# CALCULATION OF THE STRONG COUPLING CONSTANTS FOR $B_{s 1} B^{*} K$ AND $B_{s 1} B^{*} K_{0}^{*}$ VERTICES 

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#### Abstract

The structure of the $B_{s 1}(5830)$ meson has not yet been exactly known in the quark model. In this paper, the strong form factors and coupling constants of $B_{s 1}$, as a conventional $b \bar{s}$ meson, are investigated. The coupling constants $g_{B_{s 1} B^{*} K}$ and $g_{B_{s 1} B^{*} K_{0}^{*}}$ are calculated in the framework of the three-point QCD sum rules. Our findings are compared with the results of the light-cone sum rules (LCSR) and heavy-chiral unitary approaches.


В кварковых моделях структура $B_{s 1}(5830)$-мезона не вполне определена. В этой работе вычислены сильные формфакторы и константы связи $B_{s 1}$-мезона в модели, где этот мезон есть состояние $b s$. Константы связи $g_{B_{s 1} B^{*} K}$ и $g_{B_{s 1} B^{*} K_{0}^{*}}$ вычислены в правилах сумм КХД. Полученные результаты сравниваются с результатами правил сумм на световом конусе и в киральной модели с тяжелыми кварками.

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## INTRODUCTION

During the last decade, there have been made continuous efforts towards the understanding of the strong form factors and coupling constant of meson vertices in the context of QCD. The motivation comes from the fact that hadronic processes in strong interactions are explained in terms of form factors and coupling constants rather than cross sections. In addition to that there was facilitated the investigation of bottomed and charmed pseudoscalar and axial vector mesons. More accurate determination of these coupling constants plays an important role in understanding the final state interaction in the hadronic decays of $B$ meson, gets knowledge about the nature and structure of the new hadron states such as $B_{s 0}, B_{s 1}, D_{s 0}$, and $D_{s 1}$ and in the production of the charmonium states like $\psi / J, \ldots, \psi(2 s)$, which are useful sources of information in heavy-ion collisions.

Moreover, the following some coupling constants have been investigated via different approach in the literature: $D^{*} D \pi$ [1], $D D \rho$ [2], $D D J / \psi$ [3], $D^{*} D J / \psi$ [4], $D^{*} D^{*} \pi$ [5], $D^{*} D^{*} J / \psi$ [6], $D_{s} D^{*} K, D_{s}^{*} D K$ [7], $D D \omega$ [8], $D^{*} D^{*} \rho$ [9], $D^{*} D \rho$ [10], $B_{s 0} B K$ [11], $D_{s 0} D K, D_{s 1} D^{*} K$ [12], and $B_{s 0} B K, B_{s 1} B^{*} K$ [13].

[^0]Nonperturbative approaches are also required in order to calculatethe form factors corresponding to such a transition. Among these approaches, QCD sum-rules method might be the most popular one due to the fact that it is based upon QCD Lagrangian and has been successfully applied to various problems [14-17].

The structure of the bottom-strange meson $B_{s 1}$ with the spin-parity $\left(J^{P}=1^{+}\right)$has not been resolved, yet, and has been debated in the quark model. In the bottom sector, the recent observations of the $J^{p}=1^{+} B_{s 1}(5830)$ by the CDF Collaboration [18] and the $J^{p}=2^{+}$ $B_{s 2}(5840)$ by the CDF and D0 Collaborations $[18,19]$ enrich the spectrum of the bottomstrange system and estimulate our interest in the possible interpretation of these states as multiquark states. There are already some predictions for the $B_{s 1}$ mass assuming it is a $B^{*} K$ bound state [20], as a $b \bar{s}$ state [21], and as a mixture of a $b \bar{s}$ and a $b q(\bar{s} \bar{q})$ states [22]. The masses of the $B_{s 1}$ meson have been estimated with the potential quark models, heavy-quark effective theory and lattice QCD [23-28], the values differ from each other.

In this article, we take the bottom-strange meson $B_{s 1}$ as the conventional $b \bar{s}$ states, and calculate the values of the strong coupling constants $g_{B_{s 1} B^{*} K}$ and $g_{B_{s 1} B^{*} K_{0}^{*}}$ with the threepoint QCD sum rules, as well as study the possibility of the hadronic dressing mechanism in the bottomed channels. The strong coupling constant $g_{B_{s 1} B^{*} K}$ has been calculated with other approaches such as light-cone QCD sum rules [13] and heavy-chiral unitary approach [20], before.

