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BEAM STABILITY IN SYNCHROTRONS WITH DIGITAL TRANSVERSE FEEDBACK SYSTEMS IN DEPENDENCE ON BEAM TUNES

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The beam stability problem in synchrotrons with a digital transverse feedback system (TFS) is studied. The TFS damper kicker (DK) corrects the transverse momentum of a bunch in proportion to its displacement from the closed orbit measured at the location of the beam position monitor (BPM). It is shown that the area and configuration of the beam stability separatrix depend on the beam tune, the feedback gain, the phase balance between the phase advance from BPM to DK and the phase response of the feedback chain at the betatron frequency.

Приводятся результаты исследования устойчивости пучка в синхротронах с системой подавления (СП) когерентных поперечных колебаний в зависимости от частоты бетатронных колебаний. СП обеспечивает коррекцию поперечного импульса сгустков на каждом обороте с помощью дефлектора ДК с учетом данных о смещении центра тяжести пучка, измеренных датчиком положения ДП. Показано, что площадь и конфигурация сепаратрисы для стабильного пучка зависят от частоты бетатронных колебаний, коэффициента передачи цепи обратной связи, баланса фаз между набегом фазы бетатронных колебаний от ДП до ДК и сдвигом фазы соответствующего сигнала в цепи обратной связи.

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INTRODUCTION

A classical transverse feedback system (TFS) in synchrotrons consists of a beam position monitor (BPM), a damper kicker (DK), and an electronic feedback path with an appropriate signal transmission from BPM to DK [1, 2]. The damper kicker corrects the transverse momentum of a bunch in proportion to its displacement from the closed orbit measured at the BPM location. The total delay $\tau_{\rm delay}$ in the signal processing of the feedback loop from BPM to DK is adjusted to be equal to $\tau_{\rm PK}$, the particle time of the flight from BPM to DK, plus an additional delay of $\hat{q}$ turns:

$$\tau_{\rm delay} = \tau_{\rm PK} + \hat{q}\,T_{\rm rev}, \qquad (1)$$

where $T_{\rm rev}$ is the revolution period of a particle. BPM and DK are located at the fixed positions in the synchrotron. The particle betatron phase advance from BPM to DK and the phase response of the feedback loop to the corresponding beam signal should be adjusted for damping of particle oscillations. These both phases depend on the beam tune that is a tuneable parameter in synchrotrons. Beam stability conditions in dependence on the beam tune are studied below.

BASIC NOTIONS

Following the matrix description of the free oscillation of a particle in synchrotrons, the matrix equation for its states at the BPM location $s_{\rm P}$ at the $(n+1)$ and nth turns after a small kick by the DK is given by [3, 4]

$$\widehat{X}[n+1, s_{\rm P}] = \widehat{X}[n, s_{\rm P} + C_0] = \widehat{M_0}\,\widehat{X}[n, s_{\rm P}] + \widehat{B}\,\Delta\widehat{X}_{\rm K}[n, s_{\rm K}], \tag{2}$$

where elements of the column matrix $\widehat{X}[n, s]$ are the particle displacement $x[n, s]$ and the angle $x'[n, s]$ of its trajectory, $\widehat{M_0}$ is the revolution matrix, $\widehat{B}$ is a transfer matrix from the point $[n, s_{\rm K}]$ on the closed orbit at the DK location to the point $[n, s_{\rm P} + C_0]$ at the BPM position at the nth turn, C_0 is the synchrotron's circumference. The first element of column matrix $\Delta\widehat{X}_{\rm K}[n, s_{\rm K}]$ is zero, but the second one equals the kick value $\Delta x'[n, s_{\rm K}]$. Let the kick be in proportion to the particle displacement at the BPM location at the same turn:

$$\Delta x'[n, s_{\rm K}] = \frac{g}{\sqrt{\hat\beta_{\rm K}\hat\beta_{\rm P}}}\, x[n, s_{\rm P}]\,, \tag{3}$$

where g is a feedback gain, $\hat\beta_{\rm s} \equiv \hat\beta(s)$ is the betatron amplitude function at the point s. Substituting (3) into (2), one can obtain the difference equation in a matrix form:

