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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

PERTURBATIVE QCD

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These lectures, intended mainly for undergraduates, contain a very short introduction to perturbative QCD. We introduce the QCD Lagrangian, outline how QCD is quantized and renormalized, and as an example describe the calculation of the elastic two-gluon scattering. We then discuss the physics behind the parton model of hadron collisions, with the important issues of the infrared safety and factorization. We end with a short description of how different hadronic final states are searched for and studied at hadron colliders.

Эти лекции, ориентированные в основном на студентов, содержат очень краткое введение в пертурбативную КХД. Мы вводим лагранжиан КХД, подчеркиваем вопрос его квантования и перенормировки в КХД и в качестве примера описываем вычисление упругого двухглюонного рассеяния. Затем мы обсуждаем физику за рамками партонной модели в столкновениях адронов, обращая особое внимание на инфракрасную расходимость и факторизацию. Наконец, мы кратко обсуждаем вопрос о том, как исследуются на адронных коллайдерах различные конечные адронные состояния.

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1. INTRODUCTION: WHO NEEDS QCD?

The coming decade in the high-energy physics will undoubtedly be dominated by new results from the Large Hadron Collider at CERN. It has a very broad research program, but its focus is placed on study of the dynamics of the electroweak symmetry breaking and on search for possible manifestations of physics beyond the Standard Model. With all this attention towards New Physics, a student might ask a legitimate question: Who now needs the old good perturbative QCD, with its laws known since decades?

There are several answers to this question. First, it is the LHC itself. A collision at the LHC starts and often ends with the strong interaction. Although the information the LHC detectors are trying to decipher might be in the electroweak/New Physics domain, their experimental signatures are convoluted with the QCD processes. A firm knowledge of the partonic distributions and understanding how the scattered quarks and gluons materialize as hadrons is an important ingredient of any New Physics process calculation at hadronic colliders.

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In addition, all these potentially interesting processes are immersed into the huge background of pure QCD processes. It is indispensable to have a good understanding of the rates and properties of these processes to be able to efficiently discriminate between them and the rare interesting events. In short, without the expertise in QCD one would be unable to make discovery claims at the LHC.

In addition to this «pragmatic» purpose, the QCD remains interesting on its own. It serves as a prototypical Yang–Mills theory, where several different theoretical approaches can be tested in experiment in various kinematical situations. In addition to the huge unresolved problem of confinement, which is a truly nonperturbative effect, there are many other hot issues such as how far nonperturbative effects penetrate into the perturbative domain, the origin of the mathematical structures one finds in multiparticle amplitudes, properties of the theory in the limit of large number of colors, etc.

All that shows that the QCD, and specifically the perturbative QCD, remains one of the cornerstones of theoretical high-energy physics in the LHC era.

In these lectures, no attempts are made to appropriately cite every important result in QCD. Instead, we give two general references to an annotated resource letter of the QCD literature [1] and to a very detailed lecture course [2].

2. THE QCD LAGRANGIAN AND QUANTIZATION

2.1. Gauge Principle. In quantum chromodynamics (QCD), strong interactions are described as non-Abelian gauge interactions. Therefore, before going to the QCD Lagrangian, it is useful to briefly review the gauge principle on its own with a much simpler example of quantum electrodynamics, QED.

The basic idea is that if you start with a quantum field theory of free matter fields and promote a global symmetry to the local one, then you automatically get interaction between the matter fields which is carried by spin-1 bosons. Consider the free Dirac field described by the Lagrangian:

$$\mathcal{L} = i\bar{\psi}\partial^\mu\gamma_\mu\psi - m\bar{\psi}\psi. \tag{1}$$

By construction, it possesses the global $U(1)$ symmetry: $\psi(x) \to \exp(i\alpha)\psi(x)$, $\alpha = \mathrm{const}$. This symmetry reflects the fact that the overall phase of the field is not observable and can be rotated as we wish without any consequences for theory.

