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VOID FLUCTUATION STUDY OF COMPOUND HADRONS: SIGNATURES OF QUARK–HADRON PHASE TRANSITION

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Signatures of quark–hadron phase transition are probed by the investigation of event-by-event fluctuations of hadronic patterns in heavy-ion collisions. This study is intended to measure the event-to-event fluctuation of voids (nonhadronic regions) for the experimental data sets of compound hadrons of ^{32}S–AgBr interactions at $200A$ GeV. The bin-size dependence of voids is numerically evaluated with two different moments $\langle G_q \rangle$ and S_q defined by R. C. Hwa and Q. H. Zhang. The scaling behaviour of the voids provides an efficient way to use the scaling exponents γ_q and σ_q to characterize the various interesting properties of the hadronic phase transition.

В работе рассматривается поиск фазового перехода кварк–адрон при исследовании флуктуаций событие-за-событием адронных образований в столкновениях тяжелых ионов. Целью исследования является измерение флуктуаций пустот (неадронных областей) событие-за-событием на основе экспериментальных данных для составных адронов во взаимодействиях ^{32}S–AgBr при $200A$ ГэВ. Зависимость пустот от размера бина вычисляется численно для двух различных моментов $\langle G_q \rangle$ и S_q, введенных в работах Р. Ц. Хва и К. Х. Чжан. Скейлинговое поведение пустот позволяет эффективно использовать скейлинговые экспоненты γ_q и σ_q для описания различных интересных свойств адронного фазового перехода.

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INTRODUCTION

After the observations of unexpectedly large local multiplicity fluctuations in JACEE events [1], numerous attempts have been made to investigate the nonstatistical fluctuations in high-energy hadronic and nuclear collisions [2, 3]. Fluctuations measured in experiments, in general, depend on the property of the system under study and may contain important information about the system [4]. Studies involving correlations and fluctuations in relativistic nucleus–nucleus collisions are of special interest, because of the fact that the geometry of the collisions and the number of participating nucleons are believed to play an important role in multiparticle production resulting in fluctuations and random emission [5, 6]. The interest in the studies involving event-by-event fluctuations in hadronic and heavy-ion nucleus–nucleus collisions at relativistic energies is primarily connected with the idea that correlations and fluctuations of dynamical nature are believed to be associated with the critical phenomena of phase transition from the quark–gluon plasma to the normal hadronic matter and leads to the local and global differences between the events produced under similar initial conditions [7]. Furthermore, the presence of dynamical fluctuations in such collisions is expected to arise due to the occurrence of a phase transition from the quark–gluon plasma to the normal hadronic matter. Event-by-event fluctuations and correlations in hadronic and nucleus–nucleus collisions have generally been investigated by using several different approaches [2, 3, 8–13], for instance, normalized factorial moments, multifractal analysis, erraticity, power spectrum analysis, wavelet transform, k-order rapidity spacing, transverse momentum spectra, etc. Yet, another method in this context is the study of void probability in limited phase space, which can lead to some conclusive remarks regarding the production dynamics [14, 15]. Such studies would provide means to examine the validity of the linked-pair ansatz [16].

Various analyses on nuclear collisions have been reported in terms of produced pions. Although very little work has been done with the medium energy (30–400 MeV) knocked out target protons, which are also supposed to carry information about the interaction dynamics, because the time scale of emission of these particles is the same ($\approx 10^{-22}$ s) as that of the produced particles. These target protons, which are known as grey tracks in nuclear emulsion, are the low-energy part of the intranuclear cascade formed in high-energy interactions. If a number of these fast target recoil protons are combined with produced pions, in a collision, a new parameter, named "compound multiplicity" ($n_c = n_g + n_s$, where n_c is compound multiplicity, n_g — number of grey tracks, and n_s — number of shower tracks) is formed. Those particles (pions + fast target recoil protons) are called "compound hadrons". It would be interesting to study the behaviour of the compound hadron data using the available tools, since it may reveal more information about the inner dynamics of the particle production in high-energy nuclear interactions. So far, only limited attempts have been made to work with this parameter [17–21].

The present paper is intended to study the signature of quark–hadron phase transition of second order for the experimental data sets of compound hadrons in high-energy nucleus–nucleus collisions (^{32}S–AgBr interactions at 200A GeV) following the approach proposed by Hwa and Zhang [22, 23], where fluctuation of spatial patterns is analyzed with the help of an observable measure of the voids that exhibits scaling properties characteristics of any critical phenomena. Numerical values of the scaling exponents have been found to judge whether hadronization is via second order quark–hadron phase transition.

