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KINETIC THEORY OF THE QUANTUM FIELD SYSTEMS WITH UNSTABLE VACUUM

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The description of quantum field systems with meta-stable vacuum is motivated by studies of many physical problems (η-meson sector of the Witten–Di Vecchia–Veneziano model, disoriented chiral condensate, false vacuum decay in inflation cosmological models, etc.). A nonperturbative approach based on the kinetic description within the framework of the quasiparticle representation is proposed. We restrict ourselves to scalar field theory with self-interaction potential having a nontrivial set of vacuum states. If the vacuum state is not uniquely defined, the quasiparticle definition is introduced by some physical reason. As a result, we obtain the self-consistent system of the kinetic equation for quasiparticle distribution function and the equation of motion for the background field. Two examples — the quantum field system with ϕ^4 and double-well type potential — are considered.

Описание квантово-полевых систем с метастабильным вакуумом важно при изучении многих физических проблем (η-мезонный сектор модели Виттена–Ди Веккья–Венециано, неориентированный киральный конденсат, ложный вакуумный распад в космологических моделях инфляции и т. д.). Предложено непертурбативное приближение, основанное на кинетическом описании в рамках квазичастичного представления. Мы ограничиваемся скалярной теорией поля с потенциалом собственного взаимодействия, имеющим нетривиальный набор вакуумных состояний. Если вакуумное состояние не определяется однозначно, квазичастичное представление вводится из некоторых физических соображений. В итоге мы получаем самосогласованную систему кинетических уравнений для функции распределения квазичастиц и уравнение движения для поля фона. Рассмотрены два примера: квантово-полевая система с ϕ^4 и потенциал типа двойной стенки.

INTRODUCTION

Separating of the classical background field is a standard procedure of different nonperturbative approaches in QFT [1–3]. In the framework of this procedure the quantum fluctuations can be described by perturbation theory.

There is a class of problems, in which the strong background field produces particles, that in turn influence the background field (the back-reaction problem). It is worthy to mention such problems as decay of disoriented chiral condensate [4], the resonant decay of CP-odd metastable states [5, 6], QGP pre-equilibrium evolution [7], phase transition in systems with broken symmetry [8], etc.

The construction of general kinetic theory for various potential types is shown in Sec. 1. We will derive the closed system of equations for the self-consistent description of back-reaction (BR) problem, including the kinetic equation (KE) with nonperturbative source term

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describing particle creation in the quasi-classical background field and equation of motion for this background field. We use the oscillator representation (OR) to derive the KE. The efficiency of this method was demonstrated in the framework of scalar QED [9]. As illustrative examples, in Sec. 2 one-component scalar theory with ϕ^4 and double-well potential, are considered. In these examples, we study some features of proposed approach. In particular, the problem of stable vacuum state definition and possibility of emerging tachyonic regimes are discussed. The similar analysis was done for some other models of such kind (e.g., [5, 6, 10, 11]). Finally, Sec. 3 summarizes the article.

We use the metric $g^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$ and assume that $\hbar = c = 1$.

1. THE SYSTEM OF BASIC EQUATION

Let us consider the scalar particle Lagrangian with a self-interaction potential $V[\Phi]$:

$$\mathcal{L}[\Phi] = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} m_0^2 \Phi^2 - V[\Phi], \tag{1}$$

where m_0 is the bare mass. In general case, the potential $V[\Phi]$ is an arbitrary continuous nonmonotonous function with at least one minimum (this is necessary for correct definition of the vacuum state). It is assumed that the field Φ can be decomposed into the quasi-classical space homogeneous background field $\phi_0(t)$ and fluctuations $\phi(x)$

$$\Phi(x) = \phi_0(t) + \phi(x). \tag{2}$$

In accordance with definition of fluctuations, we have $\langle \phi \rangle = 0$ and $\langle \Phi \rangle = \phi_0$, where symbol $\langle \ldots \rangle$ denotes some averaging procedure. The background field $\phi_0(t)$ can be treated as quasi-classical one at the condition [14]:

$$|\dot{\phi}_0| \gg \frac{\sqrt{\hbar c}}{(c\Delta t)^2}, \tag{3}$$

where Δt is the characteristic time of the field averaging.

