EFFECTS OF FINAL-STATE INTERACTIONS
IN $B^+ \rightarrow D^+_s K^0$ DECAY

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We investigate the effects of final-state-interactions (FSI) contributions in the nonleptonic two-body $B^+ \rightarrow D^+_s K^0$ decay; however, the hadronic decay of $B^+ \rightarrow D^+_s K^0$ is analyzed by using «QCD factorization» (QCDF) method and final-state interaction (FSI). First, the $B^+ \rightarrow D^+_s K^0$ decay is calculated via QCDF method and only the annihilation graphs exist in that method. Hence, the FSI must be seriously considered to solve the $B^+ \rightarrow D^+_s K^0$ decay and the $D^0 \pi^+(D^{0*} \rho^0)$ and $D^+ \eta_c(D^{+*} J/\psi)$ via the exchange of $K^{(*)}$, $K^{0(*)}$ and $D^{(*)}$ mesons are chosen for the intermediate states. To estimate the intermediate states amplitudes, the QCDF method is again used. These amplitudes are used in the absorptive part of the diagrams. The experimental branching ratio of $B^+ \rightarrow D^+_s K^0$ decay is less than $8 \cdot 10^{-4}$ and the predicted branching ratio is $0.23 \cdot 10^{-9}$ in the absence of FSI effects and it becomes $6.74 \cdot 10^{-4}$ when FSI contributions are taken into account.

In the $B^+ \rightarrow D^+_s K^0$ decay the $B^+$ meson has $u$ quark and $\bar{b}$ antiquark and the $D^+_s$, $K^0$ mesons are produced from pair quarks of $c\bar{s}$ and $s\bar{d}$ respectively, so no form factor can be produced between the $B^+$ meson and final states $D^+_s$, $K^0$ mesons, hence this process is pure annihilation type of the decay. For this reason the mentioned decay has a tiny branching ratio in the QCDF approach. However, the hadronic decay of $B^+ \rightarrow D^+_s K^0$ is analyzed by using

\[ E^\text{eff} = \frac{2 \cdot 3 \cdot 4 \cdot 5}{2 \cdot 3 \cdot 4 \cdot 5} \]

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INTRODUCTION

In the $B^+ \rightarrow D^+_s K^0$ decay the $B^+$ meson has $u$ quark and $\bar{b}$ antiquark and the $D^+_s$, $K^0$ mesons are produced from pair quarks of $c\bar{s}$ and $s\bar{d}$ respectively, so no form factor can be produced between the $B^+$ meson and final states $D^+_s$, $K^0$ mesons, hence this process is pure annihilation type of the decay. For this reason the mentioned decay has a tiny branching ratio in the QCDF approach. However, the hadronic decay of $B^+ \rightarrow D^+_s K^0$ is analyzed by using

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Effects of Final-State Interactions in $B^+ \to D_s^+ K^0$ Decay

The next-to-leading-order low-energy effective Hamiltonian is used for the weak-interaction matrix elements and the FSI. The importance of the FSI in hadronic processes has been identified for a long time. Recently, its applications in $D$ and $B$ decays have attracted extensive interest and attention of theorists.

Since the hadronic matrix elements are fully controlled by nonperturbative QCD, the most important problem is how to evaluate them properly. Factorization method enables one to separate the nonperturbative QCD effects from the perturbative parts and to calculate the latter in terms of the field theory order by order. Several factorization approaches have been proposed to analyze $B$-meson decays, such as the naive factorization approach, the QCD factorization approach, the perturbative QCD approach, and Soft-Collinear-Effective Theory; none provided an estimate of the FSI at the hadronic level. These approaches successfully explain many phenomena; however, there are still some problems which are not easy to describe within this framework.

These may be some hints for the need of FSI in $B$ decays. FSI effects are nonperturbative in nature [1]. FSI is one of the ways to solve the nonperturbative QCD for the long-distance case. In many decay modes, the FSI may play a crucial role [2]. In this approach the CKM’s most favored two-body intermediate states are the only ones that have been taken into consideration [3].

The FSI can be considered as a soft rescatter style for certain intermediate two-body hadronic channel $B^+ \to D^0 \pi^+$ decay [4]. Therefore, the FSI are estimated via the one-particle-exchange processes at the hadronic loop level (HLL) as explained in Sec. 3.

