Письма в ЭЧАЯ. 2014. Т. 11, № 7(191). С. 1525–1528

KALUZA–KLEIN REDUCTION OF SUPERSYMMETRIC FIERZ–PAULI

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We motivate the idea of constructing a supersymmetric model of New Massive Gravity (NMG) without higher derivatives. We show how to construct explicitly the linearized, massive, off-shell, spin-2, three-dimensional $\mathcal{N}=1$ supermultiplet. We comment about its massless limit.

PACS: 11.30.Pb; 12.60.Jv

1. MOTIVATION

New Massive Gravity (NMG) is a higher-derivative extension of three-dimensional (3D) Einstein–Hilbert gravity that describes massive gravitons. It is interesting to study this model because it describes unitarily two massive degrees of freedom of helicity $+2$ and -2.

At the linearized level, using auxiliary fields, the action of this NMG model can be explicitly split into a (non-propagating) massless spin-2 Einstein–Hilbert theory and a 3D massive spin-2 Fierz–Pauli model. The introduction of auxiliary fields helps to lower the number of derivatives.

It is also possible to construct a linearized supersymmetric NMG model (SNMG) without higher derivatives. For this case we need a 3D massless and a 3D massive spin-2 supermultiplet. The massless multiplet is already known [1], but the massive multiplet, the one that is constructed in detail just recently [2], was not.

We will show the procedure explicitly for the easier spin-1 case in Sec. 2. In Sec. 3 we will show the results for the Fierz–Pauli case. Along the construction of the multiplets we will look in detail at the massless limit. Finally, we conclude in the final section.

2. SUPERSYMMETRIC PROCA

The starting point is the four-dimensional (4D) $\mathcal{N}=1$ Maxwell multiplet which consists of a vector, a 4-component Majorana spinor and a real auxiliary scalar $\{\hat{V}_{\hat{\mu}}, \hat{\psi}, \hat{F}\}$. We put hats on the fields to signify that they are in 4D. The supersymmetry rules of these fields are

$$\delta\hat{V}_{\hat{\mu}} = -\bar{\epsilon}\Gamma_{\hat{\mu}}\hat{\psi} + \partial_{\hat{\mu}}\hat{\Lambda}, \quad \delta\hat{\psi} = \frac{1}{8}\Gamma^{\hat{\mu}\hat{\nu}}\hat{F}_{\hat{\mu}\hat{\nu}}\epsilon + \frac{1}{4}i\Gamma_5\hat{F}\epsilon, \quad \delta\hat{F} = i\bar{\epsilon}\Gamma_5\Gamma^{\hat{\mu}}\partial_{\hat{\mu}}\hat{\psi}, \tag{1}$$

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where $\hat{F}_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}}\hat{V}_{\hat{\nu}} - \partial_{\hat{\nu}}\hat{V}_{\hat{\mu}}$, ϵ is the supersymmetry spinor parameter and $\hat{\Lambda}$ is the gauge parameter. Now we wish to reduce the theory to 3D by compactifying one of the coordinates on a circle, to do so, we will split the 4D coordinates as $x^{\hat{\mu}} = (x^{\mu}, x^3)$, and expand all the fields of the Maxwell multiplet as a Fourier series. The compactified coordinate will be x^3 and the unhatted coordinates and fields are in the lower-dimensional space-time. After doing the expansion, one gets an infinite number of fields in 3D labelled by the Fourier mode number n. The modes with $n = 0$ are massless, while the modes with $n \neq 0$ are associated with massive fields. Here we will be mainly interested in the first massive modes with $n = 1$.

Imposing the reality condition on the 4D bosonic fields and on the gauge parameters, the modes with numbers 1 and -1 are complex conjugates of each other, so one can write the real and the imaginary parts of them as a combination (the sum and the difference) of the first massive mode and its complex conjugate. In a similar way, the Majorana condition of the 4D spinor $\hat{\psi}$ for the first mode gives two Majorana spinors which are in 3D and have 4 components. Notice that, after doing the Fourier expansion, we doubled the number of fields; however, we are going to truncate the resulting multiplet to an $\mathcal{N} = 1$ multiplet. The truncation is given by setting half of the fields to zero and also half of the symmetries.