We consider the case of rather small fluctuations in the neighborhood of the background field. Therefore, the potential energy expansion in powers of $\phi(x)$ can be performed

$$V[\Phi] = V[\phi_0] + R_1 \phi + \frac{1}{2} R_2 \phi^2 + V_r[\phi_0, \phi], \tag{4}$$

where

$$R_1 = R_1[\phi_0] = \frac{dV[\phi_0]}{d\phi_0}, \;\; R_2 = R_2[\phi_0] = \frac{d^2 V[\phi_0]}{d\phi_0^2} \tag{5}$$

and $V_r[\phi_0, \phi]$ is a residual term containing the high orders to be neglected in current article (nondissipative approximation). The decomposition (4) can be finite (for polynomial theories) or infinite. After field decomposition (2) the equation of motion

$$\partial_\mu \partial^\mu \Phi + m_0^2 \Phi + \frac{dV[\Phi]}{d\Phi} = 0 \tag{6}$$

can be rewritten in the following form:

$$(-\partial_\mu \partial^\mu - m^2)\phi = Q[\phi_0, \phi], \tag{7}$$

where

$$m^2(t) = m^2[\phi_0] = m_0^2 + R_2[\phi_0] \tag{8}$$

is the time-dependent in-medium mass and

$$Q[\phi_0, \phi] = \ddot{\phi}_0 + m_0^2 \phi_0 + R_1[\phi_0] + Q_2[\phi_0, \phi], \tag{9}$$

$$Q_2[\phi_0, \phi] = \frac{1}{2} \frac{dR_2[\phi_0]}{d\phi_0} \phi^2. \tag{10}$$

Here and throughout the article a dot denotes the derivative with respect to time.

As a result of averaging Eq. (7), the equation of motion for background field is obtained

$$\ddot{\phi}_0 + m_0^2 \phi_0 + R_1[\phi_0] + \langle Q_2[\phi_0, \phi] \rangle = 0. \tag{11}$$

The time independence of the averaging procedure is taken into account.

The assumption about space-homogeneity means that the function $\langle Q_2[\phi_0, \phi] \rangle$ in Eq. (11) can only depend on time. As follows from Eqs. (10) and (11), the source term in the right side of Eq. (7) is exclusively defined by the fluctuations,

$$Q[\phi_0, \phi] = Q_2[\phi_0, \phi] - \langle Q_2[\phi_0, \phi] \rangle. \tag{12}$$

On the other hand, the field function $\phi(x)$ in nonstationary situation allows the decomposition:

$$\phi(x) = \int [dk] \{ \phi^{(+)}(\mathbf{k}, t) \mathrm{e}^{-\imath \mathbf{k} \bar{x}} + \phi^{(-)}(\mathbf{k}, t) \mathrm{e}^{\imath \mathbf{k} \bar{x}} \}, \tag{13}$$

$[dk] = (2\pi)^{(-3/2)} d^3 k$, $\phi^{(\pm)}(\mathbf{k}, t)$ are the positive and negative frequency solutions of the equation of motion

$$\ddot{\phi}^{(\pm)}(\mathbf{k}, t) + \omega^2(\mathbf{k}, t) \phi^{(\pm)}(\mathbf{k}, t) = -Q[\phi_0, \phi | \pm \mathbf{k}], \tag{14}$$

where

$$\omega^2(\mathbf{k}, t) = m^2(t) + \mathbf{k}^2 \tag{15}$$

and $Q[\phi_0, \phi|\mathbf{k}]$ is the Fourier image of the function $Q[\phi_0, \phi]$,

$$Q[\phi_0, \phi] = \int [dk] Q[\phi_0, \phi|\mathbf{k}] \mathrm{e}^{-\imath \mathbf{k} \bar{x}}. \tag{16}$$

The function $Q[\phi_0, \phi|\mathbf{k}]$ contains nonlinear contribution to Eq. (14). We suppose a finite limit $\lim\limits_{t \to -\infty} \phi^{(\pm)}(\mathbf{k}, t) = \phi_-^{(\pm)}(\mathbf{k})$ in the infinite past, and assume that the solutions $\phi^{(\pm)}(\mathbf{k}, t)$ become asymptotically free $\phi^{(\pm)}(\mathbf{k}, t) \to \mathrm{e}^{\pm i \omega_- t}$, where $\omega_- = \lim\limits_{t \to -\infty} \omega(\mathbf{k}, t)$. The existence of the last limit is based on adiabatic hypothesis about switching off of self-interaction in Eq. (8).