Here by using operator product expansion (OPE), the corresponding correlation functions are calculated for the perturbative and nonperturbative parts when either $B_{s 1}$ and $K\left(K_{0}^{*}\right)$ are considered to be off-shell. Double dispersion relation is used for perturbative section and double Borel transform is performed for the perturbative and nonperturbative parts.

This paper is organized as follows. In Sec.1, we calculate the form factors and strong coupling constants $g_{B_{s 1} B^{*} K}$ and $g_{B_{s 1} B^{*} K_{0}^{*}}$ within 3PSR method. Finally, Sec. 2 is devoted to the numerical results and discussions.

## 1. THE THREE-POINT QCD SUM-RULES METHOD

We start our discussion, considering the sufficient correlation functions responsible for the $B_{s 1} B^{*} K$ and $B_{s 1} B^{*} K_{0}^{*}$ vertices when both $B_{s 1}$ and $K\left(K_{0}^{*}\right)$ can be off-shell. We write the three-point correlation function associated with $B_{s 1} B^{*} K$ and $B_{s 1} B^{*} K_{0}^{*}$ vertices which is given by:

$$
\begin{equation*}
\Pi_{\nu \mu}^{B_{s 1}}\left(p, p^{\prime}\right)=i^{2} \int d^{4} x d^{4} y \mathrm{e}^{i\left(p^{\prime} x-p y\right)}\langle 0| \mathcal{T}\left\{j_{\nu}^{B^{*}}(x) j_{\mu}^{B_{s 1} \dagger}(0) j^{K\left(K_{0}^{*}\right)^{\dagger}}(y)\right\}|0\rangle \tag{1}
\end{equation*}
$$

for off-shell $B_{s 1}$ meson, and:

$$
\begin{equation*}
\Pi_{\nu \mu}^{K\left(K_{0}^{*}\right)}\left(p, p^{\prime}\right)=i^{2} \int d^{4} x d^{4} y \mathrm{e}^{i\left(p^{\prime} x-p y\right)}\langle 0| \mathcal{T}\left\{j_{\nu}^{B^{*}}(x) j^{K\left(K_{0}^{*}\right)^{\dagger}}(0) j_{\mu}^{B_{s 1} \dagger}(y)\right\}|0\rangle \tag{2}
\end{equation*}
$$

for off-shell $K\left(K_{0}^{*}\right)$ meson. Here $j^{K}=\bar{s} \gamma_{5} d, j^{K_{0}^{*}}=\bar{d} s, j_{\nu}^{B^{*}}=\bar{d} \gamma_{\nu} b$, and $j_{\mu}^{B_{s 1}}=\bar{s} \gamma_{\mu} \gamma_{5} b$ are interpolating currents of $K, K_{0}^{*}, B^{*}, B_{s 1}$ mesons, respectively, and have the same quantum numbers of the associative mesons. Also $\mathcal{T}$ is the time ordering product, $p$ and $p^{\prime}$ are the four-momenta of the initial and final mesons, respectively (see Fig. 1).


Fig. 1. Perturbative diagrams for off-shell $B_{s 1}(a)$ and off-shell $K\left(K_{0}^{*}\right)(b)$
Equations (1) and (2) can be calculated in two different ways: In physical or phenomenological part, the representation is in terms of hadronic degrees of freedom which is responsible for the introduction of the form factors, decay constants and masses. In QCD or theoretical representation, we evaluate the correlation function in quark-gluon language and in terms of QCD degrees of freedom like quark condensate, gluon condensate, etc., with the help of the Wilson operator product expansion (OPE).
1.1. The OPE Side. With the help of the operator product expansion (OPE) in Euclidean region, where $p^{2}, p^{2} \rightarrow-\infty$, we calculate the QCD side of the correlation function (Eqs. (1) and (2)) containing perturbative and nonperturbative parts.

Let us calculate the perturbative part as shown in Fig. 1. Using the double dispersion relation for each coefficient of the Lorentz structures appearing in correlation functions (Eqs. (1) and (2)), we get:

$$
\begin{equation*}
\Pi^{(\mathrm{per}) \mathrm{M}}\left(p^{2}, p^{\prime 2}, q^{2}\right)=-\frac{1}{4 \pi^{2}} \int d s \int d s^{\prime} \frac{\rho^{M}\left(s, s^{\prime}, q^{2}\right)}{\left(s-p^{2}\right)\left(s^{\prime}-p^{\prime 2}\right)}+\text { subtraction terms } \tag{3}
\end{equation*}
$$

where $\rho^{M}\left(s, s^{\prime}, q^{2}\right)$ is spectral density, and $M$ stands for $B_{s 1}, K\left(K_{0}^{*}\right)$ off-shell meson. We calculate spectral densities in terms of the usual Feynman integrals with the help of the Cutkosky rules.

