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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

THE COUPLING CONSTANTS $g_{V\sigma\gamma}$ IN QCD SUM RULES

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We review and update previous calculations of the coupling constants $g_{V\sigma\gamma}$ and also determine new variables including the magnetic susceptibility of the quark condensate in QCD sum rules. Our estimates are consistent with the values obtained in the literature.

Представлены обзор и обновленная версия опубликованных ранее результатов вычислений констант связи $g_{V\sigma\gamma}$, а также определение новых переменных, включая магнитную восприимчивость кваркового конденсата в правилах сумм КХД. Полученные оценки согласуются со значениями, имеющимися в литературе.

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INTRODUCTION

Quantum Chromodynamics (QCD) is the true theory for the strong interactions. However, little is known concerning the structure of a meson since QCD is nonperturbative at the hadronic scale. The method of QCD sum rules including the nonperturbative QCD physics has been established by Shifman, Vainstein and Zakharov [1]. This method is very successful for calculation of various quantities in low-energy hadron physics. The electromagnetic decay of vector mesons was treated in the vector meson dominance model [2] or in chiral perturbation theory [3]. Here, we reported the application of QCD sum rules to the calculation of the coupling constant of the transition $V \to \sigma\gamma$, $g_{V\sigma\gamma}$, where V denotes the ρ and ω meson.

The nature of the lightest scalar mesons has been controversial for long time. Whether they are conventional $q\overline{q}$ states [4], multiquark states [5] or $K\overline{K}$ molecules [6] is a fundamental question in particle physics. There are a number of theoretical and experimental analyses on the σ pole position [7]. In the experimental side, there is a very clear signal for a light σ meson from the $J/\psi \to \sigma\omega \to \pi\pi\omega$ decay channel in BES experiment [8] and from the $D \to \sigma\pi \to 3\pi$ decay studied by the E791 collaboration, it is clearly seen as the dominant peak [9].

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FORMALISM

The transition $V \to \sigma\gamma$ within QCDSR has been considered before [10–13]. Our aim in this work is to re-analyze coupling constants $g_{V\sigma\gamma}$ by taking into account the contribution of the magnetic susceptibility. To analyze the coupling constants $g_{V\sigma\gamma}$, where V denotes the ρ and ω meson in QCD sum rules, we consider the three-point correlation function as follows:

$$\prod_{\alpha\beta}(p,p') = \int d^4x\; d^4y\, \mathrm{e}^{ipy}\, \mathrm{e}^{-ipx}\; \langle 0|T\{J^{\gamma}_{\alpha}(0)J^{V}_{\beta}(x)J_{\sigma}(y)\}|0\rangle, \tag{1}$$

where J^{γ}_{α} is the electromagnetic current, and J^{V}_{β} and J_{σ} are the interpolating currents for vector meson and σ meson, respectively. The physical part of the sum rules can be determined by considering a double dispersion relation for the vertex function $\prod_{\alpha\beta}$. However, we neglect possible subtraction terms since they will not make any contributions in the vector and scalar channels, after Borel transformation. We can therefore obtain the physical part,

$$\prod_{\alpha\beta}(p,p') = \frac{\langle 0|J^{V}_{\beta}|V(p)\rangle\langle V(p)|J^{\alpha}_{\gamma}|\sigma(p')\rangle\langle\sigma(p')|J_{\sigma}|0\rangle}{(p^2-m_V^2)(p'^2-m_\sigma^2)} + \dots \tag{2}$$