After the decompositions (2) and (4) the Hamiltonian density is

$$H[\Phi] = H[\phi_0] + H_1[\phi_0, \phi] + H_2[\phi_0, \phi] + V_r[\phi_0, \phi], \tag{17}$$

where $H_0[\phi_0]$ is the background field Hamiltonian and H_1 and H_2 are the Hamiltonian functions of the first and second order with respect to the fluctuation field,

$$H[\phi_0] = H_0[\phi_0] + V[\phi_0] = \frac{1}{2}\dot{\phi}_0^2 + \frac{1}{2}m_0^2\phi_0^2 + V[\phi_0], \tag{18}$$

$$H_1[\phi_0, \phi] = \dot{\phi}_0\dot{\phi} + (m_0^2\phi_0^2 + R_1[\phi_0])\phi, \tag{19}$$

$$H_2[\phi_0, \phi] = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2. \tag{20}$$

One can built the Hilbert space on the system of basic functions $\phi^{(\pm)}(\mathbf{k}, t)$ defined by Eq. (14) and proceed to the Fock representation. After the decomposition (4), H_2 (20) has a nondiagonal form with respect to creation and annihilation operators and hence does not allow quasiparticle interpretation [1]. In order to diagonalize Hamiltonian it is necessary to apply either the Bogoliubov transformation [1] or OR [9].

Following the idea of OR, let us write the free field decomposition of $\phi(x)$ and $\dot{\phi}(x)$ in the absence of background field ϕ_0 with the substitution of the free particle energy $\omega(\mathbf{k}) = (m_0^2 + \mathbf{k}^2)^{1/2}$ into the time-dependent one (15)

$$\phi(x) = \int \frac{[dk]}{\sqrt{2\omega(\mathbf{k}, t)}} \left\{ a^{(+)}(\mathbf{k}, t)\mathrm{e}^{-\imath\mathbf{k}\bar{x}} + a^{(-)}(\mathbf{k}, t)\mathrm{e}^{\imath\mathbf{k}\bar{x}} \right\}, \tag{21}$$

$$\pi(x) = \imath \int [dk] \frac{\sqrt{2\omega(\mathbf{k}, t)}}{2} \left\{ a^{(+)}(\mathbf{k}, t)\mathrm{e}^{-\imath\mathbf{k}\bar{x}} - a^{(-)}(\mathbf{k}, t)\mathrm{e}^{\imath\mathbf{k}\bar{x}} \right\}, \tag{22}$$

where $\pi(x)$ plays a role of the canonical momentum. The remarkable feature of the decompositions (21) and (22) is the fulfillment of the standard commutative relations

$$[\pi(x), \phi(x)]|_{t=t'} = -\imath\delta(\bar{x} - \bar{x}'), \tag{23}$$

self-considered with the commutative relations for time-dependent creation and annihilation operators

$$[a^{(-)}(\mathbf{k}, t), a^{(+)}(\mathbf{k}, t)] = \delta(\mathbf{k} - \mathbf{k}') \tag{24}$$

(the rest commutators are equal to zero). It is not less important that the Hamiltonian (20) after the replacement $\dot{\phi} \to \pi$ has the diagonal form,

$$H_2[\phi_0, \phi] = \int d^3k\ \omega(\mathbf{k}, t)a^{(+)}(\mathbf{k}, t)a^{(-)}(\mathbf{k}, t) \tag{25}$$

and hence OR is the quasiparticle representation [9].

