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## RELATIVISTIC CONTRIBUTION OF THE FINAL-STATE INTERACTION TO DEUTERON PHOTODISINTEGRATION

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The contribution of the final-state interaction to the differential cross section of deuteron photodisintegration at laboratory photon energies below 50 MeV is analyzed in the framework of Bethe–Salpeter formalism with a phenomenological rank-one separable interaction. The approximations made are the neglect of two-body exchange currents, negative-energy components of the bound-state vertex function and the scattering  $T$  matrix. It has been found that the gross effect of the final-state interaction with  $J \leq 1$  comes from the net contributions of the spin-triplet final states and spin-singlet  $^1S_0^+$  state. The relativistic results are compared with the nonrelativistic ones in every partial-wave channel. It is found that the relativistic effects change the magnitude of the final-state interaction from several percent to several tens of percent.

В работе анализируется вклад взаимодействия в конечном состоянии в дифференциальное сечение фоторасщепления дейтрона при энергиях фотона менее 50 МэВ в лабораторной системе. Анализ проводится в рамках формализма Бете–Солпитера с феноменологическим сепарабельным ядром взаимодействия первого ранга. Сделаны следующие предположения: не учитываются вклады релятивистских двухчастичных токов и компоненты вершинной функции дейтрона и  $T$ -матрицы с отрицательной энергией. Основной эффект взаимодействия в конечном состоянии с полным угловым моментом  $J \leq 1$  происходит от триплетных и синглетного  $^1S_0^+$  конечных состояний. Релятивистские результаты сравниваются с нерелятивистскими в каждом парциальном канале. Найдено, что релятивистские эффекты изменяют вклад взаимодействия в конечном состоянии от нескольких процентов до нескольких десятков процентов.

### INTRODUCTION

The investigation of deuteron photodisintegration is a direct way of probing the structure of light nuclei. High-precision experimental data on polarization observables of the reaction can give information about the nucleon–nucleon ( $NN$ ) interaction. On the one hand, the important issue related to the nuclear force is the relativistic effects, which may play an important role in the nuclear two-body problem (the deuteron-bound state and the final-state interaction (FSI) of the final neutron–proton ( $np$ ) state). On the other hand, in deuteron photodisintegration, the electromagnetic (EM) form factors of nucleons are probed as well.

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Knowledge of the deuteron-bound state, the final-state interaction in two-nucleon scattering states and two-body currents can help to estimate the half on-mass-shell behavior of the nucleon EM form factors.

The present work is devoted to a comprehensive study of contributions to the differential cross section caused by the relativistic structure of  $np$  scattering states. These are analyzed in terms of the field-theoretical nonperturbative theory, based on the Bethe–Salpeter (BS) equation. The development of previous works [1, 2] is necessary to test the sensitivity of observables to the FSI due to the relativistic nuclear force and relativistic effects of kinematical origin. To this end, we employ a separable interaction kernel of the  $NN$  interaction to solve the BS equation for two Dirac particles in Minkowski space. As a result, we obtain the relativistic deuteron vertex function in the coupled  ${}^3S_1$ – ${}^3D_1$  channel and the scattering  $T$  matrix of the elastic  $NN$  scattering for the positive-energy partial-wave channels with the total angular momentum  $J = 0, 1$  at laboratory energies  $T_{\text{lab}} \leq 100$  MeV [3, 4].

This paper is organized as follows: in Sec. 1, we present the basic information on the kinematics of the reaction, the invariant transition amplitude and the differential cross section. The relativistic approach to the  $NN$  interaction is briefly described in Sec. 2. The rank-one separable interaction kernel is introduced in Sec. 3. In Sec. 4, we go into several points of calculating the invariant transition amplitudes within the BS formalism. Discussions of principle results and final remarks are in Sec. 5.

## 1. KINEMATICS OF THE REACTION

If we denote by  $|F(P_f, p_f, \zeta_f)\rangle$  any particular neutron–proton final state, the invariant transition amplitude  $\mathcal{M}_{fi}$  for deuteron photodisintegration in the c.m. frame being the rest frame of the  $np$  pair

$$\gamma(q, \varepsilon) + d(K_i, \xi_i) \rightarrow F(P_f, p_f, \zeta_f) \quad (1)$$

is given by the matrix element of the hadronic EM four-current operator  $\widehat{J}^\mu(x)$  between the initial deuteron state and final  $np$ -pair state:

$$\mathcal{M}_{fi} = \varepsilon^\mu(\lambda) \langle F(P_f, p_f, \zeta_f) | \widehat{J}_\mu(0) | d(K_i, \xi_i(m_d)) \rangle, \quad (2)$$

where  $P_f = (\sqrt{s}, \mathbf{0})$  and  $p_f = (0, \hat{\mathbf{p}})$  are total and asymptotic relative four-momenta of the pair;  $K_i$  and  $q = P_f - K_i$  are the deuteron and photon four-momenta, respectively. Four polarization four-vectors  $\varepsilon(\lambda)$ ,  $\xi_i(m_d)$  and  $\zeta_f$  (with  $\lambda = \pm 1$  and  $m_d = 0, \pm 1$ ) describe the internal degrees of freedom of the photon, deuteron and final  $np$  system, related to the angular momentum. All states in Eq. (2) are understood to be normalized in the covariant manner. It should be also noted that the definition of the outgoing  $np$  pair is such that, whether the final-state interaction is switched off, it is given by an antisymmetrized product of two positive-energy Dirac spinors.