Now we want to impose a stronger requirement that the observables be independent of the arbitrary phase rotation at each spatial point, i.e., with $\alpha(x) = e\Gamma(x)$. The Lagrangian (1) is not invariant under such transformations due to the presence of a derivative which acts on the phase $\alpha(x)$ as well. However, we could think of a «minimal modification» of the theory:

$$\mathcal{L}_{\mathrm{QED}} = i\bar{\psi}D^\mu\gamma_\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad D^\mu \equiv \partial^\mu - ieA^\mu(x). \tag{2}$$

Here the usual derivative ∂^μ is replaced by another operator D^μ, a derivative plus shift, which is called the «extended derivative», or covariant derivative. This «shift» is not fixed but changes in accord with the phase transformation of the matter field:

$$\psi(x) \to \mathrm{e}^{ie\Gamma(x)}\,\psi(x), \quad A^\mu(x) \to A^\mu(x) - \partial^\mu\Gamma(x). \tag{3}$$

It is straightforward to check that the Lagrangian (2) is now invariant under the gauge transformations (3). $A^\mu(x)$ is then promoted to the dynamic field and needs its kinetic term $\propto F_{\mu\nu}F^{\mu\nu}$, with $F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$. The resulting Lagrangian (2) defines the theory called quantum electrodynamics (at the classical level so far), and it reproduces the Maxwell equations for the electromagnetic field.

2.2. QCD as a Gauge Theory. The key starting point of the whole construction was the presence of an internal degree of freedom (the phase of a complex field), which was associated with a global transformation. One can generalize this approach by postulating matter fields with other internal degrees of freedom. This is precisely the origin of *color* in QCD. One postulates that the fundamental strongly interacting matter fields called quarks can be represented by points in a complex three-dimensional color space; the basis vectors in this space ψ_i, $i = 1, 2, 3$, are called «red», «green» and «blue» quarks.

The quark fields thus realize the fundamental representation of the $SU(3)$ group. Composite objects made of quarks and antiquarks transform as various higher representations of the color group. Among them, color singlets (combinations which are left-invariant under the entire $SU(3)$ group) appear as well. It is with these objects that we identify the physical hadrons we observe in our world. To demonstrate that *color is confined* and only color singlet states can have finite energy is a big unresolved problem in QCD.

In order to associate the color with a gauge interaction, we assume that the $SU(3)$ transformations of the quark fields are local:

$$\psi_i(x) \to U_{ij}(x)\psi_j(x), \quad U_{ij}(x) = \exp\left[-igt^a_{ij}\theta^a(x)\right], \quad a = 1, \ldots, 8$$

with 8 generators t^a satisfying the commutation rules $[t^a, t^b] = if^{abc}t^c$. The derivative must again be replaced by the covariant derivative, $D_\mu = \partial_\mu - igA_\mu(x)$, with g being the strong interaction coupling and the «shift» given by the *gluon* field $A_\mu(x) = A^a_\mu(x) \cdot t^a$.

The (classical) QCD Lagrangian is

$$\mathcal{L}_{\rm QCD} = i\bar\psi_i D^\mu_{ij}\gamma_\mu\psi_j - m\bar\psi_i\psi_i - \frac{1}{2}\,{\rm Tr}\,F_{\mu\nu}F^{\mu\nu}.$$

Since the $SU(3)$ group is non-Abelian, the QCD is a non-Abelian gauge theory. Its key feature is that the field strength of the gluon field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

is not gauge-invariant on its own and carries a color charge as well, in contrast to QED. The $F_{\mu\nu}F^{\mu\nu}$ is gauge-invariant, but it contains cubic and quartic terms in the gluon field, which means that gluons are self-interacting so that there is no superposition principle at work. All these complications are consequences of the non-Abelian nature of the theory and have a profound effect on its structure.

2.3. Quantization of QCD. Quantizing QCD, we get quarks and gluons as quantum particles. However, quantization of non-Abelian gauge theories involves several subtle features. Some of them are common to all gauge theories. For example, the photon field $A_\mu(x)$ in QED possesses too many degrees of freedom, some of them being unphysical, which creates obstacles in quantizing the theory. The problem is solved by introducing a gauge-fixing term $\mathcal{L}_{\rm gf}$ in the Lagrangian, which explicitly violates the gauge symmetry but makes the quantization possible. Several gauge-fixing procedures can be implemented, but the final results must not depend on this choice.

