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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

ESTIMATION OF MAGNETIC FIELD GROWTH AND CONSTRUCTION OF ADAPTIVE MESH IN CORNER DOMAIN FOR MAGNETOSTATIC PROBLEM

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A magnetostatic problem arises in searching for the distribution of the magnetic field generated by magnet systems of many physics research facilities, e.g., accelerators. The domain in which the boundary-value problem is solved often has a piecewise smooth boundary. In this case, numerical calculations of the problem require the consideration of the solution behavior in the corner domain. In this paper an upper estimate is obtained for the maximum possible growth of the magnetic field in the corner domain of vacuum. Based on this estimate, we propose a method of condensing the differential grid near the corner domain of vacuum. An example of the modeling problem calculation in the corner domain is given.

Постановка задачи магнитостатики возникает при поиске распределения магнитного поля, создаваемого магнитной системой, входящей в состав многих физических установок, например, ускорителей. Часто бывает, что область, в которой решается краевая задача магнитостатики, имеет кусочно-гладкую границу. В таких случаях при численном нахождении решения задачи необходимо учитывать характер его поведения в окрестности «угловой точки».

В данной работе дается верхняя оценка допустимого роста магнитного поля в окрестности «угловой точки» в области вакуума; на основании полученной оценки предлагается метод сгущения разностной сетки вблизи «угловой точки» в области вакуума. Приводится пример расчета модельной задачи в области, содержащей угловую точку.

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INTRODUCTION

Many physics research facilities use magnet systems of various configurations. An example is a system of spectrometric magnets. It is very important to know with a good accuracy the distribution of the magnetic field generated by this system. The problem is actually reduced to formulation of a magnetostatic problem of finding the distribution of the magnetic field generated by the magnet system under consideration. Since the magnetic system has a complicated configuration, the solution of the problem is usually sought using numerical methods. The domain in which the boundary-value problem is solved during calculations of a particular magnet system often has a piecewise smooth boundary. In this case the solution of the problem or the derivative solutions can have a singularity. Therefore, the numerical search for the solution requires the use of special methods.

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1. ESTIMATION OF THE MAGNETIC FIELD GROWTH

Let us show that the magnetic field in the corner domain of vacuum Ω_v of a ferromagnetic satisfies the condition

$$H(s) \leqslant C_0 \ln \frac{1}{r_s} + w(s), \tag{1}$$

where C_0 is a constant; $w(s)$ is a bounded function, and r_s is the distance to the corner and $s \in \Omega_v$. The integral formulation of the magnetostatic problem [1, 2] allows the magnetic field to be represented as (on the assumption that a solution exists)

$$\mathbf{H}(s) = \mathbf{H}_C(s) - \nabla_s \int\limits_{\Omega_f} (\mathbf{M}(p), \nabla_P \Psi(s, p))\, dv_p, \tag{2}$$

where $\mathbf{H}_C$ is the field from the current sources, $\mathbf{M}$ is the ferromagnetic magnetization vector, the function $\Psi(s, p)$ is equal to $\dfrac{1}{4\pi r_{sp}}$ or $\dfrac{1}{2\pi} \ln r_{sp}$ for the three-dimensional and the two-dimensional case, respectively, and Ω_f is the ferromagnetic domain. The magnetization vector is defined as $\mathbf{M} = \mu_0 \chi(H) \mathbf{H} = \mu_0 (\mu(H) - 1) \mathbf{H}$, where μ_0 is a constant, $\chi(H)$ is the magnetic susceptibility, and $\mu(H)$ is the permeability of the ferromagnetic. Given high fields ($H \to \infty$), the representation [3, 4] $\mu(H) = 1 + \dfrac{A}{H} - \dfrac{B}{H^2}$ when $H \to \infty$ is valid, where A and B are positive constants. Consequently, when $H \to \infty$, $M = |\mathbf{M}|$ is limited by a constant $M_0 = \mu_0 A$. Let us consider the 2D case. From (2) we obtain

$$\mathbf{H}(s) = \mathbf{H}_C(s) - \frac{1}{2\pi} \nabla_s \int\limits_{\Omega_f} \left(\mathbf{M}(p), \frac{\mathbf{r}_{sp}}{r_{sp}^2} \right) dv_p.$$

