Письма в ЭЧАЯ. 2016. Т. 13, № 1(199). С. 101–115

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

LORENTZ INVARIANCE VIOLATION AND GENERALIZED UNCERTAINTY PRINCIPLE

A. Tawfik [a, b, 1], *H. Magdy* [a], *A. Farag Ali* [c]

[a] Egyptian Center for Theoretical Physics, Modern University for Technology and Information, Cairo

[b] World Laboratory for Cosmology and Particle Physics, Cairo

[c] Physics Department, Faculty of Science, Benha University, Benha, Egypt

There are several theoretical indications that the quantum gravity approaches may have predictions for a minimal measurable length, maximal observable momentum, and thereby a generalization for the Heisenberg uncertainty principle. The generalized uncertainty principle (GUP) is based on a momentum-dependent modification in the standard dispersion relation which is conjectured to violate the principle of Lorentz invariance. From the resulting Hamiltonian, the velocity and time of flight of relativistic distant particles at the Planck energy can be derived. The first comparison is made with recent observations for the Hubble parameter in redshift dependence in early-type galaxies. We find that Lorentz invariance violation (LIV) has two types of contributions to the time-of-flight delay Δt comparable with these observations. Second, although the OPERA measurement on faster-than-light muon neutrino anomaly, Δt, and the relative change in the speed of muon neutrino Δv in dependence on redshift z turn to be wrong, we utilize its main features to estimate Δv. Accordingly, the results could not be interpreted as LIV. The third comparison is made with the ultra high-energy cosmic rays (UHECR). It is found that an essential ingredient of the approach combining string theory, loop quantum gravity, black hole physics, and doubly spacial relativity is the one assuming a perturbative departure from the exact Lorentz invariance. Fixing the sensitivity factor and its energy dependence are essential inputs for a reliable confronting of our calculations to UHECR. The sensitivity factor is related to the special time-of-flight delay and the time structure of the signal. Furthermore, the upper and lower bounds to the parameter α, that characterizes the generalized uncertainty principle, have to be fixed in related physical systems such as gamma-ray bursts.

Некоторые из развитых в настоящее время моделей квантовой гравитации могут предсказать величины минимальной измеряемой длины, максимального наблюдаемого импульса и таким образом помочь сформулировать обобщенный принцип неопределенности Гейзенберга. Обобщенный принцип неопределенности основывается на модификации зависящего от импульса стандартного дисперсионного соотношения, которое гипотетически может нарушать принцип лоренц-инвариантности. С помощью получаемого на основании подобного дисперсионного соотношения гамильтониана можно вычислить скорость и время пролета удаленных друг от друга релятивистских частиц при планковских энергиях.

В представленной работе делается сравнение получаемых результатов со значениями параметра Хаббла, определенными недавно в ходе измерений величины красного смещения для галактик, сформировавшихся на ранних этапах рождения Вселенной. Показано, что нарушение лоренц-инвариантности приводит к двум типам вкладов в задержку времени пролета Δt, что соответствует наблюдаемым значениям.

Несмотря на то, что результаты OPERA по обнаружению мюонных нейтрино, движущихся со скоростями, большими скорости света, Δt и зависимость скорости нейтрино Δv от величины

[1] http://atawfik.net/; E-mail: a.tawfik@eng.mti.edu.eg

красного смещения z оказались неверными, мы используем их для оценки Δv. Соответственно, эти результаты нельзя интерпретировать как следствие нарушения лоренц-инвариантности.

Рассмотрены результаты исследований космических лучей сверхвысоких энергий. Установлено, что для их понимания необходимо скомбинировать теорию струн, квантовую гравитацию с петлями, физику черных дыр и теорию относительности, а также предположить пертурбативное отклонение от точной лоренц-инвариантности. Кроме того, нужно зафиксировать фактор чувствительности и его зависимость от энергии. Фактор чувствительности связан с задержкой времени пролета и временной структурой сигнала. Верхний и нижний пределы значений параметра α, который характеризует обобщенный принцип неопределенности, должны быть зафиксированы в тех физических системах, для которых делаются вычисления, таких как вспышки гамма-излучения.

PACS: 04.60.-m; 11.30.Cp; 95.85.Pw

INTRODUCTION

The combination of the Heisenberg uncertainty principle and finiteness of speed of light c is assumed to lead to creation and annihilation processes, especially when studying the Compton wavelength of the particle of interest [1]. Another consequence of the space-time foamy structure at small scales is the Lorentz invariance violation (LIV). For completeness, we mention that the foamy structure at short distances combines quantum mechanics with general relativity. Different approaches for quantum gravity [2], the yet-to-be-built quantum theory of gravity, have been proposed [1,3]. They provide a set of predictions for a minimal measurable length, maximal observable momentum, and thereby an essential modification of the Heisenberg uncertainty principle. The corresponding effective quantum mechanics would be based on the generalized uncertainty principle (GUP) [4]. According to string theory, loop quantum gravity, and black hole physics, GUP is found proportional to a quadratic momenta [5]. Based on doubly spacial relativity, a proportionality to the first-order moments (linear) has been suggested. As introduced in [6, 7], both approaches can be integrated. The resulting one is obviously consistent with doubly special relativity (linear momenta), string theory, and black hole physics (quadratic momenta). In this regards, one could combine quantum mechanics and special relativity and hope to reveal serious difficulties in describing the one-particle theories. The quantum field theory (QFT) is a perfectly well-defined theoretical framework involving renormalization. According to Wilson, Weinberg and others, there is nothing wrong with it, i.e., QFT provides precise predictions that are successfully tested in experiments.

The roots of LIV are originated in the suggestion that Lorentz invariance (LI) may represent an approximate symmetry of nature which dates back to about four decades [8]. A self-consistent framework for analyzing possible violation of LI was suggested by Coleman and Glashow [9, 10]. In gamma-ray bursts (GRB), the energy-dependent time offsets are investigated in different energy bands assuming standard cosmological model [11]. A kind of weak indication for the redshift dependence of the time delays suggestive of LIV has been found. A comprehensive review on the main theoretical motivations and observational constraints on the Planck scale suppressed Lorentz invariance violation are given in [12] and references therein. Recently, the Planck scale itself turns to be accessible in quantum optics [13].

Various implications of GUP have been studied so far. References [14, 15] give up-to-date reviews. Effects of GUP on atomic, condensed matter systems, quark–gluon plasma, preheating phase and inflationary era of the Universe, black hole production at LHC [7,16–22] have been investigated. The implications on the Saleker–Wigner inequality, compact stars, and modified Newton's law of gravitation have been reported [23–25].

The present paper discusses GUP that potentially leads to observable experimental effects related to the Lorentz invariance violation. Computations in the model characterized by linear modifications are presented and the data are compared with some experimental results. Following the proposal of utilizing astrophysical objects to search for the energy-dependent time of the arrival delays [26], we present an estimation for the time-of-flight delays and the relative change in the velocity. We compare the results with the observations of the Hubble parameter in early-type galaxies in redshift dependence in Subsec. 2.1. Also, we compare the results of muon neutrino based on GUP approach in Subsec. 2.2. Subsection 2.3 is devoted to the calculations which are confronted with the ultra high-energy cosmic ray (UHECR) observations. The conclusions are addressed in Sec. 3.

