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ФИЗИКА И ТЕХНИКА УСКОРИТЕЛЕЙ

STABILITY OF AN ION BEAM IN SYNCHROTRONS WITH DIGITAL FILTERS IN THE FEEDBACK LOOP OF A TRANSVERSE DAMPER

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The stability of an ion beam in synchrotrons with digital filters in the feedback loop of a transverse damper is treated. A transverse feedback system (TFS) is required in synchrotrons to stabilize the high-intensity ion beams against transverse instabilities and to damp the beam injection errors. The TFS damper kicker (DK) corrects the transverse momentum of a bunch in proportion to its displacement from the closed orbit at the location of the beam position monitor (BPM). The digital signal processing unit in the feedback loop between BPM and DK ensures a condition to achieve optimal damping. Damping rates of the feedback systems with digital filters are analyzed in comparison with those in an ideal feedback system.

Приводятся результаты теоретического исследования устойчивости ионного пучка в синхротронах в зависимости от параметров цифровых фильтров в цепи обратной связи систем подавления (СП) когерентных поперечных колебаний пучка. СП применяются в синхротронах с целью создания условий для предотвращения развития когерентных поперечных неустойчивостей пучка, а также для демпфирования остаточных колебаний частиц после инжекции. СП обеспечивают коррекцию поперечного импульса сгустков на каждом обороте с помощью дефлектора с учетом данных о смещении центра тяжести пучка, измеренных датчиком положения. Для достижения оптимальных условий демпфирования поперечных колебаний сгустков используются цифровые методы обработки сигналов в цепи обратной связи между датчиком положения и дефлектором. Приводятся данные о зависимостях темпа подавления когерентных колебаний для различных параметров цифровых фильтров.

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INTRODUCTION

Transverse feedback systems (TFS) are used widely in synchrotrons for damping of coherent oscillations. A classical bunch-by-bunch feedback (see Fig. 1) consists of a beam position monitor (BPM), a damper kicker (DK) and an electronic feedback path with appropriate signal transmission from the BPM to the DK [1]. The damper kicker corrects the transverse momentum of a bunch in proportion to its displacement from the closed orbit at the BPM location. The digital signal processing unit (DSP) ensures the suppression of all the revolution harmonics in the signal from BPM, the adjustment of the signal's phase and the betatron phase advance from BPM to DK in order to achieve optimal damping. The total delay $\tau_{\rm delay}$ in the signal processing from BPM to DK is adjusted to be equal to $\tau_{\rm PK}$, the particle time of flight from BPM to DK, plus an additional delay of q turns:

$$\tau_{\rm delay} = \tau_{\rm PK} + qT_{\rm rev}, \tag{1}$$

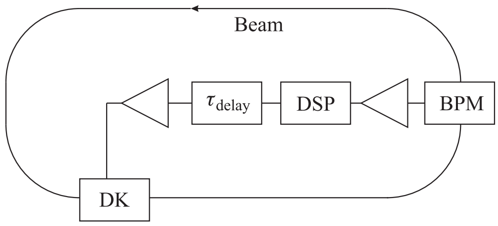

Fig. 1. Layout of a classical transverse feedback system

where $T_{\rm rev}$ is the revolution period of a particle in the synchrotron. Values of $q = 0$ or $q = 1$ are used in practice for TFS [2].

The damping rates of the TFS can be obtained from the characteristic equation [3]

$$z_k^2 - \left[2\cos(2\pi\widetilde{Q}) + ga_0 z_k^{-q} H(z_k)\sin(2\pi\widetilde{Q} - \psi_{\rm PK})\right] z_k + 1 - ga_0 z_k^{-q} H(z_k)\sin\psi_{\rm PK} = 0, \quad (2)$$

where $\widetilde{Q}$ is the beam tune; $\psi_{\rm PK}$ is the betatron oscillation phase advance from BPM to DK; $g > 0$ is the feedback gain; $H(z)$ is the Z-transform of the DSP transfer function, and a_0 is defined for $z_{\rm Q} = \exp(-j2\pi\,{\rm Re}\,\widetilde{Q})$ such that

$$|a_0 z_Q^{-q} H(z_Q)| = 1, \qquad a_0 \sin\left(\arg\left(z_Q^{-q} H(z_Q)\right) + {\rm Re}\,\psi_{\rm PK}\right) > 0. \quad (3)$$