In order to obtain the equations of motion for the operators $a^{(\pm)}(\mathbf{k}, t)$, let us write the corresponding action with the Hamiltonian (18)–(20)

$$S[\phi] = \int d^4x\ \{\pi\dot{\phi} - H_1 - H_2 - V_r\}. \tag{26}$$

After substitution of decompositions (21) and (22), we get

$$S[\phi] = \int dt\, d^3k \Big\{ \frac{\imath}{2}[a^{(+)}(\mathbf{k},t)\dot{a}^{(-)}(\mathbf{k},t) - a^{(-)}(\mathbf{k},t)\dot{a}^{(+)}(\mathbf{k},t)] - \\ - \frac{\dot{\omega}(\mathbf{k},t)}{2\omega(\mathbf{k},t)}[a^{(+)}(\mathbf{k},t)a^{(+)}(-\mathbf{k},t) - a^{(-)}(-\mathbf{k},t)a^{(+)}(\mathbf{k},t)] - \\ - \frac{1}{2}\omega(\mathbf{k},t)[a^{(+)}(\mathbf{k},t)a^{(-)}(\mathbf{k},t) + a^{(-)}(\mathbf{k},t)a^{(+)}(\mathbf{k},t)] - V_{r\mathbf{k}}[\phi_0,\phi] \Big\} + S_1[\phi], \quad (27)$$

where $S_1[\phi]$ is the part of the action corresponding to the Hamiltonian (19) and $v_{\mathbf{k}}[\phi_0,\phi]$ is the Fourier image of the residual potential term. Variation with respect to $a^{(\pm)}(\mathbf{k},t)$ and subsequent transition to the occupation number representation lead to the Heisenberg-type equations of motion $(\mathbf{k} \neq 0)$

$$\dot{a}^{(\pm)}(\mathbf{k},t) = W(\mathbf{k},t)a^{(\mp)}(\mathbf{k},t) + \imath[H_2 + V_r, a^{(\pm)}(\mathbf{k},t)], \quad (28)$$

where

$$W(\mathbf{k},t) = \frac{\dot{\omega}(\mathbf{k},t)}{2\omega(\mathbf{k},t)} = \frac{\dot{R}_2[\phi_0]}{4\omega^2} \quad (29)$$

is the vacuum transition amplitude of a particle production. In Eq. (28), the condensate contribution generated by the action part $S_1[\phi]$ is omitted because it corresponds to $\mathbf{k} = 0$ (the connection mechanism of the condensate state with $\mathbf{k} = 0$ and excitations is absent in the present model).

For the first time, equations of the type (28) were obtained in [12] in QED in the framework of the Bogoliubov transformations. They are the basis for nonperturbative derivation of KE [7]. We will use now this procedure in the present statement of the problem. Let us introduce the distribution function of quasiparticles (it is convenient to do in discrete momentum representation and subsequent transition to infinite volume of the system in the resulting KE, $L^3 \to \infty$)

$$L^3 f(\mathbf{k},t) = \langle \mathrm{in} \mid a^{(+)}(\mathbf{k},t)a^{(-)}(\mathbf{k},t) \mid \mathrm{in} \rangle, \quad (30)$$

where the averaging procedure is carried out over the in-vacuum state. We will consider nondissipative approximation $V_r[\phi_0,\phi] \to 0$. Using the method of works [7] and basic Eq. (28), it is not difficult to get the KE

$$\frac{df(\mathbf{k},t)}{dt} = 2W(\mathbf{k},t)\int_{-\infty}^{t} dt' W(\mathbf{k},t')[1+2f(\mathbf{k},t)]s\left[2\int_{t'}^{t} d\tau\omega(\mathbf{k},\tau)\right] \quad (31)$$

where $\omega(\mathbf{k},t)$ is defined by Eq. (15). KE (31) can be transformed to a system of ordinary differential equations, which is convenient for numerical analysis [7]

$$\dot{f} = Wv, \qquad \dot{v} = 2W(1+2f) - 2\omega z, \qquad \dot{z} = 2\omega v. \quad (32)$$

To rewrite Eq. (11) for background field in nondissipative approximation, one has to calculate the averaging value $\langle \mathrm{in} \mid \phi^2(x) \mid \mathrm{in} \rangle$. Using Eq. (21) and the relations (in the limit $L^3 \to \infty$)

$$\langle \mathrm{in} \mid a^{(+)}(\mathbf{k},t)a^{(-)}(\mathbf{k}',t) \mid \mathrm{in} \rangle \quad = \quad (2\pi)^3 f(\mathbf{k},t)\delta(\mathbf{k}-\mathbf{k}'), \quad (33)$$

in the space homogeneous case one can obtain

$$\langle \mathrm{in} \mid \phi^2(x) \mid \mathrm{in} \rangle = \frac{1}{2} \int \frac{d^3k}{\omega(\mathbf{k},t)} [1 + 2f(\mathbf{k},t) + v(\mathbf{k},t)]. \tag{34}$$