The coordinate system is defined by the incoming three-momentum of the photon,  $\mathbf{q} = (0, 0, \omega)$  with  $|\mathbf{q}| = \omega$ , which is along the  $z$  axis, and transverse polarizations  $\varepsilon_{\lambda=\pm 1} = (\mp 1, -i, 0)/\sqrt{2}$ . The asymptotic relative three-momentum  $\hat{\mathbf{p}}$  is characterized by the spherical angles  $\theta_{\hat{\mathbf{p}}}$  and  $\varphi_{\hat{\mathbf{p}}}$  in the chosen coordinate system. The  $z$  axis is also the quantization axis for the total spin  $S = 0, 1$  of the final  $np$  system, as well as for the polarization four-vector  $\xi_i(m_d)$  of the deuteron.

The c.m. energy squared  $s$  of the final  $np$  pair is related to the relative three-momentum  $\hat{\mathbf{p}}$ ,  $s = 4(\hat{\mathbf{p}}^2 + m^2)$ , and to the photon energy  $E_\gamma$  in the laboratory system, being the rest frame of the deuteron,  $s = M_d^2 + 2E_\gamma M_d$ , where  $m$  and  $M_d$  are the nucleon and deuteron masses, respectively. The nucleon laboratory energy  $T_{\text{lab}} = (2/m)\hat{\mathbf{p}}^2$  is connected to the laboratory photon energy as  $E_\gamma \cong (1/2)T_{\text{lab}}$ .

The differential cross section, in the case of the unpolarized initial configuration, can be written in the c.m. frame as follows:

$$\frac{d\sigma}{d\Omega_{\hat{\mathbf{p}}}} = \frac{\alpha}{16\pi s} \frac{|\hat{\mathbf{p}}|}{\omega} \overline{|\mathcal{M}_{fi}|^2}, \quad (3)$$

where  $\alpha = e^2/4\pi$  is the fine structure constant and the line over  $|\mathcal{M}_{fi}|^2$  denotes the incoherent summation over  $\lambda$ ,  $m_d$  and  $m_s$ , being the spin projection of the final  $np$  system in spin-triplet channels, on the quantization axis.

We assume in this work that the dynamical model of the EM current operator in Eq. (2) is the relativistic impulse approximation not constrained by gauge invariance. For simplicity, we do not modify this approximation by applying Siegert's theorem. This is a major drawback of the present development, preventing us from comparing theoretical predictions of the relativistic theory and experimental results for deuteron photodisintegration at low energies.

All details concerning the structure of the EM current operator and the initial deuteron state could be found in Ref. [1,2], where numerical calculations of the angular distribution in the plane wave approximation (PWA) have been done. Thus, in determining the differential cross section (3), we are focused on the relativistic structure of the final two-nucleon system.

## 2. THE TWO-BODY PROBLEM IN MOMENTUM SPACE

Within the relativistic field theory, the elastic  $NN$  scattering can be described by the scattering  $T$  matrix, which satisfies the inhomogeneous BS equation. In momentum space, the BS equation for the  $T$  matrix reads (in terms of the relative four-momenta  $p'$  and  $p$  and the total four-momentum  $P_f$ )

$$T(p', p; P_f) = V(p', p; P_f) + \frac{i}{4\pi^3} \int d^4k V(p', k; P_f) S_2(k; P_f) T(k, p; P_f), \quad (4)$$

where  $V(p', p; P_f)$  is the interaction kernel and  $S_2(k; P_f)$  is free two-particle Green's function:

$$S_2^{-1}(k; P_f) = \left( \frac{1}{2} P_f \cdot \gamma + k \cdot \gamma - m \right)^{(1)} \left( \frac{1}{2} P_f \cdot \gamma - k \cdot \gamma - m \right)^{(2)}.$$

To perform the partial-wave decomposition of the BS equation (4), we introduce relativistic two-nucleon basis states  $|aM\rangle \equiv |\pi, {}^{2S+1}L_J^{\rho} M\rangle$ , where  $S$  denotes the total spin,  $L$  is the orbital angular momentum and  $J$  is the total angular momentum with the projection  $M$ ; relativistic quantum numbers  $\rho$  and  $\pi$  refer to the total energy-spin and relative-energy parity with respect to the change of sign of the relative energy, respectively. Then the partial-wave decomposition of the  $T$  matrix in the c.m. frame has the following form:

$$T_{\alpha\beta,\gamma\delta}(p', p; P_{f(0)}) = \sum_{JMab} (\mathcal{Y}_{aM}(-\mathbf{p}') U_C)_{\alpha\beta} \otimes (U_C \mathcal{Y}_{bM}^\dagger(\mathbf{p}))_{\delta\gamma} t_{ab}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s), \quad (5)$$

where  $U_C = i\gamma^2\gamma^0$  is the charge conjugation matrix. Greek letters  $(\alpha, \beta)$  and  $(\gamma, \delta)$  in Eq. (5) refer to spinor indices and label particles in the initial and final states, respectively. It is convenient to represent the two-particle states in terms of matrices. To this end, the Dirac spinors of the second nucleon are transposed. At this stage,  $T$  is  $16 \times 16$  matrix in spinor space which, sandwiched between Dirac spinors and traced, yields the corresponding transition matrix elements between  $SLJ$  states.

