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ON THE PROTON SPECTRUM IN FREE NEUTRON β-DECAY

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1. Introduction. Formulation of the general task.

Long since, the treatment of the $\beta-$decay of free neutrons has been rightly conceived to provide the straightforward way to inquire with full confidence into weak interactions in general.

Nowadays, it has been well realized that the characteristics of weak interactions are to be acquired with a precision better than 1% in order to judge definitively the validity of the general principals of the modern elementary particle theory. Rendering, properly speaking, the simplest semi-leptonic process, the neutron $\beta-$decay is evidently relevant best of all for the most reliable and precision investigation of the underlying semi-leptonic interactions. Being far more complicated, all the other phenomena used to ascertain the main features of the semi-weak interactions - the nuclei $\beta-$decay, the hyperon semi-weak decay, the muon-capture, and so on - are known to involve the manifold ambiguities which preclude to attain the desirable high-precision results. That is why within the past decade the considerable effort has been directed towards the inquiry into the various phenomena associated with the $\beta-$decay of free neutrons. Amongst these, what is to point out, as presenting the special interest and advantage to-day, is just the precision study of the recoil proton distribution; that is what the presented work deals with.

In actual fact, to try our best in studying genuine semi-weak interactions, we have got to-day no option, but to confront the high-precision experimental data with the results of the theoretical calculations based on the trustworthy effective Lagrangian descending from the general field theory. As charged particles are involved in the phenomena, electromagnetic interactions are to be allowed for, and this effective Lagrangian proves to be of the form (see, for instance, Refs. [1–3])

$$L_{int} = L_{BfBiw} + L_{e\gamma} + L_{B\gamma} \ , \tag{1.1}$$

where

$$L_{BfBiw}(x) = \frac{G_{if}}{\sqrt{2}}(\bar{\psi}_e(x)\gamma^\alpha(1+\gamma^5)\psi_\nu(x)) \times$$
$$\bar{\Psi}_{Bf}(x)[\gamma_\alpha g_V^B(q) + g_{WM}^B\sigma_{\alpha\nu}q_\nu + g_{IS}^B q_\alpha + \gamma^5(\gamma_\alpha g_A^B(q) + g_{IP}^B q_\alpha + g_{IT}^B\sigma_{\alpha\nu}q_\nu)]\Psi_{Bi}(x) \tag{1.2}$$

renders the $(V - A)$ baryon-lepton weak interaction, q being the four-momentum transferred in the β-decay process. The expression

$$L_{e\gamma}(x) = -e\bar{\psi}_e(x)\gamma^\mu\psi_e(x) \cdot A_\mu(x) \tag{1.3}$$

describes the electromagnetic-field interaction with leptons, and $L_{B\gamma}$ similarly stands for the interaction with baryons. In (1.2), (1.3), the notations are alike ones in Ref. [1], the index B specifies the various kinds of baryons, and the system of units $h = c = 1$ is adapted; $\Psi_{Bi}(x), \Psi^+_{Bf}(x)$ render the baryon fields in the initial and final states, and ψ_e, ψ_ν, A_μ stand for the electron (positron), (anti)neutrino, and electromagnetic fields, respectively.

As to the $g_V^B(0)$ value, we adapt, alike in Ref. [2], $g_V^n(0) = 1$ for the neutron decay (and $g_V^\Sigma(0) = 0$ for the strangeness-conserving decay $\Sigma^\pm \longrightarrow \Lambda^0 + e^\pm + \nu(\bar{\nu}) + \gamma$). Then, for various semi-weak decays associated with definite $i{\longrightarrow}f$ quarks transitions, $u{\longrightarrow}d$, $s{\longrightarrow}d$, $b{\longrightarrow}d$, the amplitudes G_{if} in Eq. (1.2) are known [1–3] to be represented as

$$G_{if} = G_F \cdot |V_{if}|. \tag{1.4}$$

Here, $G_F = 1.16639(2) \cdot 10^{-5} \cdot GeV^{-2}$ is fixed by the muon lifetime [4], and the Cabibbo-Kobayashi-Maskawa, CKM, [5] quark-mixing matrix elements V_{if} satisfy the unitarity identity

$$|V_{ud}|^2 + |V_{sd}|^2 + |V_{bd}|^2 = 1\,. \tag{1.5}$$

The validity of this relation is known to be warranted mainly by the contribution from the $u{\longrightarrow}d$ transition; $|V_{ud}| \approx 0.9744 \pm 0.0010$ as asserted, for instance, in Refs. [4, 6]. Henceforward, we treat the free neutron β-decay solely.

