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THE NUCLEAR MATTER EQUATION OF STATE INCLUDING LIGHT CLUSTERS

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The equation of state (EOS) of nuclear matter at moderate temperature and density with various proton fractions is considered, in particular the region of medium excitation energy given by the temperature range $T \leqslant 30$ MeV and the baryon density range $\rho_B \leqslant 10^{14.2}$ g/cm^3. In addition to the mean-field effects, the formation of few-body correlations, in particular, the light bound clusters up to the alpha particle ($1 \leqslant A \leqslant 4$), is of interest. Calculation based on the relativistic mean-field theory with the parameter set TM1 is presented. We show results for different values of the asymmetry parameter, and β equilibrium is considered as a special case. The medium modification of the light clusters is described by self-energy and Pauli blocking effects, using an effective nucleon–nucleon interaction potential.

Рассматривается уравнение состояния ядерной материи при средних температуре и плотности с различным содержанием протонов, в частности в области промежуточной энергии возбуждения $T \leqslant 30$ МэВ и барионной плотности в диапазоне $\rho_B \leqslant 10^{14,2}$ г/см^3. В дополнение к эффектам среднего поля представляет интерес образование многочастичных корреляций, в частности легких связанных кластеров вплоть до альфа-частиц ($1 \leqslant A \leqslant 4$). Представлен расчет, основывающийся на релятивистской теории среднего поля с набором параметров TM1. Демонстрируются результаты для различных величин параметра асимметрии, а β-равновесие рассматривается в качестве частного случая. Некоторая модификация легких кластеров описывается внутренней энергией и эффектами блокировки Паули с использованием эффективного потенциала нуклон-нуклонного взаимодействия.

INTRODUCTION

An important issue of nuclear theory is the nuclear matter equation of state (EOS), the composition and the possible occurrence of phase transitions in nuclear matter. Experimental data obtained from nuclei in the ground state as well as in excited states give some benchmarks for this quantity which is also of great interest in astrophysics and cosmology. Experiments on heavy-ion collisions, performed over the last decades, gave new insight into the behavior of nuclear systems in a broad range of densities and temperatures. The observed cluster abundances, their spectral distribution and correlations in momentum space can deliver information about the state of dense, highly excited matter. A particularly interesting topic is the possible existence of a new state, the quark–gluon phase. Matter under such extreme conditions occurs in compact objects such as neutron stars and in supernova explosions. Furthermore, the EOS

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of hot and dense matter is needed for cosmological models like the hot Big Bang as an essential input. References can be found in a recent monograph on the nuclear EOS [1].

We will restrict ourselves to matter in equilibrium at temperatures $T \leqslant 30$ MeV and baryon number densities $n_B \leqslant 0.2$ fm^{-3}, where the quark substructure and the excitation of internal degrees of freedom of the nucleons (protons p and neutrons n) are not of relevance and the nucleon–nucleon interaction can be represented by an effective interaction potential. In this region of the temperature–density plane, we will investigate how the quasiparticle picture can be improved if few-body correlations are taken into account, in particular the formation of deuteron (d), triton (t), ^{3}He (h), and ^{4}He (α) clusters. The influence of cluster formation on the EOS is calculated for different situations, and the occurrence of phase instabilities is investigated. Instead of the full spectral function, the concept of composition will be introduced as an approximation to describe correlations in dense systems. Another interesting point is the formation of quantum condensates, in particular, pairing and quartetting.

A quantum statistical approach to the thermodynamic properties of nuclear matter can be given using the method of thermodynamic Green functions [2]. In general, within the grand canonical ensemble, the EOS $n(T, \mu)$ relating the particle number density n to the chemical potential μ is obtained from the single-particle spectral function, which can be expressed in terms of the self-energy. Then, thermodynamic potentials such as the pressure $p(T, \mu) = \int_{-\infty}^{\mu} n(T, \mu') d\mu'$ or the density of free energy $f(T, n) = \int_0^n \mu(T, n') dn'$ are obtained by integration.

