# GROUND STATE CORRELATIONS AND STRUCTURE OF ODD SPHERICAL NUCLEI 

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#### Abstract

It is well known that the Pauli principle plays a substantial role at low energies because the phonon operators are not ideal boson operators. Calculating the exact commutators between the quasiparticle and phonon operators one can take into account the Pauli principle corrections. Besides, the ground state correlations due to the quasiparticle interaction in the ground state influence the single-particle fragmentation as well. In this paper, we generalize the basic equations of the quasiparticle-phonon nuclear model to account for both effects mentioned. As an illustration of our approach, calculations on the structure of the low-lying states in ${ }^{133} \mathrm{Ba}$ have been performed.


Хорошо известно, что принцип Паули оказывает существенное влияние на структуру состояний при низких энергиях, так как операторы фононов не являются идеальными бозонными операторами. Вычислив точные коммутаторы между квазичастичными и фононными операторами, можно учесть поправки из-за принципа Паули. Кроме того, на фрагментацию одночастичных состояний влияют корреляции в основном состоянии, возникающие из-за взаимодействия квазичастиц. В данной работе основные уравнения квазичастично-фононной модели обобщены на случай учета обоих упомянутых эффектов. В качестве примера рассчитана структура низколежащих состояний B ${ }^{133} \mathrm{Ba}$.
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## INTRODUCTION

In the forthcoming period there will be an increasing activity in the domain of unstable nuclei studies due to the start of operation of several major facilities, producing intensive beams of radioactive nuclei, far away from the valley of stability. These studies are motivated by the significant changes that take place in the structure of these nuclei. Along with the changes in the shell structure within the mean field approximation, the many-body effects increase their role as we move away from the magic numbers. In particular, the interaction between the single-particle states with energies near the Fermi level with the vibrating core must be treated within an enlarged configuration space, which takes account of the correlations in the ground state. Here and further, by ground state correlations (GSC) we imply correlations due to the quasiparticle-phonon interaction in the ground state.

[^0]A fairly good theoretical description of the ground state correlations (GSC) can be archived within an extended version of the quasiparticle-phonon nuclear model (QPM) [1]. The QPM is widely used for the description of the energies and fragmentation of nuclear excitations. The different versions of the QPM equations for odd-even spherical nuclei are given in [2-4]. It has been shown in $[2,5]$ that corrections due to the action of the Pauli principle are very important for the determination of the energies of some states. However, in these investigations the ground state correlations effects have not been taken into account. Later, in the study of Van der Sluys et al. [6], it was proved that the GSC influence the singleparticle fragmentation as they shift the strength to higher excitation energies. In their study, the operators of quasiparticles and phonons have been taken as the commuting ones, thus neglecting the Pauli principle.

In this paper, we generalize the basic QPM equations for odd-even spherical nuclei to take account of the effects due to the GSC and the Pauli principle. We treat long-range ground state correlations by including backward-going quasiparticle-phonon vertices using the equation of motion method [7] with explicitly taking into account the Pauli principle. Numerical calculations of the structure of the low-lying states in ${ }^{133} \mathrm{Ba}$ within the developed approach have been performed.

## 1. FORMULATION OF THE MODEL

We employ the QPM-Hamiltonian including an average nuclear field, described by the Woods-Saxon potential, pairing interactions, isoscalar particle-hole residual forces in separable form with the Bohr-Mottelson radial dependence [8]:

$$
\begin{equation*}
H=\sum_{\tau}^{(n, p)}\left\{\sum_{j m}\left(E_{j}-\lambda_{\tau}\right) a_{j m}^{\dagger} a_{j m}-\frac{1}{4} G_{\tau}^{(0)}:\left(P_{0}^{\dagger} P_{0}\right)^{\tau}:-\frac{1}{2} \sum_{\lambda \mu} \kappa^{(\lambda)}:\left(M_{\lambda \mu}^{\dagger} M_{\lambda \mu}\right):\right\} \tag{1}
\end{equation*}
$$