In the current study, our tenet is, starting with the Lagrangian (1.1)-(1.3), to evaluate upright and consistently the values of the quantities measured in the correspondent trustworthy experiments, that is, to express with a high accuracy the experimental data immediately in terms of the effective interaction (1.1)-(1.3). Subsequently, confronting the results of calculations with the experimental data, we can acquire the values of the quantities $G_{if}, |V_{if}|, g_A, ...$ in (1.2) and even get in position to judge the validity of the very general form of the effective interaction (1.1)-(1.3) itself. For instance, we might judge an feasible admixture to (1.2), having the same transformation properties as (1.2) has, but differing from (1.2) via replacing $\gamma^5 \rightarrow -\gamma^5$ (see Refs. [7–9]).

Though, by now, the terms with g_{IS} and g_{IT} in (1.2) cannot be excluded absolutely (see, for instance, [2, 10]), and their possible availability, strictly speaking, must be investigated as well, we abandon these terms in our study once and for all. In the common simplified treatment, when the electromagnetic interaction (1.3) turned out, the nucleon mass is presumed to be infinite, $M_N \rightarrow \infty$, and, consequently, the terms with g_{WM}, g_{IP} disappear and the $q-$dependence of $g_V(q), g_A(q)$ is negligible, only the very terms with $g_V(0), g_A(0)$ in (1.2) cause the bulk of the $\beta-$decay probability. With accounting for the electromagnetic interaction (1.3) as well as for the finiteness of the nucleon mass, the calculations provide the correspondent, relatively small, corrections to that main quantity. Surely, the high-precision allowance for these corrections in any given case is a matter of the desirable accuracy to which we try to specify the peculiarities of the effective interaction (1.1)-(1.3).

From the very first, it is instructive to recall that starting with the effective interaction of the form (1.1)-(1.3) we inevitably encounter (see, for instance, [1–3]) the ultraviolet divergencies in evaluating radiative corrections. Consequently, the ad hoc effective cut-off parameter Λ emerges to preclude these divergencies. At first, this cut-off mass Λ was thought to be of the order of the nucleon mass $\Lambda \approx M_N$ [11, 12]. Afterwards, as weak interactions were ascertained to be mediated by heavy vector bosons [13], the masses of these mesons, $W-$boson, or $Z-$boson, were realized to stand for the effective cut-off mass, $\Lambda = M_V \gg M_N$. The profound investigations of Sirlin's [14], carried out in the framework of the $SU(2)_L \times U(1)$-gauge modal [15], are known to have confirmed this previous handy prescription, to all intents and purposes. In the work presented, we don't contemplate plunging into the general treatment of the ultraviolet divergency, but merely pursue the way paved by the aforecited investigations [13–15], the dependence of our calculations on the Λ-value being offered in due course.

2. The agenda of the neutron $\beta-$decay study to-day.

So far the final state after the neutron $\beta-$decay involves a proton, an electron, an antineutrino and $\gamma-$radiation, the probability of the polarized neutron $\beta-$decay, upon summarizing the absolute square $|M_{if}|^2$ of the transition amplitude over the polarizations of all the particles in the final state, is obviously put into the following well-known general form

$$dW(\mathbf{p_e},\mathbf{P},\mathbf{p}_\nu,\mathbf{k},\boldsymbol{\xi}) = (2\pi)^4\delta(M_n - E_P - \omega_\nu - \varepsilon - \omega)\delta(\mathbf{P}+\mathbf{p_e}+\mathbf{p}_\nu+\mathbf{k}) \times \frac{1}{2M_n}\sum|M_{if}|^2\frac{d\mathbf{P}d\mathbf{p_e}d\mathbf{p}_\nu d\mathbf{k}}{(2\pi)^{12}\,2E_p\,2\varepsilon\,2\omega_\nu\,2\omega} = w(\mathbf{p_e},\mathbf{P},\mathbf{p}_\nu,\mathbf{k},\boldsymbol{\xi})\,d\mathbf{P}d\mathbf{p_e}d\mathbf{p}_\nu d\mathbf{k}\delta(M_n - E_P - \omega_\nu - \varepsilon - \omega)\delta(\mathbf{P}+\mathbf{p_e}+\mathbf{p}_\nu+\mathbf{k}), \tag{2.1}$$