The spin-angular momentum functions  $\mathcal{Y}_{aM}(\mathbf{p})$  are expressed in terms of the positive- and negative-energy Dirac spinors  $u_m^{\rho=\pm 1/2}$ , the spherical harmonics  $Y_{Lm_L}$  and Clebsch–Gordan coefficients  $C_{j_1 m_1 j_2 m_2}^{j m}$ :

$$\begin{aligned} \mathcal{Y}_{JM:LS\rho}(\mathbf{p})U_C &= \\ &= i^L \sum_{m_L m_S m_1 m_2 \rho_1 \rho_2} C_{\frac{1}{2}\rho_1 \frac{1}{2}\rho_2}^{S\rho\rho} C_{Lm_L S m_S}^{JM} C_{\frac{1}{2}m_1 \frac{1}{2}m_2}^{S m_S} Y_{Lm_L}(\mathbf{p}) u_{m_1}^{\rho_1 (1)}(\mathbf{p}) u_{m_2}^{\rho_2 (2)T}(-\mathbf{p}). \end{aligned} \quad (6)$$

The superscripts in Eq. (6) refer to particles (1) and (2). In deriving the matrix elements between  $a$  states, the orthonormalization condition for the functions  $\mathcal{Y}_{aM}(\mathbf{p}')$  should be used:

$$\begin{aligned} \int d\varphi_{\mathbf{p}} d(\cos \theta_{\mathbf{p}}) \text{Tr} \left\{ \mathcal{Y}_{aM}^\dagger(\mathbf{p}) \mathcal{Y}_{a'M'}(\mathbf{p}) \right\} &\equiv \\ &\equiv \int d\varphi_{\mathbf{p}} d(\cos \theta_{\mathbf{p}}) (\mathcal{Y}_{aM}^\dagger(\mathbf{p}))_{\beta\alpha} (\mathcal{Y}_{a'M'}(\mathbf{p}))_{\alpha\beta} = \delta_{aa'} \delta_{MM'}, \end{aligned} \quad (7)$$

where partial states  $a$  and  $a'$  belong to the same partial channel.

The partial-wave decomposition for the interaction kernel  $V$  of the BS equation (4) can be written analogously to Eq. (5):

$$V_{\alpha\beta, \gamma\delta}(p', p; P_f(0)) = \sum_{abM} (\mathcal{Y}_{aM}(-\mathbf{p}')U_C)_{\alpha\beta} \otimes (U_C \mathcal{Y}_{bM}^\dagger(\mathbf{p}))_{\delta\gamma} v_{ab}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s). \quad (8)$$

Applying the condition (7), we can obtain a system of linear integral equations of the off-shell partial-wave amplitudes

$$\begin{aligned} t_{ab}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s) &= v_{ab}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s) + \\ &+ \frac{i}{4\pi^3} \sum_{cd} \int_{-\infty}^{+\infty} dk_0 \int_0^\infty k^2 d|\mathbf{k}| v_{ac}(p'_0, |\mathbf{p}'|; k_0, |\mathbf{k}|; s) S_{cd}(k_0, |\mathbf{k}|; s) t_{db}(k_0, |\mathbf{k}|; p_0, |\mathbf{p}|; s), \end{aligned} \quad (9)$$

where the two-particle propagator  $S_{ab}$  depends only on  $\rho$ -spin indices.

The PWA and FSI contributions can be combined by introducing the relativistic scattering amplitude for two nucleons  $\chi_{Sm_s}(p; p_f P_f)$ , which satisfies the following equation:

$$\chi_{Sm_s}(p; p_f, P_f) = \chi_{Sm_s}^{(0)}(p; p_f, P_f) + \frac{i}{4\pi^3} S_2(p; P_f) \int d^4k V(p, k; P_f) \chi_{Sm_s}(k; p_f, P_f), \quad (10)$$

with  $p_f \cdot P_f = 0$  and  $p_f^2 = -s/4 + m^2$  putting the outgoing particles onto the mass shell. The first term  $\chi_{Sm_s}^{(0)}(p; p_f, P_f)$  in Eq. (10) is the PWA amplitude, which describes the free

motion of two nucleons. Due to Pauli's principle, it is the antisymmetric combination of positive-energy Dirac spinors and isovector, isoscalar factors. Omitting isospin we can write in the c.m. frame

$$\begin{aligned}\chi_{Sm_s}^{(0)}(p; p_f, P_{f(0)}) &= \delta^{(4)}(p - p_f) \chi_{Sm_s}^{(0)}(p_f, P_{f(0)}) = \\ &= \delta^{(4)}(p - p_f) \sum_{m_1 m_2} C_{\frac{1}{2}m_1 \frac{1}{2}m_2}^{Sm_s} u_{m_1}^{(1)}(\hat{\mathbf{p}}) u_{m_2}^{(2)}(-\hat{\mathbf{p}}).\end{aligned}\quad (11)$$

It is easy to rewrite the FSI contribution to the BS amplitude, the second term in Eq. (10), from the  $T$  matrix, once the following relation is used:

$$\int d^4k V(p, k; P_f) \chi_{Sm_s}(k; p_f, P_f) = \int d^4k T(p, k; P_f) \chi_{Sm_s}^{(0)}(k; p_f, P_f).\quad (12)$$

The result is

$$\chi_{Sm_s}^{(t)}(p; p_f, P_f) = \frac{i}{4\pi^3} S_2(p; P_f) T(p, p_f; P_f) \chi_{Sm_s}^{(0)}(p_f, P_f).\quad (13)$$

The ultimate expression for the BS scattering amplitude of the  $np$  pair is solely defined by the  $T$  matrix half on the mass-shell:

$$\chi_{Sm_s}(p; p_f, P_f) = \left[ \delta^{(4)}(p - p_f) + \frac{i}{4\pi^3} S_2(p; P_f) T(p, p_f; P_f) \right] \chi_{Sm_s}^{(0)}(p_f, P_f).\quad (14)$$

The partial-wave decomposition of  $\chi_{Sm_s}^{(t)}(p; p_f, P_f)$  can be written in the form

$$\chi_{Sm_s}^{(t)}(p; p_f, P_{f(0)}) = \sum_{LmJM a} C_{LmSM_s}^{JM} Y_{Lm}^*(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \mathcal{Y}_{aM}(\mathbf{p}) \phi_{a,J:LS\rho=+1}(p_0, |\mathbf{p}|; 0, |\hat{\mathbf{p}}|; s),\quad (15)$$

where the radial function  $\phi$  is determined by the product of the transition matrix elements  $t$  and the two-particle propagator:

$$\phi_{a,J:LS\rho=+1}(p_0, |\mathbf{p}|; 0, |\hat{\mathbf{p}}|; s) = S_{+a}(p_0, |\mathbf{p}|; s) t_{a,J:LS\rho=+1}(p_0, |\mathbf{p}|; 0, |\hat{\mathbf{p}}|; s).\quad (16)$$

