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# FEMTOSCOPIC CORRELATIONS AND FINAL-STATE RESONANCE FORMATION

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The correlation femtoscopy formalism including resonance formation due to the final-state interaction and its applicability conditions are discussed. The example calculations of $\pi^+\Xi^-$ and $K^+K^-$ correlation functions are done with the account of $\Xi^*(1530)$ and $\phi(1020)$ resonances, respectively. It is shown that in these calculations the usual form of the smoothness approximation should be substituted by a more general one. A strong sensitivity of resonance formation in the final state to the position-momentum correlations at particle freeze-out is demonstrated.

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## INTRODUCTION

The momentum correlations of two or more particles at small relative momenta in their center-of-mass (c.m.) system contain important information about space-time characteristics of the production process on a femtometer level, so serving as a correlation femtoscopy tool (see, e.g., [1] and references therein). For non-interacting identical particles, like photons, or to some extent pions, these correlations result from the interference of the production amplitudes due to symmetrization requirement of quantum statistics (QS). These correlations are also influenced by the mutual final-state interaction (FSI) of produced particles. In addition to the QS correlations, the FSI effect provides important femtoscopic information, allowing for coalescence femtoscopy and correlation femtoscopy with unlike particles, including the studies of space-time asymmetries in particle production and strong interaction between specific particles.

Here we consider the correlations of non-identical particles whose mutual FSI in some quantum states is saturated by resonances. Particularly, we consider $\pi^+\Xi^-$ and $K^+K^-$ correlation functions which are dominated by the narrow $\Xi^*(1530)$ and $\phi(1020)$ $p$-wave resonances at the particle momenta in the pair c.m. systems $k \sim 150$ and $\sim 130$ MeV/$c$, respectively.

The data on $\pi\Xi$ femtoscopic correlations has been recently obtained by the STAR collaboration in Au + Au collisions at RHIC and the following preliminary observations were made at the highest nucleon–nucleon c.m. energy of 200 GeV [2, 3]: (*i*) the fit of the like- and unlike-charge $\pi\Xi$ correlation functions from $10\%$ of the most central Au + Au collisions in the region $k < 120$ MeV/$c$ (dominated by the Coulomb FSI) provided the preliminary values of the Gaussian radius of the separation $r$-distribution in the pair c.m. system $r_0 = (4.8 \pm 0.7)$ fm and a shift in the out-direction $\Delta_{\rm out} = (-5.6 \pm 1.0)$ fm; the estimated value of $r_0(\pi\Xi)$ is in agreement with the expectation $r_0(\pi\Xi) = \{[r_0^2(\pi\pi) + r_0^2(\Xi\Xi)]/2\}^{1/2}$ which, in accordance with the approximate $m_t$-scaling: $r_0 \sim m_t^{-1/2}$, is dominated by the contribution of

$r_0(\pi\pi) \sim 7$ fm; (*ii*) the $\Xi^*(1530)$ resonance shows as a peak of the unlike-charge $\pi\Xi$ correlation function at $k \approx 146$ MeV/$c$; the height of this peak rapidly decreases with the increasing centrality in qualitative agreement with the inverse volume ($r_0^{-3}$) dependence of the peak height and with the expected increase of the characteristic radius $r_0$ with increasing centrality; for 10% of the most central Au + Au collisions the observed correlation size $\mathcal{R} - 1 \sim 0.020$ in the $k$-bin of 140–150 MeV/$c$ corresponds to a peak value of $\sim 0.025$; (*iii*) the theoretical calculation performed in the framework of the simple Blast Wave model with the parameters fixed from the pion spectra and two-pion femtoscopy is in agreement with the data on both the correlation size and the out-asymmetry in the Coulomb region, it however strongly overestimates them in the $\Xi^*(1530)$ region; the simple Gaussian emission function with the parameters obtained from a fit in the Coulomb region yields the correlation size also much higher than the observed resonance peak (a reasonable description requires unrealistically large radius $r_0(\pi\Xi) > 7$ fm) and predicts even the opposite sign of the out-asymmetry in the resonance region.