The last step consists in gauging away the Stückelberg symmetries that remain due to the dimensional reduction of the gauge symmetries. Taking this into account, we obtain the final form of the supersymmetry transformation rules of the 3D, $\mathcal{N} = 1$ massive theory which corresponds to the well-known Proca theory in 3D:

$$\delta V_{\mu} = -\bar{\epsilon}\gamma_{\mu}\psi + \frac{1}{4m}\bar{\epsilon}\partial_{\mu}\chi, \quad \delta\psi = \frac{1}{8}\gamma^{\mu\nu}F_{\mu\nu}\epsilon,$$

$$\delta F = -\bar{\epsilon}\gamma^{\mu}\partial_{\mu}\chi + 4m\bar{\epsilon}\psi, \quad \delta\chi = m\gamma^{\mu}\epsilon V_{\mu} - \frac{1}{4}F\epsilon, \tag{2}$$

$$I_{\text{Proca}} = \int d^3x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - 2\bar{\psi}\,\partial\!\!\!/\,\psi - \frac{1}{8}\bar{\chi}\,\partial\!\!\!/\,\chi + \frac{1}{32}F^2 - \frac{1}{2}\,m^2 V_{\mu}V^{\mu} + m\,\bar{\psi}\chi \right).$$

To study the massless limit, we will couple the Proca system to a conjugate multiplet $\{J_{\mu}, \mathcal{J}_{\psi}, \mathcal{J}_{\chi}, J_F\}$ and sum to the Proca action an interaction part which describes the coupling between the Proca multiplet and the conjugate multiplet

$$I_{\text{int}} = V^{\mu}J_{\mu} + \bar{\psi}\,\mathcal{J}\!\!\!/_{\psi} + \bar{\chi}\,\mathcal{J}\!\!\!/_{\chi} + FJ_F. \tag{3}$$

Requiring that the interaction part is separately invariant under supersymmetry, it is possible to determine the transformation rules of the conjugate multiplet. Taking the massless limit in the complete action and in the transformation rules is non-trivial due to a factor of $1/m$ that will appear in them; therefore, it is necessary to work in the formulation where the Stückelberg symmetry is not yet fixed, so we make the redefinition

$$V_{\mu} = \tilde{V}_{\mu} + \frac{1}{m}\partial_{\mu}\phi, \tag{4}$$

to return to the multiplet and the action where the massless limit is well-defined. To fix the interaction part, we will impose some constraints over the currents. The massless limit is now everywhere well-defined and the transformation rules reduce to the rules of a massless vector and a massless scalar. The vector multiplet action is coupled to a spin-1 current multiplet:

$$I = \int d^3x \left[\left(-\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - 2\bar{\psi}\partial\!\!\!/\,\psi + \tilde{V}^{\mu}J_{\mu} + \bar{\psi}\,\mathcal{J}\!\!\!/_{\psi} \right) - \frac{1}{2}\left(\partial_{\mu}\phi\,\partial^{\mu}\phi + \frac{1}{4}\bar{\chi}\partial\!\!\!/\,\chi - \frac{1}{16}F^2 \right) \right].$$

3. SUPERSYMMETRIC FIERZ–PAULI

Now we are ready to move on to the Fierz–Pauli case. Our starting point is the off-shell 4D $\mathcal{N}=1$, massless spin-2 multiplet which consists of a supersymmetric tensor, a graviton, an auxiliary vector and two auxiliary scalars $\{\hat{h}_{\hat{\mu}\hat{\nu}},\ \hat{\psi}_{\hat{\mu}},\ \hat{A}_{\hat{\mu}},\ \hat{M}, \hat{N}\}$. The supersymmetry rules (with constant ϵ and gauge parameters $\hat{\Lambda}_\mu$ and $\hat{\eta}$) are