$$\widehat{X}[n+1, s_{\rm P}] = \widehat{M}\,\widehat{X}[n, s_{\rm P}], \qquad \widehat{M} \equiv \widehat{M_0} + \frac{g}{\sqrt{\hat\beta_{\rm K}\hat\beta_{\rm P}}}\,\widehat{B}\,\widehat{T}, \tag{4}$$

where $\widehat{T}$ is 2×2 matrix in which $T_{21} = 1$ and the other elements are zero. Consequently, the particle dynamics is determined by roots z_k of the characteristic equation:

$$\det\,(\,z_k\widehat{I} - \widehat{M}\,) = z_k^2 - [2\cos(2\pi Q) + g\sin(2\pi Q - \psi_{\rm PK})]\,z_k + 1 - g\sin\psi_{\rm PK} = 0\,, \tag{5}$$

where $\widehat{I}$ is the identity matrix, Q is the beam tune, $\psi_{\rm PK}$ is the betatron oscillation phase advance from BPM to DK. The particle motion is stable if $|z_k| < 1$ so that the damping rate is $D_k \equiv -\ln|z_k|$ and the fractional number of oscillations per turn is $\{Q\} \equiv \arg(z_k)/2\pi$.

Two eigenvalues z_1 and z_2 of the quadratic equation (5) depend on g, Q and $\psi_{\rm PK}(Q)$. Let Q_0 be the tune on the reference closed orbit in the synchrotron for particles with momentum p_0. The tune of injected particles with momentum $p_0 + \delta p$ deviates from Q_0 so that the phase advance $\psi_{\rm PK}(Q)$ for the tune $Q = Q_0 + \delta Q$ is as follows:

$$\psi_{\rm PK}(Q) \equiv \psi_{\rm PK}(Q_0 + \delta Q) = \left(1 + \frac{\delta Q}{Q_0}\right)\psi_{\rm PK}(Q_0).$$

Let us define the rate D for the maximal absolute value of z_k:

$$D = -\ln\left(\mathrm{MAX}|z_k|\right). \tag{6}$$

In the case of under-damping oscillations one can write for Eq. (5) with real coefficients:

$$1 - g\sin\psi_{\rm PK}(Q) = z_1 z_2 = \exp(-2D). \tag{7}$$

In the case of over-damping oscillations the rate D corresponds to the slowest exponential decay of oscillations. Hence, one can obtain two numbers of the gain for the fixed tune Q with the same rate D. The three-dimensional representation of beam stability data set $D(g, Q-Q_0)$ and its contours for fixed dampimg rates D_n:

$$D_0 = 0.002, \qquad D_n = n_c/80, \qquad 1 \leqslant n_c \leqslant 8 \tag{8}$$

are shown in Fig. 1 in the case of $Q_0 = 59.31$ and $\psi_{\rm PK}(Q_0) = 2\pi \times 59.25$. The contour line for the damping time $\tau = T_{\rm rev}/D$ that corresponds to D_0 is chosen for damping regime $\tau < \tau_{\rm dec}$ where the assumed decoherence time $\tau_{\rm dec} > 500\,T_{\rm rev}$. Therefore, the closed curve for D_0 can be considered as the beam stability separatix. It separates the $(g,\,\delta Q)$ space into two distinct areas. The particle motion within the separatrix corresponds to the damped oscillations, whereas the outside of the separatrix corresponds to non-damped oscillations. For example, the damping time $\tau \leqslant 10\,T_{\rm rev}$ corresponds to the internal area of the closed curve with $n_c = 8$ (the smallest area in Fig. 1, *b*), where $|\delta Q| < 0.14$ for gain $g = 0.3$. It should be