In this expression, the overlap amplitudes of the vector mesons, λ_V, and sigma meson, λ_σ, are defined as

$$\langle 0|J^{V}_{\beta}|V(p)\rangle = \lambda_V U_\beta, \quad \langle\sigma|J^{\sigma}_{\beta}|0\rangle = \lambda_\sigma, \tag{3}$$

where U_β is the polarization vector of the vector meson. The $g_{V\sigma\gamma}$ coupling constants are defined through the matrix element of the electromagnetic current,

$$\langle V(p)|J^{\gamma}_{\alpha}|\sigma(p')\rangle = -i\frac{e}{M_V} g_{V\sigma\gamma} K(q^2)(pqU_\alpha - Uqp_\alpha), \tag{4}$$

where $q = p - p'$ is the photon momentum and $K(q^2)$ is the form factor with $K(0) = 1$. The alternative parametrization for the $V\sigma\gamma$ vertex can be defined through the effective Lagrangian [14]

$$\mathcal{L}_{V\sigma\gamma} = \frac{e}{M_V} g_{V\sigma\gamma}\partial^{\alpha}V^{\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)\sigma, \tag{5}$$

which describes the coupling constants $g_{V\sigma\gamma}$. Using Eqs. (2)–(4), we get the phenomenological part of the sum rules as

$$\prod_{\alpha\beta}^{\text{phen}} = -i\frac{e}{M_V} g_{V\sigma\gamma}\frac{\lambda_\sigma\lambda_V U_\beta}{(p^2-m_V^2)(p'^2-m_\sigma^2)}(pqU_\alpha - Uqp_\alpha). \tag{6}$$

Our next step is the calculation of the theoretical part of the sum rules. For the interpolating currents of ω and ρ vector mesons and σ scalar meson, we choose the following currents:

$$\begin{aligned} J^{\omega}_{\beta} &= 1/2(\overline{u}\gamma_\beta u + \overline{d}\gamma_\beta d), \\ J^{\rho}_{\beta} &= 1/2(\overline{u}\gamma_\beta u - \overline{d}\gamma_\beta d), \\ J_{\sigma} &= 1/2(\overline{u}u + \overline{d}d), \end{aligned} \tag{7}$$

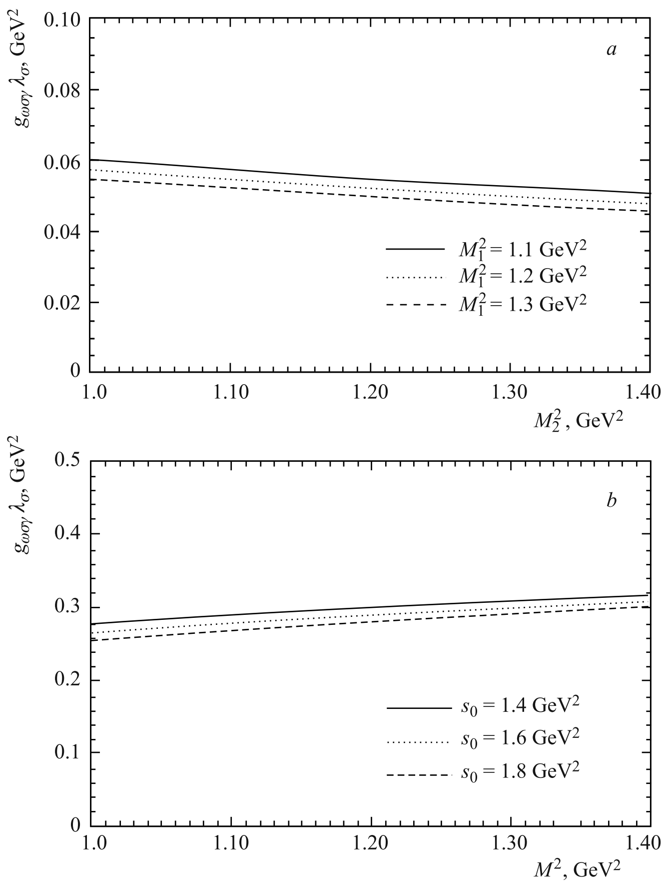

Fig. 1. The coupling constant $g_{\omega\sigma\gamma}$ as a function of the Borel parameter: *a*) M_2^2 for different values of M_1^2; *b*) M^2 for different values of the threshold parameter $s_0 = 1.4,\ 1.6,\ 1.8$ GeV^2