The single-particle states are specified by the quantum numbers $(j m) ; E_{j}$ are the singleparticle energies; $\lambda_{\tau}$ is the chemical potential; $G_{\tau}^{(0)}$ and $\kappa^{(\lambda)}$ are the strengths in the $p-p$ and $p$ - $h$ channels, respectively. The sum goes over protons ( $p$ ) and neutrons ( $n$ ) independently and the notation $\tau=\{n, p\}$ is used. The pair creation and the multipole operators entering the normal products in (1) are defined as follows:

$$
\begin{gathered}
P_{0}^{+}=\sum_{j m}(-1)^{j-m} a_{j m}^{+} a_{j-m}^{+} \\
M_{\lambda \mu}^{+}=\frac{1}{\sqrt{2 \lambda+1}} \sum_{j j^{\prime} m m^{\prime}} f_{j j^{\prime}}^{(\lambda)}\left\langle j m j^{\prime} m^{\prime} \mid \lambda \mu\right\rangle a_{j m}^{+} a_{j^{\prime} m^{\prime}}
\end{gathered}
$$

where $f_{j j^{\prime}}^{(\lambda)}$ are the single-particle radial matrix elements of the residual forces.
In what follows, we work in quasiparticle representation, defined by the canonical Bogoliubov transformation:

$$
a_{j m}^{+}=u_{j} \alpha_{j m}^{+}+(-1)^{j-m} v_{j} \alpha_{j-m}
$$

The Hamiltonian can be represented in terms of bifermion quasiparticle operators (and their conjugate ones):

$$
\begin{aligned}
B\left(j j^{\prime} ; \lambda \mu\right) & =\sum_{m m^{\prime}}(-1)^{j^{\prime}+m^{\prime}}\left\langle j m j^{\prime} m^{\prime} \mid \lambda \mu\right\rangle \alpha_{j m}^{+} \alpha_{j^{\prime}-m^{\prime}}, \\
A^{+}\left(j j^{\prime} ; \lambda \mu\right) & =\sum_{m m^{\prime}}\left\langle j m j^{\prime} m^{\prime} \mid \lambda \mu\right\rangle \alpha_{j m}^{+} \alpha_{j^{\prime} m^{\prime}}^{+} .
\end{aligned}
$$

The phonon creation operators are defined in the two-quasiparticle space in a standard fashion:

$$
Q_{\lambda \mu i}^{+}=\frac{1}{2} \sum_{j j^{\prime}}\left\{\psi_{j j^{\prime}}^{\lambda i} A^{+}\left(j j^{\prime} ; \lambda \mu\right)-(-1)^{\lambda-\mu} \varphi_{j j^{\prime}}^{\lambda i} A\left(j j^{\prime} ; \lambda-\mu\right)\right\},
$$

where index $\lambda=0,1,2,3, \ldots$ denotes multipolarity and $\mu$ is its $z$-projection in the laboratory system. The normalization of the one-phonon states reads

$$
\langle |\left[Q_{\lambda \mu i}, Q_{\lambda^{\prime} \mu^{\prime} i^{\prime}}^{+}\right]\left\rangle=\delta_{\lambda \lambda^{\prime}} \delta_{\mu \mu^{\prime}} \delta_{i i^{\prime}} .\right.
$$

In terms of quasiparticles and phonons, the Hamiltonian is rewritten

$$
\begin{gathered}
H=h_{0}+h_{p p}+h_{Q Q}+h_{Q B}, \quad h_{0}+h_{p p}=\sum_{j m} \varepsilon_{j} \alpha_{j m}^{+} \alpha_{j m}, \\
h_{Q Q}=-\frac{1}{8} \sum_{\lambda \mu i i^{\prime}} \mathcal{A}\left(\lambda i i^{\prime}\right)\left(Q_{\lambda \mu i}^{+}+(-)^{\lambda-\mu} Q_{\lambda-\mu i}\right)\left(Q_{\lambda-\mu i^{\prime}}^{+}+(-)^{\lambda+\mu} Q_{\lambda \mu i^{\prime}}\right), \\
h_{Q B}=-\frac{1}{2 \sqrt{2}} \sum_{\lambda \mu i j j^{\prime}} \frac{\pi_{j}}{\pi_{\lambda}} \Gamma\left(j j^{\prime} \lambda i\right)\left((-)^{\lambda-\mu} Q_{\lambda \mu i}^{+}+Q_{\lambda-\mu i}\right) B\left(j j^{\prime} ; \lambda-\mu\right)+\text { h.c. },
\end{gathered}
$$