The inclusion of intermediate partial-wave channels with the negative-energy Dirac spinors leads to large sets of two-dimensional equations in the system (9). Its solution determines the relativistic wave function, as well as the phase shifts from the  $T$  matrix. For simplicity, we switch off all partial-wave channels with negative-energy states and focus only on the physical channels with the total angular momentum  $J \leq 1$ . Splitting the channels with  $J = 0$  and  $J = 1$ , the partial-wave decomposition for the conjugate BS amplitude has the form

$$\begin{aligned}\bar{\chi}_{Sm_s}^{(t)}(p; p_f, P_{f(0)}) &= \\ &= \sum_{LmL'S'} C_{LmSM_s}^{00} Y_{Lm}(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \bar{\mathcal{Y}}_{00:L'S'}(\mathbf{p}) \phi_{0:LS,0:L'S'}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) + \\ &+ \sum_{MLmL'S'} C_{LmSM_s}^{1M} Y_{Lm}(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \bar{\mathcal{Y}}_{1M:L'S'}(\mathbf{p}) \phi_{1:LS,1:L'S'}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s),\end{aligned}\quad (17)$$

Table 1. Spin-angular momentum matrices  $\bar{\mathcal{G}}_{aM}$  for  $J \leq 1$ .  $p_1 = (E_{\mathbf{p}}, \mathbf{p})$ ,  $p_2 = (E_{\mathbf{p}}, -\mathbf{p})$  are on-mass-shell four-momenta

${}^1S_0^+$	$\gamma_5$
${}^3P_0^+$	$-\frac{1}{2 \mathbf{p} } (p_1 - p_2) \cdot \gamma$
${}^3S_1^+$	$-\xi_f^*(M)$
${}^1P_1^+$	$\frac{\sqrt{3}}{ \mathbf{p} } p \cdot \xi_f^*(M) \gamma_5$
${}^3P_1^+$	$\sqrt{\frac{3}{2}} \gamma_5 \left[ \frac{1}{2} (p_1 - p_2) \cdot \gamma \xi_f^*(M) \cdot \gamma - p \cdot \xi_f^*(M) \right] \frac{1}{ \mathbf{p} }$
${}^3D_1^+$	$-\frac{1}{\sqrt{2}} \left[ \xi_f^*(M) \cdot \gamma + \frac{3}{2} (p_1 - p_2) \cdot \gamma p \cdot \xi_f^*(M) \right] \frac{1}{ \mathbf{p} ^2}$

where the spin-angular momentum functions  $\bar{\mathcal{Y}}_{aM}(\mathbf{p})$  for the positive-energy states can be written in the matrix form

$$\bar{\mathcal{Y}}_{aM}(\mathbf{p}) = \frac{1}{\sqrt{8\pi}} \frac{1}{4E_{\mathbf{p}}(E_{\mathbf{p}} + m)} (m - p_2 \cdot \gamma) \bar{\mathcal{G}}_{aM} (1 + \gamma_0) (m + p_2 \cdot \gamma), \quad (18)$$

where  $E_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$  and explicit expressions for the matrices  $\bar{\mathcal{G}}_{aM}(\mathbf{p})$  are given in Table 1.

Further, we explicitly factor in contributions of the spin-singlet states  ${}^1S_0^+$  and  ${}^1P_1^+$ , uncoupled spin-triplet states  ${}^3P_0^+$  and  ${}^3P_1^+$ , and the coupled spin-triplet states  ${}^3S_1^+ - {}^3D_1^+$  in Eq. (17). The result is presented in a lengthy form

$$\begin{aligned} \bar{\chi}_{Sm_s}^{(t)}(p; p_f, P_f) &= \frac{i}{4\pi^3} S_{++}(p_0, |\mathbf{p}|; s) \times \\ &\times \left[ \delta_{S0} \delta_{m_S0} \frac{1}{\sqrt{4\pi}} \bar{\mathcal{Y}}_{1S_0^+}(\mathbf{p}) t_{1S_0^+, 1S_0^+}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) + \right. \\ &+ \delta_{S0} \delta_{m_S0} \sum_M Y_{1M}(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \bar{\mathcal{Y}}_{1P_1^+M}(\mathbf{p}) t_{1P_1^+, 1P_1^+}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) + \\ &+ \delta_{S1} (-)^{1-m_S} \frac{1}{\sqrt{3}} Y_{1-m_S}(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \bar{\mathcal{Y}}_{3P_0^+}(\mathbf{p}) t_{3P_0^+, 3P_0^+}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) + \\ &+ \delta_{S1} \sum_{Mm} C_{1m1m_S}^{1M} Y_{1m}(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \bar{\mathcal{Y}}_{3P_1^+M}(\mathbf{p}) t_{3P_1^+, 3P_1^+}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) + \\ &+ \delta_{S1} \frac{1}{\sqrt{4\pi}} \bar{\mathcal{Y}}_{3S_1^+m_S}(\mathbf{p}) t_{3S_1^+, 3S_1^+}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) + \\ &+ \delta_{S1} \sum_M C_{2M-m_S1m_S}^{1M} Y_{2M-m_S}(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \bar{\mathcal{Y}}_{3S_1^+M}(\mathbf{p}) t_{3S_1^+, 3D_1^+}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) + \\ &+ \delta_{S1} \frac{1}{\sqrt{4\pi}} \bar{\mathcal{Y}}_{3D_1^+m_S}(\mathbf{p}) t_{3D_1^+, 3S_1^+}^*(0, \hat{\mathbf{p}}; p_0, |\mathbf{p}|; s) + \\ &+ \left. \delta_{S1} \sum_M C_{2M-m_S1m_S}^{1M} Y_{2M-m_S}(\theta_{\hat{\mathbf{p}}}, \varphi_{\hat{\mathbf{p}}}) \bar{\mathcal{Y}}_{3D_1^+M}(\mathbf{p}) t_{3D_1^+, 3D_1^+}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s) \right], \quad (19) \end{aligned}$$