Quantization of a non-Abelian gauge theory poses extra problems. Due to direct self-coupling of the gauge bosons, the unphysical degrees of freedom can appear in loops. In principle, with a cleverly chosen gauge fixing function (e.g., in axial gauges), the effect of unphysical degrees of freedom disappears at the expense of breaking Lorentz invariance of the intermediate calculations. If one prefers a covariant gauge, the effect of unphysical degrees of freedom is compensated by auxiliary complex scalar fields with Fermi statistics called the Faddeev–Popov ghosts. It must be stressed that these ghosts are not put by hand; it is just a clever way to rewrite the gauge-fixing restriction placed onto the gauge field degrees of freedom in the functional integral.

2.4. An Example of QCD Process. Once the quantization procedure outlined in the previous paragraphs is completed, one can generate a list of Feynman rules for the propagation and interaction of the quark, gluon and ghost fields, which allow for calculation of the QCD processes. However, even the simplest QCD processes can become almost intractable for manual calculations unless an appropriate way of calculation is chosen.

Just to give an example, consider the elastic scattering of two gluons in the color-singlet state. The amplitude in the Feynman gauge takes the form

$$M = (M_s + M_t + M_u + M_4)^{\mu\nu\mu'\nu'} e_{1\mu} e_{2\nu} e^*_{3\mu'} e^*_{4\nu'},$$

$$\begin{aligned}
M_s^{\mu\nu\mu'\nu'} &= G_s C^{\mu\sigma\nu} \frac{1}{(q_1+q_2)^2} C^{\sigma\mu'\nu'},\\
M_t^{\mu\nu\mu'\nu'} &= G_t C^{\mu\mu'\sigma} \frac{1}{(q_1-q_3)^2} C^{\sigma\nu'\nu},\\
M_u^{\mu\nu\mu'\nu'} &= G_u C^{\mu\nu'\sigma} \frac{1}{(q_1-q_4)^2} C^{\sigma\mu'\nu},\\
M_4^{\mu\nu\mu'\nu'} &= G_s(g^{\mu\nu'} g^{\nu\mu'} - g^{\mu\mu'} g^{\nu\nu'})+\\
&\qquad + G_t(g^{\mu\nu'} g^{\nu\mu'} - g^{\mu\nu} g^{\mu'\nu'}) + G_u(g^{\mu\nu} g^{\mu'\nu'} - g^{\mu\mu'} g^{\nu\nu'}),
\end{aligned}$$

where G_s, G_t, G_u are color factors and $C^{\mu\nu\rho}$ is the triple gluon vertex given by the Feynman rules.

If one tries to calculate the cross section covariantly by squaring the amplitude and summing over all indices, one will need to include ghosts flowing through the squared diagram. As a result, straightforward covariant calculation of this process will include thousands of separate terms!

Instead, you can calculate the amplitude itself for physical polarizations of the initial and final gluons $M(\lambda_1\lambda_2 \to \lambda_3\lambda_4)$, $\lambda_i = \pm 1$. In this case ghosts are not needed, and the helicity amplitude takes a simple form:

$$M(\lambda_1\lambda_2 \to \lambda_3\lambda_4) = 3\pi\alpha_s \delta^{ab}\, \delta^{a'b'} \frac{1}{tu} \left\{ \begin{array}{lll} s^2 & \text{for} & ++ \to ++,\ -- \to --,\\ u^2 & \text{for} & +- \to +-,\ -+ \to -+,\\ t^2 & \text{for} & +- \to -+,\ -+ \to +-\,. \end{array} \right. \qquad (4)$$

Here $\alpha_s \equiv g^2/4\pi$, the strong interaction analog of the fine structure constant. The cross section can be immediately obtained.

The calculation of helicity amplitudes is dramatically simpler than the covariant squaring; however, the result (4) hints at further simplification. Indeed, the total helicity conservation can be observed: $\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4$. This is not a coincidence, but a particular case of a general result which holds for any tree-level multigluon scattering: the maximally and almost maximally helicity violating (MHV) amplitudes are zero. In the present case MHV amplitudes are $++ \to --$, $-- \to ++$, almost-MHV amplitudes are $++ \to +-$, $+- \to --$, etc., so that what remains are the total helicity conserving amplitudes.

