathcal{W}_{\mu\nu} = \int \frac{d^4\xi}{(2\pi)^4}\, \mathrm{e}^{i(q\cdot\xi)}\, \mathrm{tr}\left[\gamma_\mu \hat{\Theta}(\xi)\gamma_\nu \hat{\bar{\Theta}}(\xi)\right]. \tag{68}$$

Here, the spin-dependent fragmentation function has been defined in the coordinate space as

$$\hat{\Theta}_1(\xi) \Rightarrow \sigma_{\alpha\beta}\gamma_5 P_2^\alpha S^\beta \int\limits_0^1 \frac{dz_2'}{(z_2')^2}\, \mathrm{e}^{-i(P_2\cdot\xi)/z_2'} H_{1\,T}(z_2'), \tag{69}$$

where S denotes the hadron transverse polarization, and the Collins function's analog in the coordinate space is given [15] by

$$\begin{gathered}\hat{\bar{\Theta}}_2(\xi) \Rightarrow iM\sigma_{\alpha\beta} P_1^\alpha \xi^\beta \int\limits_0^1 \frac{dz_1'}{(z_1')^2}\, \mathrm{e}^{-i(P_1\cdot\xi)/z_1'} \bar{H}_1^\perp(z_1'), \\ \bar{H}_1^\perp(z_1') = \int dk_T^2\, \frac{k_T^2}{M^2}\, \bar{H}_1^\perp(z_1', k_T^2),\end{gathered} \tag{70}$$

where ξ is the (transverse) position in coordinate space; while M, introduced due to the dimensional analysis, implies a parameter of the order of jet mass being the only dimensionful parameter in the soft part.

[1] M. Grosse Perdekamp, private communication.

The position in the coordinate space ξ (see, (70)) yields the derivative over P_1 in momentum space. Therefore, inserting (69) and (70) in (68), we get

$$\Delta\mathcal{W}_{\mu\nu} = \int \frac{dz_1'}{z_1'} \int \frac{dz_2'}{(z_2')^2} \left\{ \frac{\partial}{\partial P_1^-} \delta(q^- - \frac{P_1^-}{z_1'}) \right\} \times \\ \times \delta\left(q^+ - \frac{P_2^+}{z_2'}\right) \delta^{(2)}\left(\frac{\mathbf{P}_1^\perp}{z_1'}\right) \bar{H}_1^\perp(z_1')\, H_{1\,T}(z_2') T_{\mu\nu}(P_1, P_2, S_T), \quad (71)$$

where

$$T_{\mu\nu}(P_1, P_2, S_T) = \mathrm{tr}\left[\gamma_\mu (iM\sigma_{+-} P_1^-)\gamma_\nu (\sigma_{-T}\gamma_5 P_2^+ S_T)\right]. \quad (72)$$

Using the kinematics defined above, we derive

$$\Delta\mathcal{W}_{\mu\nu} = T_{\mu\nu}\delta^{(2)}(\mathbf{P}_1^\perp) \left[\int dz_1' \left(z_1' \bar{H}_1^\perp(z_1')\right) \delta^{(1)}(z_1' - z_1)\right] \left[\int \frac{dz_2'}{z_2'} H_{1\,T}(z_2')\delta(z_2' - z_2)\right]. \quad (73)$$

Then, calculating the averaged hadronic tensor, one has the following:

$$\overline{\Delta\mathcal{W}}_{\mu\nu} = \int d^2\,\mathbf{P}_{1\perp}\, \delta^{(2)}(\mathbf{P}_1^\perp) \Delta\mathcal{W}_{\mu\nu} = T_{\mu\nu} \left[z_1 \bar{H}_1^\perp(z_1)\right]' \left[\frac{H_{1\,T}(z_2)}{z_2}\right]. \quad (74)$$

Note that the analogous asymmetric combination corresponding to SIDIS process related to ours by crossing was also considered in [15]. In that case, the incoming quark in SIDIS is described by the transversity distribution function:

$$\hat{h}(\eta) = \sigma_{\mu\nu}\gamma_5 P_1^\mu S^\nu \int_0^1 dx\, \mathrm{e}^{ix(P_1\cdot\eta)} h(x), \quad (75)$$

where S_μ is the target polarization. Using (4) and (75), the corresponding hadronic tensor takes the form

$$\Delta\mathcal{W}^{\mu\nu} = \int \frac{d^4\xi}{(2\pi)^4}\, \mathrm{e}^{-i(q\cdot\xi)}\, \mathrm{tr}\,[\hat{h}(\xi)\gamma^\mu \hat{H}(\xi)\gamma^\nu]. \quad (76)$$