where  $S_{++}^{-1}(k_0, |\mathbf{k}|; s) = [(\sqrt{s}/2 - E_{\mathbf{k}} + i0)^2 - k_0^2]$  is the projection of the two-particle propagator onto positive-energy states.

At this stage, the contribution of the FSI to the BS amplitude of the  $np$  pair is expressed in terms of the spin-angular momentum functions and radial parts of the half off-mass-shell  $T$  matrix in  $SLJ$  representation. To perform further calculations, we need to solve the BS equation (9).

### 3. A SEPARABLE KERNEL OF THE $NN$ INTERACTION

First, we assume that the interaction kernel  $V$  conserves parity, total spin  $S$ , total angular momentum  $J$  and its projection, and isotopic spin. Because of the tensor nuclear force, the orbital angular momentum  $L$  is not conserved. Moreover, the negative-energy two-nucleon states are switched off. The partial-wave-decomposed BS equation is therefore decomposed to the following form:

$$t_{LL'}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = v_{LL'}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s) + \frac{i}{4\pi^3} \sum_{L''} \int_{-\infty}^{+\infty} dk_0 \int_0^{\infty} \mathbf{k}^2 d|\mathbf{k}| v_{LL''}(p'_0, |\mathbf{p}'|; k_0, |\mathbf{k}|; s) S_{++}(k_0, |\mathbf{k}|; s) t_{L''L}(k_0, |\mathbf{k}|; p_0, |\mathbf{p}|; s), \quad (20)$$

where  $L = L' = J$  is for spin-singlet and uncoupled spin-triplet states and  $L, L' = J \pm 1$  is for coupled spin-triplet states.

Next, we make a rank-one separable *ansatz* for the  $NN$  interaction kernel of the form

$$v_{L'L}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \lambda g^{(L')}(p'_0, |\mathbf{p}'|) g^{(L)}(p_0, |\mathbf{p}|), \quad (21)$$

where  $\lambda$  is the coupling strength and  $g^{(L)}(p_0, |\mathbf{p}|)$  are the covariant form factors, which depend only upon the zero components  $p_0, p'_0$  and magnitudes  $|\mathbf{p}'|, |\mathbf{p}|$  of the spatial components of the relative four-momenta. In Eq. (21) the partial-wave channels for a given  $J$  are labeled only by the angular momenta  $L, L'$ .

Thus, the two-fold integrals in Eq. (20) can be solved in a closed form, yielding the final expression for the  $T$  matrix elements

$$t_{L'L}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \tau(s) g^{(L')}(p'_0, |\mathbf{p}'|) g^{(L)}(p_0, |\mathbf{p}|), \quad (22)$$

where function  $\tau(s)$  has the form

$$\tau(s) = \frac{1}{\lambda^{-1} + h(s)}, \quad (23)$$

with

$$h(s) = -\frac{i}{4\pi^3} \sum_L \int_{-\infty}^{+\infty} dk_0 \int_0^{\infty} \mathbf{k}^2 d|\mathbf{k}| \frac{g^{(L)}(k_0, |\mathbf{k}|) g^{(L)}(k_0, |\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i0)^2 - k_0^2}. \quad (24)$$

The nuclear phase shifts  $\delta(s)$  are related to the fully on-mass-shell  $T$  matrix through the condition of the two-body elastic unitarity. For the spin-singlet and uncoupled spin-triplet states, the parameterizations of the on-mass-shell  $T$  matrix have the form

$$t_L(s) \equiv t_{LL}(0, |\mathbf{p}|; 0, |\mathbf{p}|; s) = -\frac{16\pi}{\sqrt{s}\sqrt{s-4m^2}} e^{i\delta_L(s)} \sin \delta_L(s), \quad (25)$$

where  $\delta_L(s) \equiv \delta_{L=J}(s)$ .

For the coupled spin-triplet states, we use the parameterization

$$t_{L'L}(s) = \frac{8\pi i}{\sqrt{s}\sqrt{s-4m^2}} \begin{pmatrix} \cos 2\epsilon_J e^{2i\delta_{<}} - 1 & i \sin 2\epsilon_J e^{i(\delta_{<} + \delta_{>})} \\ i \sin 2\epsilon_J e^{i(\delta_{<} + \delta_{>})} & \cos 2\epsilon_J e^{2i\delta_{>}} - 1 \end{pmatrix}, \quad (26)$$

in terms of the phase shifts  $\delta_{\lesseqgtr} \equiv \delta_{L=J\mp 1}$  and the mixing parameters  $\epsilon_J(s)$ .