As for the two-kaon femtoscopic correlations, they have been studied in the NA49 experiment at CERN for both like- and unlike-charge kaon pairs produced in Pb + Pb collisions at a beam energy of 158 GeV per nucleon and the following results have been reported [4]: (*i*) the 3D out-side-longitudinal fit of the like-charge two-kaon correlation functions for the kaons produced near mid-rapidity with a mean transverse mass of 0.62 GeV/$c^2$ provided the LCMS radii of 4–5 fm and 3–4 fm for 5% and 5–20% of the most central and semi-central Pb + Pb collisions, respectively; (*ii*) the theoretical $K^+K^-$ correlation functions calculated with the help of the measured LCMS radii and taking into account the Coulomb and the $s$-wave part of the short-range strong FSI describe well the measured correlation functions for $k < 120$ MeV/$c$; (*iii*) the $\phi(1020)$ resonance shows as a peak of the $K^+K^-$ correlation function at $k \sim 126$ MeV/$c$. For the most central Pb + Pb collisions the observed correlation size $\mathcal{R} - 1 \sim 0.09$ in the $k$ interval of 123–129 MeV/$c$ corresponds to a peak value of $\sim 0.10$.

In the example calculations of $\pi^+\Xi^-$ and $K^+K^-$ correlation functions with the help of simple Gaussian emission functions, we show that a good description of the correlations at small relative momenta leads to their strong overestimation in the $\Xi^*(1530)$ and $\phi(1020)$ resonance regions. We argue that a possible solution of this problem is to take into account a position-momentum correlation which is expected due to the collective flows and resonance decays.

## 1. FORMALISM

It is well known that in a production process of a small enough phase-space density (which is usually the case even in heavy-ion collisions) the correlations of two particles emitted with nearly equal four-velocities $p_1/M_1$ and $p_2/M_2$ are dominated by their mutual FSI. The two-particle correlation function, defined as the ratio of the measured momentum distribution of the two particles to the reference one (the latter obtained, e.g., by mixing particles from different events of a given class), normalized to unity at sufficiently large relative momenta, can be calculated as a weighted sum of the squares of the wave functions $\psi_{-\mathbf{k}}^{\mathcal{J}'\mathcal{J}}(\mathbf{r})$ describing the transitions from the intermediate channels $\mathcal{J}' = \{\alpha' m'\}$ with the flavor quantum numbers $\alpha'$ and spin projections $m'$ to the final one $\mathcal{J} = \{\alpha m\}$, averaged over the spatial separation $\mathbf{r}$ of the particle freeze-out points in the c.m. system of the detected particle pair. Note that the detected channel $\mathcal{J}$ enters in the usual solutions of the multichannel scattering problem as the entrance scattering channel with the opposite sign of the channel three-momentum $\mathbf{k}$.

This formalism implies [1] (*i*) smooth behavior of single-particle spectra in a narrow region of the correlation effect (smoothness assumption); (*ii*) sufficiently small difference of the emission times $|t|$ in the pair c.m. system, $|t| \ll M_i r^2$ (the equal-time approximation $t = 0$ is usually valid better than to a few percent even for particles as light as pions); (*iii*) the production time of the particles much shorter than their interaction time in the final state, i.e., for $s$-waves, — the particle momentum in the two-particle c.m. system $k$ substantially smaller than a typical momentum transfer of a few hundreds of MeV in the production process.

Requiring also the intermediate states close to the threshold, i.e., $M_1'+M_2'+\ldots \approx M_1+M_2$, they practically reduce to the two-particle states with the intermediate particles belonging to the same isospin multiplets as the detected particles. The number of the intermediate channel flavors then reduces to one or two; particularly, for $\pi^+\Xi^-$ or $K^+K^-$ correlation functions, $\alpha' = \{\alpha, \beta\} = \{\pi^+\Xi^-, \pi^0\Xi^0\}$ or $\{K^+K^-, K^0\bar{K}^0\}$, respectively.