respectively, and also, $J_\alpha^\gamma = (e_u\overline{u}\gamma_\alpha u + e_d\overline{d}\gamma_\alpha d)$ is the electromagnetic current with the quark charges e_u and e_d. In order to obtain the theoretical part, we consider the perturbative contribution and the power corrections from operators of different dimensions to the three-point correlation function. To obtain the perturbative contribution, we consider the lowest order bare loop as shown in Fig. 3, a. Besides, the power corrections from the operators of different dimensions $\langle qq\rangle$, $\langle q\sigma Gq\rangle$ and $\langle (qq)^2\rangle$ are considered. The gluon condensate contribution proportional to $\langle G^2\rangle$ is not considered because it is estimated to be negligible for light quark system. In Fig. 3, we present the relevant Feynman diagrams for the calculations of the coupling constants.

We study in the flavor $SU(2)$ sector with $m_u = m_d = m_q$ and in the limit $m_q = 0$. In our calculations, the terms $\langle\overline{q}q\rangle$, $\langle q\sigma Gq\rangle$ only make contributions, and we do not consider the perturbative bare-loop diagram which does not make any contribution in the limit $m_q = 0$.

In order to phase out the theoretical side, we take into account the contribution which is derived from Fig. 3, d. This contribution has not been considered so far in similar calculations

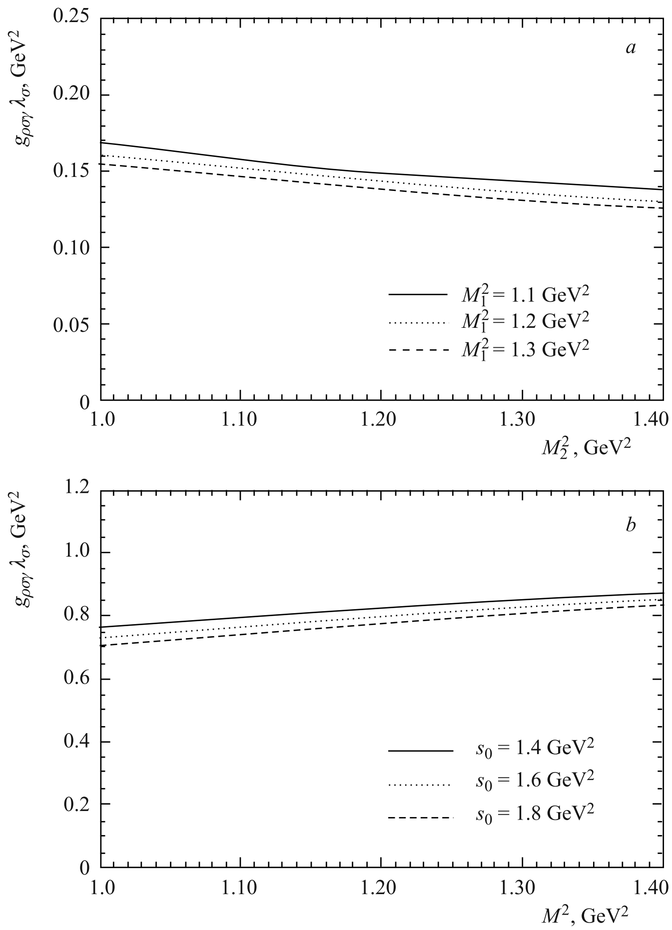

Fig. 2. The coupling constant $g_{\rho\sigma\gamma}$ as a function of the Borel parameter: *a*) M_2^2 for different values of M_1^2; *b*) M^2 for different values of the threshold parameter $s_0 = 1.4$, 1.6, 1.8 GeV^2

of the coupling constant. Using the definition of magnetic susceptibility,

$$\int d^4z\, \mathrm{e}^{iqz}\ \langle 0|T\{J^\mu_{\mathrm{el}}(z)\overline{q}(x)\sigma_{\alpha\beta}q(0)\}|0\rangle = ie_q\langle\overline{q}q\rangle(g_{\mu\alpha}q_\beta - g_{\mu\beta}q_\alpha)\chi + O(x), \qquad (8)$$

we obtain this contribution from Eq. (1) as follows:

$$\prod_{\mu\alpha} = -i\chi N_c e_q\langle\overline{q}q\rangle\frac{1}{p^2}(p_\alpha q_\mu - g_{\mu\alpha}pq), \qquad (9)$$

where χ is the magnetic susceptibility of the quark condensate. When applying double Borel transformations, we have $\prod_{\mu\alpha} = 0$. Hence, this contribution of the magnetic susceptibility cannot be obtained in this way. To find the contribution shown in Fig. 3, d corresponds to