In numerical calculations of the phase shifts, we consider the covariant relativistic generalizations of the *Yamaguchi*-type form factors

$$g^{(L=0)}(k_0, |\mathbf{k}|) = \frac{1}{k_0^2 - \mathbf{k}^2 - \beta_0^2 + i0}, \quad (27)$$

$$g^{(L=1)}(k_0, |\mathbf{k}|) = \frac{\sqrt{|-k_0^2 + \mathbf{k}^2|}}{(k_0^2 - \mathbf{k}^2 - \beta_1^2 + i0)^2}, \quad (28)$$

$$g^{(L=2)}(k_0, |\mathbf{k}|) = \frac{C(k_0^2 - \mathbf{k}^2)}{(k_0^2 - \mathbf{k}^2 - \beta_2^2 + i0)^2}. \quad (29)$$

The nuclear phase shifts  $\delta$ , mixing parameters  $\epsilon$ , as well as the low-energy  $NN$  parameters (the scattering length and the effective range) and the deuteron static properties (the binding energy, a  $D$ -state probability and the quadrupole moment) can be computed in terms of internal parameters  $\lambda$ ,  $C$  and  $\beta_{0,1,2}$  through a specially designed procedure. Technical details can be found in Ref. [4]. Values of the parameters are given in Table 2.

*Table 2. Kernel parameters for partial-wave channels with  $J \leq 1$*

	$^1S_0^+$	$^3S_1^+ - ^3D_1^+$	$^3P_0^+$	$^1P_1^+$	$^3P_1^+$
$\lambda$ , GeV <sup>2</sup>	-0.28554	-0.50269	-0.016123	0.091535	0.084724
$\beta_0$ , GeV	0.221858	0.25124			
$\beta_1$ , GeV			0.21861	0.27673	0.28398
$\beta_2$ , GeV		0.29399			
$C$		1.6471			

#### 4. CALCULATION OF THE TRANSITION AMPLITUDES

In the framework of the BS formalism, the invariant transition amplitude, given by Eq. (2), can be written as

$$\mathcal{M}_{fi}(q, K_i) = \int d^4p d^4k \bar{\chi}_{S m_s}(p; p_f, P_f) \varepsilon(\lambda) \tilde{J}(p, k; P_f, K_i) S_2(k, K_i) \Gamma_{m_d}(k, K_i), \quad (30)$$



where subscripts  $f$  and  $i$  imply the polarization quantum numbers of the final ( $S_{m_s}$ ) and initial ( $\lambda m_d$ ) states, respectively; the BS amplitude  $\bar{\chi}_{S_{m_s}}(p; p_f, P_f)$  of the  $np$  pair is given by Eq. (17);  $\Gamma_{m_d}(k, K_i)$  is the deuteron vertex function, which is the solution of the homogeneous BS equation with the same interaction kernel  $V$ ;  $\tilde{J}(p, k; P_f, K_i)$  is Mandelstam's EM vertex operator, describing the interaction of the photon with the hadronic system. The structure of the EM current operator is specified by the microscopic model such as the impulse approximation (IA).

The invariant transition amplitude of Eq. (30) can be cast into the form

$$\mathcal{M}_{fi} = \mathcal{M}_{fi,IA}^{\text{PWA}} + \mathcal{M}_{fi,IA}^{\text{FSI}} + \mathcal{M}_{fi,TB}, \quad (31)$$

where  $\mathcal{M}_{fi,IA}^{\text{PWA}}$  is the transition amplitude in the plane-wave impulse approximation;  $\mathcal{M}_{fi,IA}^{\text{FSI}}$  determines contributions of FSI to the impulse approximation, and  $\mathcal{M}_{fi,TB}$  accounts for two-body exchange currents. In the present work we consider only the first two terms in Eq. (31), and their explicit expressions are expressed by

$$\begin{aligned} \mathcal{M}_{fi,IA}^{\text{PWA}} = & -\varepsilon(\lambda) \text{Sp} \left\{ \bar{\chi}_{S_{m_s}}^{(0)}(p_f, P_f) \left[ \Gamma^{(1)}(q) + (-1)^{S+1} \Gamma^{(2)}(q) \right] \Lambda(\mathcal{L}) \times \right. \\ & \left. \times S \left( \mathcal{L}^{-1} \left( \frac{1}{2}(K_i - q) + p_f \right) \right) \Gamma_{m_d} \left( \mathcal{L}^{-1} \left( p_f - \frac{1}{2}q \right); K_{i(0)} \right) \Lambda(\mathcal{L})^{-1} \right\}, \quad (32) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{fi,IA}^{\text{FSI}} = & -\varepsilon(\lambda) \int d^4p \text{Sp} \left\{ \bar{\chi}_{S_{m_s}}^{(t)}(p; p_f, P_f) \left[ \Gamma^{(1)}(q) + (-1)^{S+1} \Gamma^{(2)}(q) \right] \Lambda(\mathcal{L}) \times \right. \\ & \left. \times S \left( \mathcal{L}^{-1} \left( \frac{1}{2}(K_i - q) + p \right) \right) \Gamma_{m_d} \left( \mathcal{L}^{-1} \left( p - \frac{1}{2}q \right); K_{i(0)} \right) \Lambda(\mathcal{L})^{-1} \right\}, \quad (33) \end{aligned}$$

where  $\Gamma^{(1,2)}$  refers to the  $\gamma NN$  vertex operator of particle (1) or (2); the Dirac operator  $\Lambda(\mathcal{L})$  corresponds to the Lorentz boost transformation  $K_{i(0)} = \mathcal{L}^{-1}K_i$ , which places the deuteron at rest. The  $np$  amplitudes  $\bar{\chi}_{S_{m_s}}^{(0)}$  and  $\bar{\chi}_{S_{m_s}}^{(t)}$  and the deuteron vertex function  $\Gamma_{m_d}$  are  $4 \times 4$  matrices in spinor space.