It is important to note that in the case that the FSI at a given channel angular momentum and isospin is dominated by a narrow resonance, the interaction time at the resonance energy becomes very large and allows for a factorization of the resonance FSI irrespective of the distance from the threshold. Of course, the application of standard formalism, implying the neglect of the space-time coherence [1], still requires a closeness to the threshold. This limitation can be however overcome with the help of a more general form of the smoothness approximation [5].

In the following, we will consider complex multiparticle processes (like those in heavy-ion collisions) implying the equilibration of spin and isospin projections. The emission function is then independent of these projections and, since particles in the intermediate and final channels are supposed to be the members of the same isospin multiplets, also of the channel flavors $\alpha'$ [1]. At small $k$, we will characterize it by a Gaussian radius parameter $r_0$ and, to test the effect of a possible $\mathbf{k}$-dependence, by a $\mathbf{r}-\mathbf{k}$ correlation parameter $b$:

$$W_P(\mathbf{r},\mathbf{k}) = \exp\left(-\frac{r^2}{4r_0^2} + b\mathbf{k}\mathbf{r}\right), \quad \mathcal{N}(p_1,p_2) = 8\pi^{3/2} r_0^3 \exp\left(b^2 k^2 r_0^2\right), \tag{1}$$

where the normalization factor $\mathcal{N}(p_1,p_2) = \int d^3\mathbf{r} W_P(\mathbf{r},\mathbf{k})$.

For the considered systems with one spin-zero particle and the other one with spin $j = 1/2$ or 0, the usual smoothness approximation for the correlation function becomes

$$\mathcal{R}(p_1,p_2) = \mathcal{N}^{-1}(p_1,p_2) \int d^3\mathbf{r}\, W_P(\mathbf{r},\mathbf{k}) \sum_{\alpha' m'} \left|\psi_{-\mathbf{k}}^{\alpha' m';\alpha j}(\mathbf{r})\right|^2, \tag{2}$$

implying a negligible $\mathbf{k}$-dependence of the separation $\mathbf{r}$-distribution $W_P(\mathbf{r},\mathbf{k})$. To overcome this limitation, one may split the wave functions in the pure Coulomb one and those describing the waves scattered due to the short-range strong interaction: $\psi_{-\mathbf{k}}^{\alpha' m';\alpha m} = \psi_{-\mathbf{k}}^{\alpha;\mathrm{coul}} \delta_{\alpha'\alpha}\delta_{m'm} + \Delta\psi_{-\mathbf{k}}^{\alpha' m';\alpha m;\mathrm{str}}$, and, introducing the corresponding partial FSI weights: $w^{\alpha;\mathrm{coul}} = |\psi_{-\mathbf{k}}^{\alpha;\mathrm{coul}}(\mathbf{r})|^2$, $w^{\alpha;\mathrm{interf}} = 2\Re[\psi_{-\mathbf{k}}^{\alpha;\mathrm{coul}*}(\mathbf{r})\Delta\psi_{-\mathbf{k}}^{\alpha j;\alpha j;\mathrm{str}}(\mathbf{r})]$, $w^{\alpha';\mathrm{str}} =$

$\sum_{m'} |\Delta\psi_{-\mathbf{k}}^{\alpha' m';\alpha j;\mathrm{str}}(\mathbf{r})|^2$, arrive at a more general form of the smoothness approximation [5]:

$$\mathcal{R}(p_1,p_2) = \mathcal{N}^{-1}(p_1,p_2) \int d^3\mathbf{r} \Big[ W_P(\mathbf{r},\mathbf{k}) w^{\alpha;\mathrm{coul}}(\mathbf{r},\mathbf{k}) + \\ + W_P\left(\mathbf{r}, \frac{\mathbf{k}-k\hat{\mathbf{r}}}{2}\right) w^{\alpha;\mathrm{interf}}(\mathbf{r},\mathbf{k}) + \sum_{\alpha'} W_P(\mathbf{r},\, -k_{\alpha'}\hat{\mathbf{r}}) w^{\alpha';\mathrm{str}}(\mathbf{r},\mathbf{k}) \Big]. \quad (3)$$