The integration region in Eq. (3) is obtained by requiring that the argument of three-delta function vanishes simultaneously. The physical region in the $s$ and $s^{\prime}$ plane is described by the following inequalities:

$$
\begin{equation*}
-1 \leqslant \frac{2 s s^{\prime}+\left(s+s^{\prime}-q^{2}\right)\left(m_{1}^{2}-s-m_{3}^{2}\right)+\left(m_{3}^{2}-m_{2}^{2}\right) 2 s}{\lambda^{1 / 2}\left(m_{1}^{2}, s, m_{3}^{2}\right) \lambda^{1 / 2}\left(s, s^{\prime}, q^{2}\right)} \leqslant+1 \tag{4}
\end{equation*}
$$

where $\lambda(a, b, c)=a^{2}+b^{2}+c^{2}-2 a b-2 a c-2 b c$. From this inequality, the lower limit of the integration over $s$ in Eq. (3) is obtaining by expressing $s$ in terms of $s^{\prime}$ as follows:

$$
\begin{equation*}
s_{L}=\frac{\left(m_{3}^{2}+q^{2}-m_{1}^{2}-s^{\prime}\right)\left(m_{1}^{2} s^{\prime}-q^{2} m_{3}^{2}\right)}{\left(m_{1}^{2}-q^{2}\right)\left(m_{3}^{2}-s^{\prime}\right)} \tag{5}
\end{equation*}
$$

The general expression for the $B_{s 1} B^{*} K$ vertex has five independent Lorentz structures. In principle, we can work with any structure. But we must choose those which have less ambiguities in the QCD sum-rules approach, which means less influence of the condensates of higher dimension and a better stability as a function of the Borel mass.

In this paper, we use the structure $p_{\mu} p_{\nu}^{\prime}$, which presents a better behavior; the spectral density when $B_{s 1}$ meson is off-shell appears as:

$$
\begin{equation*}
\rho^{B_{s 1}}\left(s, s^{\prime}, q^{2}\right)=4 i N_{c} I_{0}\left[A\left(2 m_{d}-2 m_{s}\right)+B_{1}\left(m_{d}-m_{s}\right)+B_{2}\left(m_{b}+m_{d}\right)+m_{d}\right] \tag{6}
\end{equation*}
$$

and when $K$ meson is off-shell:

$$
\begin{equation*}
\rho^{K}\left(s, s^{\prime}, q^{2}\right)=4 i N_{c} I_{0}\left[A\left(2 m_{d}-2 m_{s}\right)+B_{1}\left(m_{b}-m_{s}\right)+B_{2}\left(m_{b}+m_{d}\right)+m_{b}\right] . \tag{7}
\end{equation*}
$$

In the $B_{s 1} B^{*} K_{0}^{*}$ vertex, we have only structure $\epsilon^{\alpha \beta \mu \nu} p_{\alpha} p_{\beta}^{\prime}$; the spectral density when $B_{s 1}$ meson is off-shell turns out to be:

$$
\begin{equation*}
\rho^{B_{s 1}}\left(s, s^{\prime}, q^{2}\right)=4 i N_{c} I_{0}\left[B_{1}\left(m_{s}+m_{d}\right)+B_{2}\left(m_{d}-m_{b}\right)+m_{d}\right] \tag{8}
\end{equation*}
$$

when $K_{0}^{*}$ meson is off-shell:

$$
\begin{equation*}
\rho^{K_{0}^{*}}\left(s, s^{\prime}, q^{2}\right)=4 i N_{c} I_{0}\left[B_{1}\left(m_{s}+m_{b}\right)+B_{2}\left(m_{b}-m_{d}\right)+m_{b}\right] \tag{9}
\end{equation*}
$$

