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CALCULATION OF HADRONIC CONTRIBUTION TO THE ANOMALOUS MAGNETIC MOMENTUM OF MUON $(g-2)_\mu$ FROM LIGHT-BY-LIGHT SCATTERING DIAGRAM IN NONLOCAL CHIRAL QUARK MODEL

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The correction to anomalous magnetic momentum muon from the light-by-light scattering diagram with intermediate pion is calculated in the framework of the nonlocal chiral quark model. To fix the model parameters, it is suggested to use the values of mass and two-photon decay width of the neutral pion. The value of the correction is in the region $a_\mu^{\pi^0,\mathrm{LbL}} = (5.05 \pm 0.3) \cdot 10^{-10}$ for different set of model parameters.

В рамках нелокальной киральной кварковой модели вычислен вклад в аномальный магнитный момент мюона от диаграмм рассеяния света на свете с промежуточным пионом. Для фиксирования параметров модели предложено использовать массу нейтрального пиона и его ширину двухфотонного распада. Для различных параметризаций нелокальной модели данный вклад оценивается в промежутке $a_\mu^{\pi^0,\mathrm{LbL}} = (5{,}05 \pm 0{,}3) \cdot 10^{-10}$.

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The study of the anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ is one of the hot topics of nowadays elementary particle physics. At the present time, it is measured with the very high precision at the experiment E821 at the BNL [1] $a_\mu^{\mathrm{exp}} = 11\,659\,208.0(6.3) \cdot 10^{-10}$. The precision of this measurement is so high that one can use it not only for checking of the Standard Model but also for searching of new physics. However, the extraction of the contribution from new physics is not possible without precise calculations of the contributions from electromagnetic, weak and strong forces in the framework of the Standard Model. The main theoretical uncertainty comes from the contribution of strong forces. It can be divided into two parts: hadronic vacuum polarization (Fig. 1, a), and processes of the light-by-light scattering involving strong interaction (Fig. 1, b).

Especial difficulties emerge with the theoretical estimations of the light-by-light contribution. This contribution cannot be extracted from the experimental data in contrast to the

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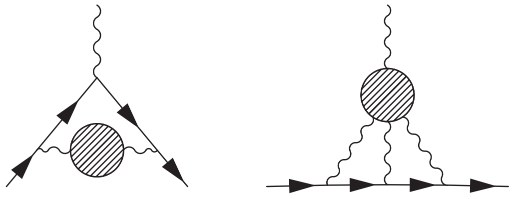
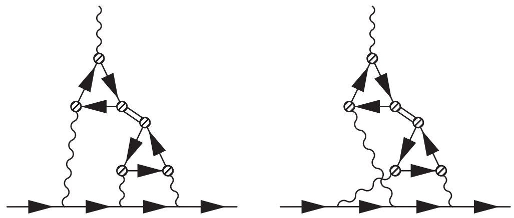


a *b*

Fig. 1. Hadronic vacuum polarization (*a*), light-by-light scattering (*b*)



a *b*

Fig. 2. Diagrams of the light-by-light scattering with the pion resonance

hadronic vacuum polarization. It is also not possible to calculate this contribution in the framework of QCD perturbation theory.

In this paper we consider one of the possible subgraphs of the light-by-light scattering processes. Namely, the contribution from diagram involving an intermediate pion resonance, Fig. 2, is estimated in the framework of nonlocal model. Details of the nonlocal model can be found in papers [2, 3]. Note that the model has the three parameters: current m_c and dynamical m_d quark masses and the nonlocality parameter Λ. The four-quark interaction constant G depends on the three parameters mentioned above by the gap equation[1]

$$G = \frac{m_d}{8N_c} \int \frac{d_E^4 k}{(2\pi)^4} \frac{f_k^2 m_k}{D_k}, \tag{1}$$

where m_k is the momentum-dependent quark mass $m_k = m_c + m_d f_k^2$, $D_k = k^2 + m_k^2$, f_k is nonlocal form factor in the form of Gauss $\exp(-k^2/2\Lambda^2)$ or Lorentzian $1/(1 + k^2/\Lambda^2)$ functions. The mass of neutral pion M_π and its two-photon decay width are used for fixing the model parameters. As a result, we have a set of model parameters as a function of m_d.

