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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ ФИЗИКИ

TIME-MODULATED ENTANGLED STATES OF LIGHT

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We investigate time-modulated EPR entangled states and intensity quantum correlation of twin light beams in application to time-resolved quantum communication. As a proper device generating such states the nondegenerate optical parametric oscillator driven by time-modulated pump field is considered.

Модулированные по времени перепутанные состояния света, а также квантовые корреляции интенсивностей световых пучков исследованы в применении к проблемам квантовых коммуникаций с временным разрешением. Для генерации таких состояний предложен и исследован оптический параметрический осциллятор под действием модулированного по времени поля возмущения.

INTRODUCTION

Research of novel protocols in the area of quantum information is presently very active. In that field, lots of interest has arisen recently towards the use of quantum continuous variables (CV). Moreover, entanglement and squeezing appear as fundamental resources for quantum communication and universal computing with CV. Up to now, most of quantum protocols have been experimentally performed in the spectral domain by using the continuous waves generated in steady state regimes. In addition, the characterization of CV entanglement and squeezing were performed in the spectral domain, rather than in the time domain, by using the technique of homodyne spectral measurements [1]. Nevertheless, it seems that the analysis of these protocols should be very easy in terms of information transfer that usually takes place in the classical information theory. Such a situation can be realized at least for communication schemes operating in a pulsed regime because, in this case, we are able to manipulate individually each quantum state involved in the exchange. In this direction, novel quantum key distribution (QKD) schemes using the quadrature components of amplitude and phase modulated coherent states have been recently proposed and experimentally demonstrated [2]. The efficient setups have also been proposed for generation and characterization of quadrature-squeezed pulses as well as quadrature-entangled pulses [3] in the time domain. Thus, we conclude that an important issue for time-resolved quantum communication is to investigate CV entanglement for various time-modulated regimes. As a realization of this program, in this paper we investigate time-dependent entanglement for light beams generated

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in time-modulated nondegenerate optical parametric oscillator (NOPO). Another nonclassical phenomenon in this area is intensity quantum correlation of twin light beams [4]. Correlated twin light beams are elegant candidates for verifying the foundations of quantum physics and are of increasing interest for quantum information protocols. For instance, a QKD scheme based on utilizing the intensity quantum correlation of optical twin beams has recently been proposed [5]. In this scheme, the homodyne detection is not needed, so the system is significantly simplified with respect to most proposed CV QKD protocols. In this direction, we analyze sub-shot-noise intensity correlations of time-modulated twin beams generated in NOPO.

1. TIME-MODULATED NOPO

We consider a type-II phase-matched NOPO with triply resonant optical ring cavity under the action of pump field with periodically varying amplitude. The semiclassical and quantum theories of time-modulated NOPO are given in [7] and are briefly recalled here. The interaction Hamiltonian of the system within the framework of rotating wave approximation and in the interaction picture is

$$H = i\hbar f\left(t\right)\left(\mathrm{e}^{i(\Phi_L-\omega_L t)}\,a_3^{+} - \mathrm{e}^{-i(\Phi_L-\omega_L t)}\,a_3\right) + i\hbar k\left(\mathrm{e}^{i\Phi_k}\,a_3 a_1^{+} a_2^{+} - \mathrm{e}^{-i\Phi_k}\,a_3^{+} a_1 a_2\right), \quad (1)$$

where a_i are the boson operators for cavity modes at the frequencies ω_i. The pump mode a_3 is driven by an amplitude-modulated external field at the frequency $\omega_L = \omega_3$ with time-periodic, real valued amplitude $f(t+T) = f(t)$. The constant $k\,\mathrm{e}^{i\Phi_k}$ determines an efficiency of the down-conversion process $\omega_L \to \frac{\omega_L}{2}(\uparrow) + \frac{\omega_L}{2}(\to)$ in $\chi^{(2)}$ medium. We take into account the cavity damping rates γ_i of the modes and consider the case of high cavity losses for the pump mode ($\gamma_3 \gg \gamma$, $\gamma_1 = \gamma_2 = \gamma$) when the pump mode is eliminated adiabatically. In this case, the stochastic equations for complex c-number variables $\alpha_{1,2}$ and $\beta_{1,2}$ corresponding to operators $a_{1,2}$ and $a_{1,2}^{+}$ are derived as

$$\frac{d\alpha_1}{dt} = -(\gamma + \lambda\alpha_2\beta_2)\alpha_1 + \varepsilon\left(t\right)\beta_2 + W_{\alpha_1}(t), \quad (2)$$

$$\frac{d\beta_1}{dt} = -(\gamma + \lambda\alpha_2\beta_2)\beta_1 + \varepsilon\left(t\right)\alpha_2 + W_{\beta_1}(t). \quad (3)$$

Here: $\varepsilon(t) = f(t)k/\gamma_3$, $\lambda = k^2/\gamma_3$ and equations for α_2, β_2 are obtained from (2), (3) by exchanging the subscripts (1) $\to$ (2) and (1) $\leftarrow$ (2). Our derivation is based on the ITO stochastic calculus; and the nonzero stochastic correlations are: $\langle W_{\alpha_1}(t)\,W_{\alpha_2}(t')\rangle = (\varepsilon(t) - \lambda\alpha_1\alpha_2)\,\delta(t-t')$, $\langle W_{\beta_1}(t)\,W_{\beta_2}(t')\rangle = (\varepsilon(t) - \lambda\beta_1\beta_2)\,\delta(t-t')$.