Here the first term is limited, and we therefore estimate the second term

$$\left| \nabla_s \int\limits_{\Omega_f} \left(\mathbf{M}(p), \frac{\mathbf{r}_{sp}}{r_{sp}^2} \right) dv_p \right| \leqslant \left| \int\limits_{\Omega_f} \frac{\partial}{\partial x_s} \frac{M^{(x)}(p)(x_p - x_s) + M^{(y)}(p)(y_p - y_s)}{(x_p - x_s)^2 + (y_p - y_s)^2} dv_p \right| +$$

$$+ \left| \int\limits_{\Omega_f} \frac{\partial}{\partial y_s} \frac{M^{(x)}(p)(x_p - x_s) + M^{(y)}(p)(y_p - y_s)}{(x_p - x_s)^2 + (y_p - y_s)^2} dv_p \right| =$$

$$= \left| \int\limits_{\Omega_f} \frac{M^{(x)}\left(\bar{x}^2 - \bar{y}^2\right) + 2M^{(y)}\overline{xy}}{\left(\bar{x}^2 + \bar{y}^2\right)^2} dv_p \right| + \left| \int\limits_{\Omega_f} \frac{M^{(y)}\left(\bar{y}^2 - \bar{x}^2\right) + 2M^{(y)}\overline{xy}}{\left(\bar{x}^2 + \bar{y}^2\right)^2} dv_p \right| \leqslant$$

$$\leqslant 2 \int\limits_{\Omega_f} \frac{2r_{sp}^2 \left|M^{(x)}\right| + 2r_{sp}^2 \left|M^{(y)}\right|}{r_{sp}^4} dv_p \leqslant 8M_0 \int\limits_{\Omega_f} \frac{1}{r_{sp}^2} dv_p,$$

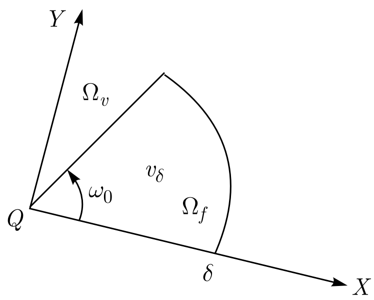

Fig. 1. The angular sector

where $\bar{x} = x_p - x_s$ and $\bar{y} = y_p - y_s$. We calculate the integral

$$\int\limits_{\Omega_f} \frac{1}{r_{sp}^2} d\upsilon_p = \int\limits_{\upsilon_\delta} \frac{1}{r_{sp}^2} d\upsilon_p + \int\limits_{\Omega_f/\upsilon_\delta} \frac{1}{r_{sp}^2} d\upsilon_p,$$

where $\upsilon_\delta = \Omega_f \cap S_\delta(Q)$ is the angular sector at the corner point Q (see Fig. 1).

The integral over the domain Ω_f/υ_δ will be limited, and we therefore consider only the integral over the domain υ_δ

$$\int\limits_{\Omega_f} \frac{1}{r_{sp}^2} d\upsilon_p = \int\limits_0^{\omega_0} d\varphi_p \int\limits_0^{\delta} \frac{r_p\, dr_p}{r_p^2 + r_s^2 - 2r_p r_s \cos\varphi_{sp}} =$$
$$= \int\limits_0^{\omega_0} d\varphi_p \left(\int\limits_0^1 \frac{t\, dt}{1 + t^2 - 2t\cos\varphi_{sp}} + \int\limits_1^{\delta/r_s} \frac{t\, dt}{1 + t^2 - 2t\cos\varphi_{sp}} \right),$$

where $t = r_p/r_s$. Then we use the expression for the generating function [5]

$$\frac{1}{\sqrt{1 + t^2 - 2t\cos\varphi_{sp}}} = \begin{cases} \sum\limits_{m=0}^{+\infty} P_m(\cos\varphi_{sp})\, t^m, & |t| < 1, \\ \sum\limits_{m=0}^{+\infty} P_m(\cos\varphi_{sp})\, t^{-m-1}, & |t| < 1 \end{cases}$$