where

$$
\begin{gather*}
A=\frac{1}{\lambda^{2}\left(s, s^{\prime}, q^{2}\right)}\left[4 s s^{\prime} u m_{3}^{2}+4 s s^{\prime} \Delta \Delta^{\prime}-3 s u \Delta^{\prime 2}-3 u \Delta^{2} s^{\prime}-u^{3} m_{3}^{2}+2 u^{2} \Delta \Delta^{\prime}\right] \\
B_{1}=\frac{1}{\lambda\left(s, s^{\prime}, q^{2}\right)}\left[2 s^{\prime} \Delta-\Delta^{\prime} u\right], \quad B_{2}=\frac{1}{\lambda\left(s, s^{\prime}, q^{2}\right)}\left[2 s \Delta^{\prime}-\Delta u\right] \tag{10}
\end{gather*}
$$

and

$$
\begin{gather*}
I_{0}\left(s, s^{\prime}, q^{2}\right)=\frac{1}{4 \lambda^{1 / 2}\left(s, s^{\prime}, q^{2}\right)} \\
\Delta=\left(s+m_{3}^{2}-m_{1}^{2}\right), \quad \Delta^{\prime}=\left(s^{\prime}+m_{3}^{2}-m_{2}^{2}\right), \quad u=s+s^{\prime}-q^{2}  \tag{11}\\
\lambda\left(s, s^{\prime}, q^{2}\right)=s^{2}+s^{\prime 2}+q^{4}-2 s q^{2}-2 s^{\prime} q^{2}-2 s s^{\prime}
\end{gather*}
$$



Fig. 2. Contribution of the quarkquark condensate for the $B_{s 1}$ offshell
for off-shell $B_{s 1}\left(K\left[K_{0}^{*}\right]\right)$ case, $m_{1}, m_{2}$, and $m_{3}$ stand for the masses of the $s, b(d)$ and $d(b)$ quarks, respectively. $N_{c}=3$ represents the color factor.

We proceed to calculate the nonperturbative contributions in the QCD side that contain the quark-quark condensate. The quark-quark condensate is considered for light quarks $u, d$, and $s$. The contribution of the quark-quark condensate which survives after the double Borel transform is represented in Fig. 2 for the $B_{s 1}$ off-shell case, and is given by:

$$
\begin{equation*}
\Pi_{(\text {nonper })}^{B_{s 1}}=i\langle\bar{d} d\rangle \frac{1}{\left(p^{2}-m_{s}^{2}\right)\left(p^{\prime 2}-m_{b}^{2}\right)} \tag{12}
\end{equation*}
$$

For the $K\left(K_{0}^{*}\right)$ off-shell there is no quark-quark condensate contribuition. Our calculations show that for two cases $B_{s 1}$ and $K\left(K_{0}^{*}\right)$ off-shell, the gluon and quark-gluon condensate contributions are very small and we can easily ignore their contributions in our calculations.
1.2. The Phenomenological Side. The phenomenological side of the correlation function is obtained by considering the contribution of three complete sets of $B_{s 1}, B^{*}$, and $K\left(K_{0}^{*}\right)$
mesons in Eqs. (1) and (2). In the following, we write down the definition for the strong coupling constants $g_{B_{s 1} B^{*} K}$ and $g_{B_{s 1} B^{*} K_{0}^{*}}$,

$$
\begin{align*}
\left\langle B^{*}\left(p^{\prime}, \epsilon^{\prime}\right) \mid K(q) B_{s 1}(p, \epsilon)\right\rangle & =i m_{B_{s 1}}^{2} g_{B_{s 1} B^{*} K}\left(q^{2}\right) \epsilon . \epsilon^{\prime}  \tag{13}\\
\left\langle B^{*}\left(p^{\prime}, \epsilon^{\prime}\right) \mid K_{0}^{*}(p) B_{s 1}(q, \epsilon)\right\rangle & =i g_{B_{s 1} B^{*} K_{0}^{*}}\left(q^{2}\right) \epsilon^{\alpha \beta \gamma \sigma} \epsilon_{\gamma}(q) \epsilon_{\alpha}^{\prime}\left(p^{\prime}\right) p_{\beta}^{\prime} q_{\sigma}
\end{align*}
$$