As we know, the branching ratio of $B^+ \to D_s^+ K^0$ decay has already been estimated by using the perturbative QCD approach and predicted $2.01 \times 10^{-8}$ [5]. We calculated the $B^+ \to D_s^+ K^0$ decay according to the QCDF method. This process only occurs via annihilation between $b$ and $\bar{u}$. We selected the leading-order Wilson coefficients at the scale $m_b$ [6,7] and obtained the BR ($B^+ \to D_s^+ K^0$) = $0.23 \times 10^{-9}$. The experimental result of this decay is BR ($B^+ \to D_s^+ K^0$) < $8 \times 10^{-4}$. In the FSI, rescattering amplitude can be derived by calculating the absorptive part of triangle diagrams [8]. In the FSI effects, the intermediate states are $D^0 \pi^+$, $D^+ \rho^0$, and $D^+ \eta_c (D^{\ast+} J/\psi)$. We calculated the $B^+ \to D_s^+ K^0$ decay according to the HLL method [9]. In this case, the branching ratio of $B^+ \to D_s^+ K^0$ is $6.74 \times 10^{-4}$.

This paper is organized as follows. We present the calculation of QCDF for the $B^+ \to D_s^+ K^0$ decay in Sec. 1. In Sec. 2, we calculate the amplitudes of the intermediate states. Then, we present the calculation of HLL for the $B^+ \to D_s^+ K^0$ decay in Sec. 3. In Sec. 4, we give the numerical results, and in the last section, we have a short conclusion.

1. QCD FACTORIZATION OF $B^+ \to D_s^+ K^0$ DECAY

In the QCD factorization approach, the $B^+ \to D_s^+ K^0$ decay has annihilation contribution that Feynman diagram in Fig. 1 clearly shows. According to the diagram of Fig. 1, we obtained the annihilation amplitude as

$$A_{\text{QCDF}}(B^+ \to D_s^+ K^0) = i \frac{G_F}{\sqrt{2}} f_B f_K f_{D_s} b_2 V_{ub} V_{cd}^*,$$

(1)
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Fig. 1. Feynman annihilation diagram for $B^+ \rightarrow D_s^+ \bar{K}^0$ decay, the left arrow shows that the Feynman diagram is being read from left side

where

$$b_2 = \frac{C_F}{N_c^2} C_2 A_1^i, \quad A_1^i = 6\pi\alpha_s \left[ 3 \left( X_A - 4 + \frac{\pi^2}{3} \right) + r^K r^{D_s} (X^2_A - 2X_A) \right],$$

$$X_A = (1 + \rho e^{i\phi}) \ln \frac{m_B}{\Lambda_{QCD}}, \quad r^K = \frac{2m_K}{(m_b + m_u)(m_u + m_s)}, \quad r^{D_s} = \frac{2m_{D_s}}{(m_b - m_c)(m_s + m_c)}.$$ (2)

2. AMPLITUDES OF INTERMEDIATE STATES

In this section, before analyzing FSI in the $B^+ \rightarrow D_0^+ \pi^+$ decay, we introduce the factorization approach of intermediate state in detail. In the FSI effects $D^0\pi^0(D^0\rho^0)$, $D^+\pi^0(D^+\rho^0)$ and $D^+\eta_c(D^+J/\psi)$ are chosen for the intermediate states. The effective weak Hamiltonian for $B$ decays consists of a sum of local operators $Q_i$ multiplied by QCDF coefficients $c_i$ and products of elements of the quark mixing matrix [10]. The factorization approach of heavy-meson decays can be expressed in terms of different topologies of various decays mechanism such as tree, penguin and annihilation.

2.1. $B^+ \rightarrow D_0^0\pi^+$ ($D_0^0\rho^0$) Decays. When two intermediate mesons exchange $u$ quark and two final-state mesons exchange $s$ quark, the $D_0^0$ and $\pi^0(\rho^0)$ mesons are produced for intermediate state via exchange mesons of $K^+$. Feynman diagrams for the $B^+ \rightarrow D_0^0\pi^+$ ($D_0^0\rho^0$) decays are shown in Fig. 2, and the amplitudes read

$$A(B^+ \rightarrow D_0^0\pi^+) = i \frac{G_F}{\sqrt{2}} f_\pi \{ F^{B-D}(m_\pi^2)(m_B^2 - m_D^2)n_1 V_{cb} V_{ud} + f_B f_D b_2 V_{ub} V_{cd} \},$$

$$A(B^+ \rightarrow D_0^0\rho^+) = i \frac{G_F}{\sqrt{2}} \left\{ f_\rho m_\rho \left\{ (\epsilon_1 \cdot \epsilon_2)(m_B + m_{D^*}) + A_1^{B-D^*}(m_\rho^2) \right\} a_1 V_{cb} V_{ud} + f_B f_{D^*} (\rho_1 b_2 V_{ub} V_{cd} \right\}.$$ (3)