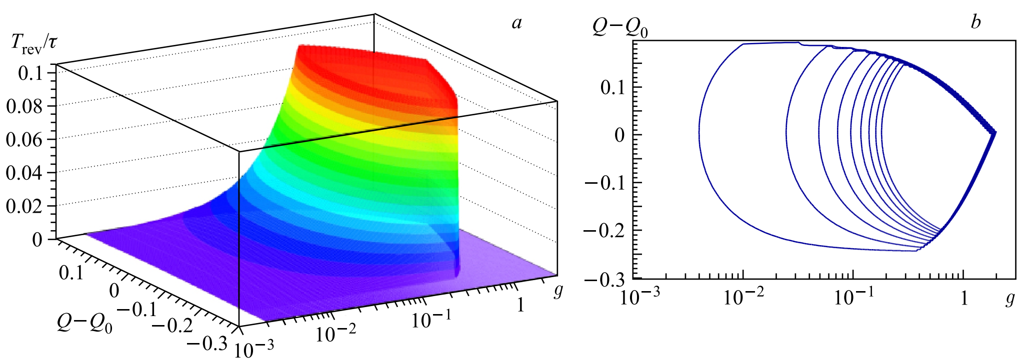

Fig. 1. Beam stability surface (*a*) and contour (*b*) plots

emphasized that in accordance with Eqs. (5) and (7) the separatrix is limited by the fractional part of the tune $\{Q\} = 0.5$ and $\delta Q > -0.25$ for $\{Q_0\} > 0.25$ (or $\{Q\} = 0$ and $\delta Q < 0.25$ for $\{Q_0\} < 0.25$), and the δQ size is maximum in the case of $|\sin\psi_{\rm PK}(Q_0)| = 1$.

DIGITAL TFS

In general, the kick value depends on the bunch displacement at the BPM location according to the structural scheme of electronics in the feedback loop. For linear time invariant feedback systems one can write

$$\Delta x'[n, s_{\rm K}] = \frac{g a_0}{\sqrt{\hat\beta_{\rm K}\hat\beta_{\rm P}}}\, u[n-\hat q] \sum_{m=0}^{n-\hat q} h[m]\, x[n-\hat q-m, s_{\rm P}]\,, \tag{9}$$

where $u[n]$ is the Heaviside step function, elements a_0 and $h[m]$ are determined by the feedback electronics, $\hat q$ is the number of turns for the delay (see Eq. (1)). Following the approach [3–5] for solving Eqs. (2) and (9) by using Z-transform, one can obtain that the particle dynamics is determined by roots z_k of the characteristic equation:

$$z_k^2 - \left[2\cos(2\pi Q) + g a_0\, z_k^{-\hat{q}} H(z_k)\sin(2\pi Q - \psi_{\rm PK})\right] z_k + 1 - g a_0\, z_k^{-\hat{q}} H(z_k)\sin\psi_{\rm PK} = 0, \tag{10}$$

where the transfer function $H(z)$ is determined by parameters $h[m]$ in (9) and a_0 is defined for $z_0 \equiv \exp(j2\pi Q_0)$ at the reference orbit such that

$$|a_0 z_0^{-\hat{q}} H(z_0)| = 1, \qquad a_0 \sin\left(\psi_{\rm PK}(Q_0) - \arg\left(z_0^{-\hat{q}}\, H(z_0)\right)\right) > 0. \tag{11}$$

If $g = 0$, then the solutions $z_{\pm}^{(0)} = \exp(\pm j2\pi Q)$ of Eq. (10) correspond to the solutions for frequencies of the betatron motion equation of a particle in synchrotrons. If the fractional part of the tune $\{Q\}$ is not close to 0 or 0.5 [5,6], then the solutions of Eq. (10) in the linear approximation with $g \ll 1$ are expressed by the following formula:

$$z_{\pm} \approx \exp\left(-\frac{g}{2}\,{\rm sgn}\,(a_0)\,\sin\Psi_{\rm PK}\right)\exp\left(\pm j\,2\pi Q \mp j\,\frac{g}{2}\,{\rm sgn}\,(a_0)\,\cos\Psi_{\rm PK}\right), \tag{12}$$

where the ${\rm sgn}\,(a_0)$ function is an odd mathematical function that extracts the sign of a_0 and

$$\Psi_{\rm PK} = \psi_{\rm PK} - \arg\left(z_{\rm Q}^{-\hat{q}}\, H(z_{\rm Q})\right), \quad z_{\rm Q} \equiv \exp(j2\pi Q). \tag{13}$$

Hence, the best damping of transverse oscillations is achieved by optimal choosing of the BPM and DK positions and the phase response of feedback electronics at the betatron frequency that provides a phase advance of $\Psi_{\rm PK}$ equal to an odd number multiplied by $\pi/2$.

To simplify further explanations, one can assume that TFS has no additional delay ($\hat{q} = 0$) so that $\Psi_{\rm PK}$ depends on the tune Q via $\psi_{\rm PK}$ and $\arg H$.