Then Eq. (11) is

$$\ddot{\phi}_0 + m_0^2 \phi_0 + R_1[\phi_0] + \frac{1}{2} \frac{dR_2}{d\phi_0} \int \frac{d^3k}{\omega(\mathbf{k},t)} \left[f(\mathbf{k},t) + \frac{1}{2} v(\mathbf{k},t) \right] = 0 \tag{35}$$

(the vacuum unit is omitted in the integral).

KE (31) (or the equivalent system of equations (32)) and Eq. (35) form the closed system of nonlinear equations describing the back-reaction problem. In the case of $v[\Phi_0, \Phi] = 0$, this system of equations is a direct nonperturbative consequence of the dynamics and the assumption (2). For the description of particle production we will use the particle density

$$n(t) = \int \frac{d^3p}{(2\pi)^3} f(\mathbf{p},t) \tag{36}$$

as well as background field energy E_{cl} and energy of produced particles E_{qu}

$$E_{\mathrm{cl}} = \frac{1}{2} \dot{\phi}_0^2 + \frac{1}{2} m_0^2 \phi_0^2 + V(\phi_0), \tag{37}$$

$$E_{\mathrm{qu}} = \int \frac{d^3p}{(2\pi)^3} \omega(\mathbf{k},t) f(\mathbf{p},t). \tag{38}$$

The conservation of the full energy of the system can be shown analytically. The proof is based on the differentiation of the total energy $E_{\mathrm{cl}} + E_{\mathrm{qu}}$ with respect to time and taking into account Eq. (35) and relations (5), (8) and (29).

The constructed formalism allows the consideration of the following initial problems at the time $t = t_0$: some initial excitation of the background field $\phi_0(t_0)$ is given with the additional condition either $f(\mathbf{p}, t_0) = 0$ or $f(\mathbf{p}, t_0) \neq 0$, where $f(\mathbf{p}, t_0)$ is some initial plasma distribution.

2. EXAMPLES: SOME POLYNOMIAL POTENTIALS

2.1. Φ^4 **Potential.** The separation of background field (2) in the potential

$$V[\Phi] = \frac{1}{4} \lambda \Phi^4, \qquad \lambda > 0 \tag{39}$$

leads to the following decomposition coefficients

$$R_1[\phi_0] = \lambda \phi_0^3, \qquad R_2[\phi_0] = 3\lambda \phi_0^2, \tag{40}$$