Fig. 2. Feynman diagrams for the $B^+ \rightarrow D_0^0\pi^+$ decay
2.2. $B^+ \rightarrow D^+\pi^0(D^{*+}\rho^0)$ Decays. If $d$ quark is exchanged between two intermediate mesons and $s$ quark is exchanged between two final-state mesons, the $D^{*+(s)}$ and $\pi^+(\rho^+)$ mesons are produced for intermediate state via $K^{0(s)}$ as the exchange mesons. Feynman diagrams for $B^+ \rightarrow D^+\pi^0(D^{*+}\rho^0)$ decays are shown in Fig. 3, and the amplitudes read

\[
A(B^+ \rightarrow D^+\pi^0) = i\frac{G_F}{\sqrt{2}}f_D \{F_{D^0\pi^+}(m_D^2)(m_B^2 - m_{\pi}^2) + f_B f_\rho b_\rho V_{cd}\},
\]

\[
A(B^+ \rightarrow D^{*+}\rho^0) = i\frac{G_F}{\sqrt{2}}f_D \{2A_{D^*\rho}(m_D^2) + \epsilon_1 \epsilon_2 (m_B + m_\rho)A_{D^*\rho}(m_D^2) - \}
\]

\[-(\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B)\frac{2A_{D^*\rho}(m_D^2)}{m_B + m_\rho}\}a_1V_{ub}V_{cd}^* + i\frac{G_F}{\sqrt{2}}f_D f_\rho b_\rho V_{cd}^*.
\]

(4)

Fig. 3. Feynman diagrams for the $B^+ \rightarrow D^+\pi^0$ decay

2.3. $B^+ \rightarrow D^+\eta_c(D^{*+}\psi)$ Decays. If $c$ quark is exchanged between two intermediate mesons and $s$ quark is exchanged between two final-state mesons, the $D^{*+(s)}$ and $\eta_c(\psi)$ mesons are produced for intermediate state via $D^*_s(\psi)$ as the exchange mesons. Feynman diagram for $B^+ \rightarrow D^+\eta_c(D^{*+}\psi)$ decays is shown in Fig. 4, and the amplitudes read

\[
A(B^+ \rightarrow D^+\eta_c) = i\frac{G_F}{\sqrt{2}}f_D f_\eta b_\eta V_{cd},
\]

\[
A(B^+ \rightarrow D^{*+}\psi) = i\frac{G_F}{\sqrt{2}}f_D f_\psi b_\psi V_{cd}^*.
\]

(5)

Fig. 4. Feynman annihilation diagram for the $B^+ \rightarrow D^+\eta_c$ decay
3. FINAL-STATE INTERACTION OF $B^+ \to D_s^+ K^0$ DECAY

When the FSI for decay is calculated, two-body intermediate states such as $D^0 \pi^0(D^0\rho^0)$, $D^+ \pi^0(D^{*+}\rho^0)$ and $D^+\eta_c(D^{*+} J/\psi)$ are produced. The absorptive part of the HLL diagrams can be calculated with the following formula:

$$\text{Abs } A(B(p_B) \to M(p_1)M(p_2) \to M(p_3)M(p_4)) = \frac{1}{2} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \times$$

$$\times (2\pi)^4 \delta^4(p_B - p_1 - p_2) A(B \to M_1 M_2) G(M_1 M_2 \to M_3 M_4),$$

for which both intermediate mesons $(M_1, M_2)$ are pseudoscalar. And

$$\text{Abs } A(B(p_B) \to M(p_1)M(p_2) \to M(p_3)M(p_4)) = i \frac{G_F}{2\sqrt{2}} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \times$$

$$\times (2\pi)^4 \delta^4(p_B - p_1 - p_2) f_V m_V V_{CKM} \left[(\epsilon^*_1 \cdot \epsilon^*_2)(m_B + m_1) A_{BM}^0 (m_2^2) - \right.$$

$$\left. - (\epsilon^*_1 \cdot p_B)(\epsilon^*_2 \cdot p_B) \frac{2 A_{BM}^0 (m_2^2)}{m_B + m_1}\right] G(M_1 M_2 \to M_3 M_4),$$

in which both mesons are vector. Also $G(M_1 M_2 \to M_3 M_4)$ involves hadronic vertices factor, which are defined as $[1,11,12]$