$$
\begin{gathered}
\delta \hat{h}_{\hat{\mu}\hat{\nu}} = \bar{\epsilon}\,\Gamma_{(\hat{\mu}}\,\hat{\psi}_{\hat{\nu})} + \partial_{(\hat{\mu}}\,\hat{\Lambda}_{\hat{\nu})},\\
\delta\hat{\psi}_{\hat{\mu}} = -\frac{1}{4}\Gamma^{\hat{\rho}\hat{\lambda}}\,\partial_{\hat{\rho}}\hat{h}_{\hat{\lambda}\hat{\mu}}\epsilon - \frac{1}{12}\Gamma_{\hat{\mu}}(\hat{M}+i\,\Gamma_5\hat{N})\,\epsilon + \frac{1}{4}i\,\hat{A}_{\hat{\mu}}\Gamma_5\epsilon - \frac{1}{12}i\,\Gamma_{\hat{\mu}}\Gamma^{\hat{\rho}}\hat{A}_{\hat{\rho}}\Gamma_5\epsilon + \partial_{\hat{\mu}}\hat{\eta},\\
\delta\hat{M} = -\bar{\epsilon}\,\Gamma^{\hat{\rho}\hat{\lambda}}\,\partial_{\hat{\rho}}\hat{\psi}_{\hat{\lambda}}, \quad \delta\hat{N} = -i\,\bar{\epsilon}\,\Gamma_5\,\Gamma^{\hat{\rho}\hat{\lambda}}\,\partial_{\hat{\rho}}\hat{\psi}_{\hat{\lambda}},\\
\delta\hat{A}_{\hat{\mu}} = \frac{3}{2}i\,\bar{\epsilon}\,\Gamma_5\,\Gamma_{\hat{\mu}}{}^{\hat{\rho}\hat{\lambda}}\,\partial_{\hat{\rho}}\,\hat{\psi}_{\hat{\lambda}} - i\,\bar{\epsilon}\,\Gamma_5\,\Gamma_{\hat{\mu}}\Gamma^{\hat{\rho}\hat{\lambda}}\,\partial_{\hat{\rho}}\hat{\psi}_{\hat{\lambda}}.
\end{gathered} \tag{5}
$$

The procedure is the same as in the spin-1 case. We first perform a harmonic expansion of all fields and local parameters, then we have to project onto the lowest Kaluza–Klein massive sector. Remembering that this procedure doubles all the fields, we will have to do a consistent truncation. The last step is to fix all the Stückelberg symmetries. Doing so, we obtain the final form of the supersymmetry rules of the 3D $\mathcal{N}=1$ off-shell massive spin-2 multiplet:

$$
\begin{gathered}
\delta h_{\mu\nu} = \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} + \frac{1}{m}\bar{\epsilon}\partial_{(\mu}\chi_{\nu)},\\
\delta\psi_\mu = -\frac{1}{4}\gamma^{\rho\lambda}\partial_\rho h_{\lambda\mu}\epsilon + \frac{1}{12}\gamma_\mu(M+P)\epsilon + \frac{1}{12m}\partial_\mu(N+\gamma^\rho A_\rho)\epsilon,\\
\delta\chi_\mu = \frac{1}{4}m\gamma^\rho h_{\rho\mu}\epsilon + \frac{1}{4}A_\mu\epsilon - \frac{1}{12}\gamma_\mu(N+\gamma^\rho A_\rho)\epsilon - \frac{1}{12m}\partial_\mu(M-2P)\epsilon,\\
\delta M = \bar{\epsilon}\gamma^{\rho\lambda}\partial_\rho\psi_\lambda - m\bar{\epsilon}\gamma^\rho\chi_\rho, \quad \delta N = -\bar{\epsilon}\gamma^{\rho\lambda}\partial_\rho\chi_\lambda + m\bar{\epsilon}\gamma^\rho\psi_\rho,\\
\delta P = \frac{1}{2}\bar{\epsilon}\gamma^{\rho\lambda}\partial_\rho\psi_\lambda + m\bar{\epsilon}\gamma^\rho\chi_\rho,\\
\delta A_\mu = \frac{3}{2}\bar{\epsilon}\gamma_\mu{}^{\rho\lambda}\partial_\rho\chi_\lambda - \bar{\epsilon}\gamma_\mu\gamma^{\rho\lambda}\partial_\rho\chi_\lambda - \frac{1}{2}m\bar{\epsilon}\gamma_\mu{}^\rho\psi_\rho + m\bar{\epsilon}\psi_\mu,
\end{gathered} \tag{6}
$$