Properties of $H(z)$ are determined by the feedback electronics. If the kick depends on the displacement in accordance with (3), then $H(z) = 1$ (the so-called ideal feedback loop). The transfer function for TFS with the notch and Hilbert filters [7] is as follows:

$$H_1(z) \equiv H_N(z)\, H_{\rm HF}(z) = \left(1 - z^{-1}\right)\left(h_0 z^{-3} + h_1 z^{-2}(1 - z^{-2}) + h_3(1 - z^{-6})\right), \tag{14}$$

where

$$h_0 = \cos(\Delta\varphi), \quad h_1 = \frac{2}{\pi}\sin(\Delta\varphi), \quad h_3 = \frac{2}{3\pi}\sin(\Delta\varphi).$$

$H(z)$ for TFS with the notch filter and the FIR filter of the first order [5] is

$$H_2(z) \equiv H_N(z)\, H_{\rm FIR}(z) = \left(1 - z^{-1}\right)\left(1 + a_2 z^{-1}\right). \tag{15}$$

The magnitude $G(Q) \equiv |a_0\, H(z_Q)|$ and phase response $\Phi(Q) \equiv \arg H(z_Q)$ graphs against the fractional part of the tune $\{Q\}$ are shown in Fig. 2 for filters with transfer functions $H_1(z)$ and $H_2(z)$ at $Q_0 = 59.31$, $\Delta\varphi = -59.33°$ and $a_2 = 0.576$ so that $G(Q_0) = 1$ and $\Phi(Q_0) = 0$. One can note for the interval of $|Q - Q_0| < 0.1$ that the deviations $|\Phi(Q) - \Phi(Q_0)| < 130°$ for the notch and Hilbert filters considerably exceed the betatron phase advance deviations $2\pi|Q - Q_0| < 36°$ comparable with the deviations $|\Phi(Q) - \Phi(Q_0)| < 25°$ for the notch filter and the FIR filter of the first order.

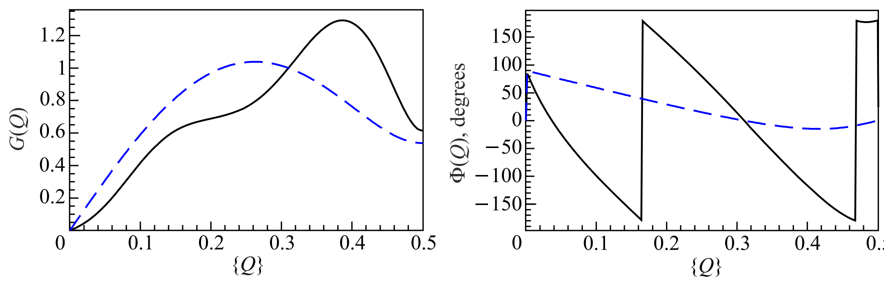

Fig. 2. The magnitude $G(Q)$ and phase response $\Phi(Q)$ graphs for the notch and Hilbert filters (solid) and for the notch filter and the FIR filter of the first order (dashed)

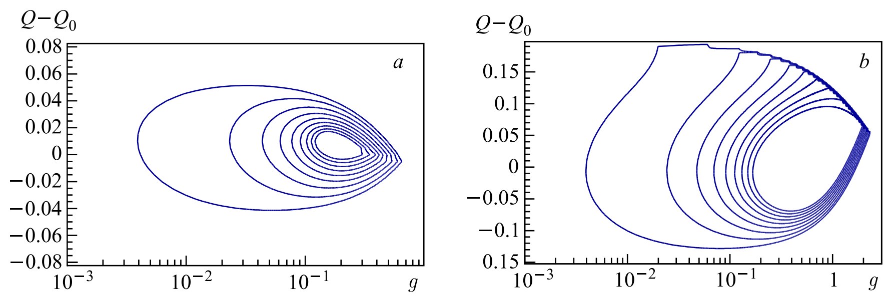

Fig. 3. Damping rate contours for TFS with transfer functions $H_1(z)$ and $H_2(z)$

Damping rate contours $D = -\ln(\mathrm{MAX}|z_k|)$ for TFS with transfer functions $H_1(z)$ and $H_2(z)$ are shown in Fig. 3 in the case of D_n from (8) and $\psi_{\mathrm{PK}}(Q_0) = 2\pi \times 59.25$. The best damping is achieved for small gains at $|\sin\Psi_{\mathrm{PK}}(Q_0)| = 1$ in agreement with Eqs. (13) and (12) due to values of $\Delta\varphi = -59.33°$ and $a_2 = 0.576$.