1. THE APPROACH

According to GUP approach, the momentum of a particle with mass M having distant origin and an energy scale comparable to Planck's one would be the subject of a slight modification [6, 7], so that the comoving momenta can be given as

$$p_\nu = p_\nu(1 - \alpha p_0 + 2\alpha^2 p_0^2), \quad p_\nu^2 = p_\nu^2(1 - 2\alpha p_0 + 10\alpha^2 p_0^2), \tag{1}$$

where p_0 is momentum at low energy. The parameter $\alpha = \alpha_0/(cM_{\rm Pl}) = \alpha_0 l_{\rm Pl}/\hbar$ [6, 7], where c, α_0, $M_{\rm Pl}(l_{\rm Pl})$ are speed of light as introduced by Lorentz and implemented in special relatively dimensionless parameter of order one and the Planck mass (length), respectively. Then in the comoving frame, the dispersion relation is given as

$$E_\nu^2 = p_\nu^2 c^2 (1 - 2\alpha p_0) + M_\nu^2 c^4. \tag{2}$$

Taking into consideration a linear dependence of p on α and ignoring the higher orders of α, the Hamiltonian is

$$\mathbf{H} = (p_\nu^2 c^2 - 2\alpha p_\nu^3 c^2 + M_\nu^2 c^4)^{1/2}. \tag{3}$$

The derivative of Eq. (3) with respect to the momentum results in a comoving time-dependent velocity, i.e., the Hamilton equation,

$$v(t) = \frac{c}{a(t)}\left(1 - 2\alpha p_0 - \frac{M_\nu^2 c^2}{2p_\nu^2} + \alpha p_0\left[\frac{M_\nu^2 c^2}{p_\nu^2} - \frac{M_\nu^2 c^4}{p_\nu^2 c^2 + M_\nu^2 c^4} + \frac{M_\nu^2 c^4}{p_\nu^2 c^2 + M_\nu^2 c^4}\frac{M_\nu^2 c^2}{2p_\nu^2}\right]\right). \tag{4}$$

The comoving momentum is related to the physical one through $p_\nu = p_{\nu_0}(t_0)/a(t)$, where a is the scale factor, which in turn can be related to the redshift z:

$$a(z) = \frac{1}{1+z}. \tag{5}$$

In the relativistic limit, $p \gg M$, the fourth and fifth terms in Eq. (4) simply cancel each other. Then

$$v(z) = c(1+z)\left[1 - 2\alpha(1+z)p_{\nu_0} - \frac{M_\nu^2 c^2}{2(1+z)^2 p_{\nu_0}^2} + \alpha\frac{M_\nu^4 c^4}{2(1+z)^3 p_{\nu_0}^3}\right]. \tag{6}$$

In getting this expression, p_0 is treated as a comoving momentum. Then, it becomes straightforward to deduce the relative change in the relative velocity

$$\frac{\Delta v(z)}{c} = \alpha\left(-2(1+z)^2 p_{\nu_0} + \frac{M_\nu^4 c^4}{2(1+z)^2 p_{\nu_0}^3}\right) - \frac{M_\nu^2 c^2}{2(1+z) p_{\nu_0}^2}. \tag{7}$$

Despite the entire results are given in Sec. 2, a few remarks can be outlined here. The curves in left panel of Fig. 2 represent the results of our approach. For a massless muon neutrino, the sign of $\Delta v(z)/c$ remains negative with increasing z. When the muon neutrino mass is taken into account, the sign turns to positive. In this case, its value nearly vanishes at large z. Accordingly, the resulting sign of the summation of the first two terms of Eq. (7) is determined by the second term, i.e., positive, at small z. Then, the sign is flipped to negative at $z \sim 0.2$, i.e., the first term becomes dominant.

The comoving redshift-dependent distance travelled by the particle of interest is defined as

$$r(z) = \int_0^z \frac{v(z)}{(1+z)H(z)}\, dz, \tag{8}$$

where $H(z)$ is the Hubble parameter depending on z. From Eqs. (6) and (8), the time of flight reads

$$t_\nu = \int_0^z \left[1 - 2\alpha(1+z)p_{\nu_0} - \frac{M_\nu^2 c^2}{2(1+z)^2 p_{\nu_0}^2} + \alpha\frac{M_\nu^4 c^4}{2(1+z)^3 p_{\nu_0}^3}\right] \frac{dz}{H(z)}, \tag{9}$$

which counts for the well-known time of flight of a prompt low-energetic photon (first term). In other words, the time of flight is invariant in the Lorentz symmetry. Furthermore, it is apparent that Eq. (9) contains a time-of-flight delay given as

$$\Delta t_\nu = \int_0^z \left[2\alpha\left((1+z)p_{\nu_0} - \frac{M_\nu^4 c^4}{4(1+z)^3 p_{\nu_0}^3}\right) + \frac{M_\nu^2 c^2}{2(1+z)^2 p_{\nu_0}^2}\right] \frac{dz}{H(z)}. \tag{10}$$

It is clear that the first and second terms are due to LIV effects stemming from GUP. Both have α parameter. The third term reflects the effects of the particle mass on the time-of-flight delay. Furthermore, the second term alone seems to contain mixed effects from LIV (GUP) and the rest mass.

In order to determine Δt_ν, Eq. (10), it is essential to find out observational results and/or a reliable theoretical model for the redshift dependence of the Hubble parameter H. What we have is that H depends on the time-dependent redshift, dz/dt,

$$H(z) = \frac{1}{a(z)}\left(\frac{da(z)}{dz}\frac{dz}{dt}\right) = -\frac{1}{1+z}\frac{dz}{dt}. \tag{11}$$

It is obvious that this expression can be deduced from Eq. (5). In general, the expansion rate of the Universe varies with the cosmological time [27–34]. It depends on the background matter/radiation and its dynamics [33]. The cosmological constant, reflecting among others the dark matter content, seems to affect the temporal evolution of H [32]. Fortunately, the redshift z itself can be measured with high accuracy through measuring the spectroscopic redshifts of galaxies having definite uncertainties ($\sigma_z \leqslant 0.001$). Based on this, a differential measurement of time, dt, at the given redshift interval automatically provides a direct and clean measurement of $H(z)$ [35–37]. These measurements can be used to derive constraints on essential cosmological parameters [38]. In the present work, we implement the measurements of the expansion rate and their constrains in evaluating the integrals given in Eq. (10).

