

## THE FLUX-TUBE PHASE TRANSITION AND BOUND STATES AT HIGH TEMPERATURES

*G. A. Kozlov*

Joint Institute for Nuclear Research, Dubna

We consider the phase transition in the dual Yang–Mills theory at finite temperature  $T$ . The phase transition is associated with a change (breaking) of symmetry. The effective mass of the dual gauge field is derived as a function of  $T$ -dependent gauge coupling constant. We investigate the analytical criterion constraining the existence of a quark–antiquark bound state at temperatures higher than the temperature of deconfinement.

Рассматривается проблема фазовых переходов в дуальной теории Янга–Миллса при конечной температуре. Фазовый переход ассоциируется с нарушением симметрии. Эффективная масса дуального калибровочного поля определена как функция от температурно-зависимой калибровочной константы взаимодействия. Получен в аналитическом виде критерий на возможность существования кварк-антикваркового связанного состояния при температурах, превышающих температуру деконфайнмента.

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### INTRODUCTION

Most expositions of dual model focus on its possible use as a framework for quark confinement in nature [1]. Rather than the confinement of color charges, here we will describe what one might call the phase transition in the dual Yang–Mills (YM) theory at finite temperature  $T$ . It is believed that the energy required grows linearly with the distance between the color charge and anticharge due to the formation of a color electric flux tube. The idea is that a charge or anticharge is a source or sink, respectively, of color electric flux, which is the analog of ordinary electric flux for the strong interactions. But unlike ordinary electric flux, the color electric flux is expelled from the vacuum and is trapped in a thin flux tube connecting the color charge and anticharge. This is very similar to the way that a superconductor expels magnetic flux and traps it in thin tubes called Abrikosov–Gorkov vortex lines.

There is a general statement that the color confinement is supported by the idea that the vacuum of quantum YM theory is realized by a condensate of monopole–antimonopole pairs [2, 7]. In such a vacuum the interacting field between two colored sources located in  $\mathbf{x}_1$  and  $\mathbf{x}_2$  is squeezed into a tube whose energy  $E_{\text{tube}} \sim |\mathbf{x}_1 - \mathbf{x}_2|$ . This is a complete dual analogy to the magnetic monopole confinement in the Type II superconductor. Since there is no monopoles as classical solutions with finite energy in a pure YM theory, it has been suggested by 't Hooft [3] to go into the Abelian projection where the gauge group  $SU(2)$

is broken by a suitable gauge condition to its (may be maximal) Abelian subgroup  $U(1)$ . It is proposed that the interplay between a quark and antiquark is analogous to the interaction between a monopole and antimonopole in a superconductor.

It is known that the topology of YM  $SU(N)$  manifold and that of its Abelian subgroup  $[U(1)]^{N-1}$  are different, and since any such gauge is singular, one might introduce the string by performing the singular gauge transformation with an Abelian gauge field  $A_\mu$  [4]

$$A_\mu(x) \rightarrow A_\mu(x) + \frac{g}{4\pi} \partial_\mu \Omega(x), \quad (1)$$

where  $\Omega(x)$  is the angle subtended by the closed space-like curve described by the string at any point  $x = (x^0, x^1)$ , and  $g = 2\pi/e$  is responsible for the magnetic flux inside the string,  $e$  being the YM coupling constant. Here, a single string in the two-dimensional world sheet  $y_x(\tau, \sigma)$ , is shown, for example. Obviously, the Abelian field-strength tensor  $F_{\mu\nu}^A = \partial_\mu A_\nu - \partial_\nu A_\mu$  transforms as

$$F_{\mu\nu}^A(x) \rightarrow F_{\mu\nu}^A(x) + \tilde{G}_{\mu\nu}(x),$$

where a new term (the Dirac strength string tensor)

$$\tilde{G}_{\mu\nu}(x) = \frac{g}{4\pi} [\partial_\mu, \partial_\nu] \Omega(x),$$

is valid on the world sheet only [5]

$$\tilde{G}_{\mu\nu}(x) = \frac{g}{2} \epsilon_{\mu\nu\alpha\beta} \iint d\sigma d\tau \frac{\partial(y^\alpha, y^\beta)}{\partial(\sigma, \tau)} \delta^4[x - y(\sigma, \tau)].$$

Actually, a gauge group element, which transforms a generic  $SU(N)$  connection onto the gauge fixing surface in the space of connections, is not regular everywhere in space–time. The projected (or transformed) connections contain topological singularities (or defects). Such a singular transformation (1) may form the worldline(s) of magnetic monopoles. Hence, this singularity leads to the monopole current  $J_\mu^{\text{mon}}$ . This is a natural way of the transformation from the YM theory to a model dealing with Abelian fields. A dual string is nothing other but a formal idealization of a magnetic flux tube in the equilibrium against the pressure of surrounding superfluid (the scalar Higgs-like field) which it displaces [6, 7].

The lattice results, e.g., [8] give the promised picture that the monopole degrees of freedom can indeed form a condensate responsible for the confinement. By lattice simulations in quantum chromodynamics (QCD), it is observed that large monopole clustering covers the entire physical vacuum in the confinement phase, which is identified as a signal of monopole condensation being responsible for confinement. The expression for the static heavy quark potential, using an effective dual Ginzburg–Landau model [9], has been presented in [10]. In paper [11], an analytic approximation to the dual field propagator without sources and in the presence of quark sources, and an expression for the static quark–antiquark potential were established.

The aim of this paper is to consider the phase transition in the four-dimensional model based on the dual description of a long-distance YM theory which shows some kind of confinement. We study the model of Lagrangian where the fundamental variables are an octet of dual potentials coupled minimally to three octets of monopole (Higgs-like) fields [12].

In the scheme presented in this work, the flux distribution in the tubes formed between two heavy color charges is understood via the following statement: the Abelian Higgs-like monopoles are excluded from the string region while the Abelian electric flux is squeezed into the string region.