The pion mass can be found from the following equation:

$$\begin{gathered} -G^{-1} + \Pi(-M_\pi^2) = 0, \\ \Pi(p^2) = 8N_c \int \frac{d_E^4 k}{(2\pi)^4} \frac{f_{k_+}^2 f_{k_-}^2}{D_{k_+} D_{k_-}} \left[(k_+ k_-) + m_{k_+} m_{k_-} \right], \; k_\pm = k \pm \frac{p}{2}, \end{gathered} \tag{2}$$

[1]The Euclidean metric is mainly used. Only in equation (4) the Minkowski space is used for the gauge-invariant expression with vectors and Levi-Civita antisymmetric tensor. Also, the short hand notation for the momentum-dependence is introduced in the form of lower index $f_k \equiv f(k^2)$.

where p is the pion momentum and $\Pi(p^2)$ is the pion polarization operator. Note that the pion propagator can be written in the form

$$g_\pi^2(p^2)\left(-(p^2+M_\pi^2)\right)^{-1} = \left(-G^{-1}+\Pi(p^2)\right)^{-1}, \qquad (3)$$

where $g_\pi(p^2)$ is the renormalization constant of pion field which depends on momentum. This leads to unique determination of the virtual pion resonance away from the mass shell.

The part of the diagrams of the light-by-light scattering process is the triangular diagram with the dynamical quarks and the external pion and photon legs with an arbitrary virtuality

$$A\left(\gamma^*(q_1,\epsilon_1)\,\gamma^*(q_2,\epsilon_2)\to\pi^0(p)\right) = -ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma F_{\pi_0\gamma\gamma}(p^2;q_1^2,q_2^2), \qquad (4)$$

where $q_1(q_2)$ and $\epsilon_1^\mu(\epsilon_2^\nu)$ are the momentum and polarization vectors of photons, p is the momentum of pion and $F_{\pi_0\gamma\gamma}(p^2;q_1^2,q_2^2)$ is the form factor of pion

$$F_{\pi_0\gamma\gamma}(p^2;q_1^2,q_2^2) = g_\pi(p^2)F_{\pi_0\gamma\gamma}(p^2;q_1^2,q_2^2),$$
$$F_{\pi_0\gamma\gamma}(p^2;q_1^2,q_2^2) = 8\int\frac{d_E^4k}{(2\pi)^4}\frac{f_{k_1}f_{k_2}}{D_{k_1}D_{k_2}D_k}\left[m_k - T_1\frac{m_{k_1}-m_k}{k_1^2-k^2} - T_2\frac{m_{k_2}-m_k}{k_2^2-k^2}\right],$$

$$T_1 = k^2 + \frac{q_2^2(kq_1)(k_1q_1)-q_1^2(kq_2)(k_1q_2)}{q_1^2q_2^2-(q_1q_2)^2},$$
$$T_2 = k^2 + \frac{q_1^2(kq_2)(k_2q_2)-q_2^2(kq_1)(k_2q_1)}{q_1^2q_2^2-(q_1q_2)^2},$$

where $k_1=k+q_1$, $k_2=k-q_2$. In the chiral limit $m_c\to 0$ it is normalized to the chiral anomaly $F_{\pi^0\gamma\gamma}(0;0,0)=\left(4\pi^2 f_\pi\right)^{-1}$, where f_π is the weak pion decay constant. The width of the pion decay into two photons is given by $\Gamma_{\pi_0\to\gamma\gamma}=\pi M_\pi^3\alpha^2F^2_{\pi_0\gamma\gamma}(-M_\pi^2;0,0)/4$.

The contribution from the light-by-light scattering diagram with an intermediate neutral pion to the muon anomalous magnetic moment can be expressed as[1] [4]

$$a_\mu^{\pi^0,\mathrm{LbL}} = \frac{2\alpha^3}{3\pi^2}\int\limits_0^\infty dq_1^2\int\limits_0^\infty dq_2^2\int\limits_{-1}^{+1}dt\sqrt{1-t^2}\frac{1}{q_3^2}\times$$
$$\times\left[2I_1\frac{F_{\pi^0\gamma\gamma}\left(q_2^2;q_1^2,q_3^2\right)F_{\pi^0\gamma\gamma}\left(q_2^2;q_2^2,0\right)}{q_2^2+M_\pi^2}+\right.$$
$$\left.+I_2\frac{F_{\pi^0\gamma\gamma}\left(q_3^2;q_1^2,q_2^2\right)F_{\pi^0\gamma\gamma}\left(q_3^2;q_3^2,0\right)}{q_3^2+M_\pi^2}\right], \qquad (5)$$

where $q_3^2=q_1^2+2(q_1q_2)+q_2^2$ and $(q_1q_2)=tq_1q_2$; I_1 and I_2 are

$$I_1 = q_1^2\left[\mathcal{A}\left(\frac{(q_1q_2)}{2}-q_2^2\left(1-t^2\right)\right)+\mathcal{E}+\mathcal{C}q_2^2\left(1-t^2\right)\left(q_2^2-2M_\mu^2\right)\right]-\frac{(q_1q_2)}{2},$$
$$I_2 = 2q_2^2\left[\mathcal{B}\left(q_1^2+(q_1q_2)\right)-\mathcal{D}-\mathcal{C}q_1^2\left(q_1^2+(q_1q_2)+M_\mu^2\left(1-t^2\right)\right)\right]+$$
$$+\mathcal{A}q_1^2(q_1q_2)-(q_1q_2), \qquad (6)$$