2. CONFRONTING WITH OBSERVATIONS AND MEASUREMENTS

First, we compare with recent observations of the early-type galaxies, which apparently provide a direct probe for the dependence of the Hubble parameter H and z. Making use of LIV contributions to $\Delta v/c$ and Δt, we study the dependence of each of these quantities on z and compare the meanwhile wrong-declared results with OPERA in Subsec. 2.2. The lesson we gain from such a comparison is that GUP can be applied even in judgement about edge-cutting observations. The ability of distant neutrinos to feel z-shift is discussed in Subsec. 2.2. Then, ultra high-energy cosmic rays (UHECR) are utilized as a laboratory to study the consequences of LIV. Following the γ-ray observations from Mrk 501, constraints on the Lorentz invariance breaking parameter based on potential departure from the exact Lorentz invariance introduced in a perturbative framework motivate the comparison with UHECR.

2.1. Early-Type Galaxies. Out of a large sample of early-type galaxies (about 11000) extracted from several spectroscopic surveys spanning over $\sim 8 \cdot 10^9$ years of cosmic look-back time, i.e., $0.15 < z < 1.42$ [36], most massive, red elliptical galaxies, passively evolving and without a signature of ongoing star formation are picked up and used as standard cosmic chronometers [38]. The differential age evolution turns to be accessible, which gives an estimation for the cosmic time and can directly probe the dependence of the Hubble parameter H and z. A list of new measurements of $H(z)$ with $5-12\%$ uncertainty is introduced in [36]. The uncertainty in these observational data seems to be comparable with our calculation for

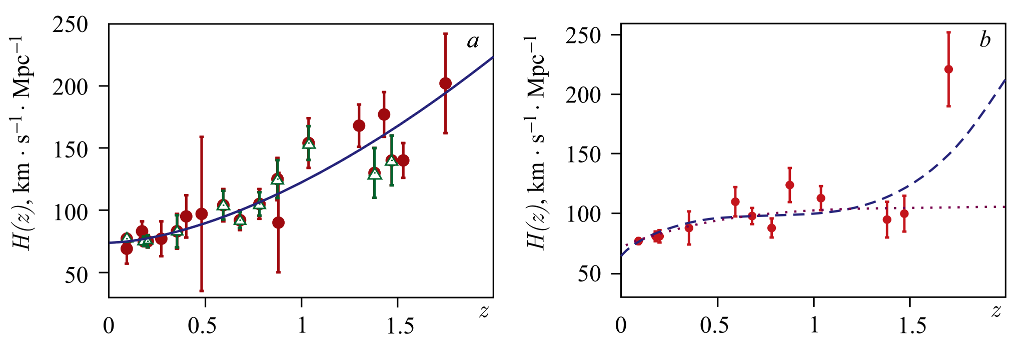

Fig. 1. *a*) Hubble parameter H calculated from BC03 model (open triangle) and in combination with CMB data constraining possible deviations from standard (minimal) flat ΛCDM model (solid circles) is given in dependence on redshift z. *b*) The results from MS model. The curves represent the fitting parameters (see the text for details)

$H(z \sim 0.2)$. Figure 1 illustrates these observations as estimated in the BC03 [39] model. They are combined with the cosmic microwave background (CMB) data and can be used to set constraints on possible deviations from the standard (minimal) flat ΛCDM model [37]. Figure 1, *b* shows a data set taken from MS model [40]. It is obvious that the results are model-dependent.

The observational measurements can be fitted as follows. For the results obtained from BC03 model [39], using a combination with CMB data and setting constraints on possible deviations from the standard (minimal) flat ΛCDM model [37], the expression

$$H(z) = \beta_1 + \gamma_1 z + \delta_1 z^2, \tag{12}$$

where $\beta_1 = 72.68 \pm 3.03$, $\gamma_1 = 19.14 \pm 5.4$, and $\delta_1 = 29.71 \pm 6.44$, fits well the observations. The solid curve in Fig. 1, *a* represents the results from this expression. For the MS model [40] measurements, we suggest two expressions:

$$H(z) = \beta_2 + \gamma_2 z + \delta_2 z^2 + \epsilon_2 z^3, \tag{13}$$

$$H(z) = \beta_3 + \gamma_3 \tanh(\delta_3 z), \tag{14}$$

where $\beta_2 = 66.78 \pm 8.19$, $\gamma_2 = 113.27 \pm 7.5$, $\delta_2 = -140.72 \pm 12.6$, $\epsilon_2 = 60.61 \pm 5.48$, $\beta_3 = 71.94 \pm 4.35$, $\gamma_3 = 33.51 \pm 7.94$, and $\delta_3 = 1.6 \pm 0.1$. The results of Eq. (13) are given by the dashed curve in Fig. 1, *b*. Equation (14) is drawn by the dotted curve, where the largest point is excluded, while the remaining points build up the ensemble used in the fitting. It is clear that the implementation of Eq. (13), which is obviously a rational function, in Eq. (10) results in a nonanalytic integral. On the other hand, implementing Eq. (14) in Eq. (10) makes the second and third integrals nonsolvable. The first term can be solved, Appendix A, where the results are also illustrated, graphically.

It is apparent that Eq. (12) simplifies the integrals given in Eq. (10). Accordingly, there are two types of LIV contributions to the time-of-flight delay. The first type is originated in finite α. Finite α appears in two terms as follows:

$$2\alpha p_{\nu_0} \int_0^z (1+z) \frac{dz}{H(z)} = \frac{\alpha}{\gamma} p_{\nu_0} \left[\ln\left[\beta_1 + z(\gamma_1 + \delta_1 z)\right] - \frac{2(\gamma_1 - 2\delta_1)}{A} \operatorname{artanh}\left(\frac{\gamma_1 + 2\delta_1 z}{A}\right)\right], \tag{15}$$

$$\begin{aligned} -2\alpha \frac{M_\nu^4 c^4}{4p_{\nu_0}^3} \int_0^z \frac{1}{(1+z)^3} \frac{dz}{H(z)} = \frac{-\alpha}{(\beta_1 - \gamma_1 + \delta_1)^3} \frac{M_\nu^4 c^4}{4p_{\nu_0}^3} \left[\frac{2(\gamma_1 - 2\delta_1)(\beta_1 - \gamma_1 + \delta_1)}{1+z} + \right. \\ + (3\gamma_1\delta_1 - \gamma_1^2 + \delta(\beta_1 - 3\delta_1)) \ln(\beta_1 + z(\gamma_1 + \delta_1 z)) - \\ - \frac{(\beta_1 - \gamma_1 + \delta_1)^2}{(1+z)^2} + 2(\gamma_1^2 - 3\gamma_1\delta_1 + \delta_1(3\delta_1 - \beta_1)) \ln(1+z) - \\ \left. - \frac{2(\gamma_1 - 2\delta_1)}{A} (\gamma_1^2 - \gamma_1\delta_1 + \delta_1(\delta_1 - 3\beta_1)) \operatorname{artanh}\left(\frac{\gamma_1 + 2\delta_1 z}{A}\right)\right], \end{aligned} \tag{16}$$

where $A = (4\beta_1\delta_1 - \gamma_1^2)^{1/2}$. Furthermore, Eq. (12) gives an exclusive estimation for the mass contribution to the time-of-flight delay,

$$\frac{M_\nu^2 c^2}{2p_{\nu_0}^2}\int\limits_0^z \frac{1}{(1+z)^2}\frac{dz}{H(z)} = \frac{1}{(\beta_1-\gamma_1+\delta_1)^2}\frac{M_\nu^2 c^2}{2p_{\nu_0}^2}\left\{\frac{\gamma_1^2 - 2\gamma_1\delta_1 + \delta_1(\delta_1-\beta_1)}{A}\times\right.$$
$$\left.\times \operatorname{artanh}\left(\frac{\gamma_1+2\delta_1 z}{A}\right) - \frac{\beta_1-\gamma_1+\delta_1}{1+z} - \frac{1}{2}(\gamma_1-2\delta_1)\ln\left[\frac{(1+z)^2}{\beta_1+z(\gamma_1+\delta_1 z)}\right]\right\}. \quad (17)$$

The results are discussed in Subsec. 2.2. Although the OPERA measurement on faster-than-light muon neutrino anomaly, Δt, and the relative change in the speed of neutrino Δv in dependence on the redshift z turn to be wrong, we utilize its main features to estimate Δv and Δt.