where $\boldsymbol{\xi}$ stands for the polarization vector of an incident neutron which is presumed resting, M_n is the neutron mass, and $p_e=(\varepsilon,\mathbf{p_e})$, $P=(E_P,\mathbf{P})$, $p_\nu=(\omega_\nu,\mathbf{p}_\nu)$, $k=(\omega,\mathbf{k})$ are the electron, proton, antineutrino and $\gamma-$ray four-momenta, receptively. The familiar expression (2.1), appropriate and handy for the following discussion, renders the momentum distribution of electrons, protons, antineutrinos and $\gamma-$rays in the final state.

Up to now, four species of experiments on the free neutron $\beta-$decay have been known to investigate the genuine form of the effective interaction (1.1)-(1.3).

On integrating the distribution (2.1) over all the momenta in the final sate and averaging over the polarizations of an original neutron, the calculation results in the total probability of neutron $\beta-$decay, i.e. the reverse lifetime, $W = 1/\tau$, which is known to be measured in experiment most precisely and trustworthy, with an accuracy of $\sim$0.3% or so [6]. Thus, by equating the measured and calculated τ values, we get the first relation to specify the effective interaction (1.1)-(1.3).

Upon integrating (2.1) over the proton and $\gamma-$ray momenta and the antineutrino energy, we arrive at the well-known electron-antineutrino distribution

$$dW(\varepsilon,\mathbf{p_e},\mathbf{n}_\nu,\boldsymbol{\xi}) = d\mathbf{w}\frac{d\mathbf{n}_\nu}{4\pi} \times \Big\{W_0(\varepsilon,\mathbf{p_e},g_V,g_A) + (\mathbf{v}\boldsymbol{\xi})\cdot W_{v\xi}(\varepsilon,\mathbf{p_e},g_V,g_A) + +(\mathbf{n}_\nu\boldsymbol{\xi})\cdot W_{\nu\xi}(\varepsilon,\mathbf{p_e},g_V,g_A) + (\mathbf{n}_\nu\mathbf{v})\cdot W_{v\nu}(\varepsilon,\mathbf{p_e},g_V,g_A)\Big\}, \tag{2.2}$$

where

$$d\mathbf{w} = \frac{G_{ud}^2}{2\pi^3}\varepsilon p\omega_{\nu 0}^2 d\varepsilon\frac{d\mathbf{n}_e}{4\pi}, \quad \omega_{\nu 0} = \Delta - \varepsilon, \quad \mathbf{n}_e = \frac{\mathbf{p_e}}{p_e}, \quad \mathbf{v} = \frac{\mathbf{p_e}}{\varepsilon}, \quad \mathbf{v}_\nu = \mathbf{n}_\nu = \frac{\mathbf{p}_\nu}{|\mathbf{p}_\nu|},$$

$$\Delta = M_n - M_p = 1.2943 MeV \ , \quad \varepsilon = \sqrt{m^2 + \mathbf{p_e}^2} \ ,$$

and $m = 0.511 MeV$, $M_n = 939.57 MeV$, $M_p = 938.28 MeV$ are the electron, neutron, and proton masses, restrictively.

Here, the dependence of the distribution (2.2) on the form of the effective interaction (1.1)-(1.3), especially on g_V, g_A, is incorporated into the coefficients $W_{0,v\xi,\nu\xi,v\nu}$. The quantities

$$A = \frac{W_{v\xi}(\varepsilon, \mathbf{p_e}, g_V, g_A)}{W_0(\varepsilon, \mathbf{p_e}, g_V, g_A)}, \quad B = \frac{W_{\nu\xi}(\varepsilon, \mathbf{p_e}, g_V, g_A)}{W_0(\varepsilon, \mathbf{p_e}, g_V, g_A)}, \quad a = \frac{W_{\nu v}(\varepsilon, \mathbf{p_e}, g_V, g_A)}{W_0(\varepsilon, \mathbf{p_e}, g_V, g_A)} \tag{2.3}$$

