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SPATIAL ENTANGLEMENT AND SUBWAVELENGTH STRUCTURES IN ATOMIC DEFLECTION

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The deflection of V-level atomic beam in two-mode quantized cavity field in a twin-photon state is investigated. Depending on the atom–light interaction scheme, two-dimensional position patterns in a subwavelength range, as well as the effects of spatial entanglement, are reported.

Исследовано рассеяние пучка V-типа атомов на двухмодовом квантованном поле в двухфотонном состоянии. В зависимости от характеристик взаимодействия атом–поле исследованы двухразмерные пространственные атомные структуры в областях меньших, чем длины волн, и эффекты пространственной перепутанности.

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INTRODUCTION

One of the basic processes of atomic optics is the deflection of atomic beams when interacting with a standing light wave inside an optical cavity. The deflection pattern of atoms from a single mode of a quantized cavity field has been investigated in detail. In particular, it has been shown that deflection of atomic beams by a one-mode cavity field is a sensitive function of the photon statistics (see, for example, [1, 2]).

In the present paper, novel effects resulting from deflection of a V-level atomic beam in a two-mode quantized cavity field are considered. The two-mode field is supposed in a twin-photon state or photon-number correlated state (PNCS) which can be generated by parametric down-conversion processes. The role of photon-number correlations is important in the proposed scheme. Indeed, it was found that the atomic deflection patterns, in the transverse plane to the direction of the center of atomic mass motion, are essentially different for the cases of independent or correlated modes, respectively. In particular, an atomic deflection pattern for the case of twin photons displays spatial correlations between atomic position variables in the transverse directions, which are understood as the spatial entanglement in the deflected pattern.

It is well known that the measurement of the state of a standing wave may lead to the localization of the position of the deflected atom [1–3]. As will be shown here, for narrow

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initial position distributions of atoms, the localization procedure for this scheme leads to producing controllable pattern structures with feature spacing smaller than a wavelength of the light in the cavity. We illustrate these effects, considering various regimes of interactions of a V-type three-level atomic beam with two crossed standing electromagnetic waves in PNCS.

1. SUBWAVELENGTH STRUCTURES IN THE ATOMIC DEFLECTIONS

Let us consider the quantum dynamics of a three-level atom with a V-type configuration of energy levels moving along the z direction and passing through a cavity electromagnetic field (see Fig.1). Our aim here is to investigate the position patterns of deflected atoms in the $x-y$ plane. We concentrate on the interaction of a three-level atom, initially prepared in a ground atomic state $|1\rangle$, with two quantized cavity modes at equal frequencies and opposite circular polarizations.

We investigate the transverse atomic motion in the $x-y$ plane, assuming that the initial transverse distribution of atoms is Gaussian in the form

$$P\,(x,y) = P(x)P(y) = \frac{1}{2\pi\Delta x\Delta y}\exp\left[-\frac{(x-\langle x\rangle)^2}{2(\Delta x)^2}\right]\exp\left[-\frac{(y-\langle y\rangle)^2}{2(\Delta y)^2}\right], \qquad (1)$$

with widths $\Delta z = \sqrt{\langle (z-\langle z\rangle)^2\rangle}$ centered at the nodes of both waves $(z = x, y)$.

Two standing quantized modes are chosen in a photon-number correlated state,

$$|\text{field}\rangle = \sum_n \mathrm{A}_n |n\rangle_2 |n\rangle_3, \qquad (2)$$

where $|n\rangle_2$, $|n\rangle_3$ are the photon states corresponding to the transitions $|1\rangle \to |2\rangle$ and $|1\rangle \to |3\rangle$, respectively, and A_n is the probability amplitude of twin photons, i.e., the amplitude of finding n-photon pairs.