Within a Green function approach, the main quantity to be evaluated is the self-energy. Different approximations can be obtained by partial summation within a diagram representation. The formation of bound states is taken into account considering ladder approximations [3], leading in the low-density limit to the solution of the Schrödinger equation. The effects of the medium can be included in a self-consistent way within the cluster mean-field approximation (see [4] for references), where the influence of the correlated medium on the single-particle states as well as on the clusters is considered in the first order with respect to the interaction. As a point of significance, the single-particle and the bound states are considered on equal footing. Besides single-particle self-energy shifts of the constituents, the bound-state energies are also modified by the Pauli blocking due to the correlated medium. An extended discussion of the two-particle problem can be found in [5]. We will also consider the medium modification of the three- and four-particle bound states (see [6]).

If a singularity in the medium-modified few-body T matrix is obtained, it may be taken to indicate the formation of a quantum condensates. Different kinds of quantum condensates are also considered [7,8]. They become obvious if the binding energy of nuclei is investigated [9]. Correlated condensates are found to give a reasonable description of near-threshold states of $n\alpha$ nuclei [10]. The contribution of condensation energy to the nuclear matter EOS would be of importance and has to be taken into account not only in mean-field approximation but also considering correlated condensates.

The relativistic EOS of nuclear matter for supernova explosions was investigated recently [11]. To include bound states such as α particles, medium modifications of the few-body states have to be taken into account. Simple concepts used there such as the excluded volume should be replaced by more rigorous treatments based on a systematic many-particle approach. We will report on results including two-particle correlations into the nuclear matter EOS. New results of calculating the effects of three- and four-particle correlations are presented.

1. IDEAL MIXTURE OF DIFFERENT COMPONENTS

In the simplest approximation, we consider an ideal quantum gas of elementary particles such as protons, neutrons, electrons and possibly neutrinos (the quark–gluon substructure will not be considered at densities and temperatures considered). Inclusion of strong, weak, electromagnetic, and gravitational interactions leads to changes in the relevant degrees of freedom and to modification of the EOS, possibly connected with phase transitions. In this way, we first discuss the formation of correlations between the nucleons at a given proton fraction. As the next step, β equilibrium is considered, so that only the baryon density as a conserved quantity is prescribed. Then electric charge neutrality also fixes the electron concentration, but will not be discussed here in detail as well as equilibrium in the gravitational field.

In the low-density limit, the most important effect of interaction with respect to the nuclear matter EOS is the formation of bound states characterized by the proton content Z_i and the neutron content N_i. We will restrict ourselves only to the light clusters with $A \leqslant 4$ since the α particle is strongly bound. Large strongly bound clusters such as iron can be described using other concepts such as the formation of another (liquid) phase in the matter, see, e.g., [11]. The occurrence of large nuclei is of importance in considering the outer crust of neutron stars.

In thermal equilibrium, within a quantum statistical approach a mass action law can be derived (see [12]). The densities of the different components are determined by the chemical potentials μ_p and μ_n and temperature T. The densities of the free protons and neutrons as well as of the bound states follow in the nonrelativistic case as

$$n_i = \frac{2\sigma_i + 1}{2\,\pi^2} \int_0^\infty dk\, k^2 \frac{1}{\exp\left[\left(E_i(k;T,\mu_p,\mu_n) - Z_i\mu_p - N_i\mu_n\right)/T\right] - (-1)^{Z_i+N_i}}, \qquad (1)$$

where for deuterons $\sigma_d = 1$, for tritons $\sigma_t = 1/2$, for helions $\sigma_h = 1/2$, and for α particles $\sigma_\alpha = 0$. In the low-density limit where the medium effects can be neglected, the energies $E_i(k;T,\mu_p,\mu_n) = m_i + k^2/(2m_i)$ can be used, where $m_i = Z_i m_p + N_i m_n + E_i^b$ and the binding energies E_i^b are given in the Table.