where

$$
\begin{gathered}
\mathcal{A}\left(\lambda i i^{\prime}\right)=\frac{X^{\lambda i}+X^{\lambda i^{\prime}}}{\sqrt{Y^{\lambda i} Y^{\lambda i^{\prime}}}}, \quad \Gamma\left(j j^{\prime} \lambda i\right)=\frac{\pi_{\lambda}}{\pi_{j}} \frac{v_{j j^{\prime}}^{(-)} f_{j j^{\prime}}^{(\lambda)}}{\sqrt{Y^{\lambda i}}}, \\
X^{\lambda i}=\sum_{j j^{\prime}} \frac{\left(f_{j j^{\prime}}^{(\lambda)} u_{j j^{\prime}}^{(+)}\right)^{2} \varepsilon_{j j^{\prime}}}{\varepsilon_{j j^{\prime}}^{2}-\omega_{\lambda i}^{2}}, \quad Y^{\lambda i}=\sum_{j j^{\prime}} \frac{\left(f_{j j^{\prime}}^{(\lambda)} u_{j j^{\prime}}^{(+)}\right)^{2} \varepsilon_{j j^{\prime}} \omega_{\lambda i}}{\left(\varepsilon_{j j^{\prime}}^{2}-\omega_{\lambda i}^{2}\right)^{2}},
\end{gathered}
$$

with $v_{j j^{\prime}}^{(-)}=u_{j} u_{j^{\prime}}-v_{j} v_{j^{\prime}}$ and $u_{j j^{\prime}}^{(+)}=u_{j^{\prime}} v_{j}+u_{j} v_{j^{\prime}}$. Here and further, we use the notation $\pi_{j}=\sqrt{(2 j+1)}$.

The model wave function of an odd-even spherical nucleus is taken in the form [6]:

$$
\begin{equation*}
\Psi_{\nu}(J M)=O_{J M \nu}^{+}| \rangle \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
O_{J M \nu}^{+}=C_{J \nu} \alpha_{J M}^{+}+\sum_{i} D_{j}^{\lambda i}(J \nu) P_{j \lambda i}^{+}(J M)-E_{J \nu} \tilde{\alpha}_{J M}-\sum_{i} F_{j}^{\lambda i}(J \nu) \tilde{P}_{j \lambda i}(J M), \tag{3}
\end{equation*}
$$

with

$$
P_{j \lambda i}^{+}(J M)=\left[\alpha_{j m}^{+} Q_{\lambda \mu i}^{+}\right]_{J M}
$$

and stands for time conjugate according to the convention: $\quad \tilde{P}_{j \lambda i}(J M)=(-1)^{J-M}$ $P_{j \lambda i}(J-M)$.

We apply the equation of motion method to the excitation operator (3):

$$
\left.\langle |\left\{\delta O_{J M \nu}, H, O_{J M \nu}^{+}\right\}\left\rangle=\eta_{J \nu}\langle |\left\{\delta O_{J M}, O_{J M}^{+}\right\}\right|\right\rangle .
$$

Following the linearization procedure [7], at the final state of calculation of the matrix elements, we consider the ground state of the even-even nucleus to be a vacuum state for both operators $\alpha_{J M}$ and $Q_{\lambda \mu i}$.

In all calculations the exact commutation relations between the quasiparticle and phonon operators are considered:

$$
\left[\alpha_{j m}, Q_{\lambda \mu i}^{+}\right]=\sum_{j^{\prime} m^{\prime}}\left\langle j m j^{\prime} m^{\prime} \mid \lambda \mu\right\rangle \psi_{j j^{\prime}}^{\lambda i} \alpha_{j^{\prime} m^{\prime}}^{+}
$$

The normalization condition of the wave function reads

$$
\begin{aligned}
\langle |\left\{O_{J M \nu}, O_{J M \nu}^{+}\right\}\rangle & =C_{J \nu}^{2}+E_{J \nu}^{2}+\sum_{j \lambda i}\left[D_{j}^{\lambda i}(J \nu)\right]^{2}+\sum_{j \lambda i}\left[F_{j}^{\lambda i}(J \nu)\right]^{2}+ \\
& +\sum_{j \lambda i j^{\prime} \lambda^{\prime} i^{\prime}}\left[D_{j}^{\lambda i}(J \nu) D_{j^{\prime}}^{\lambda^{\prime} i^{\prime}}(J \nu)+F_{j}^{\lambda i}(J \nu) F_{j^{\prime}}^{\lambda^{\prime} i^{\prime}}(J \nu)\right] \mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)=1 .
\end{aligned}
$$