The analysis shows that similar to the standard NOPO, the considered system in the semiclassical approximation also exhibits threshold behavior, which is easily described through the period-averaged pump field amplitude $\overline{f(t)} = \frac{1}{T}\int\limits_0^T f(t)dt$. The below-threshold regime with a stable trivial zero-amplitude solution is realized for $\overline{f} < f_{\mathrm{th}}$, where $f_{\mathrm{th}} = \gamma\gamma_3/k$ is the threshold value. When $\overline{f} > f_{\mathrm{th}}$, the stable nontrivial solution exists with the following properties. As for usual NOPO, the difference between phases of generated modes is undefined due to the phase diffusion, while the sum of phases is equal to $\varphi_1 + \varphi_2 = 2\pi m$, $(m = 0, \pm 1, \ldots)$. The mean photon numbers for subharmonic modes $n_{oi} = \langle a_i^+ a_i \rangle = |\alpha_i|^2$ are equal one to the other ($n_{01} = n_{02} = n_0$) due to the symmetry of the system, $\gamma_1 = \gamma_2 = \gamma$. The straightforward calculations lead to the following result for over-transient regime

$$n_0^{-1}(t) = 2\lambda \int\limits_{-\infty}^{0} \exp\left(2\int\limits_{0}^{\tau} \left(\varepsilon\left(t' + t\right) - \gamma\right) dt'\right) d\tau. \tag{4}$$

Note, that in this regime the photon number is a periodic function of time.

2. TIME-DEPENDENT EPR ENTANGLEMENT

To characterize the CV entanglement we address to both the inseparability criterion and the EPR paradox criterion. These criteria could be quantified by analyzing the variances $V_- = V\left(X_1 - X_2\right)$ and $V_+ = V\left(Y_1 + Y_2\right)$ in terms of the quadrature amplitudes of two modes $X_k = X_k\left(\Theta_k\right) = \frac{1}{\sqrt{2}}\left(a_k^+ \mathrm{e}^{-i\Theta_k} + a_k \mathrm{e}^{i\Theta_k}\right)$, $Y_k = X_k\left(\Theta_k - \frac{\pi}{2}\right)$, $k = 1, 2$, where $V(x) = \langle x^2 \rangle - \langle x \rangle^2$ is a denotation of the variance. The inseparability criterion, or weak entanglement criterion reads as $V_+ + V_- < 2$, and due to the mentioned symmetries is reduced to the following form $V = V_+ = V_- < 1$. The strong CV entanglement criterion shows that when the inequality $V_+ V_- < 1/4$ is satisfied, there arises an EPR-like paradox. We calculate the variance in above-threshold regime by using the linearizing stochastic equations as

$$V\left(t\right) = 2\int\limits_{-\infty}^{t} \exp\left(-2\int\limits_{\tau}^{t} \left(\gamma + \varepsilon\left(t'\right) + \lambda n_0\left(t'\right)\right) dt'\right) \times \\ \times \left[\gamma + \lambda n_0\left(\tau\right) + 2\gamma\lambda \int\limits_{-\infty}^{\tau} \mathrm{e}^{4\gamma\left(\tau' - \tau\right)} n_0(\tau') d\tau'\right] d\tau. \tag{5}$$

The analysis of the below-threshold regime is more simple and leads to formula (5) with $n_0 = 0$.

We consider bellow the application of these results to NOPO driven by pump field with continuously, harmonically modulated amplitude $f(t) = f_0 + f_1 \cos\left(\delta t\right)$, where δ is the modulation frequency, $\delta \ll \omega_L$. Such a modulation may be realized electronically by using the standard techniques, particularly, by an electro-optic amplitude modulator. Our analysis shows the drastic difference between the degree of two-mode squeezing/entanglement for modulated and stationary dynamics. In the case of time-modulation and for above-threshold regime, both the photon number and the variance display a time-dependent modulation with the period $2\pi/\delta$. The stationary variance near the threshold having a limiting squeezing of 0.5 is bounded by quantum inseparability criterion $V < 1$, while the variance for the case of modulated dynamics obeys the EPR criterion $V^2 < 1/4$ of strong CV entanglement for definite time intervals. The minimum values of the variance $V_{\min} = V\left(t_m\right)$ and corresponding photon numbers $n_{\min} = n_0(t_m)$ at fixed time intervals $t_m = t_0 + 2\pi m/\delta$, $(m = 0, 1, 2 \ldots)$ are shown in Fig. 1.