1. EXPERIMENTAL DETAILS

The data sets used in this present analysis are obtained by exposing Illford G5 nuclear emulsion plates to ^{32}S beam with incident energy 200A GeV at CERN SPS. A Leitz Metalloplan microscope with a 10X objective and 10X ocular lens provided with a semi-automatic scanning stage is used to scan the plates. Each plate is scanned by two independent observers to increase the scanning efficiency. For measurement, 100X oil-immersion objective is used. The measuring system fitted with it has 1 μm resolution along the X- and Y-axes and 0.5 μm resolution along the Z-axis.

For the present analysis we have considered the combination of grey and shower tracks for formation of compound multiplicity spectrum. We have taken 140 events of ^{32}S–AgBr [24] interactions at $200A$ GeV. Details of events selection criteria and classification of tracks can be found in our earlier communications [24]. The emission angle (θ) and azimuthal angle (φ) with respect to beam direction are measured for each track by taking the readings of the coordinates of the interaction point (X_0, Y_0, Z_0), the coordinates (X_1, Y_1, Z_1) at the end of the linear portion of each secondary track, and the coordinate (X_i, Y_i, Z_i) of a point on the incident beam.

Nuclear emulsion covers 4π geometry and provides very good accuracy in the measurement of emission angles due to high spatial resolution, and thus is suitable as a detector for the study of fluctuations in the fine resolution of the phase space considered.

2. METHOD OF ANALYSIS

We have defined and analyzed void in multiparticle production followed by the technique introduced by Hwa and Zhang [22, 23]. According to the technique, the two-dimensional emission angle ($\cos\theta$)–azimuthal angle (φ) space is divided into M equal blocks or bins. Bins with very low hadron density are regarded as empty bins. An empty bin or combination of the entire bins, which are connected with the empty bin with at least one side, is considered as a void region. Void is a contiguous collection of empty bins. Figure 1 illustrates a pattern of voids in a configuration generated for $M = 9^2$. An open square indicates an empty bin, while a black square contains hadrons. In that configuration there are 12 voids, the sizes of which are 6, 5, 3, 3, 3, 2... in descending order.

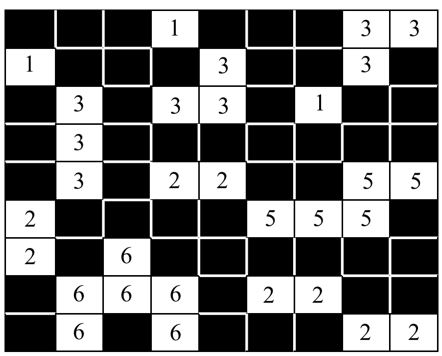

Fig. 1. Schematic representation of void pattern

Let V_k be the sum of the empty bins that are connected to one another by at least one side; k labels a particular void. One can then define a parameter x_k to be the fraction of bins on the lattice that the kth void occupies

$$x_k = \frac{V_k}{M}. \tag{1}$$

For each event we have a set $S_e = \{x_1, x_2, x_3, \ldots\}$ of void fractions that characterizes the spatial pattern. Since the pattern fluctuates from event to event, S_e cannot be used to compare patterns in an efficient way. To measure the moments properly, g_q are defined by Hwa and Zhang [22] for each event

$$g_q = \frac{1}{m}\sum_{k=1}^{m} x_k^q, \tag{2}$$

where the sum is over all voids in the event, and m denotes the total number of voids, and q is the order. Hence, the normalized G moments are defined as

$$G_q = \frac{g_q}{g_1^q}, \tag{3}$$

which depends not only on the order q, but also on the total number of bins M. Thus, by definition $G_0 = G_1 = 1$. This G_q is defined in the same sense as that in [14] for rapidity gaps, but they are not identical, because x_k (for voids) do not satisfy any sum rule. Now, G_q as defined in Eq. (3) is a number for every event for chosen values of q and M. With q and M fixed, G_q fluctuates from event to event and it is the quantitative measure of the void patterns, which, in turn, unfold the characteristic features of phase transition.

