Письма в ЭЧАЯ. 2015. Т. 12, № 3(194). С. 636–640

КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

THE DYNAMICAL CASIMIR EFFECT IN TWO-NUCLEON SYSTEMS

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Arguments for registering at the JINR synchrophasotron a quantum phenomenon similar to the theoretically predicted but still unobserved experimentally Hawking effect are given. Here an analogue of the Hawking radiation is a generalized coherent state corresponding to the dynamical symmetry group $SU(1,1)$, which describes the Bose condensate of pions in a strong external field.

Представлены аргументы в пользу регистрации на синхрофазотроне ОИЯИ квантового явления, подобного теоретически предсказанному, но пока не наблюдавшемуся экспериментально эффекту Хокинга. Аналогом излучения Хокинга в этом случае является обобщенное когерентное состояние, отвечающее группе динамической симметрии $SU(1,1)$, которое описывает бозе-конденсат пионов в сильном внешнем поле.

PACS: 25.45.De; 25.10.+s; 27.10.+h

INTRODUCTION

In [1], quasi-resonant peaks in two-proton effective mass distribution from reactions $np \to pp\ \pi^- m\pi^0$ and $np \to pp\ \pi^+\pi^-\pi^- m\pi^0$, $m = 0, 1$, were observed. Recently, it has been shown [2] that dibaryons with nearly the same masses were detected in np system from reaction $D + D \to X + D$ in the more earlier paper by A. M. Baldin et al. [3]. With the assumption that some of dibaryons were unrecognized in the experiments [1, 3], it is possible to approximate the mass spectrum within rather small, at 1–2 MeV^2 level, experimental errors by the formula

$$M_n = M_{NN} + 10.08m,\ \text{MeV},$$

where $m = 0, 1, 2, \ldots, 40$, M_{NN} is equal to the deuteron mass for deuteron excitations and to the two-proton mass for pp system. This equidistant spectrum regularity hardly can be interpreted in the frames of the 6-q bag model, which gives a different spectrum. It also cannot be assigned to some kind of oscillator consisting of quarks coupled by gluon strings [4]. For example, the state $\psi_{20}(x)$ lying in the middle of the spectrum observed in [1] has the length of about 50 Fm.

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From our point of view, it is most likely to associate the spectrum with the production of pion pairs strongly bound to compressed nucleon matter by a deep potential $-U_0$. Parity conservation requires pions to be produced in pairs (see below). Therefore, the value of energy of a single pion

$$E = \sqrt{p^2 + m^2 - U_0} \tag{1}$$

should be equal to 5.04 MeV $\equiv E_\pi$. Meson field $\varphi(\mathbf{r}, t) = \mathrm{e}^{-iEt}\varphi_E(\mathbf{r})$ in the rectangular potential well is described by the Klein–Gordon–Fock (KGF) steady-state equation

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\varphi_E(r)}{dr} + (E^2 - m^2 + U_0)\varphi_E(r) = 0,$$

which has the solution $\varphi_E(r) = A\sin pr/r$ inside the well, and $\varphi_E(r) = B\,\mathrm{e}^{-qr}/r$, $q = \sqrt{m^2 - E^2}$ outside. The requirement of continuity of logarithmic derivative at the edge of the well, $r = a$, leads to the transcendental equation

$$p\cot(pa) = \sqrt{m^2 - E^2}, \tag{2}$$

which is suitable for estimation of the relevant physical values in the interaction region. The spatial dimension corresponding to a given value of momentum transfer is

$$a = \langle r^2\rangle^{1/2} \approx \frac{\sqrt{6}}{|\mathbf{q}|} = 0.68\ \mathrm{Fm}, \quad |\mathbf{q}|^2 = -t.$$

Solving Eq. (2) with this value of a, one obtains $p \approx 0.53$ GeV, and using (1), one finds $\sqrt{U_0} \approx 0.55$ GeV.