stand for the asymmetry of the electron and antineutrino angular distributions and for the electron-antineutrino angular correlation, respectively. Surely, all the aforesaid corrections having been abandoned, the quantities (2.3) take the familiar well-known uncorrected form

$$A_0 = \frac{2g_A(1-g_A)}{1+3g_A^2}, \quad B_0 = \frac{2g_A(1+g_A)}{1+3g_A^2}, \quad a_0 = \frac{1-g_A^2}{1+3g_A^2}, \tag{2.4}$$

and the distribution (2.2) itself reduces to

$$d\mathbf{W}_0(\varepsilon, \mathbf{p_e}, \mathbf{n}_\nu, \boldsymbol{\xi}) = d\mathbf{w}\frac{d\mathbf{n}_\nu}{4\pi}(1+3g_A^2)\Big\{1 + (\mathbf{v}\boldsymbol{\xi})A_0 + B_0(\mathbf{n}_\nu\boldsymbol{\xi}) + a_0(\mathbf{n}_\nu\mathbf{v})\Big\}. \tag{2.5}$$

By now, the electron momentum distribution irrespective of the antineutrino angular distribution has been measured in several high-precision experiments [16]. Such distribution corresponds to (2.2) integrated over $d\mathbf{n}_\nu$, or to (2.1) integrated over all the momenta but $d\mathbf{p_e}$. The coefficient A prefixed to $(\mathbf{v}\boldsymbol{\xi})$ in (2.2) is immediately acquired thereby. Thus, the second relation has been gained in order to specify the form of the effective interaction (1.2), especially to size up the value of g_A.

So far antineutrinos cannot be registered in any conceivable experiments, we are to deal anyway with the general distribution (2.1), or with the distributions (2.2), (2.5) integrated over $d\mathbf{n}_\nu$. Consequently, the coefficients a, B in front of $(\mathbf{v}\mathbf{n}_\nu)$, $(\mathbf{n}_\nu\boldsymbol{\xi})$ assigned to describe the antineutrino-electron correlation and the asymmetry of the antineutrino angular distribution, respectively, are not definable immediately from such experimental measurements at all. So, there is no option for a further advance in the study of the peculiarities of neutron $\beta-$decay, but to menage and register protons and $\gamma-$rays in the final state, which is known to be many times more difficult than to measure the electron momentum distribution [16]. Whereas the $\gamma-$radiation accompanying neutron $\beta-$decay still stands beyond observations, the proton distribution has been studied by now in two kinds of experiments [17–20], and some new ones are believed to be carried out and come to fruition before long [21–24].

For more than three decade, a great deal of effort has been devoted in the series of the investigations [17, 18] to register with a high precision, better than 1%, the electron momentum $\mathbf{p}_{\mathrm{e}}$ simultaneously with the projection $P_{\|}{\equiv}P_x$ of the proton momentum $\mathbf{P}$ on an axis $\mathbf{x}$, with an incident neutron polarized along or opposite the $\mathbf{x}$-direction. Such measurements provide the distribution

$$d\mathbf{W}(\mathbf{p}_{\mathrm{e}}, P_x, \boldsymbol{\xi}) = w(\mathbf{p}_{\mathrm{e}}, P_x, \boldsymbol{\xi}) d\mathbf{p}_{\mathrm{e}} dP_x, \tag{2.6}$$

which corresponds to the general distribution (2.1), having been integrated over $d\mathbf{p}_{\nu} d\mathbf{k} dP_{\perp}$. By confronting the experimental results obtained in [18] with the calculated distribution (2.6) we arrive at the third relation to elucidate the features of the effective interaction (1.1)-(1.3). The most promising purpose of the works [17, 18] might have been to define immediately the coefficient B in the distribution (2.2) with a very high precision, better than 1%. Yet for now, such a high accuracy is seen to be not attainable in B specifying [18], so far the antineutrino momentum occurring in (2.2) can't be reconstructed precisely when the momenta of electrons and protons are observed only, without registering the $\gamma-$rays which are known to accompany $\beta-$decay unavoidably; see, for instance, Ref. [25].