2.2. Comparing Δt and Δv with Controversial OPERA Measurements. For the neutrino beam covering the distance between the source at CERN and the OPERA detector at the underground Gran Sasso Laboratory (LNGS), $\simeq$ 730 km, a time-of-flight delay of $\simeq (1045.1 \pm 11.3)$ ns is first believed to be registered [41].

As discussed in the previous section, LIV comes up with two sources of contributions to $\Delta v/c$ and Δt. The first source is stemming from finite α and vanishing mass, Eq. (15). The second source requires finite α and mass, Eq. (16). The dependence of each of these quantities on z is presented in Fig. 2, *a*. In performing these calculations, we use the same parameters of the controversial OPERA experiment in which a faster-than-light muon neutrino anomaly has been claimed [41, 42]. They are the muon neutrino mass $M_\nu = 1$ eV and beam energy $E_\nu = 17$ GeV. The comparison with our approach assumes that the muon neutrino beam has a distant origin and was witnessing a huge redshift z while the Universe expanded. Then, the time-of-flight delay Δt can be calculated in dependence on redshift z. The first two terms of Eq. (10) are calculated and drawn in Fig. 2, *b*. They are labelled by α and α, M_ν, respectively. We find that the first term, α, is one or two orders of magnitude higher than the

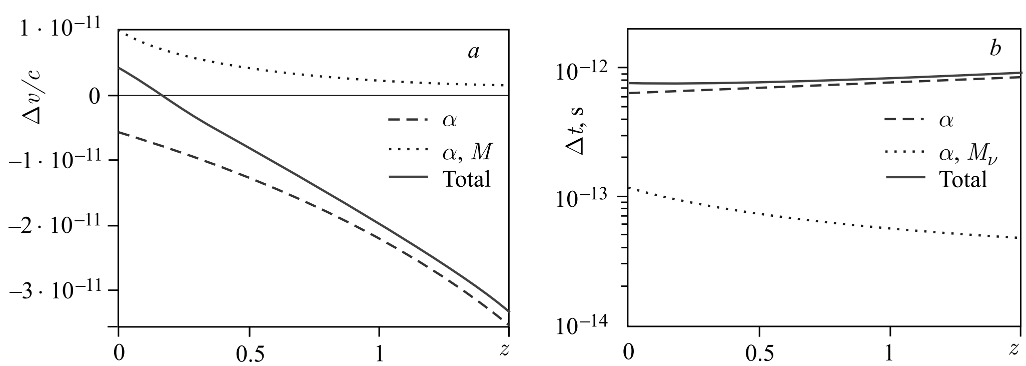

Fig. 2. Although the controversial OPERA measurement on faster-than-light muon neutrino anomaly, Δt, and the relative change in the speed of neutrino Δv in dependence on the redshift z turn to be wrong, using its main features, the relative change in the velocity of muon neutrino is given as a function of redshift z in panel *a*. Panel *b* shows the time-of-flight delay. The curves represent different contributions to $\Delta v/c$ and Δt (see the text)

second one, α, M_ν. It is apparent that the sum of these two terms combines all LIV sources. Accordingly, the time-of-flight delay can be approximated as $\Delta t \sim 10^{-12}$ s.

Disregarding its confident and statistical interpretation, the comparison with the LIV time-of-flight delay leads to the conclusion that the OPERA measurement is too huge (about six orders of magnitude) to be understood as LIV.

Furthermore, the wrong OPERA experiment suggested an increase in the speed of light by ~ 7.5 km/s (~ 25 part per millionth c) [41]. In the recent measurement [42], it is found that the difference between speed of muon neutrino and speed of light ranges from -1.8 to 2.3 part per millionth c. Figure $2, a$ presents the redshift evolution of the possible change in the velocity of muon neutrino according to LIV. It is assumed that the mass of muon neutrino $M = 1$ eV with 17-GeV energy. The first two terms of Eq. (7) are compared with each other. The results are illustrated in Fig. $2, a$.

We notice that the first term (massless muon neutrino) results in a negative speed difference. Its absolute value increases almost linearly with increasing z. The second term remains in the positive site of the ordinate. The resulting speed difference is positive. While abscissa rises, the value of the second term decreases exponentially. The upper and lower values of speed difference range between $\sim 10^{-11}$ and 0. With increasing z, the sum of these two terms changes the sign of $\Delta v(z)/c$. At small z, the second term seems to be dominant. At $z \sim 0.2$, the positive sign is switched into negative. At larger z-values, the first term becomes dominant. The average speed difference can be approximated as $\Delta v \sim -2 \cdot 10^{-11} c$. Comparing to the value measured in OPERA [41, 42], the LIV-value is about six orders of magnitude smaller.

The sign of $\Delta v(z) \equiv c_\nu - c$ is flipped, meaning that

$$\Delta v(z) = \begin{cases} \mathcal{O}(+ve), & \text{then} \quad c_\nu = c + \mathcal{O}, \\ \mathcal{O}(-ve), & \text{then} \quad c_\nu = c - \mathcal{O}, \end{cases} \tag{18}$$

where c_ν is the velocity of muon neutrino. In Eq. (18), the second case apparently follows the Lorentz invariance symmetry. The first case suggests that the speed of light would not be constant in all inertia frames. Furthermore, it would not be the maximum of travelling matter and information in the Universe. The value of the additional quantity $\mathcal{O}$ is about $\sim 10^{-11} c$, i.e., $\mathcal{O} \sim 3$ mm/s, which indicates a superluminal propagation of high-energy muon neutrino at $z \lesssim 0.2$.

It was believed that OPERA gave results comparable to MINOS [43], where the value of the relative speed change was found as $\mathcal{O} \sim 10^{-5}$. On the other hand, these measurements are not compatible with the observations of ~ 10 MeV-neutrino from supernova SN1987a [44]. In these observations, the value of $\mathcal{O}$ is estimated as $\sim 10^{-9}$. Therefore, the faster-than-light anomaly is energy-dependent. It drops rapidly, when reducing energy from GeV- to MeV-scale [10]. Nevertheless, the velocity anomaly is conjectured to reflect the propagation of all decay channels of neutrino and new physics such as LIV.

