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LEPTON PAIR PRODUCTION IN PERIPHERAL COLLISIONS OF RELATIVISTIC IONS AND THE PROBLEM OF REGULARIZATION

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The long-standing problem of multiple photon exchanges in the process of lepton pair production in the Coulomb field of two highly relativistic nuclei is considered. As was shown recently, the probability to produce lepton pairs is completely determined by the Feynman scattering matrix in the presence of two nuclei. This matrix can be expressed in terms of the scattering matrices associated with individual nuclei in the form of infinite Watson series.

We investigate the problem of infrared divergencies of separate terms of these series and show that for certain sums of these terms the numerical cancellations lead to infrared stability of the scattering matrix. The prescription is proposed permitting one to calculate the yield of lepton pairs with desirable accuracy.

Обсуждается проблема многофотонных обменов в процессе рождения лептонной пары в кулоновском поле двух релятивистских ядер. Как было показано недавно, вероятность образования лептонных пар определяется фейнмановской матрицей рассеяния двух ядер. Эта матрица может быть выражена через матрицы рассеяния отдельных ядер в форме бесконечных ватсоновских рядов.

Мы исследовали проблему инфракрасных расходимостей отдельных членов этих рядов и показали, что для определенных сумм этих членов численные сокращения приводят к инфракрасной стабильности матрицы рассеяния. Предложен метод, позволяющий вычислять выход лептонных пар с заданной точностью.

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INTRODUCTION

The interest in the process of lepton pair production in the Coulomb fields of two highly relativistic ions with charge numbers Z_1 and Z_2

$$Z_1 + Z_2 \to n(e^+e^-) + Z_1 + Z_2 \qquad (1)$$

is mainly aroused by operation of heavy ion colliders such as RHIC and LHC. At such energies the cross section of the process (1) becomes huge (tens of kilobarns at RHIC, hundreds of kilobarns at LHC energies) so that its precise knowledge becomes a pressing [1].

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For many years the process (1) has been considered in lowest order in fine structure constant α, i.e., the Born approximation [2–4]. On the other hand, in the heavy ion collisions the relevant parameter $Z\alpha$ is not small (for instance, for lead $Z\alpha \approx 0.6$), thus the multiple photon exchanges can be vital. Moreover, the multiplicity and distribution of lepton pairs produced in the Coulomb fields of two colliding relativistic heavy ions are closely connected to the problem of unitarity, which is beyond the Born approximation.

The corrections described by disconnected vacuum–vacuum diagrams are called «unitarity corrections» because they restore the unitarity of the probability of n pairs production $P_n(b)$ at given impact parameter $\mathbf{b}$ [6, 8]. As to the multiple photon exchanges between produced leptons and ions Coulomb fields, they are known as the Coulomb corrections (CC) [1].

In the last years a number of works [5–16] have been done on this issue. The problem of CC and «unitarity» corrections is considered very successively and completely in [8] (see also [12]). It was shown that the probability to produce exactly n pairs $P_n(b)$ in the process (1) is completely determined by the Feynman scattering matrix T in the presence of the two nuclei [8]. Using infinite Watson series, the matrix T can be expressed in terms of the scattering matrices T_1, T_2 of lepton scattering on individual nuclei. In the case of screened Coulomb potentials (for instance, atoms scattering), one can confine himself to finite number of terms from the Watson expansion, thus calculating the probabilities $P_n(b)$ with desired accuracy. Indeed, every item of the Watson series begins with the higher order term in α than previous one, so one can inspect the accuracy of calculations.

Nevertheless, in the case of pair production by ions whose Coulomb fields are unscreened, the problem of regularization arises. It is well known that the amplitude of lepton pair photoproduction off the unscreened Coulomb field [17] does not depend on the regularization parameter[1]. In perturbation theory [13, 16] the regularization parameter cancels in every term of certain order in fine structure constant. Unfortunately, this nice property of perturbation theory is lost when the amplitude of (1) is cast in the form of Watson series. Every term of this series depends on regularization parameter in its own way, so that bringing the parameter closer to zero leads to the oscillations making the Watson expansion meaningless. On the other hand, our experience from perturbation theory gives hope that the full Watson series must be infrared stable, i.e., does not depend on the regularization parameter.