Properties of light clusters

	Binding energy, MeV	Mass, MeV/c^2	Spin	rms radius, fm
n	0	939.565	1/2	0.34
p	0	938.783	1/2	0.87
d	–2.225	1876.12	1	2.17
t	–8.482	2809.43	1/2	1.70
h	–7.718	2809.41	1/2	1.87
α	–28.3	3728.40	0	1.63

We define the abundances of the different constituents as $X_n = n_n/n_B$, $X_p = n_p/n_B$, $X_d = 2\,n_d/n_B$, $X_t = 3\,n_t/n_B$, $X_h = 3\,n_h/n_B$, and $X_\alpha = 4\,n_\alpha/n_B$, where $n_B = n_n + n_p + 2\,n_d + 3\,n_t + 3\,n_h + 4\,n_\alpha$ is the total baryon density. Furthermore we introduce the total proton fraction as $Y_p^{\rm tot} = (n_p + n_d + n_t + 2\,n_h + 2\,n_\alpha)/n_B$. Results for the composition of nuclear matter at temperature $T = 10$ MeV with proton fraction $Y_p^{\rm tot} = 0.2$ are shown in

Fig. 1; for symmetric matter $Y_p^{\mathrm{tot}} = 0.5$, in Fig. 2. The model of an ideal mixture of free nucleons and clusters applies to the low-density limit. At higher baryon density, medium effects are relevant to calculate the composition shown in Figs. 1, 2, which are described in the following sections.

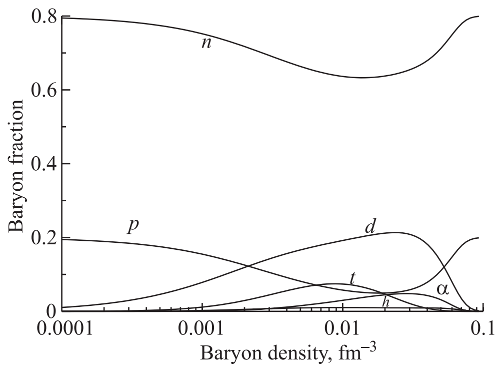

Fig. 1. Composition of nuclear matter with proton fraction 0.2 as a function of the baryon density, $T = 10$ MeV

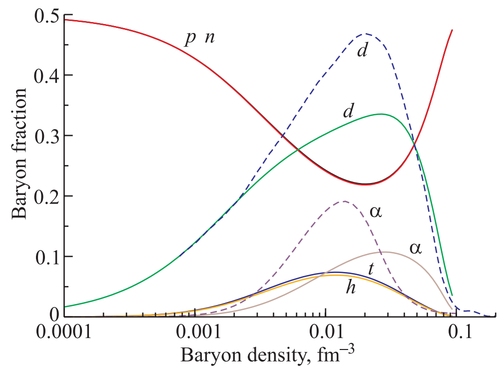

Fig. 2. Composition of nuclear matter with proton fraction 0.5 as a function of the baryon density, $T = 10$ MeV

Up to densities of about 0.001 fm^{-3}, density effects can be neglected. In this way we describe an ideal mixture in chemical equilibrium. The composition as well as the thermodynamical functions can be calculated immediately by solving the equations given above. Also the β equilibrium can be calculated describing the chemical equilibrium with respect to the weak decay $n \rightleftharpoons p + e + \bar{\nu}_e$, where one usually neglects the chemical potential of the neutrinos. For the electron chemical potential the relativistic ideal fermion gas model is used. Neglecting the formation of clusters, the corresponding results for the proton fraction as well as the thermodynamical functions are well known from the literature, see Refs. [1,13], where at higher densities a quasiparticle picture is introduced. They are used to describe nuclear matter in β equilibrium to calculate the structure of neutron stars. There is an additional relation between the chemical potentials of the proton, neutron, and electron as well as the charge neutrality condition so that $n_e = Y_p^{\mathrm{tot}} n_B$.

Also the calculation of nuclear matter in β equilibrium is improved by taking the formation of light clusters into account. The calculations within the model of an ideal mixture of different components is straightforward and will not be given here. At higher densities, also the medium effects have to be taken into account as discussed below. Notice that for the present purpose we only use empirical values without specifying the underlying nucleon–nucleon interaction.

2. RELATIVISTIC MEAN-FIELD THEORY

A description of nuclear matter as an ideal mixture of protons and neutrons, possibly in β equilibrium with electrons and neutrinos, is not sufficient to give a realistic description of dense matter. The account of the interaction between the nucleons can be performed in different ways. For instance, we have effective nucleon–nucleon interactions, which reproduce empirical two-nucleon data, e.g. the PARIS and the BONN potential. On the other hand, we have effective interactions like the Skyrme interaction, which are able to reproduce nuclear data within the mean-field approximation. The most advanced description is given by the Walecka model, which is based on a relativistic Lagrangian and models the nucleon–nucleon interactions by coupling to effective meson fields. Within the relativistic mean-field approximation, quasiparticles are introduced, which can be parameterized by a self-energy shift and an effective mass.