A few remarks on comparison with OPERA are as follows:

• The energy and mass of muon neutrino do not matter, as the applicability of GUP is not doubtable.

• The OPERA measurements are neither approved nor disapproved.

• Our GUP approaches are not biased. Therefore, we present the comparison even after withdrawing the OPERA measurements.

• It intends to illustrate trustable judgement even about edge-cutting conclusion.

Another point under study is the ability of distant neutrinos to feel z-shift. Before CMB, the extremely long interaction length of neutrinos while traversing the relic background leads to integration over cosmic time, or redshift, in order to estimate their survival probability [45, 46]. This would be considered as an indirect observation that CMB neutrinos would feel the redshift. According to standard cosmology, neutrinos should be the most abundant particles in the Universe, especially after CMB photons. Even the CMB temperature can be expressed in dependence on redshift z, $T_{\rm CMB}(z) = 2.7(1+z)$ K. While traversing the expanding Universe we live in, the effective relic UHECR neutrino density per unit redshift reads $n_{\nu 0}(1+z)/H(z)\,dz$ [45,46]. The indirect dependence on $H(z)$ means that the possibility that the observation of neutrino absorption could even reveal the thermal history of the Universe becomes high [46]. Furthermore, the GRB neutrino flux in dependence on redshift can be estimated [47]. Last but not least, we refer to the γ-ray observations from Mrk 501 which assume constraints on the Lorentz invariance breaking parameter based on potential departure from the exact Lorentz invariance introduced in a perturbative framework [9, 10]. Accordingly, we could assume that the sensitivity of neutrinos to redshift might not be negligible. Amelino-Camelia et al. [26] proposed to use astrophysical objects to look for energy-dependent time-of-arrival delays. As discussed in the section that follows, fixing the sensitivity factor and its energy dependence are essential inputs for this purpose. The sensitivity factor is related to the special time-of-flight delay and the time structure of the signal. Furthermore, a weak indication for redshift dependence of time delays suggestive of LIV has been observed by Ellis et al. [11]. They investigated the energy-dependent time offsets in different energy bands on a sample of gamma-ray bursts and assuming the standard cosmological model.

2.3. Ultra High-Energy Cosmic Rays. Following from Eqs. (9) and (10), the time of flight is conjectured to possess a delay of factor Δt. The generic ultra high-energy cosmic rays (UHECR) can be utilized as a laboratory to study the consequences of LIV. The pair production is kinematically allowed, when energy available to γ rays [10]

$$E_\gamma \geqslant m_e \left(\frac{2}{|\mathcal{D}|}\right)^{1/2}, \tag{19}$$

where the subscript e stands for electron or positron and $\mathcal{D} = (v_e - c)/c$. Depending on the values of v_e and c, $\mathcal{D}$ can be positive and negative.

Stecker and Glashow used γ-ray observations from Mrk 501 constraining the Lorentz invariance breaking parameter [10] based on potential departure from the exact Lorentz invariance introduced in a perturbative framework [9]. According to Eq. (18), we can for simplicity assume that the electron has the same energy and mass as that of the muon neutrino in the OPERA experiment. The observations of UHECR refer to the existence of electrons with energies $E_e \simeq 1 \cdot 10^{12}$ eV [48] and γ rays with energies $E_\gamma \geqslant 50 \cdot 10^{12}$ eV [49] are observed. These observation would set upper limits to $\mathcal{D}_e \simeq 1.3 \cdot 10^{-13}$ and $\mathcal{D}_\gamma \lesssim 2 \cdot 10^{-16}$, respectively. It is apparent that all these values are smaller than the values that were estimated using the GUP approach, $\Delta v \simeq 10^{-11}c$, Subsec. 2.2. Such a discrepancy would be interpreted as follows. In our calculations, the GUP approach assumes a *linear* momentum modification [6, 7]. As discussed above, this approach combines string theory, loop quantum gravity, black hole physics, and doubly spacial relativity.

Recent theoretical work on quantum gravity, especially within string theory, shows that the sensitivity factor of gamma-ray bursts (GRB) η can be related to Δt^*, the special time-of-flight delay and δt, the time structure of the signal $\eta \equiv |\Delta t^*|/\delta t$ [26]. The special time-of-flight delay is characterized by E_{qg} ($E_{\rm Pl}$), effective quantum gravity energy scale (Planck energy scale). The condition that $E_{qg} \approx E_{\rm Pl}$ means that quantum gravity energy reaches the Planck energy. At this scale, η is determined by $|\Delta t^*|/\delta t$. In the present work, Δt^* is taken equivalent to Δt. Depending on distant origin, GRB emission can reach the Earth with different time structures δt. Therefore, the time structure might be sophisticated.

On the other hand, the conventional gravitational lensing is achromatic. Therefore, the energy-dependent time delay would not be dependent on the actual emission mechanism GRB. Couple decades ago, lensed GRB was observed [50]. It can be used to estimate the sensitivity factor as $\eta \approx 10^{-6}$. This value reveals that $\delta t \approx 10^{-7}$ s. It is found that $\eta \approx 10^{-10}$ and therefore $\delta t \approx 10^{-3}$ s, when pulsars, supernovae, and other astrophysical phenomena, but not GRB, are taken into consideration [51]. The third estimation was done using neutrinos stemming from type-II supernovae, like SN1987a. In this case, $\eta \approx 10^{-4}$ and the time structure can be estimated as $\delta t \approx 10^{-9}$ s. In principle, the upper bound on α parameter, which characterizes the GUP approach, can be found by comparing the calculations with the experimental observations [7].

Confronting our calculations to UHECR requires fixing the sensitivity factor and its energy dependence. The sensitivity factor is related to the special time-of-flight delay and the time structure of the signal. To judge about the applicability of GUP on UHECR, we recall the two main scenarios of their origin. Bottom–top scenario assumes that cosmic rays are generated at low energies. Over their path to the Earth they gain energy through various mechanisms [52]. The top–bottom scenario proposes that cosmic rays are produced at much higher energies (Planck scale). Over their path to the Earth they lose energies through various mechanisms [52]. Thus, the applicability is guaranteed.

3. CONCLUSIONS AND OUTLOOK

In this paper, we introduced the calculations for the time-of-flight delays and the relative change in the velocity of muon neutrino with mass of 1 eV and energy 17 GeV. In doing this, we utilized the GUP approach, which is based on the momentum-dependent modification in the standard dispersion relation. For a particle having a distant origin and energy comparable with the Planck energy scale, the comoving momentum is given as a series of *linear* modifications on momentum. Varying the redshift, we have calculated the relative change in the speed of massive muon neutrino and its time-of-flight delays. The redshift depends on the temporal evolution of the Hubble parameter, which can be estimated from a large sample of early-type galaxies extracted from several spectroscopic surveys spanning over $\sim 8 \cdot 10^9$ years of cosmic lookback time, most massive, red elliptical galaxies, passively evolving and without a signature of ongoing star formation are picked up and used as standard cosmic chronometers giving the cosmic time directly probe for $H(z)$. The measurements, according to BC03 model and in combination with CMB data constraining the possible deviations from the standard (minimal) flat ΛCDM model, are used to estimate the z-dependence of the Hubble parameter. The measurements based on the MS model are used to show that the results are model-dependent.