This example gives just the first feeling of various nontrivial properties that QCD helicity amplitudes can have. There exists the whole machinery of how to effectively use these properties to calculate multileg and multiloop amplitudes. Note that such calculations are not just of academic interest; they are the backbone of any pQCD calculation of high-energy hadronic processes.

3. RENORMALIZATION OF QCD

When calculating diagrams beyond the tree level, one encounters integrals over loop momenta which are UV-divergent. This problem is cured by the *renormalization* of the theory. In very general words, the essence of renormalization consists in the following observation. Although we start from a Lagrangian that is written in terms of «bare fields» and contains some «bare masses» and «bare charges», the physical meaning should be assigned not to them but to their renormalized counterparts. The bare fields and parameters are represented as the physical counterparts times renormalization constants ($\psi^0 = Z_\psi \psi$, $m^0 = Z_m m$, etc.), and all the divergences are absorbed into the finite number of different Z's. The loop corrections might look divergent if expressed in terms of bare couplings, but they become finite when expressed in terms of physically observable masses and charges.

Of course, all the calculations at each step must be performed with mathematically well-defined finite quantities. Technically, this is achieved by the *regularization procedure*. First, we modify the integrals to make them finite (e.g., by a momentum cut-off Λ). All Z's are now finite but will be divergent if $\Lambda \to \infty$. The observables are then calculated at finite Λ and expressed in terms of physical masses and couplings. All the divergent terms must then cancel, and the limit $\Lambda \to \infty$ can be safely taken. Note that although several regularization procedures exist (dimensional, Pauli–Villars, lattice, etc.), all of them eventually lead to the same result.

Renormalization of QCD is accompanied by a very remarkable phenomenon of *dimensional transmutation*: although the bare Lagrangian (in the massless limit) contains no dimensional parameter, such a parameter appears after renormalization. This parameter, $\Lambda_{\rm QCD}$, actually fixes the scale at which nonperturbative effects come into full swing. This phenomenon is illustrated by the expression

$$\alpha_s^{\rm bare} = Z_i\left(\frac{\Lambda}{\mu}\right)\alpha_s\left(\frac{\mu}{\Lambda_{\rm QCD}}\right),$$

with the following argumentation.

- Regularization involves a cut-off parameter Λ with dimension of energy.
- But dimensionless Z's cannot depend on a dimensional parameter alone. Therefore, a renormalization scale μ appears.

• But the «bare» fields and couplings do not know about μ; so, μ-dependence must cancel *exactly* between Z's and the physical coupling $\alpha_s(\mu)$.

• But the dimensionless coupling $\alpha_s(\mu)$ cannot depend on μ alone. Therefore, another quantity, $\Lambda_{\rm QCD}$, emerges.

So, after renormalization we are left with $\alpha_s(\mu/\Lambda_{\rm QCD})$ which depends on an (arbitrary) *renormalization scale* μ and the «boundary condition parameter» $\Lambda_{\rm QCD}$. Note that independence of the «bare» quantities and physical observables from renormalization scale μ leads to a very powerful technique called the *renormalization group*.

The prototypical example of the application of this technique to QCD is the calculation of how α_s actually changes with the renormalization scale μ. If Z_g, Z_q, Z_A are the renormalization constants for the coupling constant g, quark and gluon fields, respectively, then

$$\alpha_s^{\rm bare} = \frac{Z_g^2}{Z_q^2\, Z_A^2}\alpha_s, \quad \frac{Z_g^2}{Z_q^2\, Z_A^2} = 1 - \alpha_s \frac{\beta_0}{4\pi} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathcal{O}(\alpha_s^2).$$

An explicit calculation gives the $\beta_0 = 11 - (2/3)n_F$, where n_F is the number of active flavors. Since $\alpha_s^{\rm bare}$ is μ-independent, we get

$$0 = \frac{d\,\log\,\alpha_s^{\rm bare}}{d\,\log\,\mu^2} = \frac{\beta_0}{4\pi}\alpha_s + \frac{d\,\log\,\alpha_s}{d\,\log\,\mu^2},$$

which gives the QCD beta function:

$$\frac{d\alpha_s}{d\,\log\,\mu^2} \equiv \beta(\alpha_s) = -\frac{\beta_0}{4\pi}\alpha_s^2 < 0.$$