We will use the so-called TM1 model which is given by the following Lagrangian, describing coupling of the nucleon field to the nonlinear sigma, omega and rho meson fields (index $i = p, n$ denotes protons or neutrons),

$$\mathcal{L} = \bar{\psi}_i[i\gamma_\mu\partial^\mu - m_i - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho_a^\mu]\psi_i + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \\ - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 - \frac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho_a^\mu. \quad (2)$$

The TM1 parameterization is given in Ref. [11].

For further details with respect to notation and the derivation of the corresponding Euler–Lagrange equations we refer to [11]. By replacing the meson fields by their expectation values one obtains an effective Dirac equation for the nucleons, which gives

$$\begin{aligned} m_i^* &= m_i + g_\sigma\,\sigma_0, \\ E_p(k;T,\mu_p,\mu_n) &= \sqrt{k^2 + m_p^{*2}} + g_\omega\,\omega_0 + g_\rho\,\rho_0, \\ E_n(k;T,\mu_p,\mu_n) &= \sqrt{k^2 + m_n^{*2}} + g_\omega\,\omega_0 - g_\rho\,\rho_0 \end{aligned} \quad (3)$$

as the expressions for the effective nucleon mass and the corresponding proton and neutron energies, respectively.

For a temperature T and chemical potentials μ_i (relative to the nucleon masses), the nucleon occupation probability reads

$$f_i(k) = \frac{1}{1 + \exp\left[\left(E_i(k;T,\mu_p,\mu_n) - \mu_i\right)/T\right]}. \quad (4)$$

The antinucleon distribution functions followed by changing the sign of the effective chemical potential [11]. Since we are interested in the low-density region, the contribution of the antinucleons can be neglected. Then, in the mean-field approximation the chemical potential is related to the nucleon number density according to

$$n_i = \frac{1}{\pi^2}\int_0^\infty dk\,k^2\,\frac{1}{\exp\left[\left(E_i(k;T,\mu_p,\mu_n) - \mu_i\right)/T\right] + 1}. \quad (5)$$

The meson fields σ_0, ω_0, and ρ_0 are found by solving a set of equations self-consistently as shown in [11]. Also expressions for the energy density, pressure and the entropy density can be found there. The empirical values of the binding energy of nuclear matter and nuclear matter density are reproduced using the above-mentioned parameterization. The nuclear matter EOS can be found expressing the chemical potentials as functions of temperature, baryon density and asymmetry. Within this relativistic mean-field approach the chemical potential as a function of density, temperature and the asymmetry parameter can be calculated. In particular, for symmetric matter an instability region is obtained below a critical temperature T_c, where a Maxwell construction has to be applied.

3. MEDIUM MODIFICATIONS OF TWO-PARTICLE CORRELATIONS

Expressions for the medium modifications of the cluster distribution functions can be derived in a quantum statistical approach to the few-body states, starting from a Hamiltonian describing the nucleon–nucleon interaction by the potential $V(12,1'2')$ (1 denoting momentum, spin and isospin). We first discuss the two-particle correlations, which have been considered extensively in the literature [5, 7]. Results for different quantities such as the spectral function, the deuteron binding energy, and wave function as well as the two-nucleon scattering phase shifts in the isospin singlet and triplet channel have been evaluated for different temperatures and densities. The composition as well as the phase instability was calculated.