We compared the results with the OPERA experiment. We conclude that the OPERA measurements for Δt and Δv are too large to be interpreted as LIV. Depending on the rest masses, the propagation of high-energy muon neutrino can be superluminal. The other possibility is not excluded. The comparison with UHECR reveals the potential discrepancy between the approach, combining string theory, loop quantum gravity, black hole physics, and doubly spacial relativity and a perturbative departure from the exact Lorentz invariance. For reliable confronting of our calculations to UHECR, we need to fix the sensitivity factor and its energy dependence. The sensitivity factor is related to the special time-of-flight delay and the time structure of the signal.

In the light of this study, we believe that GRB would be able to set an upper bound to the GUP-characterizing parameter α. Furthermore, the velocity anomaly is conjectured to reflect the propagation of all decay channels of neutrino and new physics such as LIV.

Acknowledgements. This work of AT and HM is partly supported by the German–Egyptian Scientific Projects (GESP ID: 1378).

Appendix A
TIME-OF-FLIGHT DELAY ACCORDING TO MS MODEL

It is apparent that integrating the rational expression (13) into Eq. (10) gives a numerical solution. In Fig. 3, *a*, the first (dashed curve) and second (dotted curve) terms of Eq. (10), where $H(z)$ is taken from (13), are given in dependence on z. Their summation is given by the solid curve. The time-of-flight delay Δt can be averaged as $\sim 10^{-13}$ s. This value is much smaller than the one measured in the OPERA experiment, so that the latter would not be interpreted by LIV.

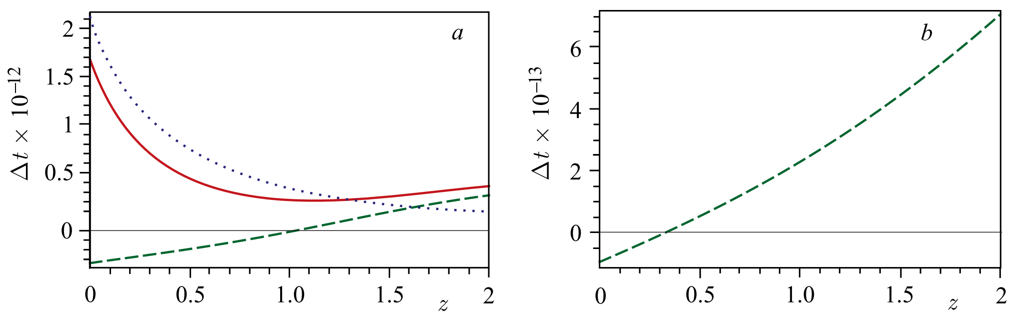

Fig. 3. *a*) The time-of-flight delay of muon neutrino (mass 1 eV and energy 17 GeV) in dependence on z, using (13) in Eq. (10). The first term is given by the dashed curve, while the dotted curve represents the second term. Panel *b* draws Eq. (A.1). Both dashed curves seem to represent comparable results

When implementing Eq. (14) into Eq. (10), the integrals in the second and third terms cannot be solved analytically. The first term can be solved as follows:

$$\Delta t(z) = 2\alpha p_{\nu_0} \int\limits_0^z (1+z)\frac{dz}{H(z)} = \frac{\alpha p_{\nu_0} B \,\mathrm{sech}\,(\delta_3 z)}{\mathrm{e}^C \beta_3 (\beta_3^2 - \gamma_3^2)\delta_3^2 [\beta_3 + \gamma_3 \tanh(\delta_3 z)]} \times \\ \times \Bigg\{ \beta_3^2 \delta_3^2 \,\mathrm{e}^C z(2+z) + \Bigg[\Bigg(\sqrt{1 - \frac{\beta_3^2}{\gamma_3^2}} - \mathrm{e}^C \Bigg) \gamma_3^2 \delta_3^2 z^2 + \beta_3 \gamma_3 \,\mathrm{e}^C \Bigg(i\pi \ln[1 + \mathrm{e}^{2\delta_3 z}] - \\ - 2C \ln(1 - \mathrm{e}^{\delta_3 z - 2C}) - i\pi \ln[\cosh(\delta_3 z)] + 2C \ln[i \sinh(C + \delta_3 z)] - \\ - 2\delta_3 \Big\{ \frac{\pi}{2} z + zC + z \ln(1 - \mathrm{e}^{-2[C + \delta_3 z]}) + \ln B \Big\} \Bigg) \Bigg] \Bigg\}, \quad \text{(A.1)}$$

where $B = \beta_3 \cosh(\delta_3 z) + \gamma_3 \sinh(\delta_3 z)$ and $C = \operatorname{artanh}(\beta_3/\gamma_3)$. The results are drawn in Fig. 3, *b*. In these calculations, only the real component of the second line of Eq. (A.1) is taken into consideration. The values of Δt can be approximated to 10^{-13} s, which is about seven orders of magnitude smaller than the time-of-flight delay measured in the OPERA experiment. With the dashed curve (first term) in Fig. 3, *a*, this term gives comparable results, qualitatively and almost quantitatively.

Appendix B
BOUNDS ON GUP PARAMETER

The GUP parameter is given as $\alpha = \alpha_0/(M_{\rm Pl}c) = \alpha_0 \ell_{\rm Pl}/\hbar$, where $c, \hbar$, and $M_{\rm Pl}$ are speed of light, the Planck constant, and mass, respectively. The Planck length $\ell_{\rm Pl} \approx 10^{-35}$ m and the Planck energy $M_{\rm Pl}c^2 \approx 10^{19}$ GeV. The α_0, the proportionality constant, is conjectured to be dimensionless [6]. In natural units $c = \hbar = 1$, α will be in GeV^{-1}, while in physical units, α should be in GeV^{-1} times c. The bounds on α_0, which were summarized in [7, 18, 20], should be the subject of precise astronomical observations, for instance, gamma-ray bursts.

• Other alternatives were provided by the tunnelling current in the scanning tunnelling microscope and the potential barrier problem [17], where the energy of the electron beam is close to the Fermi level. We found that the varying tunnelling current relative to its initial value is shifted due to the GUP effect [17, 20], $\delta I/I_0 \approx 2.7 \cdot 10^{-35}$ times α_0^2. In case of electric current density J relative to the wave function Ψ, the current accuracy of precision measurements reaches the level of 10^{-5}. Thus, the upper bound $\alpha_0 < 10^{17}$. Apparently, α tends to order 10^{-2} GeV^{-1} in natural units or 10^{-2} GeV^{-1} times c in physical units. This quantum mechanically-derived bound is consistent with the one at the electroweak scale [17, 18, 20]. Therefore, this could signal an intermediate length scale between the electroweak and the Planck scales [17, 18, 20].