and obtain

$$\int\limits_{\Omega_f} \frac{1}{r_{sp}^2} d\upsilon_p = \int\limits_0^{\omega_0} d\varphi_p \left(\int\limits_0^1 t\, dt \sum_{m,k=0}^{+\infty} P_m P_k t^{k+m} + \int\limits_1^{\delta/r_s} t\, dt \sum_{m,k=0}^{+\infty} P_m P_k t^{-(k+m+2)} \right) =$$
$$= \omega_0 \ln\frac{\delta}{r_s} + \sum_{m,k=0}^{+\infty} \frac{\alpha_{m,k}}{m+k+2} + \sum_{m+k\neq 0}^{+\infty} \frac{\alpha_{m,k}}{m+k} \left(\left(\frac{r_s}{\delta}\right)^{m+k} - 1 \right) =$$
$$= C_1 \ln\frac{1}{r_s} + w_1(s), \quad (3)$$

where $\alpha_{m,k} = \int\limits_0^{\omega_0} P_m(\cos\varphi_{sp})\, P_k(\cos\varphi_{sp})\, d\varphi_p$, C_1 is a constant, and $w_1(s)$ is a bounded function. Thus, the validity of expression (1) is ascertained.

2. METHOD OF GRID CONDENSING IN THE CORNER DOMAIN

In [6–8], there are examples of constructing a differential grid for some boundary-value problems in corner domains. The main idea is to condense the differential grid or finite elements for obtaining an admissible problem approximation error. This error involves integrals over elementary domains estimated by the quantities of the form $Ch_i^\beta \|u\|_{k,j}$, where h_i is the diameter of the ith elementary domain or grid cell, β is a positive number, $\|u\|_{k,j}$ is the norm of the function with the kth derivative in this domain, and C is a constant independent of all these factors. Then we can require, for example, that quantities $Ch_i^{\beta}\|u\|_{k,j}$ be identical in the domain under consideration. To this end, h_i^{β} can be decreased in inverse proportion to $\|u\|_{k,j}$ on approach to the singular points. We demonstrate the validity of the following statement. $\mathbf{V}(s)$ is the solution of the magnetostatic problem in the integral formulation found by a numerical method and $\mathbf{H}(s)$ is the exact solution. Then the following estimate is valid:

$$\|\mathbf{V}-\mathbf{H}\|^2_{L_2(D)} < h^2\left(c_1\ln^2 h + c_2\ln h + c_3\right), \tag{4}$$

where c_1, c_2, and c_3 are constants and h is the diameter of the domain D, which is a differential grid cell containing the ferromagnetic corner.

By virtue of (2), the following expression for $\mathbf{V}(s)$ holds:

$$\mathbf{V}(s) = \mathbf{H}_C(s) - \frac{1}{2\pi}\nabla_s\sum_{j=1}^{N}\int\limits_{\Omega_j}\left(\mathbf{M}(\mathbf{H}_j), \frac{\mathbf{r}_{sp_j}}{r^2_{sp_j}}\right)dv, \tag{5}$$

where $\mathbf{H}_j$ is the field in the cell Ω_j, $j = 1,\ldots,N$; $\bigcup\limits_{j=1}^{N}\Omega_j = \Omega_f$; and r_{sp_j} is the distance from the point s to the point $p_j \in \Omega_j$. We consider the difference

$$\mathbf{V}(s) - \mathbf{H}(s) = -\frac{1}{2\pi}\nabla_s\sum_{j=1}^{N}\int\limits_{\Omega_j}\left(\mathbf{M}(\mathbf{H}_j) - \mathbf{M}(\mathbf{H}(p_j)), \frac{\mathbf{r}_{sp_j}}{r^2_{sp_j}}\right)dv.$$