The meson-decay constants, $f_{K_{0}^{*}}, f_{K_{0}^{*}}, f_{B^{*}}$, and $f_{B_{s 1}}$, are defined by the matrix elements

$$
\begin{align*}
\langle 0| j^{K_{0}^{*}}\left|K\left(p^{\prime}\right)\right\rangle & =m_{K_{0}^{*}} f_{K_{0}^{*}}, \\
\langle 0| j^{K}\left|K\left(p^{\prime}\right)\right\rangle & =\frac{m_{K}^{2} f_{K}}{m_{s}+m_{d}}, \\
\langle 0| j_{\mu}^{B_{s 1}}\left|B_{s 1}(p, \epsilon)\right\rangle & =m_{B_{s 1}} f_{B_{s 1}} \epsilon_{\mu}(p),  \tag{14}\\
\langle 0| j_{\nu}^{B^{*}}\left|D^{*}\left(p^{\prime}, \epsilon^{\prime}\right)\right\rangle & =m_{B^{*}} f_{B^{*}} \epsilon_{\nu}^{\prime}\left(p^{\prime}\right),
\end{align*}
$$

where $q=p^{\prime}-p, \epsilon$ and $\epsilon^{\prime}$ are the polarization vectors of the $B_{s 1}$ and $B^{*}$ mesons, respectively. Saturating Eqs. (1) and (2) with the $B_{s 1}, B^{*}$ and $K\left(K_{0}^{*}\right)$ states and using Eqs. (13) and (14), the phenomenological part for the $p_{\mu} p_{\nu}^{\prime}$ structure related to the $B_{s 1} B^{*} K$ vertex, when $K\left(B_{s 1}^{*}\right)$ is the off-shell meson, is:

$$
\begin{align*}
\Pi^{K\left(B_{s 1}^{*}\right)}=- & i g_{B_{s 1} B^{*} K}^{K\left(B_{s 1}\right)}\left(q^{2}\right) \times \\
& \times \frac{m_{B_{s 1}} m_{K}^{2} f_{B_{s 1}} f_{B^{*}} f_{K}\left(m_{B_{s 1}(K)}^{2}+m_{B^{*}}^{2}-q^{2}\right)}{\left(q^{2}-m_{K\left(B_{s 1}\right)}^{2}\right)\left(p^{2}-m_{B_{s 1}(K)}^{2}\right)\left(p^{\prime 2}-m_{B^{*}}^{2}\right)\left(m_{s}+m_{d}\right) m_{D^{*}}}+\text { h.r. } \tag{15}
\end{align*}
$$

The phenomenological part for the $\epsilon^{\alpha \beta \mu \nu} p_{\alpha} p_{\beta}^{\prime}$ structure associated to $B_{s 1} B^{*} K_{0}^{*}$ vertex, when $K_{0}^{*}\left(B_{s 1}\right)$ is the off-shell meson, is as follows:

$$
\begin{equation*}
\Pi^{K_{0}^{*}\left(B_{s 1}\right)}=-i g_{B_{s 1} B^{*} K_{0}^{*}\left(B_{s 1}^{*}\right.}^{K^{*}}\left(q^{2}\right) \frac{m_{B_{s 1}} m_{K_{0}^{*}} m_{B^{*}} f_{B_{s 1}} f_{B^{*}} f_{K_{0}^{*}}}{\left(q^{2}-m_{K_{0}^{*}\left(B_{s 1}\right)}\right)\left(p^{2}-m_{B_{s 1}\left(K_{0}^{*}\right)}\right)\left(p^{\prime 2}-m_{B^{*}}^{2}\right)}+\text { h.r. } \tag{16}
\end{equation*}
$$

In Eqs. (15) and (16), h.r. represents the contributions of the higher states and continuum.
1.3. The Sum Rule. After performing the Borel transformation [29] with respect to the variables $p^{2}\left(B_{p^{2}}\left(M_{1}^{2}\right)\right)$ and $p^{\prime 2}\left(B_{p^{\prime}}^{2}\left(M_{2}^{2}\right)\right)$ on the physical (phenomenological) and QCD parts and equating these two representations of the correlations, we obtain the equation for the strong form factors as follows.

- For the $g_{B_{s 1} B^{*} K}\left(Q^{2}\right)$ form factors:
when $B_{s 1}$ meson is off-shell:

$$
\begin{gather*}
g_{B_{s 1} B^{*} K}^{B_{s 1}}\left(Q^{2}\right)=-i \frac{m_{B^{*}}\left(Q^{2}+m_{B_{s 1}}^{2}\right)\left(m_{s}+m_{d}\right)}{m_{B_{s 1}} m_{K}^{2} f_{B_{s 1}} f_{B^{*}} f_{K}\left(m_{K}^{2}+m_{B^{*}}^{2}+Q^{2}\right)} \exp \left(\frac{m_{K}^{2}}{M_{1}^{2}}\right) \exp \left(\frac{m_{B^{*}}^{2}}{M_{2}^{2}}\right) \times \\
\times\left\{-\frac{1}{4 \pi^{2}} \int_{\left(m_{d}+m_{b}\right)^{2}}^{s_{0}^{\prime}} d s^{\prime} \int_{s_{L}}^{s_{0}} d s \rho^{B_{s 1}}\left(s, s^{\prime}, Q^{2}\right) \exp \left(-\frac{s}{M_{1}^{2}}\right) \exp \left(-\frac{s^{\prime}}{M_{2}^{2}}\right)+\right. \\
\left.+B_{p^{2}}\left(M_{1}^{2}\right) B_{p^{\prime}}^{2}\left(M_{2}^{2}\right) \Pi_{(\text {nonper })}^{B_{s 1}}\left(p^{2}, p^{\prime 2}, Q^{2}\right)\right\}, \tag{17}
\end{gather*}
$$