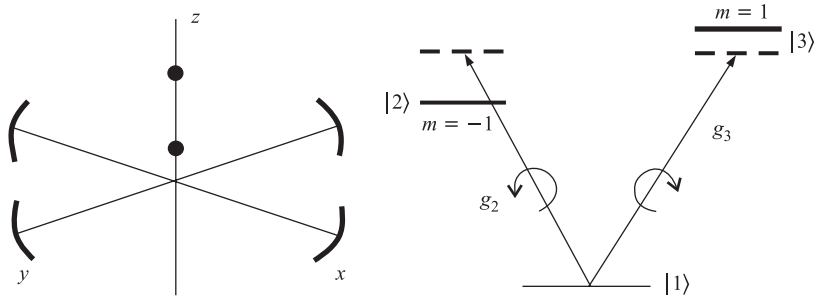

Fig. 1. Schematic diagrams showing the investigated model. *a*) The atomic flux crossing the interaction region. *b*) Energetic levels of a V-type atom interacting with two cavity modes with coupling constant g_2 and g_3

It is also assumed that the interaction time between the atom and the cavity fields is short compared to the cavity lifetime and the inverse decay rates of the excited $|2\rangle$ and $|3\rangle$ atomic states, respectively. The Hamiltonian describing such a system reads as follows:

$$H = \frac{P_z^2}{2m} + \hbar \sum_{i\in\{2,3\}} \Delta_i |i\rangle\langle i| + \hbar\,\Omega_2\, a_2^\dagger\, \sigma_{12} + \hbar\,\Omega_3\, a_3^\dagger\, \sigma_{13} + \text{h.c.}, \tag{3}$$

where P_z^2 denotes the center-of-mass motion momentum operator of the atom with mass m, while $\Delta_i = \omega_{i1} - \omega$ $(i = 2, 3)$ are the corresponding detunings of the excited atomic level frequencies $\{\omega_{31}, \omega_{21}\}$ from the cavity one ω; a_i and $a_i^\dagger$ are the annihilation and creation operators of the ith mode, while $\sigma_{12} = |1\rangle\langle 2|$ and $\sigma_{13} = |1\rangle\langle 3|$ are the corresponding transition operators of a three-level atom. The couplings of the atom to the two modes are determined by the spatial mode functions $\Omega_3 \equiv g_3 \sin(k_3 x)$ and $\Omega_2 \equiv g_2 \sin(k_2 y)$, where $\{k_3 = k_x, k_2 = k_y\}$ with $k_x = k_y = \omega/c$, while $g_2 = E_0 \mathbf{e}_+ \langle 1|\mathbf{d}|2\rangle$, and $g_3 = E_0 \mathbf{e}_- \langle 1|\mathbf{d}|3\rangle$. $E_0 = \sqrt{\omega/2v}$ is the so-called electric field per photon; $\mathbf{e}_+$ and $\mathbf{e}_-$ are the polarization vectors, while $\langle 1|\mathbf{d}|2\rangle$ and $\langle 1|\mathbf{d}|3\rangle$ are the dipole moments of the corresponding atomic transitions. The foregoing article is based on the Raman–Nath approximation; i.e., we neglect the transverse kinetic energy in comparison with the atom–field interaction energy.

In what follows, the conditional probability distribution function $W(x, y, \varphi)$ in deflection of V-type atoms when they pass through the interaction region will be analyzed, if cavity modes are in given phase states for each of the modes:

$$|\varphi\rangle = |\varphi_2\rangle|\varphi_3\rangle = \frac{1}{2\pi} \sum_{n_i} \sum_{n_j} \mathrm{e}^{i(n_i\varphi_2 + n_j\varphi_3)}\, |n_i\rangle_2 |n_j\rangle_3. \tag{4}$$