It is strongly believed in the possibility for a quark–antiquark pair to form a bound state at temperatures higher than the critical one,  $T_c$ , i.e., in the deconfinement state (see, e.g., [13] and the references therein). One of the aims of this work is to find the analytical criterion constraining the existence of bound states at  $T > T_c$ .

In the model there are the dual gauge field  $\hat{C}_\mu^a(x)$  and the scalar field  $\hat{B}_i^a(x)$  ( $i = 1, \dots, N_c(N_c - 1)/2$ ;  $a = 1, \dots, 8$  is a color index) which are relevant modes for infrared behaviour. The local coupling of the  $\hat{B}_i$ -field to the  $\hat{C}_\mu$ -field provides the mass of the dual field and, hence, a dual Meissner effect. Although  $\hat{C}_\mu(x)$  is invariant under the local transformation of  $U(1)^{N_c-1} \subset SU(N_c)$ ,  $\hat{C}_\mu = \mathbf{C}_\mu \cdot \mathbf{H}$  is a  $SU(N_c)$ -gauge-dependent object and does not appear in the real world alone ( $N_c$  is the number of colors and  $\mathbf{H}$  stands for the Cartan superalgebra). The scope of commutation relations, two-point Wightman functions and Green's functions as well-defined distributions in the space  $S(\mathfrak{R}^d)$  of complex Schwarz test functions on  $\mathfrak{R}^d$ , the monopole- and dual gauge-field propagations, the asymptotic transverse behaviour of both the dual gauge field and the color-electric field, the analytic expression for the static potential can be found in [12].

## 1. THE STRING-LIKE FLUX-TUBE PHASE TRANSITION

Phase transitions in dual models are associated with a change in symmetry or more correctly these transitions are related to the breaking of symmetry. As a starting point we assume, for simplicity, that the model is characterized by the scalar order-parameter  $\langle \hat{B}_i(x) \rangle = \hat{B}_0$  for the scalar field  $\hat{B}_i(x)$  identified in the dual model as the Higgs-like field. The classical partition function looks like

$$Z_{\text{cl}} = \int D\hat{B}_i \exp \left\{ - \int_0^\beta d\tau \int d^3\mathbf{x} L(\tau, \mathbf{x}) \right\}. \quad (2)$$

In (2) the sum is taken over fields periodic in Euclidean time  $\tau$  with period  $\beta$  in thermal (heatbath) equilibrium in space–time at temperature  $T = \beta^{-1}$ . The dual description of the YM theory is simply understood by switching on the dual gauge field  $\hat{C}_\mu(x)$  (non-Abelian magnetic gauge potentials) and the three scalar fields  $\hat{B}_i(x)$  (necessary to give the mass to the gauge field  $C_\mu^a$  and carrying color magnetic charge) in the Lagrangian density (LD)  $L$  [14]

$$L = \text{Tr} \left[ -\frac{1}{4} \hat{F}^{\mu\nu} \hat{F}_{\mu\nu} + \frac{1}{2} \left( D_\mu \hat{B}_i \right)^2 \right] - W \left( \hat{B}_i \right), \quad (3)$$

where

$$\begin{aligned} \hat{F}_{\mu\nu} &= \partial_\mu \hat{C}_\nu - \partial_\nu \hat{C}_\mu - ig[\hat{C}_\mu, \hat{C}_\nu], \\ D_\mu \hat{B}_i &= \partial_\mu \hat{B}_i - ig[\hat{C}_\mu, \hat{B}_i]. \end{aligned}$$

The Higgs-like fields develop their vacuum expectation values (v.e.v.)  $\hat{B}_{0_i}$  and the Higgs potential  $W(\hat{B}_i)$  has a minimum at  $\hat{B}_{0_i}$  of the order  $O(100 \text{ MeV})$  defined by the string tension. In the confinement phase the magnetic gauge symmetry is broken due to dual Higgs-like mechanism. All the particles become massive. The v.e.v.  $\hat{B}_{0_i}$  produce a color monopole generating current confining the electric color flux [12]:

$$J_\mu^{\text{mon}}(x) = \frac{2}{3} \partial^\nu G_{\mu\nu}(x),$$

where

$$G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu + \tilde{G}_{\mu\nu}, \quad (4)$$

$\tilde{G}_\mu = \lambda^8 C_\mu$  ( $\lambda^a$  is the generator of  $SU(3)$ );  $\tilde{G}_{\mu\nu}$  is the Dirac string tensor. The interaction of the dual gauge field with other fields (scalar nonobservable fields) is due to monopole current  $J_\mu^{\text{mon}}(x)$  in the Higgs-like condensate  $(\chi + B_0)$  in terms of the dual gauge coupling  $g$  up to divergence of the local phase of the Higgs-like field,  $\partial_\mu f(x)$ :

$$gC_\mu(x) = \frac{J_\mu^{\text{mon}}(x)}{4g(\chi + B_0)^2} + \partial_\mu f(x).$$

As a result, we obtained [12] that the dual gauge field is defined by the divergence of  $\tilde{G}_{\mu\nu}$  shifted by the divergence of the scalar Higgs-like field. For large enough  $\mathbf{x}$ , the monopole field is going to its v.e.v., while  $C_\mu(\mathbf{x} \rightarrow \infty) \rightarrow 0$  and  $J_\mu^{\text{mon}}(\mathbf{x} \rightarrow \infty) \rightarrow 8m^2 C_\mu$  with  $m$  being the mass of  $C_\mu$  field. The Higgs-like fields are associated with not individual particles but the subsidiary objects in the massive gauge theory. These fields cannot be experimentally observed as individual particles.

It is believed that LD (3) can generate classical equations of motion carrying a unit of the  $z_3$  flux confined in a narrow tube along the  $z$ -axis (corresponding to quark sources at  $z = \pm\infty$ ). This is a dual analogy to the Abrikosov [15] magnetic vortex solution.