Touching dynamics of the bound pion production, we suggest that it is induced by change of a position of walls forming the potential well in analogy with emission of electromagnetic waves due to motion of resonator's walls. This movement is capable to give energy to the virtual pions surrounding nucleons and turn them into the bound pions. Such a mechanism is known as the dynamical Casimir effect [5]. It is closely connected with the Hawking radiation phenomenon and the Unruh effect [5]. The appeal of this model is that it predicts the meson field with the vacuum quantum numbers, since the mesons are produced from the vacuum state due to the strong interaction, conserving all of them. Therefore, the pion field may be present at the ground state of deuteron, as it follows from the experimental data [3], without breaking the deuteron quantum numbers. As far as the vacuum state has positive parity, and the intrinsic parity of pion is negative, only even number of pions may be created. Similarly, isospin conservation leads to the conclusion that pions may be produced in pairs with $I = 0$, i.e., in the following vector of state:

$$\Psi_{2\pi} = \frac{1}{\sqrt{3}}(\pi_a^+\pi_b^- + \pi_a^-\pi_b^+ - \pi_a^0\pi_b^0).$$

1. DYNAMICS OF PION PRODUCTION

A picture of pion production may be depicted as follows. At some instant t_1, a potential well capable to hold a bound pion energy level of value ε is formed. Then, rather quickly, the energy level $E_\pi > \varepsilon$ is developed due to shrinkage of the potential well in the nucleon collision process. After that at moment t_2, when nucleons are moving away, the energy level returns to the value ε, and afterwards it changes again to the Yukawa vacuum, corresponding to $E = 0$ and $q = m$. From mathematical viewpoint, creation of bound pions in this framework is totally equivalent to the parametric excitation of the quantum oscillator, which appears after quantization of the field.

The time-dependent KGF equation with the evolving boundary conditions gives the wave function inside the well

$$\varphi(r,t) = \chi(t) \sin pr/r,$$

where $\chi(t)$ describes increasing amplitude of the field, which manifests itself in the pion production. It obeys the equation

$$\frac{\partial^2 \chi(t)}{\partial t^2} + (p^2 + m^2 - U_0)\chi(t) = 0,$$

which has the same form as the equation for a classical oscillator with varying frequency $\omega(t) = E(t)$. Therefore, it is possible to introduce the oscillator Hamiltonian and to quantize the field. In the Heisenberg picture, the quantized field can be expressed as

$$\hat{\varphi}(r,t) = \hat{\chi}_\omega(t) \sin pr/r = \left(\frac{\hat{a}_\omega^+(t) + \hat{a}_\omega(t)}{\sqrt{2\omega_1}} \right) \sin pr/r,$$

for any t in the range of pion production, $t_1 \leqslant t \leqslant t_2$.

Now, the field evolution may be written in the form of Bogoliubov's canonical transformation (BCT):

$$\begin{pmatrix} \hat{a}(t+dt) \\ \hat{a}^+(t+dt) \end{pmatrix} = \begin{pmatrix} u(dt) & v(dt) \\ \bar{v}(dt) & \bar{u}(dt) \end{pmatrix} \begin{pmatrix} \hat{a}(t) \\ \hat{a}^+(t) \end{pmatrix},$$

where $u(dt)$ and $v(dt)$ are usual (non-operator) functions, and the matrices of BCT generate a group under multiplication. The commutation relation requirement $[\hat{a}(t), \hat{a}^+(t)] = 1$ leads to a constraint $|u(t)|^2 - |v(t)|^2 = 1$, which means that the group is $SU(1,1)$. One may express elements of $SU(1,1)$ group through its generators, $\hat{g}(dt) = \exp\,(\beta\hat{K}_+ - \bar{\beta}\hat{K}_- - i\gamma\hat{K}_0)\,dt$. In the case of the Hamiltonian evolution, $\hat{g}(dt) = \exp\,(-i\hat{H}dt)$, so that the Hamiltonian of the field can be rewritten as a linear combination of the generators, $\hat{H} = i(\beta\hat{K}_+ - \bar{\beta}\hat{K}_- - i\gamma\hat{K}_0)$.

