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ASTROPHYSICALLY RELEVANT PROCESSES IN LORENTZ-VIOLATING QED

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We consider quantum electrodynamics with violation of Lorentz invariance. We calculate the rates of photon decay along with the cross sections of electron–positron pair production on background radiation and in the Coulomb field. The latter process is essential for detection of photon-induced air showers in the atmosphere. The results of this work are written in [1] in detail.

Рассмотрена модель квантовой электродинамики с нарушением лоренц-инвариантности. Посчитаны вероятности распада фотона и сечения рождения электрон-позитронной пары на фоновом излучении и в кулоновском поле ядра. Последний процесс является ключевым для детектирования фотонов сверхвысоких энергий в атмосфере. Более подробно результаты данной работы изложены в [1].

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INTRODUCTION

The postulate of Lorentz invariance (LI) is one of the cornerstones of modern physics. However, several approaches to quantum gravity (see [2] for a review, and [3]) suggest that deviation from LI can appear at very high energies. In the astrophysical processes the elementary particles often reach energies that vastly exceed those attained in the accelerator experiments. Therefore, these processes provide a unique probe of the particle dynamics at very high energy. The extreme energies ever observed are reached by ultra-high-energy cosmic rays (UHECR). The power of UHECR physics in constraining LV has been extensively discussed in the literature [4, 5].

Most of these studies concentrate on the kinematic effects of LV: appearance of new reactions that are kinematically forbidden in the LI case and the shift of energy thresholds of the known processes. We study in addition dynamical aspects of LV — changes in wave functions of external states, vertices and propagators. We show that for some processes this effect is essential.

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1. THE MODEL

We consider quantum electrodynamics with LV operators of dimensions four and six, assuming that CPT-parity, gauge and rotational symmetries are preserved. The Lagrangian of the model is

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\varkappa\bar{\psi}\gamma^{i}D_{i}\psi + \frac{ig}{M^2}D_{j}\bar{\psi}\gamma^{i}D_{i}D_{j}\psi + \frac{\xi}{4M^2}F_{kj}\partial_{i}^{2}F^{kj}, \quad (1)$$

where $\varkappa$, g and ξ are dimensionless parameters, covariant derivative D_μ is defined in standard way $D_\mu\psi = (\partial_\mu + ieA_\mu)\psi$. Summation over repeated indices with Minkowski metrics is understood.

Consider the free-particle states. The LV terms modify the dispersion relations for photons and electrons or positrons,

$$E_{\gamma}^{2} = k^2 + \frac{\xi k^4}{M^2}, \quad (2)$$

$$E_{e}^{2} = m^2 + p^2(1+2\varkappa) + \frac{2gp^4}{M^2}. \quad (3)$$

Note that in our conventions the velocity of low-energy photon is equal to 1.

Quantizing free electromagnetic field with LV, one obtains sums over polarizations for photons

$$\sum_{a=1,2} \varepsilon_{\mu}^{(a)}\varepsilon_{\nu}^{(a)} \simeq \mathrm{diag}\,(-E_{\gamma}^{2}/k^2, 1, 1, 1) \quad (4)$$

and, choosing nonlocal gauge, photon propagator

$$S_{\mu\nu}(k) = \quad i\left[E^2 - k^2\left(1 + \frac{\xi k^2}{M^2}\right) + i\epsilon\right]^{-1} \mathrm{diag}\left(-\left(1 + \frac{\xi k^2}{M^2}\right), 1, 1, 1\right).$$

Let us turn to electrons. It is convenient for us to use the notations

$$\tilde{p}^{0} = E\ , \qquad \tilde{p}^{i} = p^{i}\left(1 + \varkappa + \frac{gp^2}{M^2}\right). \quad (5)$$

Solving modified Dirac equation we obtain spin sums for electrons and positrons:

$$\sum_{s=1,2} u^{s}(p)\bar{u}^{s}(p) = \gamma^{\mu}\tilde{p}_{\mu} + m, \qquad \sum_{s=1,2} v^{s}(p)\bar{v}^{s}(p) = \gamma^{\mu}\tilde{p}_{\mu} - m,$$

and electron propagator

$$\hat{S}(p) = \quad \frac{i(\gamma^{\mu}\tilde{p}_{\mu} + m)}{\tilde{p}_{\mu}\tilde{p}^{\mu} - m^2 + i\epsilon}.$$