It is interesting to note that in the case that the $L$-wave FSI is saturated by a narrow spin-$J$ resonance, one can approximately calculate its angular averaged contribution to the correlation function (with precision better than $10\%$ for $\Xi^*(1530)$ and $\phi(1020)$ $p$-wave resonances) using the known integral relations [5]:

$$\Delta\mathcal{R} \approx W_P(0,\mathbf{k}) \, \frac{2L+1}{\mathcal{N}} \, \frac{2\pi}{k} \left[ \Re\delta_k f_L^{\alpha j;\alpha j} + \sum_{\alpha' m'} 2\Im \left( k_{\alpha'} f_L^{\alpha' m';\alpha j *} \delta_k f_L^{\alpha' m';\alpha j} \right) \right], \quad (4)$$

where $W_P(0,\mathbf{k}) \doteq W_P(0,0)$ ($W_P(0,\mathbf{k}) = 1$ for the $\mathbf{r}$-distribution in (1)) and the scattering amplitudes in the representation of the spin projections $f_L^{\alpha' m';\alpha m}$ are expressed through the spin-$J$ resonance amplitudes $f_L^{J;\alpha'\alpha}$ with the help of Clebsch–Gordan coefficients: $f_L^{\alpha' m';\alpha m} = C_{L0;jm}^{Jm} C_{Lm-m';jm'}^{Jm} f_L^{J;\alpha'\alpha}$. Neglecting the Coulomb FSI, the amplitudes $f_L^{J;\alpha'\alpha} = (\Gamma_{\alpha'}\Gamma_\alpha / k_{\alpha'}k_\alpha)^{1/2} M_0/(M_0^2 - E^2 - iM_0\Gamma)$ for the resonance of mass $M_0 = E_0$ and total width $\Gamma = \sum_{\alpha'} \Gamma_{\alpha'} + \Gamma'$ ($\Gamma_{\alpha'} \sim k_{\alpha'}^{2L+1}$).

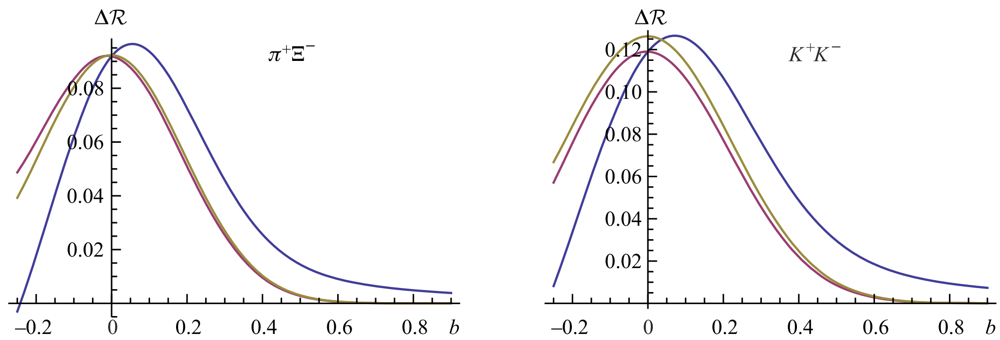
## 2. EXAMPLE CALCULATIONS

Here we discuss the results of some example calculations of $\pi^+\Xi^-$ and $K^+K^-$ correlation functions in the $k$ intervals around $\sim$ 150 and $\sim$ 130 MeV/$c$ dominated by the narrow $\Xi^*(1530)$ and $\phi(1020)$ $p$-wave resonances, respectively. In the calculations according to the emission function in (1), we put the radius parameter $r_0 \approx 5$ fm for both $\pi\Xi$ and $K^+K^-$ systems, in approximate agreement with the STAR and NA49 results for central Au + Au and Pb + Pb collisions.