The solution of this equation

$$\alpha_s = \frac{4\pi}{\beta_0\,\log\left(\dfrac{\mu^2}{\Lambda_{\rm QCD}^2}\right)}$$

decreases with μ growth. The theory is therefore *asymptotically free*, which has a profound effect on the applicability of the perturbative approach to QCD. Indeed, it states that although strong interactions are perturbatively intractable at small energies, they become tractable when the energies are large so that $\alpha_s \ll 1$. Experience shows that perturbative calculations give a reasonable description of hadronic scattering when the energies and momenta transfers exceed several GeV.

4. PARTON MODEL

4.1. Naive Parton Model. Although the proton at rest is conveniently visualized as if built of three quarks which have some dynamically generated constituent mass and are held together by gluonic forces, a wealth of experimental data in the 60s–70s led physicists to a drastically different view on the ultrarelativistic proton. Namely, such a proton can be best thought of as if made of *many* pointlike and almost collinear partons, which fly together and share among them the full charge and momentum of the proton.

An important feature of this picture is that due to relativistic time dilatation the «internal life» of these partons (how they split, merge or rescatter) can be thought of as frozen in the first approximation. Then, a hard collision of two ultrarelativistic protons can be described according to the following scheme:

hadrons $\to$ partons $\to$ hard QCD process $\to$ hadronization.

The cross section of this scattering can then be factorized at the level of probabilities, not just amplitudes:

$$\sigma_{pp\to A+\ldots}(s) = \sum_{ij} \int_0^1 dx_1\, dx_2\, f_i(x_1)\ f_j(x_2)\, \sigma_{ij\to A}(x_1 x_2 s). \quad (5)$$

Here $f_i(x)$, $i = q, \bar{q}, g$, are partonic distribution functions (PDFs) which give the probability density to find in the proton a parton i with momentum fraction x, while $\sigma_{ij\to A}$ is the partonic scattering cross section.

This is the naive parton model. Its key feature, which follows from the qualitative picture just presented, is the universality (process-independence) of PDFs. This makes the whole approach very attractive for the analysis of hadronic collisions: one just needs to determine experimentally the proton PDFs in one experiment (for example, in the deep-inelastic scattering, DIS, of an electron off the proton, which gives a clean access to the proton PDFs) and then use them in all subsequence calculations.

4.2. Infrared Stability. The problem with the naive parton model just described is that it does not survive the radiative corrections. The key issue that enters the game is the *infrared stability*.

Consider an observable that describes production of n partons: $O_n(k_1, k_2, \ldots)$. The IR stability consists in the requirement that in the *collinear limit* ($\mathbf{k}_1 || \mathbf{k}_2$) and in the *soft limit* ($k_1 \to 0$) this observable approach $O_{n-1}(k_1 + k_2, \ldots)$. «Good observables» should be IR-stable: they should be insensitive to the emission of very large wavelength gluons or to collinear splitting.

A good example of the IR-stable quantity is given by the e^+e^- annihilation into hadrons. In the first order, it proceeds via creation of the s-channel virtual photon which then decays into a $q\bar{q}$ pair. The differential cross section of $e^+e^- \to q\bar{q}$ production at fixed angles is well-behaved. One can now calculate production of more complicated states, like $q\bar{q}g$, and observe that this cross section is divergent in both the soft and collinear limit. However, at the same order of perturbation theory, one also has the loop correction to the $q\bar{q}$ production. This correction also contains the same two divergences but it has the opposite sign. As a result, when all α_s^2 corrections to the cross section are taken into account, both divergences cancel, and the resulting expression is IR-stable.