The equation of motion leads to the following system of linear equations for each state with quantum numbers $J M$ :

$$
\begin{align*}
&\left(\begin{array}{cccc}
\varepsilon_{J} & V\left(J j^{\prime} \lambda^{\prime} i^{\prime}\right) & 0 & -W\left(J j^{\prime} \lambda^{\prime} i^{\prime}\right) \\
V(J j \lambda i) & K_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime}\right) & W(J j \lambda i) & 0 \\
0 & W\left(J j^{\prime} \lambda^{\prime} i^{\prime}\right) & -\varepsilon_{J} & -V\left(J j^{\prime} \lambda^{\prime} i^{\prime}\right) \\
-W(J j \lambda i) & 0 & -V(J j \lambda i) & -K_{J}\left(j \lambda i \mid j^{\prime} \lambda i^{\prime}\right)
\end{array}\right)\left(\begin{array}{c}
C_{J \nu} \\
D_{j^{\prime}}^{\lambda^{\prime} i^{\prime}}(J \nu) \\
-E_{J \nu} \\
-F_{j^{\prime} i^{\prime}}^{\lambda^{\prime}}(J \nu)
\end{array}\right)= \\
&=\eta_{J \nu}\left(\begin{array}{c}
C_{J \nu} \\
D_{j}^{\lambda i}(J \nu)+\sum_{j^{\prime} \lambda^{\prime} i^{\prime}} D_{j^{\prime}}^{\lambda^{\prime} i^{\prime}}(J \nu) \mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right) \\
-E_{J \nu} \\
-F_{j}^{\lambda i}(J \nu)-\sum_{j^{\prime} \lambda^{\prime} i^{\prime}} F_{j^{\prime}}^{\lambda^{\prime} i^{\prime}}(J \nu) \mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)
\end{array}\right) . \tag{4}
\end{align*}
$$

The explicit expressions for the quantities entering the above formulas will be inspected one at a time

$$
\mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)=\pi_{\lambda} \pi_{\lambda^{\prime}} \sum_{j_{1}} \psi_{j_{1} j}^{\lambda^{\prime} i^{\prime}} \psi_{j_{1} j^{\prime}}^{\lambda i}\left\{\begin{array}{ccc}
j^{\prime} & j_{1} & \lambda \\
j & J & \lambda^{\prime}
\end{array}\right\},
$$

$$
\begin{aligned}
V(J j \lambda i)= & \langle |\left\{\left[\alpha_{J M}, H\right], P_{j \lambda i}^{+}(J M)\right\}\rangle= \\
& =-\frac{1}{\sqrt{2}} \Gamma(J j \lambda i)-\frac{1}{\sqrt{2}} \sum_{j^{\prime} \lambda^{\prime} i^{\prime}}\left(\mathcal{T}_{J}\left(j \lambda i ; j^{\prime} \lambda^{\prime} i^{\prime}\right)+\mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)\right) \Gamma\left(J j^{\prime} \lambda^{\prime} i^{\prime}\right)
\end{aligned}
$$

As a result of the application of the equation of motion method, the matrix elements $V(J j \lambda i)$ between quasiparticle and quasiparticle $\bigotimes$ phonon states differ from those obtained earlier [2] by an additive containing $\mathcal{T}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)$ :

$$
\mathcal{I}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)=\pi_{\lambda} \pi_{\lambda^{\prime}} \sum_{j_{1}} \psi_{j_{1} j^{\prime}}^{\lambda i} \varphi_{j_{1} j}^{\lambda^{\prime} i^{\prime}}\left\{\begin{array}{ccc}
j^{\prime} & j_{1} & \lambda \\
j & J & \lambda^{\prime}
\end{array}\right\} .
$$

$$
\begin{aligned}
W(J j \lambda i)= & \langle |\left\{\left[\alpha_{J M}^{+}, H\right], \tilde{P}_{j \lambda i}^{+}(J M)\right\}\left\rangle=-\frac{1}{4} \frac{\pi_{\lambda}}{\pi_{J}} \sum_{i^{\prime} \tau_{0}} \mathcal{A}_{\tau_{0}}\left(\lambda i i^{\prime}\right) \varphi_{J j}^{\lambda i^{\prime}}-\right. \\
& -\frac{1}{4} \sum_{\lambda^{\prime} j^{\prime} i^{\prime} i^{\prime \prime} \tau_{0}} \mathcal{A}_{\tau_{0}}\left(\lambda^{\prime} i^{\prime} i^{\prime \prime}\right) \frac{\pi_{\lambda^{\prime}}}{\pi_{J}}\left[\varphi_{J j^{\prime}}^{\lambda^{\prime} i^{\prime}} \mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime \prime}\right)-\psi_{J j^{\prime}}^{\lambda^{\prime} i^{\prime \prime}} \mathcal{T}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)\right] .
\end{aligned}
$$