The event-to-event fluctuation of G_q can be described by a probability distribution. Hwa and Zhang [22] have proposed the two lowest moments among many investigable moments of that distribution as

$$\langle G_q \rangle = \frac{1}{N} \sum_{e=1}^{N} G_g^{(e)} \tag{4}$$

and

$$S_q = \langle G_q \ln G_q \rangle, \tag{5}$$

where the superscript e denotes the eth event and N is the total number of events. The two moments $\langle G_q \rangle$ and S_q are expected to obey power law behaviour with M as follows:

$$\langle G_q \rangle \propto M^{\gamma_q}, \tag{6}$$

$$S_q \propto M^{\sigma_q}. \tag{7}$$

This scaling behaviour implies that voids of all sizes occur at phase transition. Since the moments at different q are highly correlated, one expects the scaling exponents γ_q and σ_q to depend on q in some simple way as

$$\gamma_q = c_0 + cq, \tag{8}$$

$$\sigma_q = s_0 + sq. \tag{9}$$

Thus, the values c and s (which are the slopes of slopes) are concise characterizations of the fluctuation near the critical point.

3. RESULT AND DISCUSSION

To perform the two-dimensional analysis of fluctuation of voids, we have to consider the correct partitioning condition along $\cos\theta$ and φ directions. That is accomplished with the help of the Hurst exponent, which takes care of the anisotropy of phase space [25–27]. We have considered the maximum value of the Hurst exponent, that is one, for the analysis of data of ^{32}S–AgBr interactions at $200A$ GeV.

We have divided the whole azimuthal angle (φ) space into M_φ equal divisions of width $1/M_\varphi$, where M_φ varies from 2 to 8 in steps of one, and the whole emission angle ($\cos\theta$) space into M_η divisions of width $1/M_\eta$, where M_η is related to M_φ as $M_\eta = M_\varphi^H$. Thus, the two-dimensional emission angle ($\cos\theta$)–azimuthal angle (φ) space is divided into M equal bins, where $M = M_\varphi \times M_\eta$. The detailed procedure can be found in [28].

Then, we have calculated the number of voids using connecting bin approach. If one side of an empty bin is in touch with other empty bins, then we cumulatively add the bins and consider it as a single void. Whereas, if an empty bin is connected by corner, we consider it as a separate void [22, 23]. To capture the fluctuation of voids, we have calculated the normalized G_q moments for each M by calculating void fractions x_k and the moments g_q using Eqs. (1)–(3). The normalized G_q moment is a number, which corresponds to each event for a chosen value of q and M. With q and M fixed, G_q fluctuates from event to event and is a quantitative measure of the void patterns, which, in turn, are the characteristic features of phase transition. The event-to-event fluctuation of G_q can be described by a probability distribution. To measure these fluctuations of G_q, Hwa and Zhang [22] proposed to study the two lowest moments $\langle G_q \rangle$ and S_q among many investigable moments of that distribution, which are given in Eqs. (4) and (5), respectively. We have calculated the above-mentioned two lowest moments. The variations of $\ln \langle G_q \rangle$ with $\ln M$ and $\ln S_q$ with $\ln M$ have been depicted in Figs. 2 and 3, respectively. All the plots show linear behaviour suggesting that power law behaviour of the form of Eqs. (6) and (7) is obeyed indicating that voids of all sizes occur [22]. We have performed the best linear fits to the plots of Figs. 2 and 3 and plotted the respective slope values against the order q in Figs. 4 and 5. As expected, due to the correlation among G_q moments of different order, the scaling exponents γ_q and σ_q have been observed to show linear dependence on the order q as indicated in Eqs. (8) and (9). The slopes of the linear fits (denoted by "c" and "s") corresponding to the plots of Figs. 4 and 5 have tabulated in the Table. The values of "c" and "s" are concise characterizations of the void pattern fluctuation.