In the general case, $\widetilde{Q}$ is a complex function depending on z. The real part of $\widetilde{Q}$ is the number of betatron oscillations per turn: ${\rm Re}\,\widetilde{Q} = Q$. The imaginary part of $\widetilde{Q}$ is determined by the transverse instability rise time: $2\pi|\,{\rm Im}\,\widetilde{Q}| = T_{\rm rev}/\tau_{\rm inst}$. The beam is stable if eigenvalues z_k from Eq. (2) lie inside the unit circle:

$$|z_k| < 1. \quad (4)$$

Damping rates of the coherent betatron oscillations are defined by the absolute value of z_k:

$$\frac{T_{\rm rev}}{\tau_k} = -\ln|z_k|, \quad (5)$$

where τ_k is the time constant of the betatron oscillation amplitude decay. Fractional parts $\{{\rm Re}\,\widetilde{Q}_k\}$ of the betatron frequency of a particle in the presence of TFS

$$\{{\rm Re}\,\widetilde{Q}_k\} = \frac{1}{2\pi}\arg(z_k) \quad (6)$$

are the fractional tunes ($-0.5 < \{{\rm Re}\,\widetilde{Q}_k\} \leqslant 0.5$).

In the general case a DSP unit in the feedback loop is a cascade of FIR (finite impulse response) and IIR (infinite impulse response) digital filters. Hence, the DSP transfer function $H(z)$ is a ratio of two polynomials. If $\widetilde{Q}$ depends weakly on z, then the characteristic equation (2) with the function $H(z)$ can be converted to a polynomial. It can be solved with the use of a root-finding algorithm or analytically for a polynomial of degree less than five [3]. Therefore, solving the characteristic equation (2) with different functions $H(z)$ allows one to calculate the achievable damping rates as a function of instability growth rate, feedback gain and parameters of the signal processing. It should be emphasized that the damping rates in the linear approximation with $|g| \ll 1$ are expressed by the formula [3]

$$\frac{T_{\rm rev}}{\tau_{\pm}} \approx \frac{ga_0 \exp\left(\pm \operatorname{Im} \widetilde{\Psi}_{\rm PK}\right)}{2} \sin\left(\operatorname{Re} \widetilde{\Psi}_{\rm PK}\right) \pm 2\pi \operatorname{Im} \widetilde{Q}, \tag{7}$$

where

$$\widetilde{\Psi}_{\rm PK} = \psi_{\rm PK} + 2\pi q \widetilde{Q} + \varphi, \qquad \varphi = \arg\left(H(z = \exp(-j2\pi\widetilde{Q}))\right). \tag{8}$$

The best damping of coherent transverse oscillations is achieved by optimally choosing the positions of BPM and DK yielding a phase advance of $\operatorname{Re} \widetilde{\Psi}_{\rm PK}$ equal to an odd multiple of $\pi/2$. The special case with $\varphi = 0$, $q = 0$ and the betatron phase advance of $\operatorname{Re} \psi_{\rm PK}$ equal to an odd multiple of $\pi/2$ corresponds to the *ideal* transverse feedback system which provides the best damping. Consequently, feedbacks with digital electronics should be designed with parameters close to those of the ideal TFS.

DIGITAL FEEDBACK SYSTEMS

As minimum a notch filter to suppress all the revolution harmonics (DC included) is required in the feedback loop [3]. The magnitude of the difference signal from the BPM electrodes, after passing through the notch filter, is proportional to the bunch deviation from the closed orbit. The system transfer function of the notch filter is

$$H(z) = H_{\rm NF}(z) = 1 - z^{-1}. \tag{9}$$

It is clear from (9) that the notch filter changes the gain g and the phase φ of the open loop transfer characteristics. For example, if $Q = 6.73$, then $\{Q\} = -0.27$ and $\arg\left(H_{\rm NF}(z_Q)\right) = \varphi_{\rm NF} = 41.4°$. The gain $|H_{\rm NF}| = 2|\sin(\{Q\}\pi)| = 1.5$ can be adjusted by an amplifier a_0 in the feedback loop in accordance with (3). However, according to the approximation formula (7), the damping rates for the TFS with the notch filter still change due to the phase shift $\varphi_{\rm NF}$ resulting in slower damping than for the case of the ideal TFS.