The question is what happens with the flux tube in excited matter at nonzero temperature  $T$ . At  $T \neq 0$  the oscillations of the flux tube become visible up to the energy of excitation  $e_\beta = s/\beta$  at certain entropy density  $s$ . In Eq. (2) the sum is taken over fields  $\hat{B}_i$  periodic in Euclidean time  $\tau$  with the period  $\beta$ , and the thermal equilibrium in a flat space-time at temperature  $T$  is considered. The scalar Higgs-like field is only part of the dual picture, however it is the only field that is visible in the path integral (2). The Fourier expanding of  $\hat{B}(\tau, \mathbf{x})$  in the form

$$\hat{B}(\tau, \mathbf{x}) = \hat{B}(\mathbf{x}) + \sum_{n=1} \hat{B}_n(\mathbf{x}) \exp(2\pi i n \tau / \beta) \quad (5)$$

can allow one to reflect the periodicity of  $\hat{B}(\tau, \mathbf{x})$  in imaginary time. Note, that the first term in (5) gives the zero-temperature mode while the other ones count the «heavy» high-temperature modes.

We now move to a simple physical pattern: let us define the «large» and «small» systems. It is known that in classical mechanics, the stochastic processes in a dynamic «small» system are under the weak action of a «large» system. «Small» and «large» systems are understood to mean that the number of the states of freedom of the former is less than that of the latter. The

«large» system is supposed to be in the equilibrium state (thermostat with the temperature  $T$ ). We do not exclude the interplay between two systems. The role of the «small» system is played by the restricted region of confined charges, the flux tube. The stationary stochastic processes in the deconfined state are distorted by the random source  $\tilde{G}_{\mu\nu}(x)$  in the dual field tensor  $G_{\mu\nu}(x)$  (4), and under the weak action of a «large» system described by the scalar field  $\phi$  in the Lagrangian density term  $|(\partial_\mu - igC_\mu)\phi|^2$ .

As a result, in the dual Higgs model [12] the finite energy of the piece of the isolating string-like flux tube of the length  $R$  keeps growing as  $R$

$$E(R) \simeq \frac{\mathbf{Q}_\alpha^2}{16\pi} m^2 R (12.4 - 6 \ln \tilde{\mu} R), \tag{6}$$

where  $\mathbf{Q}_\alpha = e\rho_\alpha$  is the Abelian color electric charge, while  $\rho_\alpha$  is the weight vector of the  $SU(3)$  algebra;  $\tilde{\mu}$  is the infrared mass parameter.

It is more and more attractive view of existence of (colorless) hadronic excitations even in the high-temperature phase contrary to the standard pattern of it. Certainly, it is commonly believed that the high- $T$  phase is composed of free or weakly interacting quarks and gluons (deconfinement phase). It was already shown that in the deconfinement phase, the color-Coulomb string tension does not vanish even for temperatures which exceed the critical one (see the review by D. Zwanziger in [13] and the references therein).

Let us introduce the canonical partition function

$$\begin{aligned} Z_c &= \sum_{\text{flux tube config}} \sum_{\beta} \exp[-\beta E(R)] D(|\mathbf{x}|, \beta; M) = \\ &= \sum_R \sum_{\beta} N(R) \exp[-\beta E(R)] D(|\mathbf{x}|, \beta; M) \end{aligned} \tag{7}$$

for ensembles of systems with a single static flux tube, where  $N(R)$  is the number of configurations of the flux tube of length  $R$ . Here,  $D(|\mathbf{x}|, \beta; M)$  defines the screening mass  $M(\beta)$  from the large distance exponential fall-off of correlators of gauge-invariant time-reflection odd operators  $O$  [16] at higher temperatures

$$\langle O(\tau, \mathbf{x}) O(\tau, 0) \rangle \sim \text{const } |\mathbf{x}|^a D(|\mathbf{x}|, \beta; M) \text{ as } |\mathbf{x}| \rightarrow \infty, \tag{8}$$

where

$$D(|\mathbf{x}|, \beta; M) = \exp[-M(\beta) |\mathbf{x}| \theta(T - T_c)] \tag{9}$$

and  $a$  is a constant depending on the choice for the operator  $O(\tau, \mathbf{x})$ . From the physical point of view,  $|\mathbf{x}|$  can be replaced by the characteristic scale  $L$  of the thermostat (the «large» system) and  $\theta$  in (9) is the standard step function. Actually,  $D = 1$  as  $T < T_c$ . The examples of the choice for the operator  $O$  can be found in [16, 17], where the main strategy is a nonperturbative determination of the screening mass at high temperature limit,  $T > T_c$ . In the deconfined state an existence of nonperturbative effects and even hadronic modes (as quasiparticles) having strong couplings is evident. In this case, the sum (7) does not divergent if  $T > T_c$ . The number of configurations  $N(R)$  can be considered in the discrete space of a scalar Higgs-like condensate and one requires the flux tubes to lie along the links of a

3-dimensional cubic lattice of volume  $V$  with the lattice size  $l \sim \mu^{-1} = (\sqrt{2} \lambda \hat{B}_0)^{-1}$ , where  $\mu$  is the mass of the scalar Higgs-like field and  $\lambda$  is its coupling constant. In the rest of physics, for  $l \ll R$  the number of configurations  $N(R)$  is interpreted in terms of the entropy density  $s$  of the flux tube by a fundamental formula

$$\tilde{N}(R) = e^{\tilde{s}}, \quad (10)$$

where  $\tilde{N}(R) = N(R) cl^3/V$ ,  $\tilde{s} = sR/l$ ,  $c$  is the positive constant of the order  $O(1)$  (see also [18]). The relation (10) counts the flux tubes that do not intersect the volume boundary (bound state). They are called «short» flux tubes. If the entropy density  $s$ , which was inferred from classical reasoning, is like every other entropy density that we have met, then a flux tube has a very large number of configurations, roughly  $N(R) \sim \exp[s(R/l)]$ . Can one by some sort of calculation count the number of configurations of flux tube and reproduce formula (10) for the entropy density? For this, we need a quantum theory of confinement, so at present at least dual YM theory is the only candidate. Even in this theory, the question was out of reach for the last three decades.