Explicit description of $SU(1,1)$ generators is as follows:

$$\hat{K}_+ = \frac{(\hat{a}^+)^2}{2}, \quad \hat{K}_- = \frac{\hat{a}^2}{2}, \quad \hat{K}_0 = \frac{\hat{a}\hat{a}^+ + \hat{a}^+\hat{a}}{4}$$

for $\pi^0\pi^0$, and

$$\hat{K}_+ = \hat{a}_+^+\hat{a}_-^+, \quad \hat{K}_- = \hat{a}_+\hat{a}_-, \quad \hat{K}_0 = \frac{1}{2}(\hat{a}_+^+\hat{a}_+ + \hat{a}_-^+\hat{a}_- + 1)$$

for $\pi^+\pi^-$. In fact, operators $\hat{K}_0$ do not lead to the change of number of particles and it is possible to omit them. Thus, the evolution operator may be defined as an element of $SU(1,1)$

group of a kind $\hat{S}(t) = \exp\,(\xi\hat{K}_+ - \bar{\xi}\hat{K}_-)$. Respectively, the state of system at moment t is $|\psi_t\rangle = \exp\,(\xi\hat{K}_+ - \bar{\xi}\hat{K}_-)\,|0\rangle$.

It is possible to notice a similarity of this state to the Glauber coherent state $|\psi_G\rangle = \exp\,(\alpha a^+ - \bar{\alpha} a)\,|0\rangle$, which leads to the Poisson distribution for probability to find n particles in $|\psi_G\rangle$. Similarly, the state $|\psi_t\rangle$ reads

$$|\psi_t\rangle = (1 - |\eta|^2)^k \sum_{m=0}^{\infty} \left(\frac{\Gamma(m+2k)}{m!\Gamma(2k)} \right)^{1/2} \eta^m \,|k, k+m\rangle\,.$$

Here k describes representations of $SU(1,1)$, $k = 1/4$ for $\pi^0\pi^0$, and $k = 1/2$ for $\pi^+\pi^-$, m is the number of pion pairs created, $\eta = \sqrt{\rho}\,\mathrm{e}^{i\varphi}$. A value of ρ may be expressed through coefficients $u(t_2)$ and $v(t_2)$ of BCT at the end of pion production, $\rho = |v|^2 / |u|^2$, and $\mathrm{e}^{i\varphi}$ is a phase factor unessential here. Probability to find $n = 2m$ particles in the state is equal to

$$w_n = |\langle n|\psi_t\rangle|^2 = \sqrt{1-\rho}\frac{n!}{2^n\left[(n/2)!\right]^2}\rho^{n/2} \tag{3}$$

for $\pi^0\pi^0$ system. For $\pi^+\pi^-$, it is

$$w_n = |\langle n|\psi_t\rangle|^2 = (1-\rho)\rho^{n/2}. \tag{4}$$

The value of ρ can be estimated in the framework of a certain scattering problem for a quantum mechanical particle [6]. We only display the final formula corresponding to the scattering by a rectangular potential well with depth $E_\pi^2 - \varepsilon^2$, which simulates the changing boundary conditions described above:

$$\rho = \frac{1}{1+\delta^2}, \quad \delta = \frac{2\varepsilon E_\pi}{(E_\pi^2 - \varepsilon^2)\sin\,E_\pi(t_2 - t_1)},$$

where $t_2 - t_1 \sim 1/\Gamma$, Γ is the dibaryon width, ε is the only unknown parameter, which can be found in further experiments. The data accuracy in [1] does not permit us to estimate ε, but it allows concluding that ρ is very close to 1, see (4) for the registered value of $n = 80$. The distribution (3) rapidly decreases with n, therefore only bound $\pi^+\pi^-$ pairs contribute to the heavy dibaryon tail, observed in [1].

CONCLUSION

Analysis of the data [3] reveals the presence of the pion Bose condensate (PBC) in the ground state of deuteron. According to the data [3], hard DD collisions can induce changes of the state of the bound pions in deuteron. PBC can also appear in the compressed pp system subjected to a proper cooling [1]. It is reasonable to ask whether PBC arises in compressed n-nucleon systems, for $n > 2$. If this is true, PBC can impact essentially on collective flows at the final stage of high-energy nuclear collisions, especially on the sideflow.

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