$$\langle D_s(p_3) K^*(\epsilon_2, p_2) | i \mathcal{L} | D(p_1) \rangle = - ig_{DD_K^*} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle \pi(p_3) K^*(\epsilon_2, p_2) | i \mathcal{L} | K(p_1) \rangle = - ig_{KK_K^*} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle D_s(p_3) K(p_2) | i \mathcal{L} | D^*(\epsilon_1, p_1) \rangle = - ig_{DsD} \epsilon_1 \cdot p_2,$$

$$\langle K(p_3) \rho(\epsilon_2, p_2) | i \mathcal{L} | K(p_1) \rangle = - ig_{KK} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle D_s(p_3) K^*(\epsilon_2, p_2) | i \mathcal{L} | D^*(\epsilon_1, p_1) \rangle = - ig_{DD_K^*} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle \pi(p_3) K^*(\epsilon_2, p_2) | i \mathcal{L} | K(p_1) \rangle = - ig_{KK_K^*} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle D_s(p_3) \rho(\epsilon_2, p_2) | i \mathcal{L} | D_s(p_1) \rangle = - ig_{DD_s} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle K(p_3) \psi(\epsilon_2, p_2) | i \mathcal{L} | K(p_1) \rangle = - ig_{KK_s} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle D_s(p_3) \psi(\epsilon_2, p_2) | i \mathcal{L} | D_s(p_1) \rangle = - ig_{DD_s} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle \psi(\epsilon, p_3) K(p_2) | i \mathcal{L} | D^*(\epsilon_1, p_1) \rangle = - ig_{DD_s} \epsilon_2 \cdot (p_1 + p_3),$$

$$\langle D_s(\epsilon - 3, p_3) K(p_2) | i \mathcal{L} | D^*(\epsilon_1, p_1) \rangle = - ig_{DD_s} \epsilon_2 \cdot (p_1 + p_3).$$


The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [1,13]

\[
\text{Dis } M(m_B^2) = \frac{1}{\pi} \int_{s}^{\infty} \frac{\text{Abs } M(s')}{s' - m_B^2} ds',
\]

(9)

where \(s'\) is the square of the momentum carried by the exchanged particle and \(s\) is the threshold of intermediate states, in this case \(s \sim m_B^2\). Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integrations.

3.1. Final-State Interaction in the \(B^+ \rightarrow D^0 \pi^+ \rightarrow D_s^+ \bar{K}^0\) Decay. The quark model and the hadronic level diagrams for the \(B^+ \rightarrow D^0 \pi^+ \rightarrow D_s^+ \bar{K}^0\) decay are shown in Figs. 5 and 6.

In this framework we choose the \(t\)-channel one-particle-exchange processes. The absorptive part of the diagram (a) shown in Fig. 6, the amplitude of the mode \(B^+ \rightarrow D^0(p_1)\pi^+(p_2) \rightarrow D_s^+(p_3)\bar{K}^0(p_4)\) with the exchange of the \(K^{*+}\), is given by

\[
\text{Abs } (6a) = \frac{1}{2} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2)(-ig_{DD,K^*}) \epsilon_q \cdot (p_1 + p_3) \times (-ig_{KK^*}) \epsilon_q \cdot (p_2 + p_4) A(B^+ \rightarrow D^0\pi^+) \frac{F^2(q^2,m_i^2)}{T} =
\]

\[
= -\int \frac{|p_1| d(\cos \theta)}{16\pi m_B} g_{DD,K^*} g_{KK^*} A(B^+ \rightarrow D^0\pi^+) \frac{F^2(q^2,m_i^2)}{T} H, \tag{10}
\]

Fig. 5. Quark level diagram for \(B^+ \rightarrow D^0 \pi^+ \rightarrow D_s^+ \bar{K}^0\) decay

Fig. 6. HLL diagrams for long-distance \(t\)-channel contribution to \(B^+ \rightarrow D^0 \pi^+(D^0\rho^+) \rightarrow D_s^+ \bar{K}^0\)
where $\theta$ is the angle between $p_1$ and $p_3$, $q$ and $m_i$ are the momentum and mass of the exchange $K^{++}$ meson, respectively, and

$$H = \epsilon_q \cdot (p_1 + p_3) \epsilon_{q'} (p_2 + p_4) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_K^2}\right) (p_1 + p_3) \mu (p_2 + p_4) \nu =$$

$$= -p_1 \cdot p_2 - p_1 \cdot p_4 - p_2 \cdot p_3 - p_3 \cdot p_4 + \frac{(m_1^2 - m_3^2)(m_4^2 - m_2^2)}{m_K^2},$$

$$T = q^2 - m_i^2 = m_D^2 + m_{D_s}^2 - m_{K^*}^2 - 2E_D E_{D_s} + 2|\mathbf{p}_D||\mathbf{p}_{D_s}| \cos \theta,$$

and $F(q^2, m_i^2)$ is the form factor defined to take care of the off-shell character of the exchange particles, defined as [14]

$$F(q^2, m_i^2) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2}\right)^n. \quad (12)$$