The account of two-particle correlations in nuclear matter can be performed considering the two-particle Green function in ladder approximation. The solution of the corresponding Bethe–Salpeter equation taking into account mean-field and Pauli blocking terms is equivalent to the solution of the wave equation

$$\left[E^{\mathrm{MF}}(1)+E^{\mathrm{MF}}(2)-E_{nP}\right]\psi_{nP}(12)+ \\ +\left[1-f_1(1)-f_1(2)\right]\sum_{1'2'}V(12,1'2')\,\psi_{nP}(1'2')=0, \quad (6)$$

$E^{\mathrm{MF}}(1)=p_1^2/2m+\sum_2 V(12,12)_{\mathrm{ex}}f_1(2)$ are the Hartree–Fock single-particle energies; $f_1(1)=\{\exp[E^{\mathrm{MF}}(1)/T-\mu_1/T]+1\}^{-1}$. From the solution of this in-medium two-particle Schrödinger equation or the corresponding T matrix the scattering and possibly bound states are obtained. Due to the self-energy shifts and the Pauli blocking, the binding energy of deuteron $E_d(P,T,\mu_p,\mu_n)$ as well as the scattering phase shifts $\delta_{\tau_2}(E,P,T,\mu_p,\mu_n)$ in the isospin singlet or triplet channel τ_2, respectively, will depend on the temperature and the chemical potentials. For a separable interaction $V(12,1'2')$ like the PEST4 potential [14], an analytical solution of Eq. (6) can be found in the low-density limit, and the results for the shift of the binding energy and the medium modification of the scattering phase shifts are discussed extensively, see [5,7]. We will discuss the medium shift of the binding energy in perturbation theory.

An important phenomenon is the Mott effect. At a given temperature T and total momentum P, the binding energy of the deuteron bound state vanishes at the density $n_d^{\mathrm{Mott}}(P,T)$ due to the Pauli blocking. As a consequence, the virial expansion of the EOS (generalized Bethe–Uhlenbeck formula)

$$n_B(T,\mu_p,\mu_n)=n_1(T,\mu_p,\mu_n)+n_2(T,\mu_p,\mu_n) \quad (7)$$

constitutes the single-particle contributions $n_1=n_p^f+n_n^f$, where $n_\tau^f(T,\mu_\tau)=2/(2\pi\hbar)^3\times\int d^3p f_\tau(E_\tau(p))$ describes the free-quasiparticle contributions of protons ($\tau=p$) or neutrons

($\tau = n$), respectively, and the two-particle contributions $n_2 = n_2^{\text{bound}} + n_2^{\text{scat}}$ containing the contribution of deuterons (spin factor 3)

$$n_2^{\text{bound}}(T, \mu_p, \mu_n) = 3 \int_{P > P_{\text{Mott}}} \frac{d^3P}{(2\pi\hbar)^3} \, f_d(E_d) \tag{8}$$

with $f_d(E_d) = [\exp(E_d(P, T, \mu_p, \mu_n)/T - \mu_p/T - \mu_n/T) - 1]^{-1}$, and scattering states of the isospin singlet and triplet channel τ_2 (degeneration factor γ_{τ_2})

$$n_2^{\text{scat}}(T, \mu_p, \mu_n) = \sum_{\tau_2} \gamma_{\tau_2} \int \frac{d^3P}{(2\pi\hbar)^3} \int_0^\infty \frac{dE}{2\pi} \; f_{\tau_2}(\Delta E_d^{\text{SE}}(P) + E) \sin^2 \delta_{\tau_2} \frac{d}{dE} \delta_{\tau_2}, \tag{9}$$

$\Delta E_d^{\text{SE}}(P)$ is the shift of the continuum edge (self-energies at momentum $P/2$).

The EOS (7) shows some interesting features: (i) In the low-density limit, a mass action law is obtained describing an ideal mixture of free nucleons and deuterons. We stress that a quasiparticle picture is not able to reproduce this important limiting case correctly. (ii) With increasing density, the single-particle properties as well as the two-particle properties are simultaneously modified by the medium. In particular, the bound states are dissolved at high densities. (iii) There is also a contribution from scattering states to the two-particle density. As a consequence of the Levinson theorem, the contribution of the disappearing bound states is replaced by a contribution from the scattering states (resonances) at the Mott density so that the total two-particle density n_2 behaves smoothly. (iv) Due to the pole of the Bose distribution function at low temperatures, pairing can occur in n_2. A smooth transition from Bose–Einstein condensation of deuterons at low densities to Cooper pairing at high densities is observed [7].