background field potential

$$V[\phi_0] = \frac{1}{4} \lambda \phi_0^4, \tag{41}$$

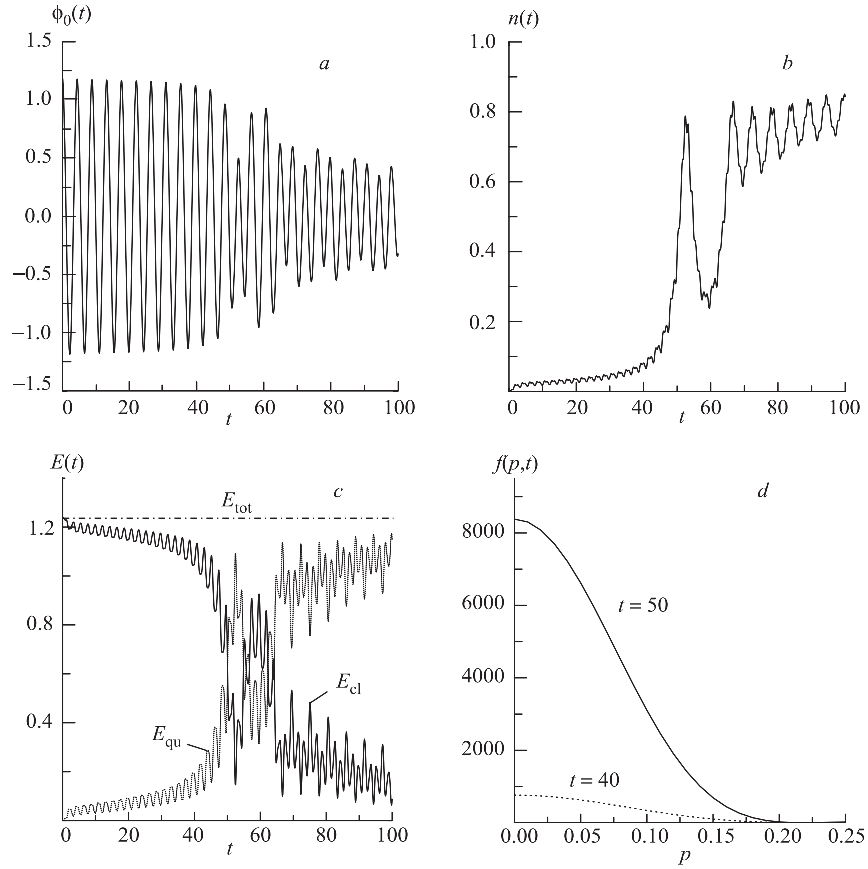

Fig. 1. Time evolution for the symmetric Φ^4 potential. Parameters are: $m_0 = 1$, $\phi_0(0) = 1.2$, $\lambda = 1$. *a*) Evolution of the mean field; *b*) evolution of the particle density; *c*) evolution of the energy; *d*) momentum spectrum of particles at time $t = 40$ and $t = 50$

and residual potential

$$V_r[\phi_0, \phi] = \lambda(\phi_0 + \phi/4)\phi^3. \tag{42}$$

Thus, the time-dependent quasiparticle mass of fluctuating field (8) is equal to

$$m^2(t) = m_0^2 + 3\lambda\phi_0^2. \tag{43}$$

If $\lambda < 0$ and the excitation level is large enough, it is possible that the tachyonic mode will arise, that corresponds to unstable state [13]. The mass (43) determines the vacuum transition amplitude (29)

$$W(\mathbf{k}, t) = \lambda \frac{2\phi_0\dot{\phi}_0}{\omega^2(\mathbf{k}, t)}. \tag{44}$$

KE (31) with this amplitude is correct in the nondissipative approximation, which corresponds to the neglecting of the residual potential (42).

Let us write also the equation of motion for the background field (35) in the same approximation:

$$\ddot{\phi}_0 + M^2(t)\phi_0 + \lambda\phi_0^3 = 0 \tag{45}$$

with the corresponding mass equal to

$$M^2(t) = m_0^2 + 3\lambda \int \frac{d^3k}{\omega(\mathbf{k},t)} \left[f(\mathbf{k},t) + \frac{1}{2}v(\mathbf{k}) \right]. \tag{46}$$

In numerical calculations we apply zero initial conditions for the distribution function and nonzero one for the background field $\phi_0(t_0) = 1.2$. The choice of parameters ($\lambda = 1$ and $m_0 = 1$) as well as initial conditions here and in Subsec. 2.2 is motivated by the desire to make a qualitative comparison between our work and [8], where the authors offered the alternative method for description of quantum systems under strong background field action (the so-called Cornwall–Jackiw–Tomboulis (CJT) method [15]).

As can be seen in Fig. 1, at the early evolution stage all the energy is mainly concentrated in the field oscillation. For $t < 50$ we have a slow growing of the number density. However it drastically increases at $t \sim 50$, and after this time the quantum energy dominates over classical one.

The case $\lambda < 0$ (absolutely unstable potential) is associated with tachyonic regime, that is realized for enough high excitation level, when the initial amplitude $\phi_0(t_0)$ satisfies the condition $m_0^2 + 3\lambda\phi_0^2(t_0) \leqslant 0$. The low excitation level $m_0^2 + 3\lambda\phi_0^2(t_0) > 0$ corresponds to pretachyonic regime. The vacuum particle production in the pretachyonic region is characterized by developing instability, i.e., by unrestricted growth at a tendency of $\phi_0(t)$ to reach the critical value $\phi_0^{(c)} = m_0(3|\lambda|)^{-1/2}$.