when $K$ meson is off-shell:

$$
\begin{align*}
& g_{B_{s 1} B^{*} K}^{K}\left(Q^{2}\right)=-i \frac{m_{B^{*}}\left(Q^{2}+m_{K}^{2}\right)\left(m_{s}+m_{d}\right)}{m_{B_{s 1}} m_{K}^{2} f_{B_{s 1}} f_{B^{*}} f_{K}\left(m_{B_{s 1}}^{2}+m_{B^{*}}^{2}+Q^{2}\right)} \exp \left(\frac{m_{B_{s 1}}^{2}}{M_{1}^{2}}\right) \times \\
& \times \exp \left(\frac{m_{B^{*}}^{2}}{M_{2}^{2}}\right)\left\{-\frac{1}{4 \pi^{2}} \int_{\left(m_{b}+m_{d}\right)^{2}}^{s_{0}^{\prime}} d s^{\prime} \int_{s_{L}}^{s_{0}} d s \rho^{K}\left(s, s^{\prime}, Q^{2}\right) \exp \left(-\frac{s}{M_{1}^{2}}\right) \exp \left(-\frac{s^{\prime}}{M_{2}^{2}}\right)\right\} \tag{18}
\end{align*}
$$

- For the $g_{B_{s 1} B^{*} K_{0}^{*}}\left(Q^{2}\right)$ form factors: when $B_{s 1}$ meson is off-shell:

$$
\begin{align*}
& g_{B_{s 1} B^{*} K_{0}^{*}}^{B_{s 1}}\left(Q^{2}\right)=i \frac{\left(Q^{2}+m_{B_{s 1}}^{2}\right)}{m_{B_{s 1}} m_{B^{*}} m_{K_{0}^{*}} f_{B_{s 1}} f_{B^{*}} f_{K_{0}^{*}}} \exp \left(\frac{m_{K_{0}^{*}}^{2}}{M_{1}^{2}}\right) \exp \left(\frac{m_{B^{*}}^{2}}{M_{2}^{2}}\right) \times \\
& \times\left\{-\frac{1}{4 \pi^{2}} \int_{\left(m_{d}+m_{b}\right)^{2}}^{s_{0}^{\prime}} d s^{\prime} \int_{s_{L}}^{s_{0}} d s \rho^{B_{s 1}}\left(s, s^{\prime}, Q^{2}\right) \exp \left(-\frac{s}{M_{1}^{2}}\right) \exp \left(-\frac{s^{\prime}}{M_{2}^{2}}\right)+\right. \\
& \left.\quad+B_{p^{2}}\left(M_{1}^{2}\right) B_{p^{\prime}}^{2}\left(M_{2}^{2}\right) \Pi_{(\text {nonper })}^{B_{s 1}}\left(p^{2}, p^{\prime 2}, Q^{2}\right)\right\} \tag{19}
\end{align*}
$$

when $K_{0}^{*}$ meson is off-shell:

$$
\begin{align*}
& g_{B_{s 1} B^{*} K_{0}^{*}}^{K_{0}^{*}}\left(Q^{2}\right)=i \frac{\left(Q^{2}+m_{K_{0}^{*}}^{2}\right)}{m_{B_{s 1}} m_{B^{*}} m_{K_{0}^{*}} f_{B_{s 1}} f_{B^{*}} f_{K_{0}^{*}}} \exp \left(\frac{m_{B_{s 1}}^{2}}{M_{1}^{2}}\right) \exp \left(\frac{m_{B^{*}}^{2}}{M_{2}^{2}}\right) \times \\
& \left.\quad \times\left\{-\frac{1}{4 \pi^{2}} \int_{\left(m_{b}+m_{d}\right)^{2}}^{s_{0}^{\prime}} d s^{\prime} \int_{s_{L}}^{s_{0}} d s \rho^{K_{0}^{*}}\left(s, s^{\prime}, Q^{2}\right) \exp \left(-\frac{s}{M_{1}^{2}}\right) \exp \left(-\frac{s^{\prime}}{M_{2}^{2}}\right)\right\}\right\} \tag{20}
\end{align*}
$$

where $Q^{2}=-q^{2}, s_{0}$ and $s_{0}^{\prime}$ are the continuum thresholds, and $s_{L}$ is the lower limit of the integral over $s$ presented in Eq. (5).