Since the quantity $|\mathbf{M}| < M_0$ is limited, it follows that $|\mathbf{M}(\mathbf{H}_j) - \mathbf{M}(\mathbf{H}(p_j))| < 2M_0$ for $j = 1,\ldots,N$. Thus, we obtain

$$|\mathbf{V}(s) - \mathbf{H}(s)| < \frac{8M_0}{\pi}\sum_{j=1}^{N}\int\limits_{\Omega_j}\frac{dv}{r^2_{sp_j}} = \frac{8M_0}{\pi}\sum_{j=1}^{N}\int\limits_{\Omega_j\cap S_\delta(Q)}\frac{dv}{r^2_{sp_j}} + \frac{8M_0}{\pi}\sum_{j=1}^{N}\int\limits_{\Omega_j/S_\delta(Q)}\frac{dv}{r^2_{sp_j}}.$$

As a result, using the estimate obtained above, we arrive at the expression

$$|\mathbf{V}(s) - \mathbf{H}(s)| < C_2\ln\frac{1}{r_s} + w_2(s). \tag{6}$$

It remains to estimate $\|\mathbf{V}-\mathbf{H}\|^2_{L_2(D)}$, where the domain D is the $S_\delta(Q)$ — δ-domain of the corner point Q. Using (6), we obtain

$$\|\mathbf{V}-\mathbf{H}\|^2_{L_2(D)} = \int\limits_{D}|\mathbf{V}(s) - \mathbf{H}(s)|^2\,dv < h^2\left(c_1\ln^2 h + c_2\ln h + c_3\right),$$

where $h = 2\delta$, and c_1, c_2, and c_3 are constants. We propose a differential grid condensing method

$$\int\limits_{0}^{h_1}\left|\ln\frac{1}{x}\right|dx = d_0, \quad \int\limits_{x_{m-1}}^{x_m}\left|\ln\frac{1}{x}\right|^2 dx = d_0, \quad x_m - x_{m-1} = h_m, \quad m = 1,2,\ldots,M.$$

Here d_0 is a constant; M is the number of partitions along the coordinate axis (OX or OY) in the corner domain; h_m is the grid spacing, and x_m is the coordinate of the grid node along the OX or OY axis (the origin of the coordinates is at the corner point), $|x| < 1$. In model numerical calculation tasks with a corner point on the course, building a mesh grid of the differential in accordance with (6) has yielded good results.

3. THE SOLENOID-TYPE MAGNETIC FIELD DETECTOR MODELING

Magnetic systems are very important parts [9, 10]. To create the necessary configuration of magnetic field, the repeated solution of nonlinear boundary value problem of magnetostatics is needed. In the present work, we consider the problem of creation of homogeneous map of magnetic system of solenoidal type (see Fig. 2). As a result of optimization, the geometric parameters of magnetic system were chosen in such a way so as to get maximal size of the domain of homogeneity of the magnetic field.