2.2. Double-Well Potential

$$V[\Phi] = \frac{1}{4}\lambda\Phi^4 - \frac{1}{2}\mu^2\Phi^2, \qquad \lambda > 0 \tag{47}$$

leads to the same Eq. (45) for the background field with new mass (we set here $m_0 = 0$ and $\mu^2 > 0$)

$$M^2(t) = -\mu^2 + 3\lambda \int \frac{d^3k}{\omega(t)} \left[f(\mathbf{k},t) + \frac{1}{2}v(\mathbf{k},t) \right]. \tag{48}$$

The vacuum transition amplitude (29) is equal here to

$$W(\mathbf{k},t) = \lambda \frac{3\phi_0\dot{\phi}_0}{2\omega^2(\mathbf{k},t)}, \tag{49}$$

where now the quasiparticle frequency (15) contains the mass

$$m^2(t) = -\mu^2 + 3\lambda\phi_0^2. \tag{50}$$

The neighborhood of the central point $\phi_0(t) = 0$ is the instability region. In this region the group velocity $v_g = d\omega(k)/dk = k/\omega(k)$ is either superluminous ($v_g > 1$ and $\mathrm{Im}\,\omega = 0$ for $k > k_c$, where k_c is the root of the equation $\omega(k,t) = 0$) or indefinite ($\mathrm{Re}\,\omega(k) = 0$ for $k < k_c$). Thus, it is tachyonic region.