Unfortunately, the naive parton model is not IR-stable. This can be checked by the following calculations. Consider again DIS and calculate the next-order correction to the $\gamma^* p \to X$ cross section:

$$\sigma_p(s) = \int dx\, f(x)\, [\sigma_q^{(0)}(xs) + \sigma_q^{(1)}(xs) + \ldots], \quad (6)$$

where $\sigma_q^{(1)}(xs)$ is the full (real + virtual) correction. Here x is the proton's momentum fraction carried by the struck quark. The virtual correction accounts for emission and reabsorption of a virtual gluon by this quark, while the real correction takes into account the possibility that the struck quark emits a gluon reducing its momemtum fraction from x to zx just before collision with the photon. The expression for $\sigma^{(1)}$ reads

$$\sigma^{(1)}(xs) = \frac{C_F\alpha_s}{2\pi}\int\frac{dk_\perp^2}{k_\perp^2}\int\limits_0^1 dz\frac{1+z^2}{1-z}\left(\sigma^{(0)}(zxs)-\sigma^{(0)}(xs)\right) = \\ = \frac{\alpha_s}{2\pi}\int\frac{dk_\perp^2}{k_\perp^2}\int\limits_0^1 dz\, P_+(z)\sigma^{(0)}(zxs), \quad (7)$$

where

$$P(z) = C_F\frac{1+z^2}{1-z}, \qquad \int dz\, P_+(z)g(z) \equiv \int dz\, P(z)[g(z)-g(1)].$$

The $k_\perp^2$ integration in (7) goes from 0 to $\sim Q^2$ (photon's virtuality) since for $k_\perp^2 > Q^2$, partonic cross section strongly decreases. We see that the soft singularities $z \to 1$ in the real and virtual contributions cancel, while the collinear singularity ($k_\perp^2 \to 0$ at finite z) persists. This proves that the naive parton model is not IR-stable.

4.3. Updating the Parton Picture. The persistent collinear singularity calls upon a redefinition of the partonic picture. It turns out that this singularity can be factorized from the scattering process. The technical procedure here is very reminiscent of renormalization. We first regularize the transverse momentum integral by a small but finite λ:

$$\int\limits_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2} \to \int\limits_{\lambda^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2} = \log\frac{Q^2}{\lambda^2} = \log\frac{Q^2}{\mu_F^2} + \log\frac{\mu_F^2}{\lambda^2},$$

with some *factorization scale* μ_F. Denoting the two logarithms as L_1 and L_2, we schematically represent the cross section as

$$1+\alpha_s L = 1+\alpha_s L_1+\alpha_s L_2 = (1+\alpha_s L_1)(1+\alpha_s L_2)+\mathcal{O}(\alpha_s^2).$$

This trick allows us to absorb the «bad» log (containing $\lambda^2 \to 0$) into the definition of the partonic densities. The cross section (6) is then represented as the integral of the factorization scale-dependent PDF $f(x;\mu_F)$ and partonic cross section $\sigma(xs;\mu_F)$, where

$$\begin{aligned} f(x;\mu_F) &= f(x) + \frac{\alpha_s}{2\pi}\log\frac{\mu_F^2}{\lambda^2}\int\frac{dz}{z}P_+ f(x/z), \\ \sigma(xs;\mu_F) &= \sigma^{(0)}(xs) + \frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu_F^2}\int dz P_+\sigma^{(0)}(xzs). \end{aligned} \quad (8)$$

Note that the collinear divergence is now fully absorbed into the definition of $f(x;\mu_F)$. Both $f(x;\mu_F)$ and $\sigma(xs;\mu_F)$ must be taken/calculated at the same μ_F, which is in principle arbitrary, but it is convenient to choose $\mu_F^2 \approx Q^2$.

This exercise offers a novel interpretation of the partonic densities. PDFs now include all partonic splittings with transverse momenta up to μ_F, while partonic splittings with $k_\perp > \mu_F$

are considered as the true pQCD corrections to the cross section. One can also make an important observation concerning PDFs: although they cannot be calculated from pQCD, one can calculate their μ_F-evolution just from (8):

$$\frac{\partial f_i(x;\mu_F)}{d\,\log\,\mu_F^2} = \frac{\alpha_s}{2\pi}\int\limits_0^1 \frac{dz}{z} P_{ij} f_j\left(\frac{x}{z};\mu_F\right). \tag{9}$$

Here, $P_{ij}(z)$ are the splitting functions of parton j into parton i with momentum fraction z, which are calculable in pQCD. Equations (8) are known as the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) equations.