For the case of statistically independent modes the probability amplitude of having two-photons factorizes, such that the position distribution is also factorized $W(x, y) = W(x)W(y)$; i.e., it is represented as a product of two independent probability distributions for x and y, respectively. For the case of a PNCS of two modes, i.e., Eq. (2), the conditional probability distribution reads as

$$W(x, y, \varphi) = \frac{P(x)P(y)}{(2\pi)^2} \left| \sum_n A_n \mathrm{e}^{\mathrm{in}\left(F(x,y) - \varphi\right)} \right|^2, \tag{5}$$

where $\varphi = \varphi_2 + \varphi_3$, while A_n is the probability amplitude of finding n correlated photon pairs in the modes, and

$$F(x, y) = \chi_2 \sin^2(k_2 y) + \chi_3 \sin^2(k_3 x), \tag{6}$$

where $\chi_2 = g_2^2 t/\Delta_2$ and $\chi_3 = g_3^2 t/\Delta_3$. Thus, the probability distribution (5) is not factorizable function, i.e., $W(x, y) \neq W(x)W(y)$, and this could be understood as spatial entanglement of the deflected particles.

The photon-pair distribution $P_n = |A_n|^2$ has been theoretically and experimentally investigated mainly for different regimes of parametric down-conversion processes. Here we use a quasi-phenomenological approach to describe these states relying on both theoretical

and experimental considerations; that is, for concrete calculations we use the PNCS with a Poissonian distribution [4–6]

$$A_n = \mathrm{e}^{-N/2}\frac{\alpha^n}{\sqrt{n!}} \text{ and hence } P_n = \frac{N^n\ \mathrm{e}^{-N}}{n!}, \tag{7}$$

where $\alpha = \sqrt{N}\mathrm{e}^{i\theta}$, $N = |\alpha|^2$ is the averaged value of the total number of photon pairs generated by a single pump pulse. We note that an average of 0.9 photon pairs per pulse was experimentally obtained in a periodically poled lithium niobate waveguide [7].

The typical results for the atomic position patterns due to interaction of atomic beam with twin-photon number modes are shown in Fig. 2 for the cases of narrow initial position distribution of atoms, when the width of the distribution (1) is smaller than the wavelength of the cavity modes, and for $\varphi = \theta$ which gives the best localization. Below we give an interpretation of these results. If we take the width of the atomic wave packet to be much smaller than the atomic transition wavelength and redirect it at a node of the field, we can then replace the sinusoidal potentials in Eq. (5) with its linear expansion, i.e., $\sin(k_i z) \approx k_i z$. Then, according to formulae (5), (6), the maximal interference is realized for

$$|\chi_2|(k_2 y)^2 \pm |\chi_3|(k_3 x)^2 = 2\pi m, \tag{8}$$

with $k_2 y \ll 1$, $k_3 x \ll 1$ and $m = 0, 1, 2, \ldots$ The upper sign stands for identical detunings, that is, $(\Delta_2, \Delta_3) > 0$ or $(\Delta_2, \Delta_3) < 0$, while the lower one for opposite detunings, i.e., $\Delta_2 > 0, \Delta_3 < 0$ or vice versa. It is not difficult to realize that the maxima of the deflection pattern, which is characterized by $W(x, y)$, adopt the form of a sequence of circles in the $x-y$ plane for the same detunings and of crossed lines for opposite detunings. Figure 2 depicts the corresponding probability distributions $W(x, y)$ for initially Gaussian atomic wave packets. In particular, Fig. 2, *a* shows the corresponding deflection pattern with correlated beams that are identically detuned from the resonance, while Fig. 2, *b* describes the respective case with opposite detunings. One can observe here that the sections parallel to the plane $x - y$ in Fig. 2 are qualitatively described by Eq. (8). We see that the position distribution $W(x, y)$ focused on the case $\varphi = \theta$ is indeed peaked at the nodes of the two modes. By increasing