• On the other hand, for a particle with mass m, electric charge e affected by a constant magnetic field $\mathbf{B} = B_{\hat{z}} \approx 10$ T, vector potential $\mathbf{A} = Bx\hat{y}$, and cyclotron frequency $\omega_c = eB/m$, the Landau energy is shifted due to the GUP effect [17, 20] by

$$\frac{\Delta E_{n(\rm GUP)}}{E_n} = -\sqrt{8\,m}\alpha(\hbar\omega_c)^{1/2}\left(n + \frac{1}{2}\right)^{1/2} \approx -10^{-27}\alpha_0. \tag{B.1}$$

Thus, we conclude that if $\alpha_0 \sim 1$, then $\Delta E_{n(\rm GUP)}/E_n$ is too tiny to be measured. But with the current measurement accuracy of 1 in 10^3, the upper bound on $\alpha_0 < 10^{24}$ leads to $\alpha = 10^{-5}$ in natural units or $\alpha = 10^{-5}$ times c in physical units.

• Similarly, for the hydrogen atom with Hamiltonian $H = H_0 + H_1$, where standard Hamiltonian $H_0 = p_0^2/(2m) - k/r$ and the first perturbation Hamiltonian $H_1 = -\alpha p_0^3/m$, it can be shown that the GUP effect on the Lamb shift [17, 20] reads

$$\frac{\Delta E_{n(\rm GUP)}}{\Delta E_n} \approx 10^{-24}\alpha_0. \tag{B.2}$$

Again, if $\alpha_0 \sim 1$, then $\Delta E_{n(\rm GUP)}/E_n$ is too small to be measured, while the current measurement accuracy gives 10^{12}. Thus, we assume that $\alpha_0 > 10^{-10}$.

In the light of this discussion, we should assume that the dimensionless α_0 has the order of unity in natural units, then α equals to the Planck length $\approx 10^{-35}$ m. The current experiments seem unable to register discreteness smaller than about 10^{-3}-th fm, $\approx 10^{-18}$ m [17, 20]. We conclude that the assumption that $\alpha_0 \sim 1$ seems to contradict various observations and experiments [17,20]. Therefore, such an assumption should be relaxed to meet the accuracy of the given experiments. Accordingly, the lower bounds on α range from 10^{-10} to 10^{-2} GeV^{-1}. This means that α_0 ranges between $10^9 c$ and $10^{17} c$.

REFERENCES

1. *Garay L. J.* Quantum Gravity and Minimum Length // Intern. J. Mod. Phys. A. 1995. V. 10. P. 145–166; arXiv:gr-qc/9403008.
2. *Kiefer C.* // Compend. Quant. Phys. 2009. P. 565–572.
3. *Isham C. J.* // Lecture Notes Phys. 1994. V. 434. P. 1–21.
4. *Maggiore M.* Quantum Groups, Gravity and the Generalized Uncertainty Principle // Phys. Rev. D. 1994. V. 49. P. 5182;
   Kempf A. // Symp. at the 21st Intern. Colloquium on "Group Theoretical Methods in Physics". Group'21, ICGTMP'96, Goslar, Germany, 1996;
   Bambi C. A Revision of the Generalized Uncertainty Principle // Class. Quant. Grav. 2008. V. 25. P. 105003;
   Adler R. J. Six Easy Roads to the Planck Scale // Am. J. Phys. 2010. V. 78. P. 925;
   Nozari K., Pedram P., Molkara M. Minimal Length, Maximal Momentum and the Entropic Force Law // Intern. J. Theor. Phys. 2012. V. 51. P. 1268.
5. *Amati D., Ciafaloni M., Veneziano G.* Classical and Quantum Gravity Effects from Planckian Energy Superstring Collisions // Intern. J. Mod. Phys. A. 1988. V. 3. P. 1615; Superstring Collisions at Planckian Energies // Phys. Lett. B. 1987. V. 197. P. 81; Higher Order Gravitational Deflection and Soft Bremsstrahlung in Planckian Energy Superstring Collisions // Nucl. Phys. B. 1990. V. 347. P. 550.
6. *Farag Ali A., Das S., Vagenas E. C.* Discreteness of Space from the Generalized Uncertainty Principle // Phys. Lett. B. 2009. V. 678. P. 497–499;
   Das S., Vagenas E. C., Farag Ali A. Discreteness of Space from GUP II: Relativistic Wave Equations // Phys. Lett. B. 2010. V. 690. P. 407–412; Erratum // Ibid. V. 692. P. 342.
7. *Farag Ali A., Das S., Vagenas E. C.* A Proposal for Testing Quantum Gravity in the Lab // Phys. Rev. D. 2011. V. 84. P. 044013.
8. *Sato H., Tati T.* Hot Universe, Cosmic Rays of Ultrahigh Energy and Absolute Reference System // Prog. Theor. Phys. 1972. V. 47. P. 1788.
9. *Coleman S., Glashow S. L.* High-Energy Tests of Lorentz Invariance // Phys. Rev. D. 1999. V. 59. P. 116008.
10. *Stecker F. W., Glashow S. L.* New Tests of Lorentz Invariance Following from Observations of the Highest Energy Cosmic Gamma Rays // Astropart. Phys. 2001. V. 16. P. 97–99.
11. *Ellis E. J. et al.* Quantum-Gravity Analysis of Gamma-Ray Bursts Using Wavelets // Astron. Astrophys. 2003. V. 402. P. 409–424;
    Boggs S. E. et al. Testing Lorentz Non-Invariance with GRB021206 // Astrophys. J. 2004. V. 611. P. L77–L80.
12. *Liberati S., Maccione L.* Lorentz Violation: Motivation and New Constraints // Ann. Rev. Nucl. Part. Sci. 2009. V. 59. P. 245–267; arXiv:0906.0681 [astro-ph.HE].