We investigate this problem and show how to deal with it. Considering the specific sets of the Watson expansion corresponding to finite number of photon exchanges attached to one of the ions (any number of exchanges with another ion), we show that, as a result of complex cancellations, the relevant amplitudes do not depend on regularization parameter. The prescription is proposed which allows one to calculate the scattering matrix on any two Coulomb centers and thus the full probability with desirable accuracy.

The following notations are used in the paper: e, m are the electron charge and mass, respectively; $A^j_\mu(p)$ is the electromagnetic vector potential created by nucleus; γ_μ are Dirac matrices and $\gamma_\pm = \gamma_0 \pm \gamma_z$. We use the light-cone definition of four momenta and coordinates $k_\pm = k_0 \pm k_z$, $x_\pm = x_0 \pm x_z$. Throughout the paper, the transverse components of momenta and coordinates are defined as two-dimensional vectors. For instance, $\mathbf{b}_j$ are the impact

[1]The amplitude of lepton scattering in the Coulomb field depends on the regularization parameter in the form of phase factor.

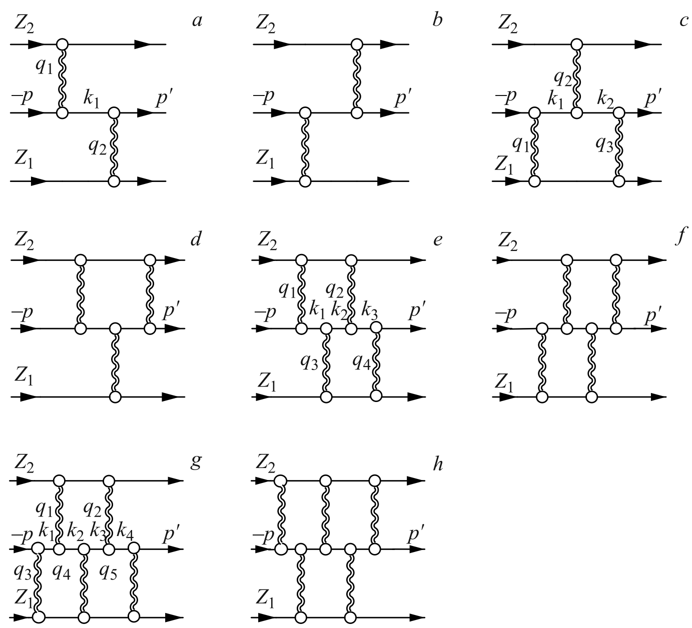
parameters of ions, whereas $\mathbf{x}_i, \mathbf{k}_i$ are transverse coordinates and momenta of leptons. The index $j = 1, 2$ is reserved for quantities attached to relevant ions Z_1, Z_2.

1. SCATTERING MATRIX IN THE PRESENCE OF TWO NUCLEI

As was shown in [8], the probability of n lepton pairs production in the process (1) $P_n(b)$ is completely determined by Feynman scattering matrix T in the presence of two nuclei. This matrix can be expressed in terms of the operators relevant to lepton scattering off separate nuclei T_1, T_2 and free Feynman propagator G_F with the help of Watson series[1]. In short notation, the Watson series for scattering on two centers reads

$$T = T_1 + T_2 - T_1 \otimes G_F \otimes T_2 - T_2 \otimes G_F \otimes T_1 + \\ + T_1 \otimes G_F \otimes T_2 \otimes G_F \otimes T_1 + T_2 \otimes G_F \otimes T_1 \otimes G_F \otimes T_2 + \dots \quad (2)$$

In the figure we depicted possible exchanges in lepton pair production in accordance with various terms of the Watson expansion[2]. The thick lines attached to ions Z_1, Z_2 represent the full set of photon exchanges between the lepton (electron or positron) and the ion.



The diagrams relevant to first six terms of the Watson series

[1]For detailed discussion of the Watson series and difference between the retarded and Feynman propagators, see [8].