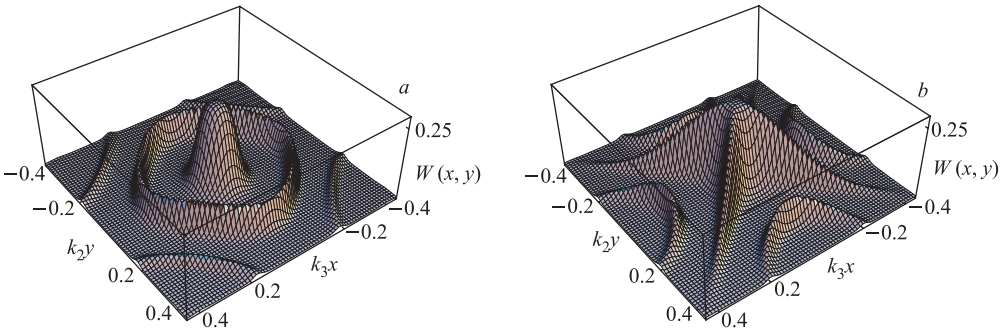

Fig. 2. The position distribution atomic pattern $W(x, y)$ (in units of $2\pi/\lambda^2$) as a function of $\{k_3 x/\pi, k_2 y/\pi\}$ for correlated modes with $\varphi = \theta$. Here $N = 1$, $|\chi_2| = |\chi_3| = 10$, and $\Delta x = \Delta y = 0.1\lambda$: *a*) $\{\chi_2, \chi_3\} > 0$; *b*) $\{\chi_2 > 0, \chi_3 < 0\}$

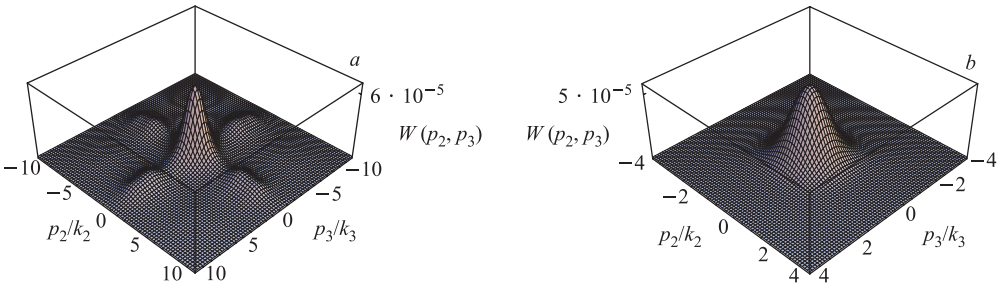

Fig. 3. The momentum distributions $W(p_2, p_3)$ (in units of λ^2) as a function of $\{p_2/k_2, p_3/k_3\}$ for the case of off-resonance interaction. Here $\chi_2 = \chi_3 = 10$, $\Delta x = \Delta y = 0.1\lambda$: *a*) the case of independent modes in coherent states with the mean photon number equal to one and with $\varphi = 0$; *b*) the case of correlated modes in PNCS with the mean twin-photon number $N = 1$

the interaction parameter χ for both modes, one can obtain smaller structures. Thus, this scheme provides the mechanism for two-dimensional localization of atoms in the presence of the photon correlation. Depending on the detuning's sign we demonstrate the formation of two types of subwavelength two-dimensional atomic structures via off-resonant interaction of atoms with light beams in PNCS.

2. LOCALIZATION IN THE MOMENTUM SPACE

In this section the localization mechanism for the case of PNCS is shortly discussed in the momentum space. For this goal the distribution $W(p_2, p_3)$, in terms of the transverse atomic momenta $p_2 = p_y$ and $p_3 = p_x$, in the deflection patterns is calculated. The results are shown in Fig. 3 for narrow initial atomic wave packets and hence, as we see, wider distributions in the momentum space are realized. Calculations show that the momentum pattern for the case of independent modes contains a set of maxima spread around the position $p_2 = p_3 = 0$ in concordance with the respective spatial distribution of the atoms. The momentum distribution for PNCS and for narrow initial Gaussian atomic wave packet centered at the position of the best localization shows the localization of the components of the momenta around the position $p_2 = p_3 = 0$ as shown in Fig. 3.

In conclusion, I believe that this approach could be of some practical interest, particularly, for problems of controllable formation of atomic nanostructures and quantum lithography.

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