13. *Pikovski I. et al.* Probing Planck-Scale Physics with Quantum Optics // Nature Phys. 2012. V. 8. P. 393–397.
14. *Hossenfelder S.* Minimal Length Scale Scenarios for Quantum Gravity // Living Rev. Rel. 2013. V. 16. P. 2; arXiv:1203.6191 [gr-qc].
15. *Tawfik A., Diab A.* Generalized Uncertainty Principle: Approaches and Applications // Intern. J. Mod. Phys. D. 2014. V. 23. P. 1430025; arXiv:1410.0206 [gr-qc].
16. *Das S., Vagenas E. C.* Universality of Quantum Gravity Corrections // Phys. Rev. Lett. 2008. V. 101. P. 221301; arXiv:0810.5333 [hep-th].
17. *Ali A. F., Das S., Vagenas E. C.* The Generalized Uncertainty Principle and Quantum Gravity Phenomenology. arXiv:1001.2642 [hep-th].
18. *Das S., Vagenas E. C.* Phenomenological Implications of the Generalized Uncertainty Principle // Can. J. Phys. 2009. V. 87. P. 233; arXiv:0901.1768 [hep-th].
19. *Elmashad I. et al.* Quantum Gravity Effect on the Quark–Gluon Plasma // SOP Trans. Theor. Phys. 2014. V. 1. P. 1–6; arXiv:1208.4028 [hep-ph].
20. *Chemissany W. et al.* Effect of the Generalized Uncertainty Principle on Post-Inflation Preheating // JCAP. 2011. V. 1112. P. 017; arXiv:1111.7288 [hep-th].
21. *Tawfik A., Magdy H., Ali A. F.* Effects of Quantum Gravity on the Inflationary Parameters and Thermodynamics of the Early Universe // Gen. Rel. Grav. 2013. V. 45. P. 1227–1246; arXiv:1208.5655 [gr-qc].
22. *Ali A. F.* No Existence of Black Holes at LHC Due to Minimal Length in Quantum Gravity // JHEP. 2012. V. 1209. P. 067; arXiv:1208.6584 [hep-th].
23. *Tawfik A.* Impacts of Generalized Uncertainty Principle on Black Hole Thermodynamics and Salecker–Wigner Inequalities // JCAP. 2013. V. 1307. P. 040; arXiv:1307.1894 [gr-qc].
24. *Farag Ali A., Tawfik A. N.* Impacts of Generalized Uncertainty Principle on Black Hole Thermodynamics and Salecker–Wigner Inequalities // Intern. J. Mod. Phys. D. 2013. V. 22. P. 1350020; arXiv:1301.6133[gr-qc].
25. *Farag Ali A., Tawfik A.* Modified Newton's Law of Gravitation Due to Minimal Length in Quantum Gravity // Adv. High Energy Phys. 2013. V. 2013. P. 126528; arXiv:1301.3508 [gr-qc].
26. *Amelino-Camelia G. et al.* Tests of Quantum Gravity from Observations of Gamma-Ray Bursts // Nature. 1998. V. 393. P. 763–765.
27. *Tawfik A. et al.* Dissipative Processes in the Early Universe: Bulk Viscosity // Uzb. J. Phys. 2010. V. 12. P. 316–321.
28. *Tawfik A.* // 12th Marcel Grossmann Meeting on General Relativity, Paris, France, 2009.
29. *Tawfik A. et al.* Viscous Quark–Gluon Plasma in the Early Universe // Ann. Phys. 2011. V. 523. P. 194–207.
30. *Tawfik A.* Thermodynamics in the Viscous Early Universe // Can. J. Phys. 2010. V. 88. P. 825–831.
31. *Tawfik A. et al.* Hubble Parameter in QCD Universe for Finite Bulk Viscosity // Ann. Phys. 2010. V. 522. P. 912–923.
32. *Tawfik A.* The Hubble Parameter in the Early Universe with Viscous QCD Matter and Finite Cosmological Constant // Ann. Phys. 2011. V. 523. P. 423–434.
33. *Tawfik A., Harko T.* Quark–Hadron Phase Transitions in Viscous Early Universe // Phys. Rev. D. 2012. V. 85. P. 084032.
34. *Tawfik A., Magdy H.* Thermodynamics of Viscous Matter and Radiation in the Early Universe // Can. J. Phys. 2012. V. 90. P. 433–440.
35. *Samushia L., Ratra B.* Cosmological Constraints from Hubble Parameter versus Redshift Data // Astrophys. J. 2006. V. 650. P. L5–L8.

36. *Moresco M. et al.* Improved Constraints on the Expansion Rate of the Universe up to $z \sim 1.1$ from the Spectroscopic Evolution of Cosmic Chronometers // JCAP. 2012. V. 1208. P. 006.
37. *Moresco M. et al.* New Constraints on Cosmological Parameters and Neutrino Properties Using the Expansion Rate of the Universe to $z \sim 1.75$ // Ibid. V. 1207. P. 053.
38. *Jimenez R., Loeb A.* Constraining Cosmological Parameters Based on Relative Galaxy Ages // Astrophys. J. 2002. V. 573. P. 37.
39. *Bruzual G., Charlot S.* Stellar Population Synthesis at the Resolution of 2003 // Mon. Not. Roy. Astron. Soc. 2003. V. 344. P. 1000.
40. *Maraston C., Strömbäck G.* Stellar Population Models at High Spectral Resolution // Mon. Not. Roy. Astron. Soc. 2011. V. 418. P. 2785; arXiv:1109.0543 [astro-ph.CO].
41. *Adam T. et al.* Measurement of the Neutrino Velocity with the OPERA Detector in the CNGS Beam // JHEP. 2012. V. 1210. P. 093.
42. *Adam T. et al. (Opera Collab.).* Measurement of the Neutrino Velocity with the OPERA Detector in the CNGS Beam Using the 2012 Dedicated Data // JHEP. 2013. V. 1301. P. 153.
43. *Adamson P. et al. (MINOS Collab.).* Measurement of Neutrino Velocity with the MINOS Detectors and NuMI Neutrino Beam // Phys. Rev. D. 2007. V. 76. P. 072005.
44. *Hirata K. et al. (Kamiokande-II Collab.).* Observation of a Neutrino Burst from the Supernova SN1987a // Phys. Rev. Lett. 1987. V. 58. P. 1490–1493;
Bionta R. et al. Observation of a Neutrino Burst in Coincidence with Supernova SN1987a in the Large Magellanic Cloud // Ibid. P. 1494;
Longo M. J. Tests of Relativity from SN1987a // Phys. Rev. D. 1987. V. 36. P. 3276.
45. *Barenboim G., Requejo O. M., Quigg Ch.* Diagnostic Potential of Cosmic-Neutrino Absorption Spectroscopy // Phys. Rev. D. 2005. V. 71. P. 083002.
46. *Kusenko A., Postma M.* Neutrinos Produced by Ultrahigh-Energy Photons at High Red Shift // Phys. Rev. Lett. 2001. V. 86. P. 1430–1433.
47. *Majumdar D.* Probing Pseudo-Dirac Neutrino through Detection of Neutrino Induced Muons from GRB Neutrinos // Pramana. 2008. V. 70. P. 51–60.
48. *Nishimura J. et al.* Emulsion Chamber Observations of Primary Cosmic Ray Electrons in the Energy Range 30 GeV–1000 GeV // Astrophys. J. 1980. V. 238. P. 394.
49. *Tanimori T. et al. (CANGAROO Collab.).* Detection of Gamma Rays up to 50 TeV from the Crab Nebula // Astrophys. J. 1998. V. 492. P. L33.
50. *Hurley K. et al.* Detection of a Gamma-Ray Burst of Very Long Duration and Very High Energy // Nature. 1994. V. 372. P. 652–654.
51. *Amelino-Camelia G. et al.* Distance Measurement and Wave Dispersion in a Liouville String Approach to Quantum Gravity // Intern. J. Mod. Phys. A. 1997. V. 12. P. 607–623.
52. *Tawfik A.* Orders of Fermi- and Plasma-Accelerations of Cosmic Rays // SOP Trans. Theor. Phys. 2014. V. 1. P. 7–9; arXiv:1010.5390 [astro-ph.HE].

Received on April 6, 2015.