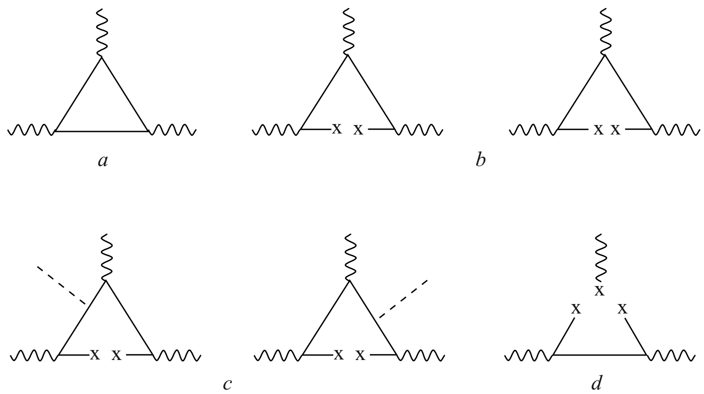

Fig. 3. Feynman diagrams for the $V\sigma\gamma$ vertex

interaction of the external field; therefore, we can follow the references [15, 16]. To obtain the contribution of the magnetic susceptibility, one can convert the expansion in Eq. (8) to an expansion on the light cone as [15, 16]

$$\int d^4z\, \mathrm{e}^{iqz}\ \langle 0|T\{J^{\mu}_{\mathrm{el}}(z)\overline{q}(x)\sigma_{\alpha\beta}q(0)\}|0\rangle =$$
$$= ie_q\langle\overline{q}q\rangle \int du\, \mathrm{e}^{iuqx}\{(g_{\mu\alpha}q_\beta - g_{\beta\mu}q_\alpha)[\chi\varphi_\gamma(u) + \ldots\}, \quad (10)$$

where e_q is the quark charge; the function $\varphi_\gamma(u)$ is the leading twist-2 photon wave function. Using this equation, we get the following expression:

$$M_\chi = ie_q\chi\langle\overline{q}q\rangle\varphi_\gamma(u_0)M^2(1 - \mathrm{e}^{-s_0/M^2}). \quad (11)$$

Equating the theoretical and the phenomenological parts of the correlation function, we obtain the expression of the coupling constant $g_{V\sigma\gamma}$ in QCD sum rules. Performing the double Borel transforms with respect to the variables $Q^2 = -p^2$ and $Q'^2 = -p'^2$ on both sides of the correlation function, and by considering the gauge-invariant structure $(pqu_\alpha - uqp_\alpha)$, we obtain the sum rules for the $g_{V\sigma\gamma}$ coupling constants:

$$\lambda_\sigma g_{V\sigma\gamma}\ \exp\left(-\frac{m_\sigma^2}{M_1^2}\right)\ \exp\left(-\frac{m_V^2}{M_2^2}\right) =$$
$$= \frac{m_V}{\lambda_V}(\mathrm{e}_u \mp e_d)\langle\overline{q}q\rangle\left[-\varphi_\gamma(u_0)\chi M^2\left(1 - \exp\left(-\frac{s_0}{M^2}\right)\right) + \frac{3}{4} + \frac{3}{32}\frac{m_0^2}{M_1^2} + \frac{5}{32}\frac{m_0^2}{M_2^2}\right], \quad (12)$$

where the term $\left(1 - \exp\left(-\dfrac{s_0}{M^2}\right)\right)$ is the function used to subtract the continuum, s_0 is the continuum threshold, and

$$u_0 = \frac{M_2^2}{M_1^2 + M_2^2}, \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}, \quad (13)$$

where M_1^2 and M_2^2 are the Borel parameters in the vector meson, ρ and ω, and σ channels, respectively. Since the masses of the ρ, ω and σ are very close to each other, we will set $M_1^2 = M_2^2 = 2M^2$ and hence $U_0 = 1/2$.