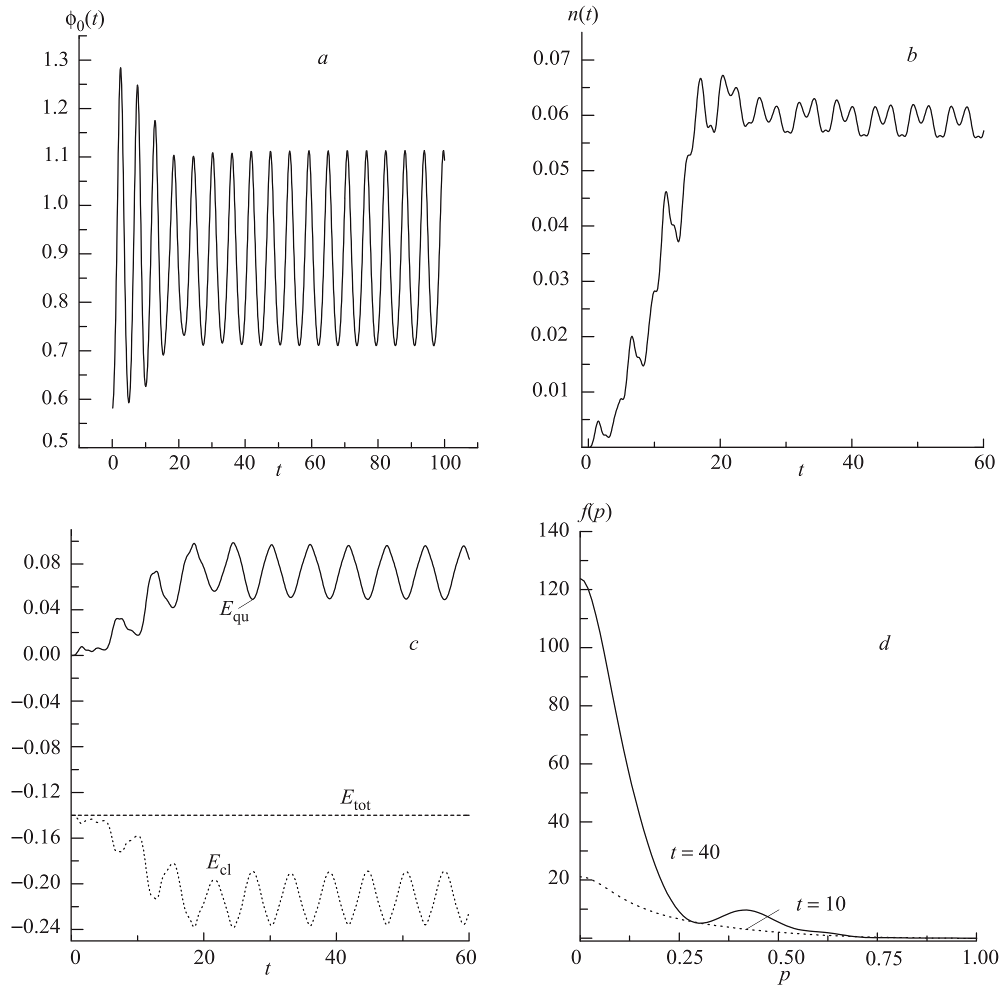

Fig. 2. Time evolution for the bistable Φ^4 potential (47). Parameters are: $\mu^2 = 1$, $\lambda = 1$, $\phi_0(0) = 0.58$. *a*) Evolution of the mean field; *b*) evolution of the particle density; *c*) evolution of the energy; *d*) momentum spectrum of particles at time $t = 10$ and $t = 40$

Let us mark the minimum points of the potential (47) as $\psi_\pm = \pm\mu/\sqrt{\lambda} = \pm\Psi_0$ and put new origin of frame of reference in one of these points, $\Phi = \Psi_\pm + \Psi$. We separate now background component ϕ_0 from the field Ψ, i.e., $\Psi = \phi_0 + \phi$. Using Eq. (6) and the methods of Sec. 2, it is easy to obtain the following system of equations of motion:

$$\ddot{\phi}_0 + 2\mu^2\phi_0 + 3\lambda\Psi_\pm\phi_0^2 + \lambda\phi_0^3 + 3\lambda(\Psi_\pm + \phi_0)\langle\phi^2\rangle + \lambda\langle\phi^3\rangle = 0, \tag{51}$$

$$-[\partial_\mu\partial^\mu + m_\pm^2(t)]\phi + 3\lambda(\Psi_\pm + \phi_0)[\langle\phi^2\rangle - \phi^2] + \lambda[\langle\phi^3\rangle - \phi^3] = 0, \tag{52}$$

where

$$m_\pm^2(t) = 2\mu^2 + 3\lambda\phi_0(\phi_0 + 2\Psi_\pm). \tag{53}$$

In the accepted nondissipative approximation, we must keep in Eq. (52) the linear terms only. That leads to KE (31) with the amplitude defined by time-dependent mass (53)

$$W_{\pm}(\mathbf{k},t) = \frac{\dot{\omega}(\mathbf{k},t)}{2\omega(\mathbf{k},t)} = \frac{3\lambda\dot{\phi}_0(2\phi_0 + \Psi_{\pm})}{2\omega^2(\mathbf{k},t)}. \tag{54}$$

The mean values $\langle\phi^2\rangle$ and $\langle\phi^3\rangle$ are calculated either in minimal order of perturbative theory (for $\lambda \ll 1$) or in RPA.

We obtain the result (34) for $\langle\phi^2\rangle$ and $\langle\phi^3\rangle = 0$. Thus, we have

$$\ddot{\phi}_0 + \lambda\phi_0\left[(2+3\phi_0)\Psi_{\pm} + \phi_0^2\right] + 3\lambda(\Psi_{\pm} + \phi_0)\int \frac{d^3k}{2\,\omega(\mathbf{k},t)}\left[2f(\mathbf{k},t) + v(\mathbf{k},t)\right] = 0. \tag{55}$$

Figure 2 presents the numerical results for parameters set $m = 1, \lambda = 6, \phi_0(0) = 0.58$, near the border of a tachyonic mode. The stationary regime is achieved faster, than in the case of symmetric potential.

Other formalism for the description of the strong field problem for the quantum field system with the potential (47) is developed by J. Baacke et al. ([8], and works cited therein).

3. SUMMARY

The general kinetic approach for the description of arbitrarily strong excited nonequilibrium states in the scalar QFT with the self-interaction admitting the existence of unstable vacuum states was developed in the present work. We restrict ourselves by the collisionless (nondissipative) approximation. However, the attempts to go beyond this aproximation were done [16]. As the concrete example, ϕ^4 and double-well potentials were investigated.

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