The matrix elements $W(J j \lambda i)$ appear after the introduction of the backward-going terms in the operator (3) and they present a central issue of this work.

$$
\begin{gathered}
I_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)=\langle |\left\{P_{j \lambda i}(J M),\left[H, P_{j^{\prime} \lambda^{\prime} i^{\prime}}^{+}(J M)\right]\right\}| \rangle, \\
I_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)+I_{J}\left(j^{\prime} \lambda^{\prime} i^{\prime} \mid j \lambda i\right)= \\
=2 \delta_{j j^{\prime}} \delta_{\lambda \lambda^{\prime}} \delta_{i i^{\prime}}\left(\omega_{\lambda i}+\varepsilon_{j}\right)+\mathcal{L}_{J}\left(j^{\prime} \lambda^{\prime} i^{\prime} \mid j \lambda i\right)\left(\varepsilon_{j^{\prime} j}+\omega_{\lambda^{\prime} i^{\prime}}+\omega_{\lambda i}\right)-\mathcal{R}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right), \\
K_{J}\left(j \lambda i \mid j^{\prime} \lambda i^{\prime}\right)=\frac{1}{2}\left[I_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)+I_{J}\left(j^{\prime} \lambda^{\prime} i^{\prime} \mid j \lambda i\right)\right] \\
\mathcal{R}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)=\frac{1}{4} \sum_{i_{1} \tau_{0}}\left[\mathcal{A}_{\tau_{0}}\left(\lambda i_{1} i\right) \mathcal{L}_{J}\left(j^{\prime} \lambda^{\prime} i^{\prime} \mid j \lambda i_{1}\right)+\mathcal{A}_{\tau_{0}}\left(\lambda^{\prime} i_{1} i^{\prime}\right) \mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i_{1}\right)\right]+ \\
+\frac{1}{4} \sum_{\lambda_{1} i_{1} i_{2} j_{1} \tau_{0}} \mathcal{A}_{\tau_{0}}\left(\lambda_{1} i_{1} i_{2}\right)\left[\mathcal{L}_{J}\left(j \lambda i \mid j_{1} \lambda_{1} i_{1}\right) \mathcal{L}_{J}\left(j^{\prime} \lambda^{\prime} i^{\prime} ; j_{1} \lambda_{1} i_{2}\right)+\right. \\
\left.+\mathcal{L}_{J}\left(j^{\prime} \lambda^{\prime} i^{\prime} \mid j_{1} \lambda_{1} i_{1}\right) \mathcal{L}_{J}\left(j \lambda i \mid j_{1} \lambda_{1} i_{2}\right)\right] .
\end{gathered}
$$

The quantities $\mathcal{L}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right), \mathcal{T}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)$ and $\mathcal{R}_{J}\left(j \lambda i \mid j^{\prime} \lambda^{\prime} i^{\prime}\right)$ vanish if the Pauli principle is not respected.

## 2. NUMERICAL RESULTS

In order to give a qualitative picture of the effect on the structure of the low-lying states, imposed by the backward-going amplitudes and the Pauli principle, numerical calculations for ${ }^{133} \mathrm{Ba}$ have been performed. This nucleus enters the transitional region where the anharmonic effects play a gradually increasing role at low and mainly at intermediate energies, and therefore the results presented in this section may lack some accuracy because the wave function (2) does not contain configurations to account for these effects. The pairing constants $G_{\tau}$ are fitted so as to reproduce the odd-even mass differences.

Our calculations include quadrupole and octupole phonons. The strength parameter $\kappa^{(2)}$ is adjusted so that the odd energy spectrum of the low-lying states is reasonably close to the experimental values, while $\kappa^{(3)}$ is fixed by the experimental energy of the first octupole state of the neighbouring even-even nucleus. As a result, the energies of the first quadrupole state in ${ }^{132} \mathrm{Ba}$ have a value that is much higher than the experimental one in the models studied. Moreover, if $\kappa^{(2)}$ is fixed within the model which takes account of the GSC, the values of $\omega_{2_{1}}$ are systematically increased of about $10 \%$ as compared to the model, where the backward amplitudes are not considered. It is worth mentioning that after the introduction of the anharmonic effects the energy of this state would decrease.