As proposed by Hwa and Zhang [23], the value of "c" ranging between 0.75 and 0.96 and "s" ranging between 0.7 and 0.9 may be regarded as the quantitative signature

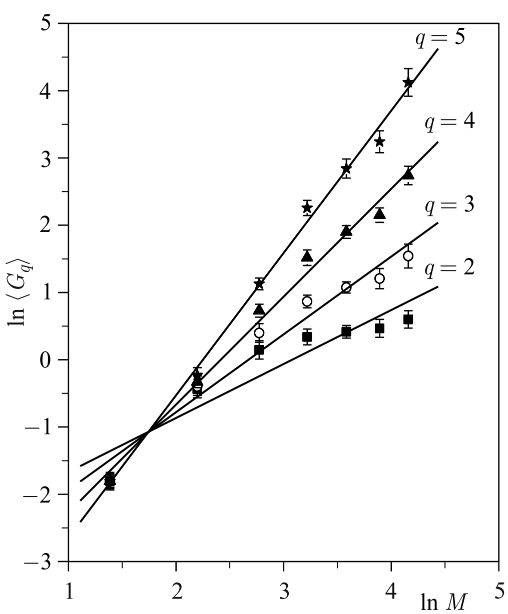

Fig. 2. Representation of the variation of $\ln \langle G_q \rangle$ with $\ln M$

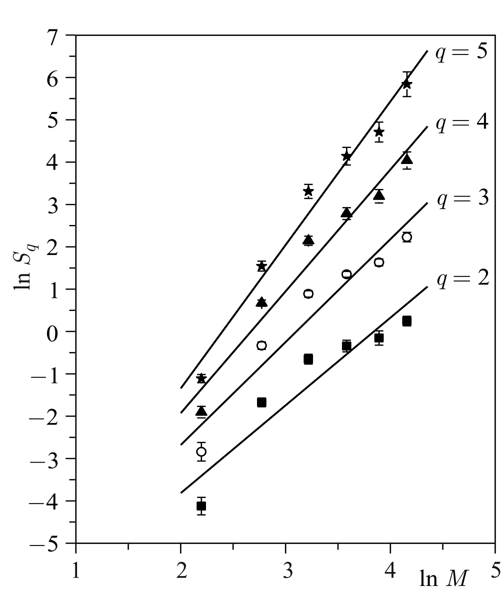

Fig. 3. Representation of the variation of $\ln S_q$ with $\ln M$

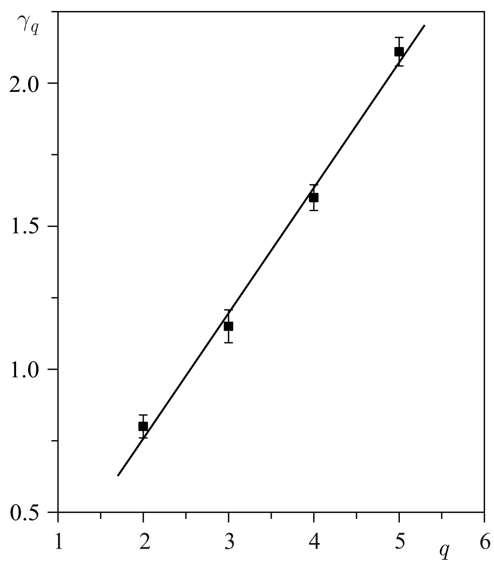

Fig. 4. Representation of the variation of scaling exponents (γ_q) with order of moment (q)

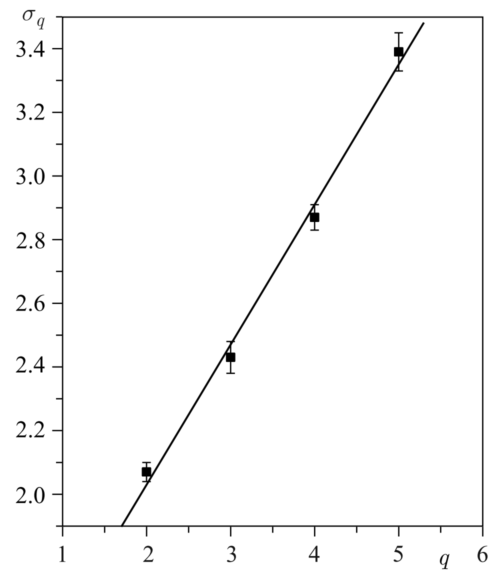

Fig. 5. Representation of the variation of scaling exponents (σ_q) with order of moment (q)

The values of slopes "c" and "s" for experimental data of ^{32}S–AgBr interactions at 200A GeV

Value of "c"	Value of "s"
0.44 $\pm$0.02	0.44 $\pm$0.02

of second order quark–hadron phase transition. It is observed from the Table, both "c" and "s" values are much less than the predicted critical values suggesting no quark–hadron phase transition of second order have taken place for the considered heavy-ion interaction at 200A GeV.

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