Calculations of the composition (n_2/n_B) of symmetric nuclear matter ($n_p = n_n$, no Coulomb interaction) are shown in Ref. [7]. At low densities, the contribution of bound states becomes dominant at low temperatures. At fixed temperature, the contribution of the correlated density n_2 is first increasing with increasing density according to the mass action law, but above the Mott line it is sharply decreasing, so that near nuclear matter density ($n_B = n_{\text{tot}} = 0.17$ fm^{-3}) the contribution of the correlated density almost vanishes. Also, the critical temperature for the pairing transition is shown there.

For a given temperature $T = 10$ MeV, the composition with respect to the two-particle correlation is shown as a function of the baryon density in [5], where it is shown that the correlated density contains the contribution of bound states as well as the contribution of scattering states. Above the so-called Mott density, where the bound states begin to disappear, according to the Levinson theorem, the continuous behavior of the correlated density is produced by the scattering states.

Since we perform only an exploratory calculation with respect to the density modification, instead of highly sophisticated parameterization of the interaction such as the PARIS and BONN potential we will use a simple, separable Yamaguchi interaction

$$\begin{gathered} \langle k, K | V | k', K' \rangle = V(k, k') \, \delta_{K,K'}, \\ V(k, k') = \sum_{i=s,t} \lambda_i w(k) w(k'), \qquad w(k) = \frac{1}{k^2 + \beta^2}. \end{gathered} \tag{10}$$

The set of parameters $\beta = 1.4488$ fm^{-1}, $\lambda_s = 4263.05$ MeV $\cdot$ fm^{-1} and $\lambda_t = 2550.03$ MeV $\cdot$ fm^{-1} is chosen in order to reproduce the binding energy of the deuteron as well as of the alpha particle. The further properties, such as the wave functions, are assumed to be reasonable approximations in evaluating the density effects.

From these considerations we see that at high densities the clusters are dissolved and should be described as weakly interacting quasiparticles. To give an optimal description of the quasiparticle energies, instead of using the Hartree–Fock approximation of the Yamaguchi model, we will adopt the well established Walecka model. In this way the self-energy effects are consistently described for the free and bound states. Since the Pauli blocking terms cannot be evaluated using the Walecka model, these expressions are computed using the Yamaguchi model. It should be stressed that in the density region where clusters are relevant both interaction models give self-energies and effective masses which are in reasonable agreement. So the mixing of these two interaction models is not contradictionary.

Thus, we take the quasiparticle energies which are described by an effective mass and a self-energy shift and solve the Schrödinger equation for the separable Yamaguchi potential. Separating the centre-of-mass motion with energy $p^2/2\,M_d^*$ from the relative motion with reduced mass $M_p^*\,M_n^*/(M_p^* + M_n^*)$, we find the binding energy E_d^{quasi}, which is density-dependent due to the effective masses. The corresponding wave function is used to evaluate the Pauli blocking term

$$\Delta E_d^{\text{Pauli}} = \sum_{1\,2\,1'\,2'} \psi(1\,2)\, V(1\,2, 1'\,2')\, [f(1) + f(2)]\, \psi^*(1'\,2') \tag{11}$$

in the first-order perturbation theory. The self-energy shift ΔE_d^{SE} is obtained simply by the sum of the quasiparticle self-energy shift of the proton and neutron, obtained by the Walecka model.

Within our exploratory calculation we use a simplified description of the contribution of correlated states, considering only the bound state with an effective shift, which reproduces the correlated density. This shift is taken as a quadratic function in the densities, where the linear term is calculated from perturbation theory and the quadratic term is fitted to reproduce the results for the composition as found by the full microscopic calculation including the contribution of scattering states.

With the deuteron wave function, the contribution of the single-particle self-energy shift to the shift of the binding energies of clusters is evaluated using the quasiparticle shifts derived within the Walecka model. This ensures that the nucleons are described consistently also above the Mott density where the bound state merges in the continuum of scattering states. For the Pauli blocking term we evaluate the average over the interaction potential, multiplied with the distribution function.