The measurement [19, 20] of the energy distribution of protons in the $\beta-$decay of unpolarized neutrons,

$$dW(E_P) = w(E_P) dE_P = w(|\mathbf{P}|) d|\mathbf{P}|, \tag{2.7}$$

is the fourth kind of the experiments carried out by now to investigate the neutron $\beta-$decay. The experimentally observed distribution (2.7) corresponds to (2.1), integrated over all the momenta but $d|\mathbf{P}|$. Thus, the fourth relationship is procured to inquire into the actual form of the effective interaction (1.1)-(1.3). As far back as in the 'seventies, the distribution (2.7) was acquired in the experiment [19, 20], but an accuracy about 5% only was attained, which is considered to be not sufficient for now. Lately, the ingenues experimental setup [21] has been thoroughly elaborated in order to measure the distribution (2.7) with a precision better than 1%. The correspondent measurements are known to come true for now [22] and the desirable high-precision results are believed to be offered before long. According to our lights, we are to try our best to acquire outright the distribution (2.7), the effective interaction (1.1)-(1.3) put to use.

3. Transition amplitude.

Pursuing the way proclaimed afore, we are now to evaluate, upright and consistently, the distribution (2.7), measured in [19, 20], starting with the effective Lagrangian (1.1)-(1.3). Upon such calculating, the experimental results come out to be presented immediately in terms of the original effective interaction (1.1)-(1.3), so that the form (1.1)-(1.3) itself and the values of the parameters involved in (1.2) can be specified thereby. As a matter of course, so far as an accuracy better than 1% has to be procured, all the peculiarities in describing $\beta-$decay became of value and must be properly allowed for. In the well-known work [26], all the corrections entailed by accounting for the finiteness of the nucleon mass have been thoroughly evaluated in studying the electron and antineutrino distribution (2.2). The results of Ref. [26] are thought to be adjusted, in properly way, but not just immediately, for acquiring the correspondent corrections to the distribution (2.7); as some simplified approach, the calculations [27] might be referred to. We are not on the point of discussing here that task, but we have solely the aim to scrutinize the effect of the electromagnetic interaction (1.3) on the distribution (2.7).

The effective interaction (1.1)-(1.3) is known to give rise to both the real $\gamma-$radiation of the charged particles involved in $\beta-$decay and to the virtual photon exchange between them. To the lowest order in electric charge e, the matrix elements of the transition amplitude to be calculated are displayed, as usually (see, for instance, [1, 28]), by the diagrams presented in Fig. 1. All the notations in the pictures need no explanations as being familiar. The diagram a) renders the uncorrected bulk of the free neutron $\beta-$decay, and b) describes the electron bremsstrahlung. As the proton bremsstrahlung displayed by the diagram c) is concerned, there is no need to take it into consideration so far the nucleon mass is as 2000 times greater as the electron one.

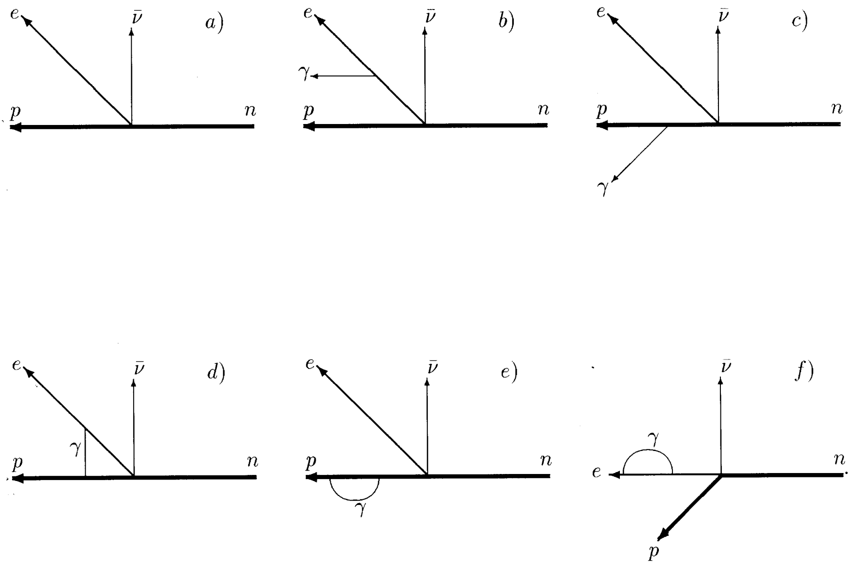

Fig. 1. The diagrams to describe the neutron β-decay to the lowest e-order.