As we move away from the magic number 82 for the neutron subsystem, the correlations in the ground state tend to increase along with the quantities $W(J j \lambda i)$. This trend is presented in Table 1, where $W(J j \lambda i)$ are evaluated only at the lowest poles. Characteristic feature of QPM is that quantities, related to both the pairing and the multipole-multipole interactions, enter the expressions for the interaction vertices, producing some competitive effects between them which are central for the understanding of the behaviour of $V(J j \lambda i)$ and $W(J j \lambda i)$ along the isotopic chain. Having the lowest quasiparticle energies, the states $1 h_{11 / 2}, 3 s_{1 / 2}$ and $2 d_{3 / 2}$ experience the greatest part of the interaction with the remaining quasiparticles in the ground state.
Table 1. Values of the matrix elements $V^{2}(J j \lambda i)$ and $W^{2}(J j \lambda i)$ for ${ }^{133}$ Ba calculated for $J^{\pi}=$ $1 / 2^{+}, 3 / 2^{+}, 11 / 2^{-}, 5 / 2^{+}, 7 / 2^{+}$at the lowest poles

| State | Pole's structure | $V^{2}$ | $W^{2}$ | $\mathcal{R}$ | $1+\mathcal{L}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $1 / 2^{+}$ | $2 d_{3 / 2} \otimes 2_{1}$ | 0.1225 | 2.1 | -0.43 | 0.932 |
| $3 / 2^{+}$ | $2 d_{3 / 2} \otimes 2_{1}$ | 0.0445 | 1.72 | 0.072 | 1.031 |
| $11 / 2^{-}$ | $1 h_{11 / 2} \otimes 2_{1}$ | 0.0493 | 3.92 | -1.25 | 0.753 |
| $5 / 2^{+}$ | $2 d_{3 / 2} \otimes 2_{1}$ | 0.047 | 0.14 | -0.068 | 0.986 |
| $7 / 2^{+}$ | $2 d_{3 / 2} \otimes 2_{1}$ | 0.118 | 0.439 | -0.68 | 0.874 |

The quantities $\mathcal{R}(J j \lambda i)$ and $\mathcal{L}(J j \lambda i)$ experience a strong dependence on the degree of collectivity of the vibrational states in the neighbouring even-even nuclei. As seen from Table 1, their values increase as we move away from the magic number of the neutron subsystem. As far as we study the low-lying states only, it is mainly the first quadrupole state that influences them.

For reasons of conciseness, we introduce the following notations, indicating the different variants of the model:

- QPM - standard model as given in [1];
- QPM_P - model, including only the Pauli principle;
- QPM_BCK - model, including backward amplitudes but not the Pauli principle, i.e., $\mathcal{L}, \mathcal{T}$ and $\mathcal{R}$ are set to zero;
- QPM_BCK_P - model, including backward amplitudes and the Pauli principle (see Eq. (4)).

$$
5 / 2^{+}
$$


$5 / 2^{+} \longrightarrow 892$


Solving the system of equations (4), one can find the structure of the wave functions (2) and the energies of the excited states. Working in a diagonal approximation for $\mathcal{L}_{J}$ and $\mathcal{T}_{J}$, this system reduces to a generalized eigenvalue problem. In the figure a comparison between the experimental values of the energies and the theoretical calculations within different versions of the model is presented.

We restrict the calculation to six neutron states $1 / 2^{+}, 3 / 2^{+}, 9 / 2^{-}, 11 / 2^{-}, 5 / 2^{+}$and $7 / 2^{+}$. As mentioned above, the values of $\kappa^{(2)}$ are determined by the spectrum of the oddeven nuclei. In order to perform a comparative study of the levels' positions, we fix the values of $\kappa^{(2)}$ in QPM_BCK_P and keep them constant in the calculations within the other versions of the model.

From this figure it can be seen that for states near the Fermi level, the first solutions, obtained after the inclusion of the backward-going terms, become closer to the first poles and consequently closer to the second solutions, thus significantly reducing the gap between the first and the second states with signatures $J^{\pi}=1 / 2^{+}, 3 / 2^{+}, 11 / 2^{-}$. For the states $5 / 2^{+}$and $7 / 2^{+}$the effect of the GSC is less important because their energies are well above the Fermi level, and the values of $W(J j \lambda i)$ are therefore small (see Table 1). It is due to the effect of the correlations in the ground state that allowed the correct level ordering of the first several states of this isotope.