Inserting (10) into Eq. (7) one gets

$$Z_c = \frac{V}{l^3} \sum_R \exp[-\beta \sigma_{\text{eff}}(\beta) R], \quad (11)$$

where

$$\sigma_{\text{eff}}(\beta) = \tilde{\sigma}_{\text{eff}}(\beta) + \sigma_D(\beta). \quad (12)$$

In Eq. (12)

$$\tilde{\sigma}_{\text{eff}}(\beta) = \tilde{\sigma}_0 - \frac{s}{l\beta} \quad (13)$$

is the order parameter of the phase transition and

$$\tilde{\sigma}_0 = \sigma_0 \left(1 - \frac{1}{4} \ln \frac{\tilde{\mu}^2}{m_R^2}\right), \quad \sigma_0 = \frac{3}{4} \alpha(Q) m^2 = \frac{3}{4} \frac{\pi}{g^2} m^2 \quad (14)$$

with  $\alpha(Q)$  being the running coupling constant. Here,  $R$  in the logarithmic function in (6) has been replaced by the characteristic length  $R_c \sim 1/m_R$  which determines the transverse dimension of the dual field concentration, while  $\tilde{\mu}$  is associated with the inverse «coherent length» and the dual field mass  $m$  defines the «penetration depth» in the Type II superconductor where  $m < \mu$ . The second term in Eq. (12)

$$\sigma_D(\beta) = b_{\text{th}} M(\beta) T, \quad \text{as } T > T_c, \quad (15)$$

is the important component of deconfined matter above the phase transition that gives the contribution to the physical properties of the strongly interacting matter. In formula (15),  $b_{\text{th}} = L/R$  is the thermostat criterion factor;  $M(\beta)$  in  $SU(N)$  for  $N_f$  quark flavors is defined perturbatively [16, 17] with the leading order screening mass  $M^{\text{LO}}(\beta)$  as

$$M^{\text{LO}}(\beta) + N\alpha T \ln \frac{M^{\text{LO}}(\beta)}{4\pi\alpha T}, \quad M^{\text{LO}}(\beta) = \sqrt{4\pi\alpha \left(\frac{N}{3} + \frac{N_f}{6}\right)} T,$$

and within the nonperturbative regime as  $4\pi\alpha c_N T$  + higher order corrections,  $c_{N=3} = 2.46 \pm 0.15$  [16]. Therefore, the result  $\sigma_D(\beta) \sim \alpha T^2$  can be shown explicitly at  $T > T_c$  with the nonperturbative regime. Formula (15) gives the evidence of magnetic component of the deconfinement phase state which is related to thermal Abelian monopoles evaporating from the magnetic condensate which is present at low  $T$ .

The spatial Wilson loop  $L_{\text{Wils}}$  has area law behaviour below and above  $T_c$ . For large enough  $L_{\text{Wils}}$  the spatial string tensor is determined by the effective action of the dual (magnetic) theory at  $T > T_c$

$$S_{\text{eff}}(L_{\text{Wils}}, T) \rightarrow L_{\text{Wils}}^2 \sigma_D(\beta), \text{ as } L_{\text{Wils}} \rightarrow \infty.$$

Hence, the thermostat characteristic scale  $L$  is given by

$$L = \lim_{L_{\text{Wils}} \rightarrow \infty} \left( \frac{S_{\text{eff}}(L_{\text{Wils}}, T)}{L_{\text{Wils}}^2} \frac{1}{M(\beta) T} \right) R, \quad M(\beta) \sim O(\alpha T).$$

At zero temperature we got  $\sigma_0 \simeq 0.18 \text{ GeV}^2$  [12] for the mass of the dual  $C_\mu$ -field  $m = 0.85 \text{ GeV}$  and  $\alpha = e^2/(4\pi) = 0.37$  obtained from fitting the heavy quark-antiquark pair spectrum [19]. The above-mentioned value  $\sigma_0$  is close to a phenomenological one (e.g., coming from the Regge slope of the hadrons). Making the formal comparison of the result obtained in the analytic form, we recall the expression of the energy per unit length of the vortex in the Type II superconductor [10, 20]

$$\epsilon_1 = \frac{\phi_0^2 m_A^2}{32 \pi^2} \ln \left( \frac{m_\phi}{m_A} \right)^2, \tag{16}$$

where  $\phi_0$  is the magnetic flux of the vortex;  $m_A$  and  $m_\phi$  are penetration depth mass and the inverse coherent length, respectively. On the other hand, the string tension in Nambu's paper (see [7]) is given by

$$\epsilon_2 = \frac{g_m^2 m_v^2}{8 \pi} \ln \left( 1 + \frac{m_s^2}{m_v^2} \right), \tag{17}$$

with  $m_s$  and  $m_v$  being the masses of scalar and vector fields and  $g_m$  is a magnetic-type charge. It is clear that for a sufficiently long string  $R \gg m^{-1}$  the  $\sim R$ -behaviour of the static potential is dominant; for a short string  $R \ll m^{-1}$  the singular interaction provided by the second term in (6) becomes important if the average size of the monopole is even smaller.