One can note that in the case of the notch and Hilbert filters the damping time $\tau \leqslant 10\,T_{\mathrm{rev}}$ corresponds to the internal area of the closed curve with $n_c = 8$ (the smallest area in Fig. 3, *a*) where $0 < \delta Q < 0.02$ for gain $g = 0.15$. The damping time $\tau = 40\,T_{\mathrm{rev}}$ corresponds to the closed curve with $n_c = 2$ (the third curve in Fig. 3, *a*) where $-0.022 < \delta Q < 0.035$ for gain $g = 0.1$. On the other hand, in the case of the notch filter and the FIR filter of the first order, the damping time $\tau \leqslant 10\,T_{\mathrm{rev}}$ corresponds to the internal area of the closed curve with $n_c = 8$ (the smallest area in Fig. 3, *b*) where $|\delta Q| < 0.065$ for gain $g = 0.3$. Hence, the area of separatrix in the case of $H_1(z)$ is much less than the same area for $H_2(z)$, which, in its turn, is less than the separatrix area for $H(z) = 1$ (see Fig. 1, *b*).

It should be emphasized that the phase advance $\Psi_{\mathrm{PK}}(Q_0)$ can be matched to optimal magnitude by choosing the digital filter parameters according to the phase advance $\psi_{\mathrm{PK}}(Q_0)$. For example, if $\psi_{\mathrm{PK}}(Q_0) = 2\pi \times 59.092$ at $Q_0 = 59.31$, then $|\sin\Psi_{\mathrm{PK}}(Q_0)| = 1$ can be achieved for $\Delta\varphi = -116.4°$ or $a_2 = 2.86$ (see Fig. 4, *a*). One can see that there is no beam stability for TFS with the notch filter ($H_N(z) = 1 - z^{-1}$) for these numbers of $\psi_{\mathrm{PK}}(Q_0)$ and Q_0. Damping times for the ideal feedback loop ($H(z) = 1$, but $|\sin\psi_{\mathrm{PK}}| < 1$) is much bigger than the same values in the case of $H_1(z)$ and $H_2(z)$ for $g < 0.25$. Damping rate contours for TFS with $H_1(z)$ and $H_2(z)$ at $\Delta\varphi = -116.4°$ and $a_2 = 2.86$ look like the contours in Fig. 3. However, D_n-contours and damping rates depend on $\Delta\varphi$. For example,

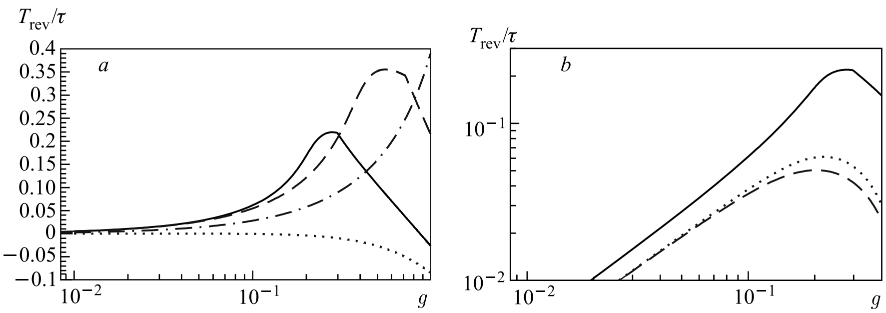

Fig. 4. Dependences of $T_{\rm rev}/\tau$ on gain g for $H(z) = 1$ (dash-dotted), $H_N(z)$ (dotted), $H_1(z)$ (solid), $H_2(z)$ (dashed) in the panel *a* and for TFS with $H_1(z)$ at $\Delta\varphi = -116.4^\circ$ (solid), $\Delta\varphi = -76.4^\circ$ (dashed), $\Delta\varphi = -156.4^\circ$ (dotted) in the panel *b*

if $\Delta\varphi = -(116.4 \pm 40)^\circ$, then the damping rates are less than those at $\Delta\varphi = -116.4^\circ$ (see Fig. 4, *b*). This dynamic behavior can be used for tuning and optimisation of the transverse feedback loop parameters.

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