RESULTS

In order to obtain the coupling constants $g_{V\sigma\gamma}$ in QCD sum rules, we work two models. Model I includes the contributions coming from the Feynman diagrams which are shown in Fig. 3, a–c. Model II is constructed by adding the contribution resulting from the Feynman diagram in Fig. 3, d to model I. It follows from Eq. (10), we need to know the parameters λ_V and λ_σ to determine the coupling constants. For the overlap amplitude of the vector mesons, ω and ρ, we use the values $\lambda_\omega = (0.16 \mp 0.01)$ GeV^2 and $\lambda_\rho = (0.17 \mp 0.01)$ GeV^2 that are determined by employing the QCD sum rules method [17]. The overlap amplitude of sigma meson was also determined using the QCD sum rules method in [10, 11], which predicts $\lambda_\sigma = (0.12 \mp 0.03)$ GeV^2 and $\lambda_\sigma = (0.2 \mp 0.02)$ GeV^2, respectively. Note that the two predictions are in disagreement by almost a factor of two.

In numerical analysis of the sum rules, we use the various parameters as follow: $m_0^2 = (0.8 \mp 0.02)$ GeV^2, $\langle \overline{u}u \rangle = -(0.014 \mp 0.002)$ GeV^2 [18]. For the mass of the vector mesons and the scalar meson, we use $m_\omega = 0.782$ GeV, $m_\rho = 0.770$ GeV and $m_\sigma = 0.5$–0.7 GeV, respectively. Then considering independent variations of the continuum threshold s_0 and the Borel parameters M_1^2 and M_2^2, we analyze the dependence of the sum rule for the coupling constants $g_{V\sigma\gamma}$ on these parameters. We study the independent variations of the Borel parameters in Model I. The limits for $M_1^2 = 1.2$ GeV^2 and for $1.0 \leqslant M_2^2 \geqslant 1.4$ GeV^2 determine allowed interval for the vector channel [19]. We observe from Fig. 1, a and Fig. 2, c that the variation of the coupling constants $g_{\omega\sigma\gamma}$ and $g_{\rho\sigma\gamma}$ as a function of the Borel parameters M_1^2 for different values of M_2^2 is quite stable. For the middle value $M_1^2 = 1$ GeV^2 of the Borel parameter, we determine the coupling constants $g_{V\sigma\gamma}$ depending on the value λ_σ as

$$g_{\omega\sigma\gamma}\lambda_\sigma = (0.064 \mp 0.01)\ \mathrm{GeV}^2,$$
$$g_{\rho\sigma\gamma}\lambda_\sigma = (0.177 \mp 0.03)\ \mathrm{GeV}^2,$$

where errors come from the values of the overlap amplitudes λ_ω, λ_ρ and the Borel parameter, the values of the vacuum condensate. Note that due to discrepancies in the predicted values for λ_σ, here we presented the product $g_{V\sigma\gamma}\lambda_\sigma$. Model II includes the parameter called the magnetic susceptibility of the quark condensate, χ, which is estimated in different frameworks. Using the OPE (Operator Product Expansion) and pion dominance in the longitudinal part of T-product of axial and two vector currents, the value of the parameter is obtained as $\chi = -1/(335\ \mathrm{MeV})^2$ [20]. Then, photon distribution amplitudes in QCD analysis predict $\chi = (-3.15 \pm 0.1)$ GeV^{-2} [21], which we use in the present work. Also, in [22] the author calculated the value of the magnetic susceptibility as $\chi = 4.32$ GeV^{-2} in the framework of the instanton liquid model. In Fig. 1, b and Fig. 2, d we present the dependence of the coupling constants $g_{\omega\sigma\gamma}$ and $g_{\rho\sigma\gamma}$ on the Borel parameter $M_1^2 = 1$ GeV^2 at the values of the continuum threshold: $s_0 = 1.4, 1.6$ and 1.8 GeV^2. Then, we obtain the coupling constants $g_{V\sigma\gamma}$ as