## 2. NUMERICAL ANALYSIS

In the present section, we numerically analyze the expressions of QCD sum rules obtained for the considered strong coupling constants. Some input parameters used in the calculations are: $m_{d}=(5.6 \pm 1.6) \mathrm{MeV}, m_{s}=(0.14 \pm 0.01) \mathrm{GeV}, m_{b}=(4.7 \pm 0.1) \mathrm{GeV}, m_{K}=$ $0.493 \mathrm{GeV}, m_{K_{0}^{*}(1430)}=(1.425 \pm 0.05) \mathrm{GeV}, m_{B^{*}}=5.325 \mathrm{GeV}[30], m_{B_{s 1}}=(5.72 \pm$ 0.09) GeV [31], $f_{K}=0.16 \mathrm{GeV}$ [32], $f_{B^{*}}=(0.17 \pm 0.02) \mathrm{GeV}$ [33], $f_{B_{s 1}}=(0.24 \pm$ 0.02) GeV [34], $f_{K_{0}^{*}(1430)}=(0.445 \pm 0.05) \mathrm{GeV}$ [35], and $\langle\bar{d} d\rangle=-(0.24 \pm 0.01)^{3} \mathrm{GeV}^{3}$ [36].

The expressions for the strong form factors and coupling constants contain also four auxiliary parameters: Borel mass parameters $M_{1}$ and $M_{2}$ and continuum thresholds $s_{0}$ and $s_{0}^{\prime}$.


Fig. 3. The $g_{B_{s 1} B^{*} K}\left(Q^{2}=1 \mathrm{GeV}^{2}\right)$ as a function of the Borel mass $M^{2}$ for two cases $B_{s 1}$ and $K$ off-shell mesons

These are mathematical objects, so the physical quantities, i.e., strong form factors and coupling constants should be independent of them. The parameters of $s_{0}$ and $s_{0}^{\prime}$ are the continuum thresholds. The values of the continuum thresholds $s_{0}=\left(m_{i}+r_{1}\right)^{2}$ and $s_{0}^{\prime}=\left(m_{0}+r_{2}\right)^{2}$, where $m_{i}$ and $m_{o}$ are the masses of the incoming and outgoing meson, respectively [1-11].

In this work we use the following relations between the Borel masses $M_{1}^{2}$ and $M_{2}^{2}$ : $M_{1}^{2} / M_{2}^{2}=\left(m_{K_{0}^{*}}^{2} / m_{B^{*}}^{2}-m_{b}^{2}\right)$ for a $B_{s 1}$ off-shell and $M_{1}^{2} / M_{2}^{2}=\left(m_{B_{s 1}}^{2} / m_{B^{*}}^{2}\right)$ for a $K\left(K_{0}^{*}\right)$ off-shell.

Using $r_{1}=0.5 \mathrm{GeV}$ and $r_{2}=0.5 \mathrm{GeV}$ for the continuum and fixing $Q^{2}=1 \mathrm{GeV}^{2}$, we found a good stability of the sum rule in the interval $10 \leqslant M_{1} \leqslant 20 \mathrm{GeV}^{2}$ for two cases $B_{s 1}$ and $K\left(K_{0}^{*}\right)$ off-shell. The dependence of the strong form factors $g_{B_{s 1} B^{*} K}$ and $g_{B_{s 1} B^{*} K_{0}^{*}}$ on Borel mass parameters for off-shell $B_{s 1}$ and $K\left(K_{0}^{*}\right)$ mesons are shown in Figs. 3 and 4, respectively.