The model presented here is characterized by a limiting temperature  $T_c$ , and it is evident that

$$T_c = \frac{3}{4} \frac{1}{s} \alpha(Q) \frac{m^2}{\mu} \left( 1 - \frac{1}{4} \ln \frac{\tilde{\mu}^2}{m_R^2} \right) \tag{18}$$

for which  $\tilde{\sigma}_{\text{eff}}(T_c) = 0$ . The vacuum expectation value  $B_0$  is the threshold energy to excite the monopole (Higgs-like field) in the vacuum. It corresponds to the Bogolyubov particle in the ordinary superconductor. In case if such excitations exist, the phase transition is expected to occur at  $T_c \sim 200 \text{ MeV}$ . The value  $B_0 \simeq 276 \text{ MeV}$  is regarded as the ultraviolet cutoff of the theory. At sufficiently high temperature QCD definitely loses confinement and the flux

tube definitely disappears. It is evident that at  $T \rightarrow T_c$  the flux tube becomes arbitrary long. As a result, the temperature-dependent mass  $m(\beta)$  of the dual gauge field  $\hat{C}_\mu$  is derived as follows:

$$m^2(\beta) = \frac{4}{3} \frac{\sigma_{\text{eff}}(\beta)}{\alpha(Q, \beta)}. \quad (19)$$

Obviously,  $m(\beta) \rightarrow m$  as  $\beta \rightarrow \infty$ , and  $m(\beta) \rightarrow 0$  as  $1/\beta \rightarrow T_c$ . The latter limit means that

$$\partial^\nu \tilde{G}_{\mu\nu} \sim (m^2 C_\mu + 4m \partial_\mu \bar{b}) \rightarrow 0 \quad (20)$$

as  $T \rightarrow T_c$  (here,  $\bar{b}$  is the Higgs-like field). On the other hand, the divergence of  $\tilde{G}_{\mu\nu}$  is just the current carried by a charge  $g$  moving along the path  $\Gamma$ :

$$\partial^\nu \tilde{G}_{\mu\nu}(x) = -g \int_{\Gamma} dz_\mu \delta^4(x - z). \quad (21)$$

Hence,  $\partial^\nu \tilde{G}_{\mu\nu}(x) \rightarrow 0$  as  $g \rightarrow 0$ . Actually, formula (19) relates the confinement to the spontaneous breaking of a magnetic symmetry induced by monopole condensation. The magnetic condensate disappears at the deconfining phase transition. And the final remark concerning the zeroth value of  $m(\beta)$ : recall that  $m^2(\beta) \sim g^2(\beta) \delta^2(0)$ , where  $\delta^2(0)$  is the inverse cross section of the flux tube. This cross section is infinitely large if  $m \rightarrow 0$ . Actually,  $\sigma_{\text{eff}}(\beta)$  is the effective measure of the phase transition when the flux tube is produced. The fact that  $\tilde{\sigma}_{\text{eff}}(\beta_c = T_c^{-1}) = 0$  means the special phase where two color charges are separated from each other by infinite distance. At  $T = T_c$  the entropy and the total energy are related to each other by  $s = E/T_c$ . The level density of a system is  $e^s$ , therefore  $s = E/T_c$  implies an exponentially rising mass spectrum if one identifies  $E$  with the mass of a quark–antiquark bound state.

It is assumed that the flux tube starts in a thermal exciting phase, a phase in which the flux tube is quasi-static and in thermal equilibrium at temperatures close to  $T_c$  or even higher than  $T_c$ . We assume that the string coupling is sufficiently small and the local space–time geometry is close to the flat over the length scale of the finite size box-block of volume  $v = r^3$ . In each block  $j$  the flux tube is homogeneous and isotropic with the energy  $E_j$ . It was shown [21] that applying the string thermodynamics to a volume  $v = r^3$  in the string gas cosmology one can get the mean square mass fluctuation in a region of radius  $r$  evaluated at the temperature close to  $T_c$ :

$$\langle (\delta\mu)^2 \rangle = \frac{r^2}{R^3} \frac{1}{\beta - \beta_c}, \quad \beta > \beta_c. \quad (22)$$

At high temperatures  $T_0 > T_c$ , the mass  $m$  disappears and the main object is the screening mass  $M(\beta)$ . Actually, in deconfined state the scale  $R$  of the real hadron at low temperature is replaced by the thermostat (heatbath) scale  $L$ . The spectrum of physical «quark–antiquark» bound states at  $T_0$  in  $SU(3)$  can be expanded as

$$E(T_0) \sim \alpha L T_0^2 \left[ \sqrt{\frac{4\pi}{\alpha} \left( 1 + \frac{N_f}{6} \right)} + 3 \ln \sqrt{\frac{1}{4\pi\alpha} \left( 1 + \frac{N_f}{6} \right)} + 4\pi c_3 + \dots \right], \quad (23)$$

where one sees the saving of the  $\alpha = \alpha(Q, T_0)$ -dependence in both perturbative (two first terms in (23)) and nonperturbative regimes.

## 2. COUPLING CONSTANT

In gauge theories at  $T \neq 0$  thermal fluctuations of the gluon act to screen the electric field component of the gluon, through the development of temperature-dependent electric mass  $m_{\text{el}} \sim gT$ . Recent studies show remarkable facts that instantons are related to monopoles in the Abelian gauge although these topological objects belong to different homotopy group. It is known that both analytical and lattice studies can show a strong correlation between instantons and monopoles in the Abelian projected theory of QCD. It can be postulated that at finite  $T$  the running coupling would be replaced by the static screened charge

$$\frac{1}{\alpha(Q, T)} = \frac{1}{\alpha(Q)} \left\{ \left[ 1 - \frac{\Pi^{00}(q^0 = 0, \mathbf{q} \rightarrow 0; \beta)}{\mathbf{Q}^2} \right] + \frac{\alpha(Q)}{6\pi} \left( \frac{11N}{2} - N_f \right) \ln \frac{\mathbf{Q}^2}{M^2} \right\} \quad (24)$$

for gauge group  $SU(N)$ , where  $M$  is the renormalization energy scale and the inverse screening length is given by the gluon self-energy  $\Pi^{\mu\nu}(q)$  at the lowest order of  $g^2$  in hot theory containing the quark fields with the mass  $m_q$  (see, e.g., [22])

$$-\Pi^{00}(q^0 = 0, \mathbf{q} \rightarrow 0; \beta) = g^2 T^2 \left[ \frac{N}{3} + \frac{N_F}{\pi^2 T^2} I_F(\beta, \bar{\mu}, m_q) \right] = m_{\text{el}}^2(\beta), \quad (25)$$

where

$$I_F = \int_0^\infty \frac{dx x^2}{\sqrt{x^2 + m_q^2}} [n_F(x^2) + \bar{n}_F(x^2)], \quad (26)$$

$$n_F(x^2) = \frac{1}{\exp \left[ \left( \sqrt{x^2 + m_q^2} - \bar{\mu} \right) \beta \right] + 1}, \quad \bar{n}_F(x^2) = \frac{1}{\exp \left[ \left( \sqrt{x^2 + m_q^2} + \bar{\mu} \right) \beta \right] + 1}, \quad (27)$$