In the low-density limit, perturbation theory gives $E_d = E_d^0 + P^2/2M_d + \Delta E_d^{\text{SE}} + \Delta E_d^{\text{Pauli}}$ with $E_d^0 = -2.22$ MeV, and

$$\Delta E_d^{\text{Pauli}} = \sum_{12,1'2'} \psi_{d,P}(12)[f_1(1) + f_1(2)] V(12, 1'2') \psi_{d,P}(1'2') \approx$$
$$\approx \psi_{d,P}^2(0)(n_p + n_n)(E_d^0 - E_d^{\text{kin}}), \tag{12}$$

where E_d^{kin} denotes the mean kinetic energy for the unperturbed deuteron. To reproduce the behavior shown in Fig. 2, we adopt the following parameterization:

$$\Delta E_d \approx 340(n_p + n_n)\ \mathrm{MeV} \cdot \mathrm{fm}^3 + 13000(n_p^2 + n_n^2)\ \mathrm{MeV} \cdot \mathrm{fm}^6. \tag{13}$$

The result of this calculation is also shown in Fig. 2 to be compared with the evaluation of the correlated density in [5]. Two-particle correlations are suppressed for densities higher than the Mott density of about 0.001 fm^{-3}, but survive to densities of the order of nuclear matter density.

4. MEDIUM MODIFICATION OF HIGHER CLUSTERS

The modification of the three- and four-particle system due to the medium can be considered in the cluster mean-field approximation. Describing the medium in quasiparticle approximation, a medium-modified Faddeev equation can be derived which was already solved for the case of three-particle bound states in [6] as well as for the case of four-particle bound states in [6, 8]. Similar to the two-particle case, due to the Pauli blocking the bound state disappears at a given temperature and total momentum at the corresponding Mott density.

For our exploratory calculation we use Gaussian-type wave functions to find optimal bound states in the three- and four-particle case. Then we are able to calculate the perturbative expression for the shift of the bound-state energy and find

$$\Delta E_t = 600(n_p + 2n_n)\ \mathrm{MeV} \cdot \mathrm{fm}^3 + 3300(n_p^2 + 2n_n^2)\ \mathrm{MeV} \cdot \mathrm{fm}^6, \tag{14}$$

$$\Delta E_h = 600(2n_p + n_n)\ \mathrm{MeV} \cdot \mathrm{fm}^3 + 3300(2n_p^2 + n_n^2)\ \mathrm{MeV} \cdot \mathrm{fm}^6, \tag{15}$$

$$\Delta E_\alpha = 709(2n_p + 2n_n)\ \mathrm{MeV} \cdot \mathrm{fm}^3 + 6500(2n_p^2 + 2n_n^2)\ \mathrm{MeV} \cdot \mathrm{fm}^6. \tag{16}$$

Now we can calculate the composition replacing the binding energies by the density-dependent ones. Results for the composition are shown in Figs. 1, 2. It is shown that, in particular, α clusters are formed in symmetric nuclear matter, but they are destroyed at about nuclear matter density. In the case of asymmetric matter, triton becomes abundant.

We conclude that not only the α particle but also the other light clusters contribute significantly to the composition. Furthermore, they also contribute to the baryon chemical potential and in this way the modification of the phase instability region with respect to the temperature, baryon density, and asymmetry can be obtained. As an example, for symmetric matter the baryon chemical potential as a function of density for $T = 10$ MeV is shown in Fig. 3. We see that the instability region is reduced if cluster formation is taken into account.

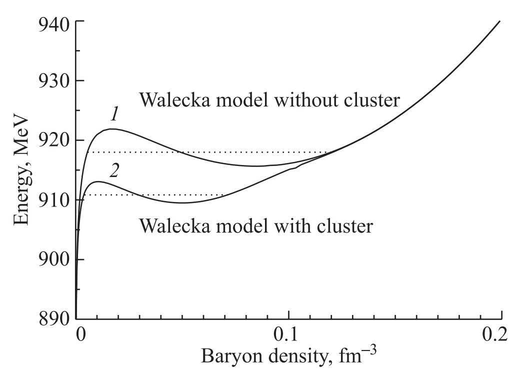

Fig. 3. Baryon chemical potential as a function of the baryon density for symmetric nuclear matter, $T = 10$ MeV. With and without the formation of light clusters

Within the approach, given here, also the β equilibrium can be calculated and the influence of the cluster formation on the proton fraction can be considered. The formation of clusters will increase the proton fraction.