and the action invariant under them

$$
\begin{aligned}
I_{m\neq 0} = \int d^3x\,\Big\{& h^{\mu\nu}G^{\text{lin}}_{\mu\nu}(h) - m^2\left(h^{\mu\nu}h_{\mu\nu} - h^2\right) -\\
& - 4\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho - 4\bar{\chi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\chi_\rho + 8m\bar{\psi}_\mu\gamma^{\mu\nu}\chi_\nu - \frac{2}{3}M^2 - \frac{2}{3}N^2 + \frac{2}{3}P^2 + \frac{2}{3}A_\mu A^\mu\Big\}.
\end{aligned}
$$

The first line is the Fierz–Pauli action. The fermionic off-diagonal mass term in the second line can be diagonalized by going to a basis in terms of the $+3/2$ and $-3/2$ helicity states formed by the sum and difference of the two vector-spinors ψ_μ and χ_μ. The "trivial" auxiliary fields M, N, P and A_μ are needed for the off-shell closure of the supersymmetry algebra.

As is well known, the massless limit $m \to 0$ of the ordinary spin-2 Fierz–Pauli theory in the presence of sources leads to linearized Einstein gravity plus an extra term coupling the matter and the scalar mode of the graviton. This phenomenon is known as the van Dam–Veltman–Zakharov (vDVZ) discontinuity.

As in the Proca case we will include a coupling to a conjugate multiplet, we will require that the interaction action is separately invariant under supersymmetry and we will go back to a formulation invariant under the Stückelberg symmetries and impose some constraints on the currents. The resulting action consists of three massless multiplets: a spin-2 multiplet, a mixed graviton-vector multiplet and a scalar multiplet:

$$I = \int d^3x \Big\{ h'^{\mu\nu} G^{\text{lin}}_{\mu\nu}(h') - 4\bar{\psi}'_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi'_\rho - 8S'^2 + h'_{\mu\nu} T^{\mu\nu} + \bar{\psi}'^\mu \mathscr{J}^\psi_\mu + S' T_S - \\ - F^{\mu\nu} F_{\mu\nu} - \frac{2}{3} N^2 + \frac{2}{3} A^\mu A_\mu - 4\bar{\chi}'_\mu \gamma^{\mu\nu\rho} \partial_\nu \chi'_\rho - 8\bar{\psi}\gamma^\mu \partial_\mu \psi + \\ + 2\left[-\partial_\mu \phi' \partial^\mu \phi' - \frac{1}{4}\bar{\chi}' \gamma^\mu \partial_\mu \chi' + \frac{1}{16} F^2 \right] + \phi' \eta^{\mu\nu} T_{\mu\nu} - \frac{1}{4} \bar{\chi}' \gamma^\mu \mathscr{J}^\psi_\mu - \frac{1}{8} F T_S \Big\}. \quad (7)$$

The spin-2 multiplet couples to the super current multiplet in the usual fashion. Unlike the Proca case, however, there is also a coupling between the scalar multiplet and the trace of the energy-momentum tensor. This is a 3D supersymmetric version of the vDVZ discontinuity.

CONCLUSIONS

We motivated and discussed the Kaluza–Klein formulation to obtain the 3D, off-shell massive spin-2 supermultiplet. This multiplet, together with the off-shell massless spin-2 multiplet which is already known, can be used to write a supersymmetric version of linearized NMG in the auxiliary field form.

In the massless limit there is a non-trivial coupling of a scalar multiplet to a current multiplet. In the usual Fierz–Pauli case, it is possible to cure vDVZ discontinuity taking into account the non-linear version of it, but a non-linear version of the SNMG is not obvious since the construction of the massive spin-2 supermultiplet is based on the truncation of the Kaluza–Klein expansion to the first massive level, which can only be performed at the linearized level. To obtain the non-linear SNMG theory, one should perhaps use superspace techniques.

Acknowledgements. L. P. would like to thank the organizers of the International Workshop "Supersymmetries and Quantum Symmetries — SQS'2013" for providing a stimulating working environment.

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