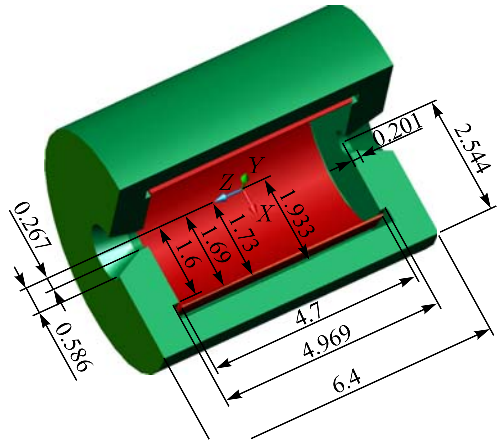

Fig. 2. Magnet geometry

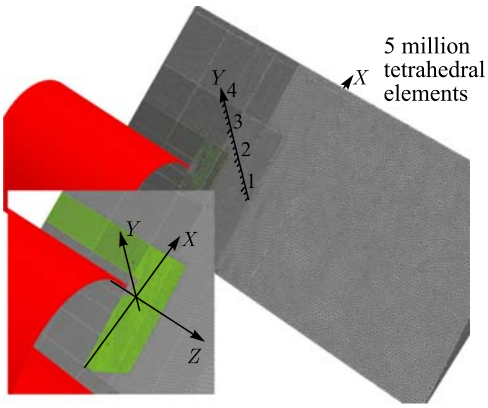

Fig. 3. Mesh

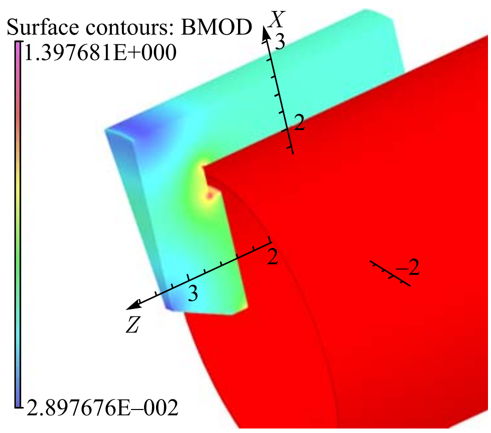

Fig. 4. Field distribution

Due to symmetry, in the modeling we use only 1/24 part of the geometry with corresponding boundary conditions. The calculations were performed (using two software products: TOSCA and native MFC) by the method of finite elements on tetrahedral mesh with 5 000 000 elements (see Fig. 3).

The distribution of the module of magnetic field on the surface of ferromagnetic is shown in Fig. 4. It is seen that maximal value of magnetic field is reached in the corner points (1.3 T).

The density of the current in winding $J = 9.956410099 \cdot 10^6$ A/m^2. The cross section of coil $S = 0.04 \times 4.7$ m. The total current $I = 1.871805098 \cdot 10^6$ A. The field in the center of magnetic system $B_{\text{center}} = 0.5$ T.

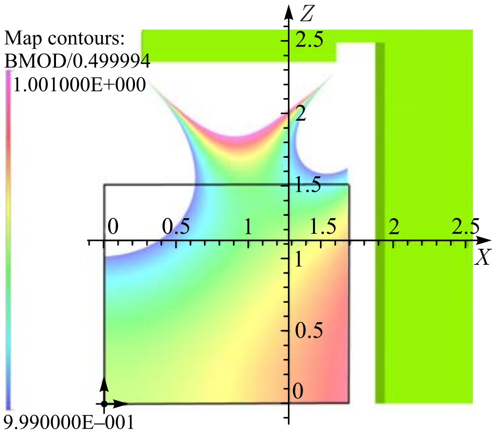

Fig. 5. Field homogeneity is ±0.1%

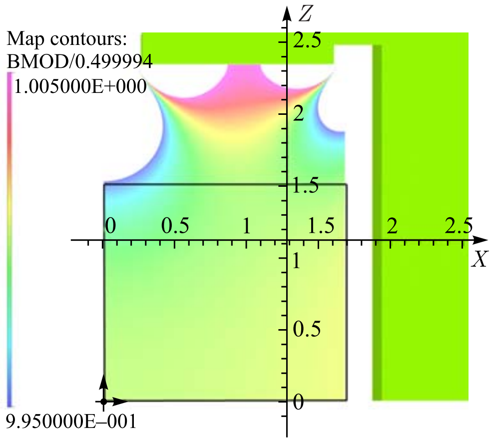

Fig. 6. Field homogeneity is ±0.5%

In Figs. 5 and 6 the domains with the degrees of homogeneity of magnetic field are 0.1% and 0.5%, correspondingly. The black continuous line shows the homogeneity of 0.1% is needed. In Fig. 5, the scale of magnetic field has site from 0.99–1.001 T, in Fig. 6 from 0.998–1.002 T.

CONCLUSIONS

1. The upper estimate for the admissible growth of the magnetic field $\mathbf{H}(p)$ in the corner domain of vacuum Ω_v $H(p) \leqslant C_0 \ln \dfrac{1}{r_p} + w(p)$, where C_0 is a constant, $w(p)$ is a bounded function, and r_p is the distance to the corner, is asymptotically obtained for the case of $\mu(H) \to 1$ when $H \to \infty$.

2. A method of condensing the differential grid in the corner domain of vacuum Ω_v is proposed, which appreciably improves the accuracy of the calculated solution.

3. As a result of optimization, the geometric parameters of the solenoid-type magnetic field detector were chosen in such a way so as to get maximal size of the domain of homogeneity of the magnetic field.

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