[2]Later on, we begin the numbering from the third term in (2), because in the process (1) the two first terms do not contribute.

The single amplitudes $T_j(p', p)$ satisfy the well-known (see, e.g., [18]) operator equations

$$T_j = V_j - V_j \otimes G \otimes T_j; \quad V_j(p, p') = e\gamma_\mu A^j_\mu(p - p'). \tag{3}$$

These equations can be solved in the case of ultrarelativistic energies. At high energies due to Lorentz contraction the Coulomb field of the nucleus looks like very thin disc, for which the Coulomb potential in moving system takes a simple form. The solution of Eq. (3) for the Feynman propagators reads [8]

$$\begin{aligned} T_1(p, p') &= (2\pi)^2 \delta(p_+ - p'_+)[\theta(p_+) f_1^+(\mathbf{p} - \mathbf{p}') - \theta(-p_+) f_1^-(\mathbf{p} - \mathbf{p}')]\gamma_+, \\ T_2(p, p') &= (2\pi)^2 \delta(p_- - p'_-)[\theta(p_-) f_2^+(\mathbf{p} - \mathbf{p}') - \theta(-p_-) f_2^-(\mathbf{p} - \mathbf{p}')]\gamma_-, \end{aligned} \tag{4}$$

$$\begin{gathered} f_j^\pm(\mathbf{q}) = \frac{i}{2\pi} \int d^2x \, \mathrm{e}^{i\mathbf{q}\mathbf{x}} [1 - S_j^\pm(\mathbf{x}, \mathbf{b}_j)]; \\ S_j^\pm(\mathbf{x}, \mathbf{b}_j) = \exp\left(\pm i\chi(\mathbf{x}, \mathbf{b}_j)\right); \quad \chi_j(\mathbf{x}, \mathbf{b}_j) = e \int\limits_{-\infty}^{\infty} \Phi_j\left(\sqrt{(\mathbf{x} - \mathbf{b}_j)^2 + z^2}\right) dz. \end{gathered} \tag{5}$$

Substituting these expressions in the Watson expansion (2), one can calculate the scattering matrix in the presence of two centers and therefore the probability of the process (1). Every consequent term in the Watson series begins with higher order in the parameter $Z\alpha$, which allows one to obtain the probabilities with a desirable precision. This is true for the screened Coulomb potential, for instance, in the case of interaction of relativistic atoms. But heavy ion colliders deal with ions whose Coulomb fields are unscreened and for which the problem of regularization demands special consideration.

2. THE REGULARIZATION OF WATSON SERIES

The integrals (5) defining the phase shifts χ_j are divergent in the case of unscreened Coulomb potential which is relevant to ion scattering. Let us consider the case of screened potentials with regularization parameter (screening radius)[1], which goes to zero in the final expressions

$$\Phi_j(r) = \lim_{\lambda_j \to 0} \frac{eZ_j \, \exp\left(-\lambda_j r\right)}{r}. \tag{6}$$

The relevant Coulomb phases read

$$\chi_j(b) = e \int \Phi_j(\sqrt{b^2 + z^2}) \, dz = 2Z_j\alpha K_0(b\lambda_j) \to -2Z_j\alpha(\ln(b\lambda_j) + C). \tag{7}$$

Substitution of the above expressions in the Watson expansion (2) leads to the products of $S^\pm$-matrix elements some of which do not depend on the regularization parameter, for instance,

$$S_j^+(\mathbf{x}) S_j^-(\mathbf{x}') = \exp\left(2iZ_j\alpha \ln \frac{|\mathbf{x}' - \mathbf{b}_j|}{|\mathbf{x} - \mathbf{b}_j|}\right). \tag{8}$$

[1]The regularization parameters can vary for different ions, thus for every ion we introduce the relevant λ_j.

However, the majority of obtained products are oscillating functions of λ_j, which makes the Watson expansion meaningless. On the other hand, our experience from pair photoproduction in the Coulomb field [17] and perturbation theory [13, 16] tells us that the amplitude of the process (1) must be infrared stable, so all oscillating products have to be cancelled in the full amplitude.