Along with the experimental energies, our calculations provide a reasonable description of the spectroscopic factors for $(d, p)$ reactions (Table 2).

Table 2. Experimental and theoretical spectroscopic factors of the lowest states with $J^{\pi}=$ $1 / 2^{+}, 3 / 2^{+}$for ${ }^{133} \mathbf{B a}$

| State | EXP | QPM | QPM_P | QPM_BCK | QPM_BCK_P |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / 2^{+}$ | 0.18 | 0.176 | 0.182 | 0.246 | 0.231 |
| $3 / 2^{+}$ | 0.3 | 0.29 | 0.297 | 0.34 | 0.34 |

The observed differences in the last four columns in this table are due to the quasiparticles in the ground state that additionally modify the single-particle occupation numbers, giving us a good idea of the removal of the backward amplitudes.

Finally, we examine the effect of the GSC and the Pauli principle on the fragmentation of the single-particle states among complex quasiparticle $\bigotimes$ phonon states. In the QPM_BCK versions of the model, the spectroscopic factors for the $(d, p)$ and $(d, t)$ reactions are written as follows:

$$
\begin{align*}
& S_{J \nu}^{(d, p)}=\left(C_{J \nu} u_{J}-E_{J \nu} v_{J}\right)^{2}, \\
& S_{J \nu}^{(d, t)}=\left(C_{J \nu} v_{J}+E_{J \nu} u_{J}\right)^{2} . \tag{5}
\end{align*}
$$

We notice that serious deviations from the expressions for these quantities within the standard QPM ( $C_{J \nu}^{2} u_{J}^{2}, C_{J \nu}^{2} v_{J}^{2}$ ) may occur due to their non-quadratic behaviour with respect to $u_{J}$ and the presence of the backward amplitudes $E_{J \nu}$. Again, in the case when the core has a magic number of nucleons, expressions (5) yield the classical quantities because of the step-wise behaviour of $u_{J}$ and $v_{J}$ in these nuclei.

We examine only levels in the vicinity of the Fermi level, as for them the interaction in the ground state is stronger than for those lying at higher energies. Furthermore, the values of
$\mathcal{R}(J j \lambda i)$, which effectively result in a shift of the poles, thus changing the gap between the pure one-quasiparticle states and quasiparticle $\otimes$ phonon states, along with the re-normalization factors $(1+\mathcal{L}(J j \lambda i))$ exert influence on the single-particle fragmentation as well.

## CONCLUSION

We generalized the basic equations of the quasiparticle-phonon nuclear model to account for effects of the ground state correlations and the Pauli principle. As an illustration of our approach, calculations on the structure of the low-lying states in ${ }^{133} \mathrm{Ba}$ have been performed. The comparison between theoretical calculations and experimental data for ${ }^{133} \mathrm{Ba}$ has shown that in order to describe the structure of the low-lying states in odd-even mass nuclei far from the magic numbers one needs to take into account the Pauli principle and the ground state correlations effects simultaneously. To improve this approach a self-consistent description of the mean field with more realistic effective nucleon-nucleon forces is desirable.

## REFERENCES

1. Soloviev V. G. Theory of Atomic Nuclei: Quasiparticles and Phonons. Bristol; Philadelphia: Inst. of Phys., 1992.
2. Chan Zuy Khuong, Soloviev V. G., Voronov V. V. // J. Phys. G: Nucl. Phys. 1981. V. 7. P. 151.
3. Vdovin A. I. et al. // Part. Nucl. 1985. V. 16. P. 245.
4. Gales S., Stoyanov Ch., Vdovin A. I. // Phys. Rep. 1988. V. 166. P. 125.
5. Alikov B. A. et al. // Izv. Akad. Nauk SSSR. Ser. Fiz. 1981. V.45. P. 2112 (in Russian).
6. Van der Sluys V. et al. // Nucl. Phys. A. 1993. V. 551. P. 210.
7. Rowe D. Nuclear Collective Motion. London: Menthuen, 1970.
8. Bohr A., Mottelson B. Nuclear Structure. N. Y.: Benjamin, 1975. V. 2.
9. Nucl. Data Sheets. 1994. V.72. P. 487.

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