Fig. 5. The strong form factors $g_{B_{s 1} B^{*} K}^{B_{s 1}}\left(Q^{2}\right)$ and $g_{B_{s 1} B^{*} K}^{K}\left(Q^{2}\right)$ as a function of $Q^{2}$


Fig. 6. The strong form factors $g_{B_{s 1} B^{*} K_{0}^{*}}^{B_{s 1}}\left(Q^{2}\right)$ and $g_{B_{s 1} D^{*} K_{0}^{*}}^{K_{0}^{*}}\left(Q^{2}\right)$ as a function of $Q^{2}$

We have chosen the Borel mass to be $M_{1}^{2}=13 \mathrm{GeV}^{2}$. Having determined $M_{1}^{2}$, we calculated the $Q^{2}$ dependence of the form factors. We present the results in Figs. 5 and 6 for the $B_{s 1} B^{*} K$ and the $B_{s 1} B^{*} K_{0}^{*}$ vertices, respectively. In this figures, the small circles and boxes correspond to the form factors in the interval where the sum rule is valid. As is seen, the form factors and their fit functions coincide together, well.

For off-shell $B_{s 1}$ meson, our numerical calculations show that the sufficient parameterization of the form factors with respect to $Q^{2}$ is:

$$
\begin{equation*}
g\left(Q^{2}\right)=\frac{A}{Q^{2}+B} \tag{21}
\end{equation*}
$$

and for off-shell $K\left(K_{0}^{*}\right)$ meson the strong form factors can be fitted by the exponential fit function as given:

$$
\begin{equation*}
g\left(Q^{2}\right)=A \mathrm{e}^{-Q^{2} / B} \tag{22}
\end{equation*}
$$

Table 1. Parameters appearing in the fit functions

| Form factor | $A$ | $B$ |
| :---: | :---: | :---: |
| $g_{B_{s 1} B^{*} K}^{K}$ | 0.84 | 4.69 |
| $g_{B_{s 1} B^{*} K}^{B_{s 1}}$ | 1614.19 | 1668.98 |
| $g_{B_{s 1} B^{*} K_{0}^{*}}^{K_{0}^{*}}$ | 1.80 | 3.51 |
| $g_{B_{s 1} B^{*} K_{0}^{*}}^{B_{s 1}}$ | 135.23 | 76.02 |

The values of the parameters $A$ and $B$ are given in Table 1.

As in our previous works [1-11], we define the coupling constant as the value of the strong coupling form factor at $Q^{2}=-m_{m}^{2}$ in Eqs. (21) and (22), where $m_{m}$ is the mass of the off-shell meson.

The errors corresponding to the values of the coupling constants are estimated by considering: a) variation of the continuum threshold, here we vary $r_{1,2}$ between $0.35 \leqslant$ $r_{1,2} \leqslant 0.65 \mathrm{GeV}$ for two cases $B_{s 1}$ and $K\left(K_{0}^{*}\right)$ offshell; b) the uncertainties in the values of the leptonic decay costants $f_{D_{s 1}}, f_{D^{*}}, f_{K}$, and $f_{K_{0}^{*}}$; c) the uncertainties in the values of other input parameters. We obtain the values of the strong coupling constants and their corresponding errors shown in Table 2. We can see that the two cases considered here, off shell $B_{s 1}$ and $K\left(K_{0}^{*}\right)$ mesons, give compatible results for the coupling constant.

Table 2. The strong coupling constants $g_{B_{s 1} B^{*} K}$ and $g_{B_{s 1} B^{*} K_{0}^{*}}$ in different approaches: 3PSR (our), light-cone sum rules (LCSR) [13], heavy chiral unitary [20], in $\mathrm{GeV}^{-1}$

| Strong <br> coupling constant | $K\left(K_{0}^{*}\right)$ (off-shell) | $B_{s 1}$ (off-shell) | Average (our) | $[13]$ | [20] |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $g_{B_{s 1} B^{*} K}$ | $0.89 \pm 0.13$ | $0.99 \pm 0.15$ | $0.94 \pm 0.14$ | $0.57 \pm 0.19$ | 0.74 |
| $g_{B_{s 1} B^{*} K_{0}^{*}}$ | $3.21 \pm 0.25$ | $3.12 \pm 0.34$ | $3.26 \pm 0.31$ | - |  |

In conclusion, the strong form factors and the coupling constants for $B_{s 1} B^{*} K$ and $B_{s 1} B^{*} K_{0}^{*}$ vertices are determined within QCD sum rules. Comparison is made for $g_{B_{s 1} B^{*} K}$ between our founding and the results of light-cone QCD sum rules and heavy chiral unitary. Experimental data for these strong form factors and the coupling constants and their comparison with the phenomenological models like QCD sum rules could give useful information about the structure of the $B_{s 1}(5830)$ axial vector meson.

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