It appears that the simple Gaussian emission function ($b = 0$) with the radius $r_0 = 5$ (4.5) fm overestimates the peak value of the $\phi(1020)$-resonance FSI contribution to the $K^+K^-$ correlation function by $\sim 20\%$ ($\sim 65\%$) and leaves no room for the direct $\phi(1020)$ production. For the $\Xi^*(1530)$-resonance FSI contribution to the $\pi^+\Xi^-$ correlation function this overestimation is even larger and for $r_0 = 5$ (5.5) fm comprises almost a factor 4 (3).

A possible reason for the overestimation of the $\phi(1020)$ and $\Xi^*(1530)$ FSI contributions in the simple Gaussian model may be the neglect of the $\mathbf{r}-\mathbf{k}$ correlation. Such a correlation may result from the collective flows and resonance decays. In heavy-ion collisions, the flows are predicted in the transport and hydrodynamic models and indicated by the experimental observations.

Accounting for the $\mathbf{r}-\mathbf{k}$ correlation with the help of the correlation parameter $b$ in (1), the resonance contribution is approximately suppressed by a factor $\sim \exp[-(bkr_0)^2]$. The




The peak values of the $p$-wave resonance FSI contributions $\Delta\mathcal{R}$ to the $\pi^+\Xi^-$ (left) and $K^+K^-$ (right) correlation functions calculated according to the separation $\mathbf{r}$-distribution in (1) with the radius parameter $r_0 = 5$ fm as functions of the $\mathbf{r}-\mathbf{k}$ correlation parameter $b$. For $b > 0.05$, the curves in decreasing order correspond to Eqs. (2), (4) and (3), respectively

hydrodynamics-motivated Blast Wave type models (see, e.g., [6]) yield the correlation parameter $b \sim 0.2$. In the case of particles with different masses, the position-momentum correlation further yields the out-asymmetry of the spatial separation of the particle emitters characterized by a shift $\Delta_{\rm out}$, leading to additional suppression of the resonance FSI contribution by a factor $\sim \exp\,[-(\Delta_{\rm out}/2r_0)^2]$ ($\sim 0.7$ for the $\pi^+\Xi^-$ system).

It may be seen from the figure that the $\mathbf{r}-\mathbf{k}$ correlation parameter $b \sim 0.25$ is sufficient, in addition to the out-asymmetry parameter $\Delta_{\rm out} \sim -6$ fm, to recover the measured value of the correlation function at the $\Xi^*(1530)$ resonance. For the $K^+K^-$ system, the corresponding boundary $b$ value is about twice smaller. The figure further demonstrates a strong violation of the usual smoothness approximation in (2) for the values of the $\mathbf{r}-\mathbf{k}$ correlation parameter $|b| > 0.2$. It also confirms that the angular averaged resonance FSI contribution can be well approximated by the simple expression in (4).

## CONCLUSIONS

We have found that the usual femtoscopic correlation formalism accounting for the resonance formation due to particle interaction in the final state should be modified in the case of a substantial position-momentum correlations at particle freeze-out. Such a correlation is expected as a consequence of strong collective flows and resonance decays. When neglected, the theoretical predictions for the $p$-wave resonance FSI contributions to the $\pi^+\Xi^-$ and $K^+K^-$ correlation functions overestimate the STAR and NA49 correlation data from the most central collisions, thus leaving no room for a direct (thermal) $\Xi^*(1530)$ and $\phi(1020)$ production. The Blast Wave type models with strong position-momentum correlations suggest a possible solution of this problem. The narrow resonance FSI correlations may thus serve as an additional tool allowing for a critical test of the production dynamics.

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