Let us finally stress here that the factorization of hadronic observables into «hard» quantities calculable in pQCD and certain (generalized) distributions is a recurring theme in QCD. Normally, one tries to prove this factorization in the kinematical situations under study ($\gamma^* p$, pp, diffractive DIS, etc.), but it is not always possible. Factorization breaking in certain situations is an equally important phenomenon and it tells us something interesting about the process.

5. FINAL STATES IN HADRONIC COLLISIONS

Let us also briefly describe what kinds of hadronic final states in pp collisions are usually studied and how perturbative QCD enters the corresponding analyses.

A typical hadron–hadron collision starts with scattering of a single parton in each of the two hadrons. Since the partonic densities are larger at small x, a typical parton–parton collision is much less energetic than the hadron–hadron collision, and in addition it is dominated by small-angle scattering. When partons try to leave the hadron, color reconnection strings arise and then break into (light) hadrons. As a result, a typical hadron–hadron collision leads to production of dozens of hadrons, which more or less evenly populate a large pseudorapidity interval. The transverse momenta p_t of these hadrons are small, of the order of 0.1–1 GeV, which is still not sufficient to safely apply perturbative QCD to calculation of such processes.

5.1. Jets. The p_t-spectrum of produced particles has a long tail: once in a while particles with large p_t are also produced. Remarkably, such high-p_t particles are usually not isolated but appear in clusters with similar rapidity and azimuthal angles. Such collimated streams of hadrons are called *hadronic jets*.

Jets have a clear QCD origin. When a parton with large p_t is produced in a hard reaction, it can evolve through several successive splittings into daughter partons. However, since the collinear and soft splitting are favored, in most of the cases they do not involve any large momentum with respect to the parent parton's direction. Thus, although a partonic shower develops, the momentum flow is still contained in a relatively narrow cone. Later on, when the partons hadronize the color reconnection strings appear and break. Since they represent a soft QCD phenomenon, they again do not introduce any large additional momentum into the final hadrons' kinematics. As a result, the partonic shower transforms into hadronic jet with roughly the same properties.

Good identification of jets is of paramount importance for the analysis of energetic hadronic reactions. It is through jets that experimentalists «see» the underlying hard scattering of partons or a possible production and decay of a heavy unstable particle and can compare their measurements with theoretical calculations. Unfortunately, jets do not always jump into your eyes: attributing a given hadron to a jet might be ambiguous, jets can overlap, etc. Therefore, a lot of effort by many groups is dedicated to build an efficient jet reconstruction algorithm. The efficiency of the algorithm is quantified in terms of stability, infrared safety, running time, etc.

Two generic schemes for jet reconstruction exist. In a sequential recombining algorithm, one finds pairs of hadrons which are close enough in the momentum space and recombines them attempting to track back the splittings. This is repeated until no sufficiently close pairs remain, and the resulting objects highlight the jets. In a cone-type algorithm, one takes another strategy. One starts from the cone itself, placing a circle of fixed radius on the rapidity-angle diagram and trying to encompass as many energetic hadrons as possible. Although many jet algorithms have already been suggested, there is still continuous progress of this field.

5.2. Diffraction. A different type of final state in energetic hadronic reaction is given by a *diffractive scattering*, which is characterized by large rapidity gaps (intervals in the rapidity devoid of hadronic activity). In brief, hadronic diffraction is a generalization of elastic scattering, when the proton survives the collision (or is excited to a low-mass system), but loses a small fraction of its momentum. This momentum is carried away by a dynamical QCD object, the Pomeron, which in the first approximation can be modelled by a pair of gluons in the color-singlet state.

Dynamics of the Pomeron and diffraction in general is extremely interesting as it offers access to another kinematical regime of QCD, the so-called Regge limit. Various theoretical descriptions of the Pomeron have been proposed, some of them based on perturbative QCD, other appealing to nonperturbative physics, but the applicability regions of these approaches remain the subject of hot debates. Since diffractive reactions boast a very clean signature in detectors, their experimental study at the LHC will have a big pay-off.

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