In order to get the correct physical behavior, medium modifications for the clusters have to be taken into account at high densities. A simple approach is the concept of an excluded volume as used in [11]. Other effects such as the modification of quasiparticles forming a bound state are not considered. For details we refer to [11]. We have also shown the result for the composition in Fig. 2 if only α-particle formation is taken into account, using the concept of the excluded volume. The abundance of α particles increases up to baryon densities of about a tenth of nuclear matter density and rapidly decreases with higher densities. In contrast, the quantum statistical approach shows a weaker decrease of the correlated density with the baryon density. In particular, two-particle correlations are present up to nuclear matter density. Discussing the difference in both approaches, we first note that the concept of a hard core, which leads to the excluded volume, overestimates the Pauli blocking, which makes the interaction potential softer. Further, in addition to the medium modification due to the Pauli blocking, the effect of the quasiparticle self-energy shift has to be taken into account.

5. QUANTUM CONDENSATES IN NUCLEAR MATTER

Nuclear matter is an example of a strongly coupled fermion systems, where bound states arise. On the other hand, it is a well-known fact that interacting fermion systems can form quantum condensates so that a superfluid state arises. It is commonly accepted that pairing occurs not only in nuclear matter but also in finite nuclei. In the low-density limit, where even-number fermionic bound states can be considered as bosons, Bose–Einstein condensation is expected to occur at low temperatures. The solution of Eq. (6) with $E_{2nP} = 2\mu$ gives the onset of pairing, the solution of the corresponding four-particle wave equation [8] with $E_{4nP} = 4\mu$ gives the onset of quartetting in (symmetric) nuclear matter. An interesting topic is the cross-over from Bardeen–Cooper–Schrieffer (BCS) pairing to Bose–Einstein condensation (BEC) (see [5]).

Due to the strong interaction of protons and neutrons in the deuteron channel, isospin singlet pn pairing is favored in symmetric nuclear matter in comparison with isospin triplet pp or nn pairing. Considering asymmetric matter with increasing difference between the chemical potentials of protons and neutrons, isospin triplet pairing will become more favored. As can be shown from the evaluation of condensation energy, coexisting isospin singlet and isospin triplet condensates are not stable in symmetric matter. It is an open question how in the ground state of nuclear matter the transition from isospin singlet pairing to isospin triplet pairing occurs if the asymmetry parameter increases.

In the recent letter [8], it has been shown from the solution of the in-medium wave equations (6) that in a certain region of density, pairing has to compete with quartetting. It has been found that in low-density symmetric nuclear matter the transition to triplet pairing, which is stronger than singlet pairing, does not occur because the quartetting transition occurs earlier. The transition temperature to quartetting has been estimated as a function of the chemical potential as well as of the density by using a variational calculation. Quartetting (α-like condensate) beats the transition to triplet pairing (deuteron-like condensate) if the density is smaller than 0.03 fm^{-3}. A more detailed investigation of the quartetting solution near its break down at about 0.03 fm^{-3} is missing up to now.

The consequences of isospin singlet pairing and α-like quartetting on the binding energies of nuclei has been investigated within the local density approach [9]. The Wigner energy in $N = Z$ nuclei was identified with the formation of an isospin singlet condensate, and for nuclei with medium atomic number $A \approx 40$ this additional contribution of nuclear binding energy is decreasing with increasing asymmetry and vanishes at $N - Z \approx 4$. Exploratory calculations for the binding energy of nuclei with $Z = N$ show that the contribution due to quartetting is small but may become large for small A or for nuclear matter at low density. In particular, alpha cluster condensation has been investigated in threshold states of self-conjugate $4n$ nuclei [10].

CONCLUSIONS

In certain regions of the density–temperature plane, a significant fraction of nuclear matter is bound into clusters. The EOS and the region of phase instability are modified. In the case of β equilibrium, the proton fraction and the occurrence of inhomogeneous density distribution are influenced in an essential way. Important consequences are also expected for nonequilibrium processes [15].

The inclusion of both three- and four-particle correlations in nuclear matter allows one not only to describe the abundances of t, h, α but also their influence on the equation of state and phase transitions. In contrast to the mean-field treatment of the superfluid phase, higher-order correlations will arise in the quantum condensate.

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