Finally, in this model interaction vertices change. New vertices are drawn in Fig. 1 and described by the following formulas (number of indices coincides with number of photon legs):

$$\mathcal{V}_{1\gamma}^{\mu} = -ie\gamma^{\mu} - ie\delta_{i}^{\mu}\left[\varkappa\gamma^{i} + \frac{g}{M^2}\left(p_{1}^{i}(p_1\cdot\gamma) + p_{2}^{i}(p_2\cdot\gamma) - (p_1\cdot p_2)\gamma^{i}\right)\right], \quad (6)$$

$$\mathcal{V}_{2\gamma}^{\mu\nu} = \frac{ige^2}{M^2}\left[\gamma^{i}(p_2 - p_1)^{j} + \gamma^{j}(p_2 - p_1)^{i} + \delta^{ij}\left((p_2 - p_1)\cdot\gamma\right)\right]\delta_{i}^{\mu}\delta_{j}^{\nu}, \quad (7)$$

$$\mathcal{V}_{3\gamma}^{\mu\nu\lambda} = -\frac{2ige^3}{M^2}\left[\delta_{i}^{\mu}\delta_{j}^{\nu}\delta_{j}^{\lambda} + \delta_{i}^{\nu}\delta_{j}^{\mu}\delta_{j}^{\lambda} + \delta_{i}^{\lambda}\delta_{j}^{\mu}\delta_{j}^{\nu}\right]\gamma^{i}. \quad (8)$$

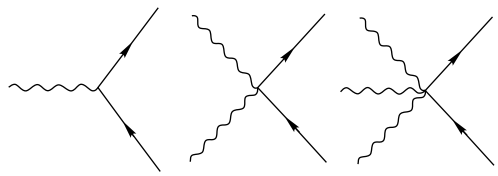

Fig. 1. Interaction vertices in the model (1)

We apply these Feynman rules to compute the rates of several reactions.

2. PROCESSES AND RATES

We start with the elementary process of vacuum photon decay. From the viewpoint of the astrophysical applications, the exact expression for the rate is unnecessary — this process is extremely fast, once kinematically allowed. However, the study of this reaction sets the stage for the study of pair production on background photon and in the Coulomb field.

2.1. Photon Decay. The photon decay

$$\gamma \to e^+ e^- \tag{9}$$

can become allowed in the presence of LV above certain threshold, see, e.g., [5] for the discussion of the kinematics of this reaction. We are interested in computing the rate of the process well above the threshold. In this regime the masses of the electron and positron can be neglected, which considerably simplifies the calculation. The matrix element has the form

$$\mathcal{M} = \bar{u}(p_1)\, \mathcal{V}^{\mu}_{1\gamma}\, v(p_2)\, \varepsilon_\mu,$$

with the vertex given by Eq. (6). The inclusive rate is obtained by taking the square of this expression, summing over the spins of the final states and averaging over the photon polarizations. We work in the frame with the following electron (p_1) and positron (p_2) momenta:

$$p_1^i = (k(1+x)/2, p_\perp, 0), \qquad p_2^i = (k(1-x)/2, -p_\perp, 0).$$

Variable x characterizes asymmetry between electron and positron momenta. It is convenient for us to introduce the following combination:

$$\omega_{\rm LV}(x) = -\varkappa k + \frac{\xi k^3}{2M^2} - \frac{g k^3}{4M^2}(1+3x^2). \tag{10}$$

Then the width of the photon is described by the formula

$$\Gamma_{\gamma\to e^+e^-} = \frac{\alpha}{4}\int dx\,(1+x^2)\,\omega_{\rm LV}(x). \tag{11}$$

Here limits of integration are defined by inequality $\omega_{\rm LV}(x) \geqslant 0$. Depending on LV parameters, several cases appear. The simplest one, $0 \leqslant 2g \leqslant \xi - 2M^2\varkappa/k^2$ or $g \leqslant 0$, $g/2 \leqslant \xi - 2M^2\varkappa/k^2$ yields

$$\Gamma_{\gamma\to e^+e^-} = \alpha k\left[-\frac{2\varkappa}{3} + \frac{k^2}{M^2}\left(\frac{\xi}{3} - \frac{11g}{30}\right)\right].$$

In any case the lifetime of photon is very small. So, for all practical purposes photon decay may be considered as happening instantaneously.

2.2. Pair Production. We now turn to more complicated reactions containing two particles in the initial state. The first reaction is production of an electron–positron pair in the collision of a high-energy photon with a soft photon from an astrophysical background,

$$\gamma\gamma_b \to e^+e^-.$$

Unlike the previous reaction, this process is kinematically allowed in the LI case. Our goal is to find how its cross section is affected by LV.