It was proposed in [4] to correct a phase shift in the feedback loop by a Hilbert filter with the system transfer function

$$H_{\rm HF}(z) = h_0 z^{-3} + h_1 z^{-2}(1 - z^{-2}) + h_3(1 - z^{-6}), \tag{10}$$

where

$$h_0 = \cos(\Delta\varphi),\ h_1 = -\frac{2}{\pi}\sin(\Delta\varphi),\ h_3 = -\frac{2}{3\pi}\sin(\Delta\varphi)$$

are the Hilbert transform impulse response coefficients. For example, the phase shift needed for compensation of $\varphi_{\rm NF} = 41.4°$ is obtained by using the Hilbert filter with $\Delta\varphi = -72.8°$ [3]. However, the Hilbert filter has six one-turn delays in its electrical circuit that increases a transition time of TFS.

The unwanted phase shift $\varphi_{\rm NF}$ can be compensated also by an all-pass filter [5] with a frequency-response magnitude that is constant but a phase advance which is variable and adjustable. The transfer function of the first-order all-pass filter is

$$H_{\rm AF}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}, \tag{11}$$

where a is a free filter parameter for the adjustment of the phase, and a^* denotes its complex conjugate. For example, the phase shift needed for compensation of $\varphi_{\rm NF} = 41.4°$ is obtained by using the all-phase filter with $a = -0.501$ [3]. However, the all-pass filter is an IIR filter, and its recursive circuit that corresponds to the denominator in (11) can be a source of an unwanted noise during a long time for any fluctuation in the BPM signal.

A cascade of two FIR filters of the first order can be used for providing damping rates close to the ideal TFS. The first FIR filter (see Fig. 2) is a notch filter, and the second one is designed with the parameter a_2 for obtaining best damping. The transfer function for the cascade of two FIR filters is

$$H_2(z) = (1 - z^{-1}) \cdot (1 + a_2 z^{-1}), \tag{12}$$

and the characteristic equation (2) is a polynomial of the fourth power that can be solved analytically. Dependences of damping rates $T_{\rm rev}/\tau_{\rm inst}$ on gain g for the ideal TFS, for the TFS with a notch filter and with two FIR filters are shown in Fig. 3 (the tune of $Q = 6.73$ was used [6], and the instability rise time of $\tau_{\rm inst} = 100 T_{\rm rev}$ was assumed).

It should be emphasized that the damping regime is obtained at $a_2 > 0$. Consequently, the gain transfer characteristic of the feedback loop with the cascade of two FIR filters (see Fig. 3) has a poor-frequency response at $f = 0.5 f_{\rm rev}$ and maximum values near betatron frequencies. Therefore, a cascade of a notch filter and an FIR filter of the first order in the case of beam stability corresponding to $a_2 > 0$ provides an additional advantage of the feedback loop for a signal to noise ratio.