The form factor (i.e., $n = 1$) is normalized to unity at $q^2 = m_i^2$. The $m_i$ and $q$ are the physical parameters of the exchange particle and $\Lambda$ is a phenomenological parameter. It is obvious that for $q^2 \to 0$, $F(q^2, m_i^2)$ becomes a number. If $\Lambda \gg m_i$, then $F(q^2, m_i^2)$ turns to be unity, whereas as $q^2 \to \infty$ the form factor approaches zero and the distance becomes small and the hadron interaction is no longer valid. Since $\Lambda$ should not be far from the $m_i$ and $q$, we choose

$$\Lambda = m_i + \eta \Lambda_{QCD}. \quad (13)$$

Likewise, for diagrams 6, b and 6, c, the amplitudes of the $B^+ \to D^{0*}(\epsilon_1, p_1) \rho^+(\epsilon_2, p_2) \to D^+_s(p_3) K^0(p_4)$ (where $K^+$ and $K^{++}$ are exchanged at t-channel) are given by

$$\text{Abs} (6b) = i \frac{G_F}{2\sqrt{2}} \int \frac{d^3 p_1}{2E_1(2\pi)^3} \frac{d^3 p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2)(-ig_{D^* D K})(\epsilon_1 \cdot q) \times$$

$$\times (-ig_{KK^*}) \epsilon_2 \cdot (p_4 + q) \left\{ f_{\rho}(m_\rho) \left[ (\epsilon_1 \cdot \epsilon_2)(m_B + m_{D^*})A_1^{B-D^*}(m_\rho^2) - \right.$$

$$\left. - (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B)\frac{2A_2^{B-D^*}(m_\rho^2)}{m_B + m_{D^*}} \right] a_1 V_{cb} V_{ud}^* + f_B f_{D^*} f_{B^*} V_{ub} V_{cd} \right\} \frac{F^2(q^2, m_\rho^2)}{T_1} =$$

$$= -i \frac{G_F}{8\sqrt{2\pi} m_B} g_{D^* D K} g_{KK^*} \int_{-1}^1 |\mathbf{p}_1| d(|\cos \theta|) \left\{ f_{\rho}(m_\rho) \left[ (m_B + m_{D^*})A_1^{B-D^*}(m_\rho^2) H_1 - \right.$$

$$\left. - \frac{2A_2^{B-D^*}(m_\rho^2)}{m_B + m_{D^*}} H_2 \right] a_1 V_{cb} V_{ud}^* + f_B f_{D^*} f_{B^*} V_{ub} V_{cd} H_3 \right\} \frac{F^2(q^2, m_\rho^2)}{T_1}, \quad (14)$$
$$H_1 = (\epsilon_1 \cdot \epsilon_2)(\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) =$$

$$= p_3 \cdot p_4 - \frac{(p_2 \cdot p_4)(p_2 \cdot p_3)}{m_2^2} - \frac{(p_1 \cdot p_3)(p_1 \cdot p_4)}{m_1^2} + \frac{(p_1 \cdot p_3)(p_1 \cdot p_2)(p_2 \cdot p_4)}{m_1^2 m_2^2},$$

$$H_2 = (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B)(\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) =$$

$$= \left[ -p_3 \cdot p_B + \frac{(p_1 \cdot p_B)(p_1 \cdot p_3)}{m_1^2} \right] \left[ -p_4 \cdot p_B + \frac{(p_2 \cdot p_B)(p_2 \cdot p_4)}{m_2^2} \right],$$

$$H_3 = (\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) = \left( \frac{E_3 |p_1| - E_1 |p_3| \cos \theta}{m_B |p_1|} \right) \left( \frac{E_4 |p_2| - E_2 |p_4| \cos \theta}{m_B |p_4|} \right),$$

$$T_1 = m_D^2 + m_\rho^2 - m_K^2 - 2E_D \cdot E_\rho + 2|p_D| |p_\rho| \cos \theta.$$
As the bridge between the dispersive part of FSI amplitude and the absorptive part, the dispersion relation is

$$\text{Dis}_S(m_B^2) = \frac{1}{\pi} \int_{s'}^\infty \frac{\text{Abs}_S(s') + \text{Abs}_D(s') + \text{Abs}_C(s')}{s' - m_B^2} ds'.$$