$N_F$  is the number of quarks, the chemical potential  $\bar{\mu}$  is defined by the baryon density  $\rho$  in the formula

$$\rho \sim \int \frac{d^3x}{(2\pi)^3} [n_F(x^2) - \bar{n}_F(x^2)]. \quad (28)$$

The first term in (25) refers to pure  $SU(N)$  gauge theory. Hence,  $\alpha^{-1}(Q, T)$  has the following expansion over  $\mathbf{Q}^2/M^2$  and  $T^2/\mathbf{Q}^2$ :

$$\frac{1}{\alpha(Q, T)} = \frac{1}{\alpha(Q)} + \frac{1}{6\pi} \left( \frac{11N}{2} - N_f \right) \ln \frac{\mathbf{Q}^2}{M^2} + 4\pi \frac{T^2}{\mathbf{Q}^2} \left[ \frac{N}{3} + \frac{N_F}{\pi^2 T^2} I_F(\beta, \bar{\mu}, m_q) \right], \quad (29)$$

where  $\alpha(Q, T) \rightarrow 0$  as  $T \rightarrow \infty$ .

Because quark confinement is considered here as the dual version of the confinement of magnetic point charges in the Type II superconductor (magnetic Abrikosov vortices), the upper limit for  $T_c$  is given by the requirement  $(m/\mu) < 1$ , i.e.,

$$T_c < \frac{3}{4} \alpha(Q) m \left( 1 - \frac{1}{4} \ln \frac{\tilde{\mu}^2}{m_R^2} \right). \quad (30)$$

Numerical estimation leads to  $T_c < 222$  MeV at  $B_0 \simeq 276$  MeV,  $\alpha = 0.37$  and  $m = 0.85$  GeV [12] for  $m_R \sim \tilde{\mu}$  and  $s \sim O(1)$ .

### 3. COULOMB POTENTIAL IN DECONFINEMENT

It is known that the confinement of quarks is explained within the instantaneous part of the potential  $V$  defined by the «time–time» component of gluon propagator  $D_{00}(x = (\mathbf{x}, t))$  (see, e.g., the paper by D. Zwanziger in [13])

$$4\pi\alpha D_{00}(x) = V(|\mathbf{x}|) \delta(t) + \text{non-instantaneous vacuum polarization term.} \quad (31)$$

At the Gribov horizon [23]  $V(R = |\mathbf{x}|)$  is caused by the long-range forces having confining properties  $V(R \rightarrow \infty) = \infty$ . One of the aims of this paper is also to understand the origin of the presence of some-range forces that confines «quarks» in deconfined phase. To proceed for this one should restore the following expression for Coulomb-like potential  $V_c(R, T)$  at finite temperature in the form:

$$V_c(R, T) = -\frac{2}{3\pi^2} \int d^3\mathbf{q} \frac{\alpha(q^2, T)}{q^2} e^{-\mathbf{q}\mathbf{R}}, \quad (32)$$

where  $\mathbf{q}$  is the difference between momenta of a particle and antiparticle confined by forces we are exploring here;  $\alpha(q^2, T)$  is given by (29). At zero temperature, or even for low  $T$ , the integral in (32) diverges at the upper limit  $|\mathbf{q}| \rightarrow \infty$ . At large  $T$ , this integral can be naturally regularized by introducing the temperature-dependent soft regularization function  $\Upsilon(q^2, T) = 1/[1 + q^2/M^2(\beta)]$  (see also [24, 25]) which has the properties:  $\Upsilon(q^2, T) \rightarrow 1$  as  $T \rightarrow \infty$  and  $\Upsilon(q^2, T) \rightarrow \Upsilon_0(q^2)$  as  $T \rightarrow 0$ . At low temperatures,  $\alpha(q^2, T)$  is rather slowly varying with  $q^2$  compared to  $\sin(qR)/(qR)$  function in one-dimensional representation of the integral in (32) (the integrating over the angles is already done). On the other hand, at high  $T > |\mathbf{q}|$  the main contribution will be done by  $T$ -dependent term in  $\alpha(q^2, T)$  expansion (29). Thus, from the mathematical point of view, the problem with divergence of the integral at the upper limit would be solved if  $\alpha(q^2, T)$  is replaced by  $\alpha(T)$ . We get ( $\bar{\alpha} = (4/3)\alpha$ ):

$$V_c(R, T) = -\frac{8\alpha(T)}{3\pi} \int_0^\infty dq \Upsilon(q^2, T) \frac{\sin qR}{qR} = \frac{\bar{\alpha}(T)}{R} \left( e^{-M(\beta)R} - 1 \right). \quad (33)$$

At short distances one gets that the Coulomb potential is consistent with a linear increase with  $R$ :

$$V_c(R, T) = \sigma_c(T) R - \alpha(T) M(\beta), \quad (34)$$

where the Coulomb string tension  $\sigma_c(T) = 0.5 \bar{\alpha}(T) M^2(\beta)$  for strongly interacting particles in deconfinement is

$$\sigma_c(T) = \frac{2}{3} [4\pi\alpha(T) T]^2 \left[ a_{N, N_f} + \sqrt{\alpha(T)} b_N \ln \left( \frac{a_{N, N_f}}{\sqrt{\alpha(T)}} \right) + \sqrt{\alpha(T)} c_N + \dots \right]^2, \quad (35)$$

where

$$a_{N, N_f} = \sqrt{\frac{1}{12\pi} \left( N + \frac{N_f}{2} \right)}, \quad b_N = \frac{N}{4\pi}.$$

We found that  $V_c(R, T)$  has the linear rising,  $\sigma_c(T) > 0$  at  $T > T_c$ , where the physical (given by the Wilson loop) string tension  $\tilde{\sigma}_{\text{eff}}(T > T_c) = 0$ . The fact that the Coulomb string tension in deconfinement increases with  $\alpha^2 T^2$  is consistent with magnetic mass having the behaviour as  $\sim \alpha T$  within the nonperturbative regime.