[1] Factor 2 in front of I_1 corresponds to two possible contributions of Fig. 2, *a*.

where M_μ is muon mass and averaging functions are [4]

$$\begin{gathered}\mathcal{A} = -\frac{1-R_1}{2M_\mu^2}, \quad \mathcal{B} = -\frac{1-R_2}{2M_\mu^2}, \quad \mathcal{C} = \frac{1}{M_\mu^2 R_{12}} \arctan\left[\frac{zx}{1-zt}\right], \\ \mathcal{D} = -(q_1q_2)\frac{(1-R_2)^2}{8M_\mu^2}, \quad \mathcal{E} = (q_1q_2)\frac{(1-R_1)^2}{8M_\mu^2}, \\ x = \sqrt{1-t^2}, \quad R_i = \sqrt{1+\frac{4M_\mu^2}{q_i^2}}, \\ z = \frac{q_1q_2}{4M_\mu^2}(1-R_1)(1-R_2), \quad R_{12} = q_1q_2x.\end{gathered} \tag{7}$$

The model predictions for the contribution of the neutral pion to the muon anomalous magnetic moment for different parametrizations are given in Table 1 and Fig. 3. It seems that the most physical region of the model parametrizations corresponds to the dynamical quark mass in the region 200 to 350 MeV. In this case, the contribution of the pion to the anomalous magnetic moment of the muon does not strongly depend on the specific form of nonlocality and model parametrization. So, our estimation is $a_\mu^{\pi^0,\mathrm{LbL}} = (5.05 \pm 0.3) \cdot 10^{-10}$.

Table 1. **Model parameters and corresponding values of the contribution of the neutral pion to the anomalous magnetic moment of muon for different parametrizations of the model**

m_d,	Gauss			Lorentz		
MeV	m_c, MeV	Λ, GeV	$a_\mu^{\pi^0,\mathrm{LbL}} \cdot 10^{10}$	m_c, MeV	Λ, GeV	$a_\mu^{\pi^0,\mathrm{LbL}} \cdot 10^{10}$
150	0.31	7.15	3.39	0.16	6.905	3.38
200	2.16	2.28	4.77	1.14	2.21	4.74
250	4.88	1.37	5.23	2.51	1.33	5.26
300	7.85	1.01	5.19	3.89	0.990	5.35
350	10.97	0.821	4.94	5.22	0.805	5.27

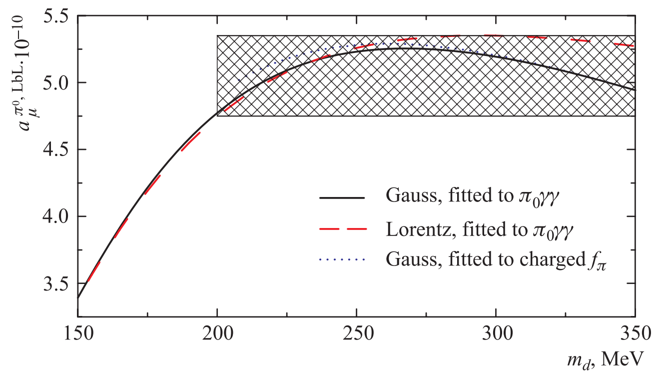


Fig. 3. Dependence of the contribution of the neutral pion to the anomalous magnetic moment of muon for different parametrizations of the model

Table 2. **The neutral pion contribution to the anomalous magnetic moment of muon obtained in different papers. Note that the result of this paper practically coincides with our previous result [12], where the model parameters are fitted to the charged pion parameters**

$a_\mu^{\rm LbL} \cdot 10^{-10}$	π^0
[5]	5.6
[6]	5.8 ± 1.0
[7]	5.8 ± 1.0
[8]	8.18 ± 1.65
[9]	7.65
[10]	6.5 ± 0.2
[11]	5.75 ± 0.69
[12]	5.1 ± 0.2
This paper	5.05 ± 0.3

In Table 2 the comparison of our results with the results of other papers is given. One can conclude that the pion contribution is a bit smaller than the results of the other papers. It can be due to full kinematic dependence of transition form factor of pion which is taken into account in the present paper.

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