(18)

### 3.2. Final-State Interaction in the $B^+ \rightarrow D^+ \pi^0(D^{**+}\rho^0) \rightarrow D_s^+ \bar{K}^0$ Decay.

The quark model diagram for the $B^+ \rightarrow D^+ \pi^0(D^{**+}\rho^0) \rightarrow D_s^+ \bar{K}^0$ decay is shown in Fig. 7 and the hadronic level diagrams are shown in Fig. 8. The amplitude of the mode $B^+ \rightarrow D^+(p_1)\pi^0(p_2) \rightarrow D_s^+(p_3)\bar{K}^0(p_4)$ (where $K^{0*}$ is exchanged at $t$-channel) is given by

$$\text{Abs}(8a) = \frac{1}{2} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2)(-ig_{DD_sK*})\epsilon_q \cdot (p_1 + p_3) \times $$

$$\times (-ig_{KK*})\epsilon_q \cdot (p_2 + p_4) A(B^+ \rightarrow D^+\pi^0) \frac{F^2(q^2, m_{K*}^2)}{T} =$$

$$= -\frac{1}{16\pi m_B} \left| p_1 |d(cos \theta)\right| g_{DD_sK*} g_{KK*} A(B^+ \rightarrow D^+\pi^0) \frac{F^2(q^2, m_{K*}^2)}{T} H, \quad (19)$$

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**Fig. 7.** Quark level diagram for the $B^+ \rightarrow D^+ \pi^0 \rightarrow D_s^+ \bar{K}^0$ decay

**Fig. 8.** HLL diagrams for long-distance $t$-channel contribution to $B^+ \rightarrow D^+ \pi^0(D^{**+}\rho^0) \rightarrow D_s^+ \bar{K}^0$
The dispersion relation is

\[ \text{Dis} \, 8(m_B^2) = \frac{1}{\pi} \int_s^\infty \frac{\text{Abs}_{8a}(s') + \text{Abs}_{8b}(s') + \text{Abs}_{8c}(s')}{s' - m_B^2} ds'. \]
3.3. Final-State Interaction in the $B^+ \rightarrow \eta_c D^+ (J/\psi D^{**}) \rightarrow D_s^+ K^0$ Decay. The quark model and the hadronic level diagrams for the $B^+ \rightarrow \eta_c D^+ (J/\psi D^{**}) \rightarrow D_s^+ K^0$ decay are shown in Figs. 9 and 10. The amplitude of the mode $B^+ \rightarrow \eta_c(p_1) D^+(p_2) \rightarrow D_s^+(p_3) K^0(p_4)$ with the exchange of the $D_s^{**}$ is given by

$$\text{Abs}(10a) = \frac{1}{2} \int \frac{d^3 p_1}{2E_1(2\pi)^3} \frac{d^3 p_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2)(-ig_D s \eta_c) \epsilon_q \cdot p_1 \times$$

$$\times (-ig_D D_s \eta_c) \epsilon_q \cdot p_4 A(B^+ \rightarrow \eta_c D^+) \frac{F^2(q^2, m_{D_s}^2)}{T_3} =$$

$$= - \frac{1}{16\pi m_B} g_{D_s \eta_c} g_{D_s K} A(B^+ \rightarrow \eta_c D^+) \frac{F^2(q^2, m_{D_s}^2)}{T_3} H_7, \quad (23)$$

where

$$H_7 = \epsilon_q \cdot p_1 \epsilon_q \cdot p_4 = \left( -g_{\mu \nu} + \frac{q_\mu q_\nu}{m_{D_s}^2} \right) \frac{p_1^\mu p_4^\nu}{m_{D_s}^2} = -p_1 \cdot p_4 + \frac{m_{D_s}^2 (p_1 \cdot p_3 - p_2 \cdot p_4)}{m_{D_s}^2},$$

$$T_3 = q^2 - m_q^2 = m_{D_s}^2 + m_{\eta_c}^2 - m_{D_s}^2 - 2E_D E_{\eta_c} + 2|p_D| |p_{\eta_c}| \cos \theta. \quad (24)$$

Fig. 9. Quark level diagram for the $B^+ \rightarrow \eta_c D^+ \rightarrow D_s^+ K^0$ decay