Next, the partial-wave decompositions of Eqs. (32) and (33) have to be carried out. Since in Refs. [1,2] that has been done in great detail for the invariant transition amplitude in PWA, here we present only the final result for the invariant transition amplitude with FSI included:

$$\begin{aligned} \mathcal{M}_{fi,IA}^{\text{FSI}} = & \sum_{J=0,1} \sum_{S=0,1} \sum_{L''=0,2} \sum_{LL' m\mu} i^L (-1)^S C_{Lm S\mu}^{Jm+\mu} Y_{Lm}(\theta_{\hat{\mathbf{p}}}, 0) \times \\ & \times \int_{-\infty}^{+\infty} dp_0 \int_0^{+\infty} \mathbf{p}^2 d|\mathbf{p}| \int_{-1}^{+1} d \cos \theta_{\mathbf{p}} \frac{t_{LL'}^*(0, |\hat{\mathbf{p}}|; p_0, |\mathbf{p}|; s)}{(\sqrt{s}/2 - E_{\mathbf{p}} + i0)^2 - p_0^2} \times \\ & \times \frac{\phi_{L''}(\mathcal{L}^{-1}(p_0 - \omega/2), |\mathcal{L}^{-1}(\mathbf{p} - \mathbf{q}/2)|)}{(P_f/2 - q + p)^2 - m^2 + i0} \Upsilon_{L'L''}^\mu(\hat{\mathbf{p}}, \mathbf{p}; q), \quad (34) \end{aligned}$$

where  $\phi_{L''}(p_0, |\mathbf{p}|)$  is the radial part of the deuteron vertex function;  $\Upsilon_{L'L''}^\mu(\hat{\mathbf{p}}, \mathbf{p}; q)$  are the matrix elements of the Dirac operators  $\Lambda(\mathcal{L})$  and the  $\gamma NN$  vertex  $\Gamma^{(1,2)}(q)$  between the spin-angular momentum functions  $\bar{\mathcal{Y}}_{L'}$  and  $\mathcal{Y}_{L''}$ . We calculate  $\Upsilon_{L'L''}^\mu(\hat{\mathbf{p}}, \mathbf{p}; q)$  using the algebra manipulation package REDUCE. What then remains to be done is a three-dimensional integration over  $p_0$ ,  $|\mathbf{p}|$  and  $\cos \theta_{\mathbf{p}}$ . The integration over  $p_0$  is performed analytically with the help of Cauchy's theorem, giving special attention to the complicated singularity structure of the integrand in Eq. (34). The remaining two-dimensional integration is done numerically, using the programming language FORTRAN.

## 5. RESULTS AND DISCUSSIONS

We should mention that we can demonstrate the importance of final-state interaction from the threshold of deuteron breakup to the laboratory photon energy  $E_\gamma \cong 50$  MeV. This is because the rank-one separable potential does not provide satisfactory fits to the  $NN$  scattering experimental data for the spin-triplet  ${}^3P_{0,1}^+$  channels above  $T_{\text{lab}} \geq 100$  MeV. Fortunately, it is sufficient to take into account final states only with  $J \leq 1$  within this energy range.

In Figs. 1, 2 we show the results of our investigation. The curves labeled by PWA and NR PWA depict the results of calculations in the plane-wave approximation with one-body current within the relativistic and nonrelativistic models. Notations FSI and NR FSI correspond to calculations in impulse approximation with final-state interaction included within the same models.

In Fig. 1, the differential cross section  $d\sigma/d\Omega_{\hat{\mathbf{p}}}$ , v.s. Eq. (3), at three different laboratory energies of 3, 20 and 50 MeV, in units of microbarns, is shown as a function of the c.m. proton angle. The sensitivity of the angular distributions to the various final-state interaction is illustrated by a computation of contributions from separate partial-wave channels of the final  $np$  system with  $J \leq 1$ . The solid curve in Fig. 1 represents the result of the plane-wave impulse approximation. Immediately, we can see that the relative value and sign of final-state contributions change with increasing photon energy  $E_\gamma$ . The tensor force in the initial bound and final-scattering  ${}^3S_1^+ - {}^3D_1^+$  states is made  $P_d = 4\%$ . On the other hand, transitions to the isovector final states  ${}^1S_0^+$  and  ${}^3P_1^+$  play a predictably significant role even at small  $E_\gamma$ , and their contributions become relatively large as the photon energy increases. The contribution of the isoscalar spin-singlet final state  ${}^1P_1^+$  is negligible for the considered range of the photon energies, because this partial-wave amplitude is proportional to small factor  $(\omega\mu_s/2m)$  with  $\mu_s = -0.12$  and relativistic effects are very small. The isoscalar spin-triplet final state  ${}^3S_1^+ - {}^3D_1^+$  becomes pronounced at about  $E_\gamma = 50$  MeV. It is confirmed by numerical results, obtained in Ref. [5], in the framework of the nonrelativistic theory. Moreover, in view of the interference between isovector and isoscalar spin-triplet final states, the contribution from the transition into  ${}^3S_1^+ - {}^3D_1^+$  final state does not vanish as  $E_\gamma$  increases. On the other hand, the  ${}^1P_1^+$  contribution is indeed small for photon energies of up to 300 MeV. An interesting question here is the role played by the relativistic corrections.

Figure 2 helps to gain some insight into the relative importance of the relativistic effects in the considered final  $np$  scattering states. We make additional calculations of the differential cross section in the nonrelativistic phenomenological treatment of deuteron photodisintegration: (1) using the radial wave function of the deuteron for the nonrelativistic Graz-II separable kernel with the deuteron  $D$ -state probability  $P_d = 4\%$  [6]; and (2) computing the relativistic