The diagrams contributing to the required matrix element are shown in Fig. 2. We make our calculations in the limit $k(\omega(1-\cos\theta)+\omega_{\rm LV}) \gg m^2$. Here we make a notation $\omega_{\rm LV} \equiv \omega_{\rm LV}(1)$ (asymmetric production of electron–positron pair). Parameter θ is a collision angle between two photon momenta. After long but straightforward calculations we obtain

$$\sigma_{\gamma\gamma\to e^+e^-} = \frac{\alpha^2\,\pi}{k\omega(1-\cos\theta)}\left[1+\left(1+\frac{2\omega_{\rm LV}}{\omega(1-\cos\theta)}\right)^2\right]\ln\left[\frac{k(\omega(1-\cos\theta)+\omega_{\rm LV})}{m^2}\right]. \quad (12)$$

Putting $\omega_{\rm LV}=0$, one obtains the standard LI result. If $\omega_{\rm LV}$ is positive, vacuum photon decay occurs significantly faster than pair production. In the case $-\omega(1-\cos\theta) < \omega_{\rm LV} < 0$ the cross section (12) differs from the standard relativistic expression by a factor of order one.

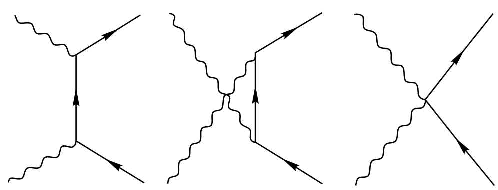

Fig. 2. The diagrams contributing to the matrix element of pair production

2.3. Pair Production in the Coulomb Field. The last reaction we consider is pair production by a high-energy photon in the Coulomb field of a nucleus,

$$\gamma Z \to Ze^+e^-.$$

Here Z denotes the charge of the nucleus. This reaction does not have a threshold and in the standard LI case dominates the interaction of the UHECR photons with the Earth's atmosphere giving rise to the electromagnetic showers that are used for the detection and identification of such photons [6]. The analysis of the changes induced by LV in the cross section of this reaction is therefore crucial to determine the detection efficiency for UHECR photons in LV models.

The process is conveniently represented as a collision of the high-energy photon with a soft virtual photon with purely spacial momentum q from the nucleus' Coulomb field. Thus, it is described by the same diagrams, shown in Fig. 2, as the two-photon collision of the previous subsection. One can distinguish two cases: if we consider pure Coulomb field, momentum q may possess arbitrary value. In a more realistic situation the nucleus is surrounded by atomic electrons that screen its Coulomb field at large distances. Thus, the momentum of virtual photon q is bounded from below by the inverse size of the atom. In both these cases we obtain with logarithmic accuracy,

$$\sigma_{\gamma Z \to Z e^+ e^-} = \frac{4Z^2\alpha^3}{3k|\omega_{\rm LV}|} \ln \frac{k|\omega_{\rm LV}|}{m^2} \begin{cases} \ln \dfrac{k}{|\omega_{\rm LV}|} - \dfrac{1}{2} \ln \dfrac{k|\omega_{\rm LV}|}{m^2} & \text{no screening,} \\ 2\ln \dfrac{1}{\alpha Z^{1/3}} + \dfrac{1}{2} \ln \dfrac{k|\omega_{\rm LV}|}{m^2} & \text{with screening.} \end{cases} \quad (13)$$

As previously, we make our calculation in the limit of small electron mass:

$$|k\omega_{\rm LV}| \gg m^2. \quad (14)$$

The expression (13) must be compared to the standard LI result [7]:

$$\sigma^{\rm LI}_{\gamma Z \to Z e^+ e^-} = \frac{28Z^2\alpha^3}{9m^2} \begin{cases} \ln \dfrac{2k}{m} - \dfrac{109}{42} & \text{no screening,} \\ \ln \dfrac{183}{Z^{1/3}} - \dfrac{1}{42} & \text{with screening.} \end{cases}$$

We see that in the regime (14), LV strongly suppresses the cross section of pair production on a nucleus.

CONCLUSIONS

We have systematically derived the Feynman rules for a model of quantum electrodynamics with violation of Lorentz invariance and have applied them to calculate the rates of several astrophysically relevant processes.

We show that even small LV either allows vacuum photon decay or strongly suppresses the pair production in Coulomb field, so in both these cases any UHE photon cannot ever be detected. Thus, possible future detection of UHE photons can make very strong bound on LV in QED sector.

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