To follow these cancellations, consider first the case where one of the ions, for instance Z_2, is light so that one can expand the amplitude in the parameter $Z_2\alpha$. In the general case, the Watson series (2) is infinite and there are no reasons to truncate it. However, it is automatically cut off if one considers the finite number of exchanged photons attached to one of the nuclei (with any number of exchanges with the other nucleus).

Denoting the transverse momenta of leptons in intermediate states by $\mathbf{k}_i$ and the transverse momenta of exchanged photons by $\mathbf{q}_i$ (see the figure), it is convenient to introduce the following notations:

$$\Omega_j(\mathbf{q},\mathbf{q}') = \frac{1}{(2\pi)^2}\int d^2x\, d^2x' \exp\left(i\mathbf{q}\mathbf{x} + i\mathbf{q}'\mathbf{x}'\right)\left(1 - S_j^+(\mathbf{x})S_j^-(\mathbf{x}')\right) =$$
$$= f_j^+(\mathbf{q})f_j^-(\mathbf{q}') - 2\pi i\delta(\mathbf{q})f_j^+(\mathbf{q}) - 2\pi i\delta(\mathbf{q}')f_j^-(\mathbf{q}'), \quad (9)$$

$$f_j^+ = -\sum_{n=1}^{\infty}\frac{i^{n+1}}{n!}f_j^{(n)}, \quad f_j^- = \sum_{n=1}^{\infty}\frac{(-i)^{n+1}}{n!}f_j^{(n)}, \quad (10)$$

$$f_j^{(n)} = \frac{1}{2\pi}\int d^2x\, \mathrm{e}^{i\mathbf{q}\mathbf{x}}\chi^n(\mathbf{b}_2 - \mathbf{x}). \quad (11)$$

The expressions (10) are nothing else than the expansion of amplitudes from (5). Notice that the combinations $\Omega_j(\mathbf{q},\mathbf{q}')$ are independent of regularization parameters.

To obtain the sum of terms from the Watson expansion (2) relevant to the first order exchange in $Z_2\alpha$ and all exchanges with Z_1, one has to calculate the terms which are linear in T_2. These terms correspond to the first three diagrams of the figure, with obvious replacement of the thick line attached to the ion Z_2 by a single-photon exchange.

Using the above expressions after a lengthy algebra, we get

$$T^{(1)} = \sum_{n=1}^{\infty} T_n^{(1)} = \frac{i}{8\pi}\int \frac{\bar{u}(p')\gamma_+\nu_1\gamma_-\nu_2\gamma_+\upsilon(p)}{\mu_1 p_+ + \mu_2 p'_+} f_2^{(1)}(q_2)\Omega_1(q_1,q_3)\, d^2k_1\, d^2k_2,$$
$$\nu_i = m - \mathbf{k}_i\boldsymbol{\gamma}; \quad \mu_i = m^2 + \mathbf{k}_i^2. \quad (12)$$

This matrix does not depend on the regularization parameter λ_1, which is a result of nontrivial cancellations among the different terms of the Watson series. Passing in this expression to the impact parameter representation upon the relevant Fourier transformations, it can be shown that it is in accordance with the results obtained in [15, 19].

As a next example of the λ_2 independence, we consider the set of terms from the Watson series corresponding to two photons attached to the ion Z_2 and any number of exchanged photons with Z_1. This contribution is provided by the first four diagrams of the figure, with obvious replacement of a set of photon exchanges attached to ion Z_2 by one- and two-photon exchanges. The result of our calculations can be cast in the form