#### 4. FLUX-TUBE SOLUTIONS

The temperature-dependent flux-tube solution for the dual gauge field along the  $z$ -axis (within the cylindrical symmetry) has the following asymptotic transverse behaviour (for details see [12] at  $T = 0$ ):

$$\tilde{C}(r, \beta) \simeq \frac{4n}{7g(\beta)} - \sqrt{\frac{\pi m(\beta)r}{2\kappa}} e^{-\kappa m(\beta)r} \left[ 1 + \frac{3}{8\kappa m(\beta)r} \right], \quad (36)$$

where  $r$  is the radial coordinate (the distance from the center of the flux tube);  $n$  is the integer number associated with the topological charge [26],  $\kappa = \sqrt{21}$ .

The color-electric field  $E$  inside the quark-antiquark bound state is given by the rotation of the dual gauge field

$$\mathbf{E} = \nabla \times \mathbf{C} = \frac{1}{r} \frac{d\tilde{C}(r)}{dr} \mathbf{e}_z \simeq E_z(r) \mathbf{e}_z, \quad (37)$$

where  $\mathbf{e}_z$  is a unit vector along the  $z$ -axis, and the  $T$ -dependent  $E_z(r, \beta)$  looks like [12]

$$E_z(r, \beta) = \sqrt{\frac{\pi m(\beta)}{2\kappa r}} e^{-\kappa m(\beta)r} \left[ \kappa m(\beta) - \frac{1}{2r} \right]. \quad (38)$$

The lower bound on  $r = r_0$  can be estimated from the relation  $r_0 > [2\kappa m(\beta)]^{-1}$  which leads to  $r_0 > 0.03$  fm at  $T = 0$ . Obviously,  $r_0 \rightarrow \infty$  as  $m(\beta) \rightarrow 0$  at  $T \rightarrow T_0$  (deconfinement).

In Fig. 1, we show the dependence of  $m$  as a function of the temperature  $T$  at different scale parameters  $M$ . No dependence found on quark current masses (we used  $m_q = 7, 10$  and 135 MeV). No essential dependence found for different  $N_f$  and  $N_F$ .

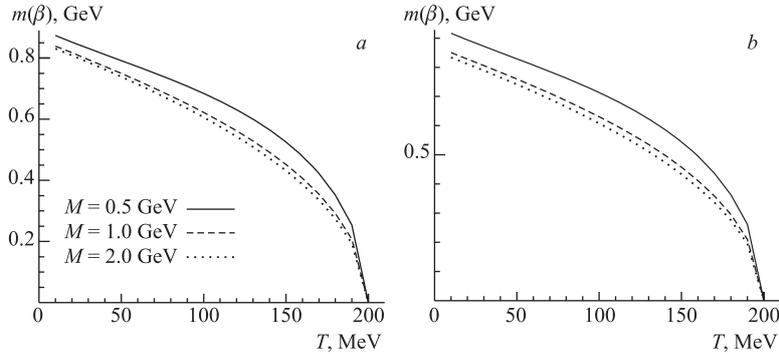


Fig. 1. The dual gauge boson mass  $m(\beta)$  shown as a function of  $T$  at different renormalization energy scales  $M$  and fixed value  $m_q = 7$  MeV with  $N_f = N_F = 3$  (a) and  $N_f = N_F = 6$  (b)

In Figs. 2 and 3, we show numerical solutions of the flux tube, namely, the profiles of the transverse behaviour of  $\tilde{C}(r, \beta)$  and the color-electric field  $E_z(r, \beta)$ , respectively, as functions of radial variable  $r$  at different temperatures. We found rather sharp increasing of  $\tilde{C}(r, \beta)$  at small values of  $r$ . No essential dependence of  $r$  emerges in the region  $r > 0.1$  fm. The field  $E(r, \beta)$  disappears when the temperature is close to  $T_c$ .

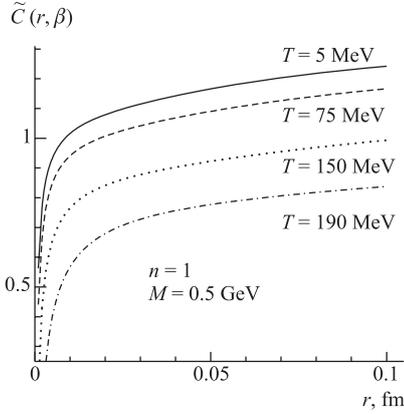


Fig. 2. The profiles of the dual gauge boson field  $\tilde{C}(r, \beta)$  shown as a function of the radial coordinate  $r$  at different  $T$

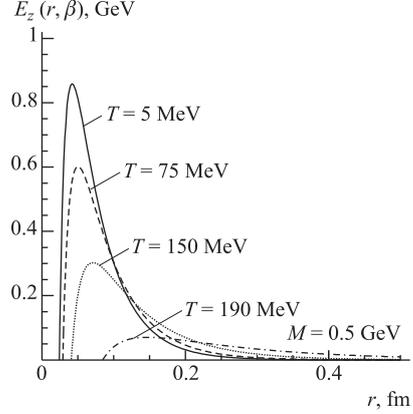


Fig. 3. The profiles of the color-electric field  $E_z(r, \beta)$  as a function of the radial coordinate  $r$  at different  $T$

## CONCLUSIONS

We based on the dual gauge model of the long-distance Yang–Mills theory in terms of two-point Wightman functions. Among the physicists dealing with the models of interplay of a scalar (Higgs-like) field with a dual vector (gauge) boson field, where the vacuum state of the quantum YM theory is realized by a condensate of the monopole–antimonopole pairs, there is a strong belief that the flux-tube solution explains the scenarios of color confinement. Based on the flux-tube scheme approach of Abelian dominance and monopole condensation, we obtained the analytic expressions for both the monopole and dual gauge boson field propagators [12]. These propagators lead to a consistent perturbative expansion of Green’s functions.