Fig. 10. HLL diagrams for long-distance $t$-channel contribution to $B^+ \rightarrow \eta_c D^+ (J/\psi D^{**}) \rightarrow D_s^+ K^0$
The amplitudes of the mode \( B^+ \to J/\psi(\epsilon_1, p_1)D^{+*}(\epsilon_2, p_2) \to D_s^+(p_3)\bar{K}^0(p_4) \) with the exchange of the \( D_s^+ \) and \( D_s^{++} \) mesons are given by

\[
\text{Abs}(10b) = \frac{G_F}{2\sqrt{2}} \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{F^2(q^2, m_{D_s}^2)}{2T_4} \delta(p_B - p_1 - p_2) (-ig_{D_s D_s} \psi)(-\epsilon_1) \cdot (p_1 - q) (-ig_{D_s D_s} \psi) \epsilon_2 \cdot p_4 f_B f_{J/\psi} b_2 V_{ub} V_{cd}^* = \]

\[
= \frac{i G_F}{16\sqrt{2\pi} m_B} g_{D_s D_s} \epsilon_{G} D_s \epsilon_{J/\psi} b_2 V_{ub} V_{cd}^* \int_{-1}^{1} |p_1| |d(\cos \theta)| \frac{F^2(q^2, m_{D_s}^2)}{2T_4} H_8, \tag{25}
\]

where

\[
H_8 = (\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) \left( \frac{E_3 |p_1| - E_1 |p_3| \cos \theta}{m_B |p_1|} \right) \left( \frac{E_4 |p_2| - E_2 |p_4| \cos \theta}{m_B |p_4|} \right),
\]

\[
T_4 = q^2 - m_i^2 = m_{D_s}^2 + m_{J/\psi}^2 - m_{D_s}^2 - 2E_{D_s} E_{J/\psi} + 2|p_{D_s}||p_{J/\psi}| \cos \theta.
\]

The dispersion relation is

\[
\text{Dis} 10(m_B^2) = \frac{1}{\pi} \int_{s}^{\infty} \frac{\text{Abs}_{10a}(s') + \text{Abs}_{10b}(s') + \text{Abs}_{10c}(s')}{s' - m_B^2} ds'. \tag{29}
\]

The decay amplitude of \( B^+ \to D_s^+ \bar{K}^0 \) via the HLL diagrams is

\[
A(B^+ \to D_s^+ \bar{K}^0) = \text{Abs}(6a) + \text{Abs}(6b) + \text{Abs}(6c) + \text{Dis} 6 + \text{Abs}(8a) + \text{Dis} 8 + \text{Abs}(10a) + \text{Abs}(10b) + \text{Abs}(10c) + \text{Dis} 10. \tag{30}
\]
4. NUMERICAL RESULTS

The Wilson coefficients $c_i$ have been calculated in different schemes. In this paper we will use consistently the naive dimensional regularization (NDR) scheme. The values of $c_i$ at $\mu = m_b$ with the next-to-leading order (NLO) QCD corrections are given by [7, 15]

$$
\begin{align*}
  c_1 &= 1.117, & c_2 &= -0.257, \\
  c_3 &= 0.017, & c_4 &= -0.044, \\
  c_5 &= 0.011, & c_6 &= -0.056, \\
  c_7 &= -1 \cdot 10^{-5}, & c_8 &= 5 \cdot 10^{-4}, \\
  c_9 &= -0.010, & c_{10} &= 0.002.
\end{align*}
$$

(31)