$$T^{(2)} = \sum_{n=1}^{\infty} T_n^{(2)} = -\frac{i}{(4\pi)^2}\int \frac{\bar{u}(p')\gamma_+\nu_1\gamma_-\nu_2\gamma_+\upsilon(p)}{\mu_1 p_+ + \mu_2 p'_+} f_2^{(2)}(q_2)\Omega_1(q_1,q_3)\ln\left(\frac{\mu_1 p_+}{\mu_2 p'_+}\right)\times$$
$$\times\, d^2k_1\, d^2k_2 - \frac{i}{(4\pi)^3}\int \frac{\bar{u}(p')\gamma_+\nu_1\gamma_-\nu_2\gamma_+\nu_3\gamma_-\upsilon(p)}{\mu_1\mu_3 + \mu_2 p'_+ p_-} f_2^{(1)}(q_2) f_2^{(1)}(q_4)\,\Omega_1(q_1,q_3)\times$$
$$\times\left[\ln\left(\frac{\mu_1\mu_3}{\mu_2 p'_+ p_-}\right) + i\pi\right] d^2k_1\, d^2k_2\, d^2k_3 - \frac{i}{(4\pi)^3}\int \frac{\bar{u}(p')\gamma_-\nu_1\gamma_+\nu_2\gamma_-\nu_3\gamma_+\upsilon(p)}{\mu_1\mu_3 + \mu_2 p'_- p_+}\times$$
$$\times\, f_2^{(1)}(q_1) f_2^{(1)}(q_3)\Omega_1(q_2,q_4)\left[\ln\left(\frac{\mu_1\mu_3}{\mu_2 p'_- p_+}\right) + i\pi\right] d^2k_1\, d^2k_2\, d^2k_3. \quad (13)$$

As in the previous case, this expression does not depend on the regularization parameter λ_1.

We do not cite here the next sets of Watson terms corresponding to three and four photons attached to the ion Z_2 in view of their inconvenience, but we verified that they also do not depend on the regularization parameter λ_1. It is obvious that, operating in the same way, one obtains the independence from λ_2, choosing the sets relevant to finite number of photon exchanges with ion Z_1.

To investigate the problem of regularization in the general case, we consider the first six terms of the Watson expansion (2) (diagrams *a–f* in the figure). This set consists of the infrared stable term T_s and the term T_u depending on λ_j:

$$\sum_{n,m=1}^{\infty} T_n^m = T_s + T_u. \quad (14)$$

We calculated the stable part T_s with the following result:

$$T_s = \frac{i}{(4\pi)^3}\int d^2k_1 d^2k_2 d^2k_3 \bar{u}(p')\times$$
$$\times\left[\gamma_+\nu_1\gamma_-\nu_2\gamma_+\nu_3\gamma_-\Omega_1(q_1,q_3)\Omega_2(q_2,q_4)\frac{\ln\left(\frac{\mu_1\mu_3}{\mu_2 p'_+ p_-}\right) + i\pi}{\mu_1\mu_3 + \mu_2 p'_+ p_-} + \right.$$
$$\left. + \gamma_-\nu_1\gamma_+\nu_2\gamma_-\nu_3\gamma_+\Omega_2(q_1,q_3)\Omega_1(q_2,q_4)\frac{\ln\left(\frac{\mu_1\mu_3}{\mu_2 p'_- p_+}\right) + i\pi}{\mu_1\mu_3 + \mu_2 p'_- p_+}\right]\upsilon(p). \quad (15)$$

As to the unstable part T_u, it turns out to be of order $(Z_1 Z_2 \alpha^2)^3$, i.e., a higher order in fine structure constant than the stable one. This unstable part has to be exactly cancelled, when one considers the next terms of the Watson series.

CONCLUSIONS

The problem of multiple photon exchanges in the process of lepton pair production in the Coulomb field of two highly relativistic nuclei turns out to be enough complex, and thus discrepancies on this issue existing in literature are not surprising. The progress in this area due to the recent investigation [8] stimulated us to consider the important problem of Watson series regularization, the issue that always arises when one considers the interaction with unscreened Coulomb potential.

The Feynman scattering matrix in the presence of two ions can be constructed in the form of infinite the Watson series. We show that the specific sets of the Watson series corresponding to finite number of photon exchanges with one of the ions and all possible exchanges with the other ion do not depend on regularization parameter relevant to the ion with infinite exchanges. Moreover, it is shown that the first six terms of the Watson series can be presented as infrared stable part and unstable part which is of higher order in parameter $Z_1 Z_2 \alpha^2$ than the stable part. This observation allows one to construct the infrared stable sets from the Watson series and therefore calculate the full probability of any number pairs production in peripheral collisions of relativistic ions with high accuracy.

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