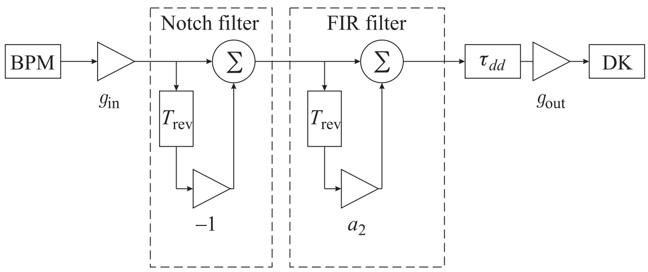

Fig. 2. Block diagram of feedback loop with a cascade of two FIR filters

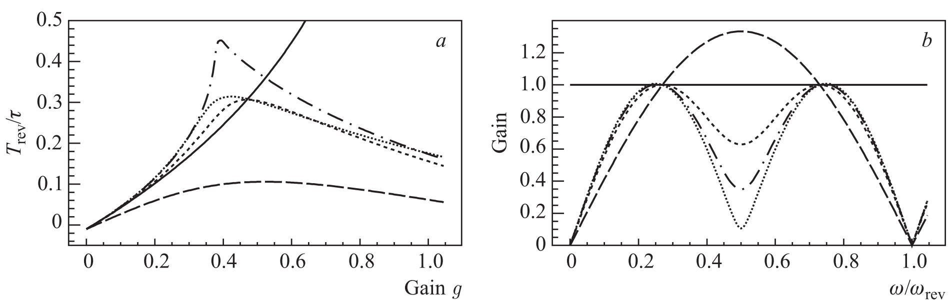

Fig. 3. Dependences of damping rates $T_{\rm rev}/\tau_{\rm inst}$ on gain g (a) and gain on frequency (b) for the ideal TFS (solid curve), for the TFS with a notch filter (dashed curve) and with two FIR filters in the case of $a_2 = 0.5$ (short-dashed curve), $a_2 = 0.7$ (dash-dotted curve), $a_2 = 0.9$ (dotted curve)

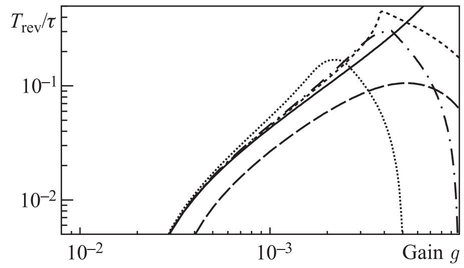

Fig. 4. Dependences of damping rates $T_{\rm rev}/\tau_{\rm inst}$ on gain g for the ideal TFS (solid curve), for the TFS with a notch filter (dashed curve), two FIR filters ($a_2 = 0.7$, short-dashed curve), an all-pass filter ($a = 0.501$, dash-dotted curve) and a Hilbert filter ($\Delta\varphi = -72.8^\circ$, dotted curve)

Dependences of damping rates $T_{\rm rev}/\tau_{\rm inst}$ on gain g for the ideal TFS, for the TFS with a notch filter, two FIR filters, an all-pass filter and a Hilbert filter are shown in Fig. 4 in the case of optimal damping regimes mentioned above at $Q = 6.73$ and $\tau_{\rm inst} = 100T_{\rm rev}$. It is shown that for small gains of the feedback loop the optimum damping characteristics of the ideal TFS can be restored in the presence of a notch filter using a first-order all-pass filter or a six-order Hilbert filter or a cascade of notch and FIR filters with optimized parameters. However, the widest beam stability range for TFS with digital filters discussed corresponds to a cascade of a notch filter and an FIR filter.

The damping time τ_d of TFS must be shorter than the instability rise time $\tau_{\rm inst}$ to suppress instability: $\tau_d < \tau_{\rm inst}$. In addition to that the damping time must be chosen to limit the emittance growth due to the beam injection errors. If $e_{\rm inj}$ is the maximum assumed amplitude of a beam deviation from the closed orbit due to displacement and angular errors at injection, then the relative emittance growth $\Delta\varepsilon/\epsilon$ is [7]

$$\frac{\Delta\epsilon}{\epsilon} = \frac{e_{\rm inj}^2}{2\sigma^2}F_a^2, \qquad F_a = \left(1 + \frac{\tau_{\rm dec}}{\tau_d} - \frac{\tau_{\rm dec}}{\tau_{\rm inst}}\right)^{-1}, \tag{13}$$

where σ is the initial RMS beam size and $\tau_{\rm dec}$ is the beam decoherence time. As a rule, $F_a < 0.1$ is assumed that corresponds to $\tau_d \approx 40T_{\rm rev}$ for $\tau_{\rm inst} > 100T_{\rm rev}$ and $\tau_{\rm dec} > 500T_{\rm rev}$. The damping time $\tau_d = 40T_{\rm rev}$ is used commonly as the design specification of TFS for damping of ion beams in synchrotrons [2, 3]. It should be emphasized that the gain g of TFS with the notch filter only in accordance with dependences in Fig. 4 must be ≈ 1.3 times higher in the case of $\tau_d = 40T_{\rm rev}$ than, for example, for TFS with the cascade of the notch and FIR filters with optimized parameters. Thus, tuning of digital filters for obtaining zero phase advance on the betatron frequency leads to the optimum damping characteristics of TFS.

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