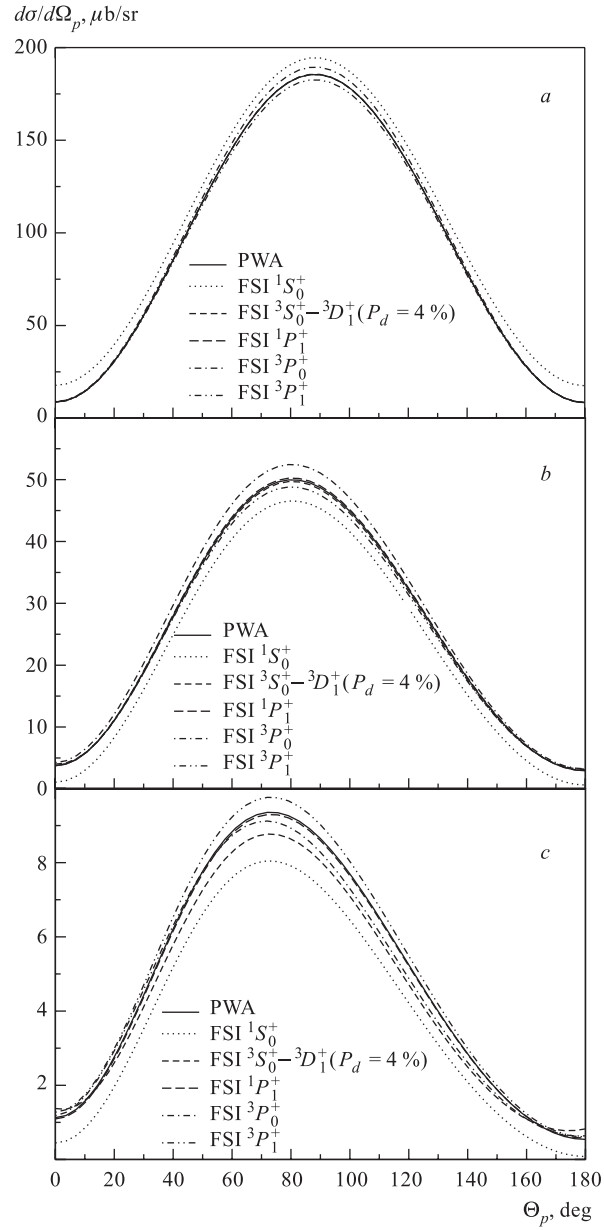


Fig. 1. Contribution of the separate partial-wave channels of the final  $np$  pair into the angular distribution of c.m. differential cross section at different laboratory photon energies: a)  $E_\gamma = 3$  MeV; b)  $E_\gamma = 20$  MeV; c)  $E_\gamma = 50$  MeV

matrix elements  $t_{LL'}(0, |\hat{\mathbf{p}}; p'_0, |\mathbf{p}'|; s)$  at  $p'_0 = 0$ . This provides us an opportunity to make term-by-term comparisons of various final-state contributions.

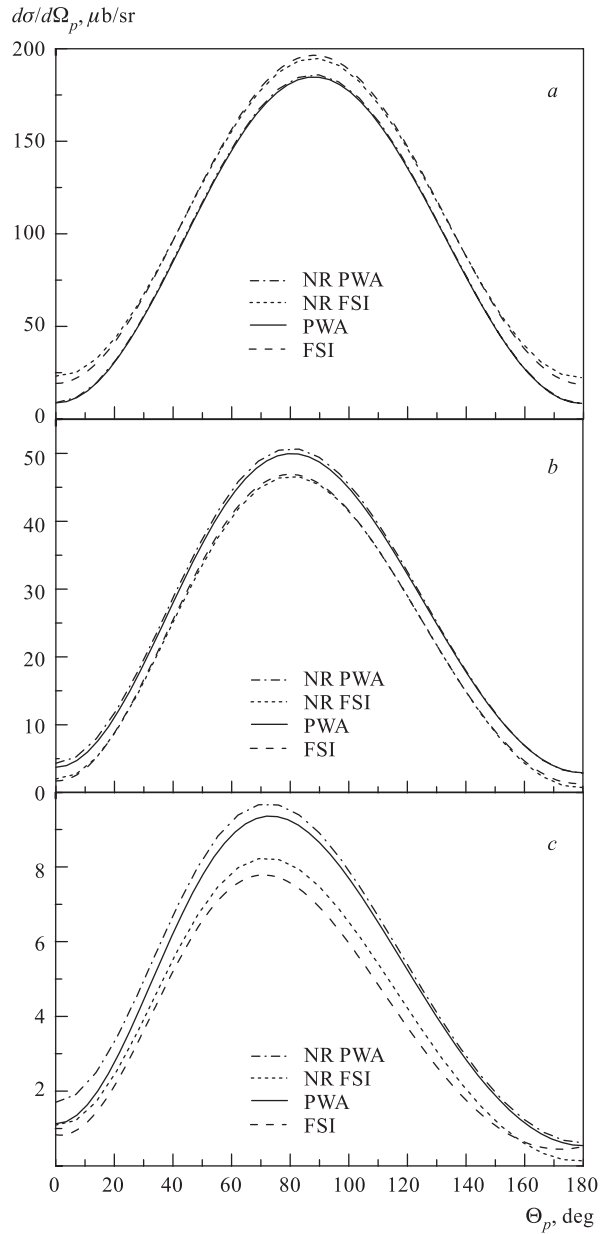


Fig. 2. C.m. differential cross section at different laboratory photon energies: a)  $E_\gamma = 3$  MeV; b)  $E_\gamma = 20$  MeV; c)  $E_\gamma = 50$  MeV

Our analysis shows that contribution of final-state interaction is changed by several percent at  $E_\gamma = 5$  MeV and up to twenty percent at  $E_\gamma = 50$  MeV, when computed in the framework of the relativistic model. We conclude that the relativistic effects stemming from the final-

state contributions should be incorporated in the development of a rigorous theory of deuteron photodisintegration at higher energies. Unfortunately, our results cannot be compared to the experimental data. The differential cross section is much too low because we neglect the two-body exchange currents and negative-energy components of the bound-state vertex function and the scattering  $T$  matrix.

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