The relevant input parameters used are: $m_B = 5.279$, $m_K = 0.49$, $m_{K^*} = 0.89$, $m_\rho = 0.78$, $m_{\pi} = 0.139$, $m_{D} = 1.87$, $m_{D^*} = 2.01$, $m_{D_s} = 1.97$, $m_{D_{s1}} = 2.11$, $m_{J/\psi} = 3.1$, $m_{\eta_c} = 3.0$, $m_\phi = 4.20$, $m_\omega = 1.27$, $m_\eta = 0.104$, $f_B = 0.176$, $f_K = 0.16$, $f_{\pi} = 0.13$, $f_D = 0.223$, $f_{\eta_c} = 0.35$, $f_{J/\psi} = 0.416$, $f_\rho = 0.216$, $f_{D^*} = 0.23$, $f_{D_s} = 0.294$ [16] (values of masses and decay constants are in GeV); $r^K_\rho = 1.09$, $r^{D_s}_D = 1.93$, $\phi = -55^\circ(PP)$, $\phi = -20^\circ(PV)$, $\phi = -70^\circ(VP)$, $\rho = 0.5$, $\Lambda_{QCD} = 0.225$ GeV, $C_F = 4/3$, $\alpha_s = 0.2244$, $N = 3$, $G_F = 1.166 \cdot 10^{-5}$, $V_{ub} = 0.0043$, $V_{ud} = 0.974$, $V_{cb} = 0.042$, $V_{cd} = 0.230$ [16]; $F_{B^{*}\pi} = 0.33$, $A_{1}^{B^{-}\rho} = 0.28$ [17]; $F_{B^{*}D} = 0.6$, $A_{1}^{B^{-}D^*} = 0.8$, $A_{2}^{B^{-}D^*} = 0.99$ [15]; $g_{DD,K^*} = \sqrt{\frac{m_D}{m_D}}g_{DD\rho} = 3.83$, $g_{D^*D,K^*} = \sqrt{\frac{m_D}{m_D}}g_{D^*D\rho} = 2.79$, $g_{D^*D,K} = \sqrt{\frac{m_D}{m_D}}g_{D^*D\pi} = 3$, $g_{D^*D,K^*} = \sqrt{\frac{m_D}{m_D}}g_{D^*D^*\pi} = 6.6$, $g_{D^*D,K} = \sqrt{\frac{m_D}{m_D}}g_{D^*D^*\pi} = 12.83$ [1] ($g_{D^*D^*\pi} = 12.5$ [3, 8]); $g_{KK,K^*} = 4.6$, $g_{KK^*} = 4.28$ [18]; $g_{KK^*\rho} = 6.48$ [19]; $g_{DD^*,\eta_c} = m_{\eta_c}/f_{\eta_c} = 8.57$ [20]; $g_{DD,\psi} = 7.71$, $g_{DD^*,\psi} = 8.64$ [14]. Using the parameters relevant for the $B^+ \rightarrow D^+_s \bar{K}^0$ decay, we get flavor averaged branching ratio for the QCDF method as

$$
\text{BR}(B^+ \rightarrow D^+_s \bar{K}^0) = 0.23 \cdot 10^{-9}.
$$

(32)

After calculating the amplitudes of the intermediate states, we obtain:

$$
\begin{align*}
  A(B^+ \rightarrow D^0 \pi^+) &= 0.56 \cdot 10^{-6}, \\
  A(B^+ \rightarrow D^0 \pi^0) &= 0.74 \cdot 10^{-7}, \\
  A(B^+ \rightarrow D^+ \eta_c) &= 0.37 \cdot 10^{-7}.
\end{align*}
$$

(33)

Now, according to FSI, we can obtain the branching ratios of the $B^+ \rightarrow D^+_s \bar{K}^0$ decay with different values of $\eta$ which are shown in table.

The branching ratio of the $B^+ \rightarrow D^+_s \bar{K}^0$ decay with $\eta = 1$–2.5 and experimental data (in units of $10^{-4}$)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>Experiment [16]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.36</td>
</tr>
<tr>
<td>1.5</td>
<td>3.36</td>
</tr>
<tr>
<td>2</td>
<td>6.74</td>
</tr>
<tr>
<td>2.5</td>
<td>$&lt; 8$</td>
</tr>
</tbody>
</table>
CONCLUSIONS

In this work, we have calculated the contribution of the $t$-channel FSI, i.e., inelastic rescattering processes to the branching ratio of the $B^+ \to D_s^+ K^0$ decay. For evaluating the FSI effects, we have only considered the absorptive and dispersive parts of the HLL, because both the hadrons which were produced via the weak interaction were on their mass shells.

We have calculated the branching ratio of the $B^+ \to D_s^+ K^0$ decay by using QCDF and FSI. The experimental result of this decay is $\text{BR}(B^+ \to D_s^+ K^0) < 8 \cdot 10^{-4}$ [16]. The branching ratio of this decay in [5] was $2.01 \cdot 10^{-8}$. According to QCDF and FSI, our results are $\text{BR}(B^+ \to D_s^+ K^0) = 0.23 \cdot 10^{-9}$ and $6.74 \cdot 10^{-4}$, respectively.

There exist some phenomenological parameters in our calculations on FSI such as $\eta$ in (13) and many other sources of uncertainties, for example, the coupling constant $g_{D^* D_s k}$ etc., the neglected subdominant contributions in the FSI, the estimate of pure QCDF contribution, etc. We have introduced the phenomenological parameter $\eta$; that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates. For a given exchanged particle, we have used $\eta = 1 - 2.5$. If $\eta = 2.5$ is selected, the branching ratio of the $B^+ \to D_s^+ K^0$ decay approaches the upper bound of experimental result.

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