The monopole condensation causes the strong and long-range interplay between heavy quark and antiquark, which gives the confining force, through the dual Higgs mechanism. The analytic expression for the static potential at large distances grows linearly with the distance  $R$  apart from logarithmic correction.

We observed that the flux tube can be produced abundantly when the phase transition emerges at the temperature  $T = T_c$ , obeying the condition  $\tilde{\sigma}_{\text{eff}}(T = T_c) = 0$ . We found that the phase transition temperature essentially depends on  $\alpha(Q)$  and the mass of the dual gauge field  $m$ . The analytic criterion constraining the existence of a quark–antiquark bound state at  $T > T_c$  is obtained (see (12) and (15)). We find that the Coulomb string tension for strongly interacting particles in deconfinement increases with  $\alpha^2 T^2$ , and at short distances the Coulomb potential has the linear rising.

It is observed [16] that in wide range of higher temperatures,  $T_c < T < 100 T_c$ , the nonperturbative screening mass  $M(\beta)$  is rather constant for both the  $SU(2)$  and  $SU(3)$  cases, and this mass is defined by the leading order perturbative result  $M(\beta) \simeq 3M^{\text{LO}}(\beta)$ . This means an essential role of  $\sigma_D(\beta)$  and the existence of heavy quark–antiquark bound states at temperatures above the critical ones in the framework of the dual gauge theory. It is the

only question, whether this result can modify the standard picture of finite-temperature gauge theory relevant to understanding of the quark–hadron phase transition and existence of strong QCD effect in deconfinement state.

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#### REFERENCES

1. Ripka G. Dual Superconductor Models of Color Confinement. Springer, 2004, and ref. therein.
2. Mandelstam S. // Phys. Rep. C. 1976. V. 23. P. 245; Phys. Rev. D. 1979. V. 19. P. 2391;  
Polyakov A.M. // Nucl. Phys. B. 1977. V. 120. P. 429;  
't Hooft G. // Nucl. Phys. B. 1978. V. 138. P. 1.
3. 't Hooft G. // Nucl. Phys. B. 1981. V. 190. P. 455.
4. Nepomechie R. I., Rubin M. A., Hozotani Y. // Phys. Lett. B. 1981. V. 105. P. 457.
5. Dirac P. A. M. // Phys. Rev. 1948. V. 74. P. 817.
6. Nielsen H. B., Olesen P. // Nucl. Phys. B. 1973. V. 61. P. 45.
7. Nambu Y. // Phys. Rev. D. 1974. V. 10. P. 4262.
8. Di Giacomo A. et al. // Phys. Rev. D. 2000. V. 61. P. 034503.
9. Suzuki T. // Prog. Theor. Phys. 1988. V. 80. P. 929; 1989. V. 81. P. 752;  
Maedan S., Suzuki T. // Ibid. P. 229.
10. Suganuma H., Sasaki S., Toki H. // Nucl. Phys. B. 1995. V. 435. P. 207.
11. Baker M. et al. // Phys. Rev. D. 1998. V. 58. P. 034010.
12. Kozlov G. A., Baldicchi M. // New J. Phys. 2002. V. 4. P. 16.1.
13. Simonov Yu. A. // Phys. Lett. B. 2005. V. 619. P. 293;  
Zwanziger D. Equation of State of Gluon Plasma from Local Action. hep-ph/0610021.
14. Baker M. et al. // Phys. Rev. D. 1996. V. 54. P. 2829.
15. Abrikosov A. A. // Sov. Phys. JETP. 1957. V. 32. P. 1442.
16. Kajantie K. et al. // Phys. Rev. Lett. 1997. V. 79. P. 3130.
17. Arnold P., Yaffe L. G. // Phys. Rev. D. 1995. V. 52. P. 7208.
18. Copeland E., Holbraad S., Rivers R. Fluctuations and the Cosmic String Phase Transitions // Proc. of the 2nd Workshop on Thermal Field Theories and Their Applications, Tsukuba, Japan, July 23–27, 1990 / Ed. by H. Ezawa, T. Arimitsu, Y. Hashimoto. Elsevier Sci. Publ. B.V., 1991. P. 295.
19. Baker M., Ball J. S., Zachariasen F. // Phys. Rev. D. 1995. V. 51. P. 1968.
20. Lifshitz E. M., Pitaevski L. P. Statistical Physics. Course of Theoretical Physics. V. 19. Oxford: Pergamon, 1981.

21. *Nayeri A., Brandenberger R. H., Vafa C.* // Phys. Rev. Lett. 2006. V. 97. P. 021302.
22. *Gao S., Liu B., Chao W. Q.* // Phys. Lett. B. 1996. V. 378. P. 23.
23. *Gribov V. N.* // Nucl. Phys. B. 1978. V. 139. P. 1.
24. *Kozlov G. A. et al.* // J. Phys. G: Nucl. Part. Phys. 2004. V. 30. P. 1201.
25. *Pauli H.-C.* The Hadronic Potential at Short Distances. hep-ph/0312198.
26. *Koma Y., Suganuma H., Toki H.* // Phys. Rev. D. 1999. V. 60. P. 074024.

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