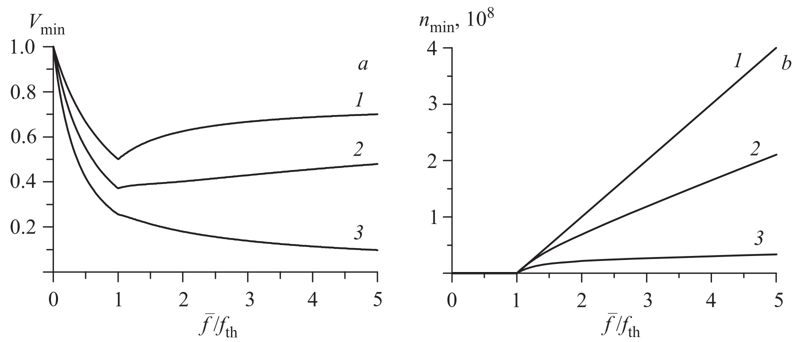

Fig. 1. The minimum level of the variance (*a*) and the corresponding mean photon number (*b*) versus $\overline{f}/f_{\rm th}$ for three levels of modulation: $f_1 = 0$ (*1*), $f_1 = 0.75\overline{f}$ (*2*) and $f_1 = 2\overline{f}$ (*3*). The parameters are: $k/\gamma = 5 \cdot 10^{-4}$, $\gamma_3/\gamma = 25$, $\delta/\gamma = 2$

3. TIME-MODULATED PHOTON-NUMBER CORRELATION

In this section, we consider intensity quantum correlation on the base of the level of quantum noise in the photon-number difference $\Delta(t) = \left\langle (a_1^+ a_1 - a_2^+ a_2)^2 \right\rangle - (\langle n_1 \rangle - \langle n_2 \rangle)^2$. This quantity can be expressed through the stochastic variables $n_+ = \alpha_1\beta_1 + \alpha_2\beta_2$, $n_-^2 = (\alpha_1\beta_1 - \alpha_2\beta_2)^2$ as $\Delta(t) = \langle n_+(t) \rangle + \left\langle n_-^2(t) \right\rangle$.

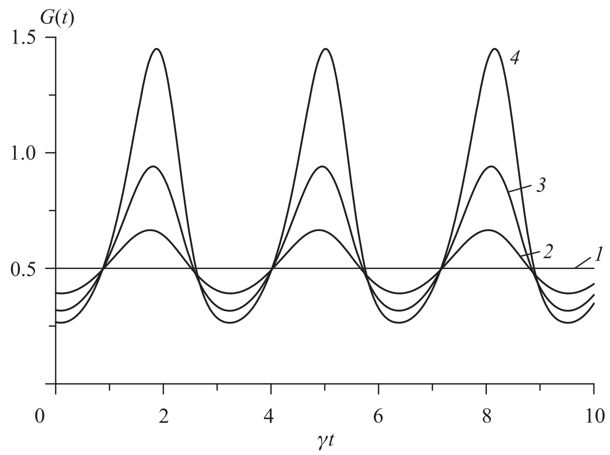

Fig. 2. The normalized variance of the photon-number difference for time-modulated twin beams versus dimensionless time for the parameters: $\lambda/\gamma = 10^{-8}$, $\delta/\gamma = 2$, $\overline{f} = 1.2 f_{\rm th}$: *1* — $f_1 = 0$; *2* — $f_1 = 0.5 f_{\rm th}$; *3* — $f_1 = f_{\rm th}$; *4* — $f_1 = 1.5 f_{\rm th}$

Surprisingly, a general expression for the variance can be derived from the Eqs. (2), (3) without resorting to a linear treatment of quantum fluctuations. The result reads as

$$\Delta(t) = 4\gamma \int_{-\infty}^{0} \mathrm{e}^{4\gamma\tau} \langle n(t+\tau) \rangle \, d\tau. \tag{6}$$

For the case of an ordinary NOPO with stationary pump amplitude, without any modulation, $f(t) = f_0 = \mathrm{const}$, we have $\Delta(t) = n_s = (f_0 - f_{\mathrm{th}})/k$, where n_s is the stationary mean photon number of the modes. Thus, the variance normalized to the level of fluctuations for the coherent state reaches only 50% relative to SNL, $G(t) = \Delta/(\langle n_1 \rangle + \langle n_2 \rangle) = 1/2$, in agreement with the result of the paper [7].

For the case of periodical time-modulation, the normalized variance goes beyond the limit of 50% which is established for an ordinary twin beams, i.e. $G(t) < 1/2$ for a periodic sequence of the definite time intervals. The result is illustrated in Fig. 2 for the different levels of modulation. As we can see, time-modulation of twin beams essentially improves the degree of photon-number quantum correlation. This fact makes the time-dependent twin beams be very preferable for quantum measurements and quantum information.

CONCLUSION

In summary, we have investigated CV entanglement as well as photon-number correlation in the time domain in addition to many analogous investigations performed in the spectral domain. We have demonstrated that time-modulation of twin beams essentially improved the degree of CV entanglement doing it beyond the standard limit established for CV entanglement. We believe that the obtained results can be applied to time-resolved quantum information technology.

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