To study the k_T (or $\mathbf{P}_{2\perp}$) distributions in SIDIS and the related asymmetries, we should consider the weighted hadronic tensor which projects out the corresponding moment of the Collins function:

$$\Delta_n \overline{\mathcal{W}}^{\mu\nu} = \int d^4 P_2 \delta(P_2^2)(P_2\cdot n_\perp)\delta\left(\frac{P_1\cdot P_2}{P_1\cdot q} - z\right) \Delta\mathcal{W}^{\mu\nu}, \quad (77)$$

where $n_\perp$ is the unit transverse 4-vector ($n_\perp \cdot P_1 = n_\perp \cdot q = 0$, $n_\perp^2 = -1$) which defines the transverse direction. Using (4), (75) and (77), one can see that the derivative $\partial^\alpha \equiv \partial/\partial P_2^\alpha$ in

$$\Delta_n \overline{\mathcal{W}}^{\mu\nu} = iM \int d^4 P_2\, (P_2\cdot n_\perp)\, \delta(P_2^2)\, \delta\left(\frac{P_1\cdot P_2}{P_1\cdot q} - z\right) \times \\ \times \int dx\, dz' \partial^\alpha \delta\left(xP_1 + q - \frac{P_2}{z'}\right) h(x)(z' I(z'))\mathrm{tr}\,[\gamma_5 \hat{P}_1 \hat{S}\gamma^\mu [\hat{P}_2\gamma_\alpha]\gamma^\nu] \quad (78)$$

should act only on $(P_2 \cdot n_\perp)$, so that the result [15] is equal to the standard expression for the contribution of Collins function, except that the role of the direction of intrinsic transverse momentum is played by the auxiliary transverse vector $n_\perp$:

$$\mathrm{tr}\,[\hat{p}_1 \hat{S} \gamma_5 \gamma^\mu \hat{p}_3 \hat{n}_\perp \gamma^\nu] \Longrightarrow \mathrm{tr}\,[\hat{p}_1 \hat{S} \gamma_5 \gamma^\mu \hat{p}_3 \hat{k}_T \gamma^\nu]. \tag{79}$$

This substitution does not change the azimuthal dependence, as the weighted integration corresponds to azimuthal average:

$$\langle d\sigma(\phi_h) \cos(\phi_h - \phi_n) \rangle = \cos\phi_n \langle d\sigma(\phi_h) \cos(\phi_h) \rangle + \sin\phi_n \langle d\sigma(\phi_h) \sin(\phi_h) \rangle. \tag{80}$$

As a result, the azimuthal dependence of the cross section is transferred to the dependence on the angle ϕ_n, and $I(z)$ corresponds to the *moment* of the Collins function:

$$I(z) \sim \int dk_T^2 \frac{k_T^2}{M^2} H_1(z, k_T^2). \tag{81}$$

Thus, to describe the Collins effect we suggest the k_T-dependent fragmentation function to be written in the coordinate space, where no specification of intrinsic k_T is required. Therefore, the calculation of radiation corrections and evolution may be performed [**?**] in the same way as in this paper.

Moreover, this approach may be applied [15] for the description of higher weighted moments in k_T and p_T. Say, choosing the term of order ξ^2 in the expansion of unpolarized fragmentation function in the coordinate space and taking the p_T moment of the hadronic tensor weighted with p_T^2 corresponds to the account of the width of the k_T-dependent unpolarized fragmentation function. The exponential shape of the latter, in turn, corresponds to partial resummation [15] of the infinite series of higher twists, analogous to that leading to the appearance of non-local vacuum condensates, when vacuum rather than hadronic averages are considered. The expansion in coordinate space accompanied by taking the respective weighted moments of cross sections provides a complementary definition of observables, accounting for the shape of k_T-dependent fragmentation (and distribution) functions.

CONCLUSIONS

We described a method which allows us to prove a factorization of the process with two fragmentation functions. We would like to point out that the presented method can be applied for any two-current process. The difficulties of factorization for such kind of processes emerge for the case when the kinematical transversalities inside the hadron are rather small. It leads to the problem in the definition of what is the hard subprocess for the process.

Following the idea of the paper [14], we showed that the corresponding δ functions in the hadronic tensors should be treated as the hard parts. It is based on the observation that these δ functions can be associated with the imaginary parts of the effective propagators related to the well-defined hard subprocess. As a result, we finally have the completely factorized expression for the hadronic tensor with the evolved fragmentation functions.

In this paper, the proposed method has been tested in the simplest case when the differential cross section of e^+e^- annihilation is related to the spin-independent integrated fragmentation functions. In a future work, we will extend our approach to the study of the spin-dependent structures and k_T-dependent fragmentation functions (in particular, Collins function) [?]. This will allow us to perform the leading-order QCD fits of relevant experimental data.

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