$$g_{\omega\sigma\gamma}\lambda_\sigma = (0.219 \mp 0.04)\ \mathrm{GeV}^2,$$
$$g_{\rho\sigma\gamma}\lambda_\sigma = (0.60 \mp 0.13)\ \mathrm{GeV}^2,$$

where in addition to previous uncertainty, errors come from the variation of the continuum threshold s_0 and the magnetic susceptibility of the quark condensate. These values are different from the previous values of the coupling constants, $g_{\omega\sigma\gamma}$ and $g_{\rho\sigma\gamma}$. These differences arise from the term including the parameter χ whose contribution is dominant in Model II. It should also be noted that if we use the values of the input parameters given in [11, 12], we get the values of the coupling constants as $g_{\omega\sigma\gamma}\lambda_\sigma = (0.17 \mp 0.04)$ GeV^2 and $g_{\rho\sigma\gamma}\lambda_\sigma = (0.48 \mp 0.14)$ GeV^2 (moreover, $g_{\omega\sigma\gamma} = 0.80 \mp 0.02$ and $g_{\rho\sigma\gamma} = 2.70 \mp 0.45$ for the value $\lambda_\sigma = 0.12$), respectively. Differences between the results come from the different sign in the equation obtained for the coupling constants.

Comparison between different predictions and QCD sum rules

Ref.	$g_{\omega\sigma\gamma}$	Ref.	$g_{\rho\sigma\gamma}$
[12]	0.78 ∓ 0.14	[10]	2.2 ∓ 0.4
[13]	-0.72 ∓ 0.08	[11]	3.2 ∓ 0.6
[23]	1.58 and -1.73	[14]	2.71
[23]	0.13 and -0.27	[25]	8.45 ∓ 1.77
[24]	0.11 ∓ 0.01	[25]	-6.96 ∓ 1.78
[24]	-0.21 ∓ 0.02	[26]	3.0
Model I	$(0.064 \mp 0.01)/\lambda_\sigma$	Model I	$(0.177 \mp 0.03)/\lambda_\sigma$
$\lambda_\sigma = 0.12$	0.540 ∓ 0.093	$\lambda_\sigma = 0.12$	1.482 ∓ 0.265
$\lambda_\sigma = 0.2$	0.324 ∓ 0.056	$\lambda_\sigma = 0.2$	0.889 ∓ 0.160
Model II	$(0.219 \mp 0.04)/\lambda_\sigma$	Model II	$(0.60 \mp 0.13)/\lambda_\sigma$
$\lambda_\sigma = 0.12$	1.828 ∓ 0.382	$\lambda_\sigma = 0.12$	5.019 ∓ 1.086
$\lambda_\sigma = 0.2$	1.097 ∓ 0.229	$\lambda_\sigma = 0.2$	3.011 ∓ 0.752

Finally, we would like to present a comparison of our results for the coupling constants $g_{V\sigma\gamma}$ with the results of different studies in the literature. In table, we denote the predictions from phenomenological approach [23–25], three-point QCD sum rules [11, 12], light-cone QCD sum rules [10, 13], the effective Lagrangian approach [14], ρ-meson photoproduction [26] and what we have obtained in this work. For the coupling constant $g_{\omega\sigma\gamma}$ the results shown in table indicate that the QCD sum rules for Model II yield predictions which are in better agreement with the values in [12, 23, 26]. However, the results in Model II of the coupling constant $g_{\rho\sigma\gamma}$ are in a very good agreement with the values in [10, 11, 14, 26]. Therefore, the term including magnetic susceptibility from Fig. 3, d should be considered in the studies of these coupling constants.

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