With accounting for the diagrams $e), f)$, the uncorrected, zeroth-order in e transition amplitude displayed by the diagram $a)$,

$$M_0 = \frac{G_{ud}}{\sqrt{2}} \cdot \bar{u}_e(\mathbf{p}_e) l_0^\lambda u_\nu(\mathbf{p}_\nu) \cdot \bar{U}_p(\mathbf{p}_p) h_{0\lambda} U_n(\mathbf{p}_n), \tag{3.1}$$

$$l_0^\lambda = \gamma^\lambda(1+\gamma^5), \quad h_{0\lambda} = \gamma_\lambda(g_V + g_A\gamma^5), \tag{3.2}$$

is known to be replaced by the renormalized one (see, for instance, [1, 28])

$$\tilde{M}_0 \approx M_0 + M_R, \quad M_R = M_0(Z_e^{(1)} + Z_p^{(1)})/2, \tag{3.3}$$

$$Z_i^{(1)} = -\frac{\alpha}{2\pi}[\ln(\Lambda/m_i) + 9/4 + 2\ln(\lambda/m_i)], \quad m_i = m, M_p\,.$$

In (3.1), $p_e, p_\nu, p_n, p_p = P$ are the electron, antineutrino, neutron and proton four-momenta, respectively, and u_e, u_ν, U_n, U_p indicate their Dirac spinors. The photon mass λ is introduced, as usually, to treat the soft (infrared) γ-radiation (see, for instance Refs. [1, 28, 29]). The contribution (3.3) M_R to the transition amplitude from the diagrams e), f) is known to diverge logarithmically [1–3, 28] owing to integrating over the four-momenta of virtual photons. Consequently, the cut-off mass Λ [11–14] has emerged in (3.3) to prevent this divergency, as was expounded afore, see Sec. 1.

The matrix element presented by the diagram d), involving an internal photon line, takes the form

$$M_{2\gamma} = \frac{i}{(2\pi)^4} \cdot \frac{e^2 G_{ud}}{\sqrt{2}} \cdot \int d^4k (\bar{u}_e(\mathbf{p}_e) \mathcal{P}^{\mu\lambda} u_\nu(\mathbf{p}_\nu)) \cdot (\bar{U}_p(\mathbf{p}_p)(h_\gamma)_{\mu\lambda} U_n(\mathbf{p}_n)) \cdot F(k). \tag{3.4}$$

The following notation has been introduced in (3.4):

$$\begin{aligned} F(k) = 1/[(p^2 - m^2 + i0)(Q^2 - M_p^2 + i0)(k^2 - \lambda^2 + i0)] \\ (h_\gamma)^{\mu\lambda} = \gamma^\mu(\hat{Q} + M_p)\gamma^\lambda(g_V + g_A\gamma^5) \\ \mathcal{P}^{\mu\lambda} = \gamma^\mu(\hat{p} + m)\gamma^\lambda(1 + \gamma^5) \\ p = p_e - k, \; Q = P + k, \; \hat{O} \equiv O_\alpha \gamma^\alpha. \end{aligned} \tag{3.5}$$

Obviously, to the first order in the fine-structure constant α, the expression (3.4) incorporates all the effects of the electromagnetic interactions between the charged particles involved in β-decay. In particular, the so-called "Coulomb corrections" are not separated as against what have been assumed in a number of the previous papers [27, 30–36] in evaluating the electromagnetic corrections to the distribution (2.7). Let us emphasize that though outer nucleons are non-relativistic and even have got negligible velocities, a virtual proton in the intermediate state in the diagram d) must be described by the relativistic propagator

$$G(Q) = \frac{\hat{Q} + M_p}{Q^2 - M_p^2 + i0} \tag{3.6}$$

because integrating over d^4k in (3.4) involves arbitrarily large values of the virtual photon momentum k and, consequently, of the virtual proton momentum Q. If anything, it might be pertinent to point out that if we had replaced the function G (3.6) by the non-relativistic value, the calculation of the radiative corrections to the transition amplitude and, subsequently, to the β-decay probability could have reduced, to all intents and purposes, to handling their, so called, "model independent" parts only, as was presumed, for instance, in Refs. [30, 31, 33–36] accordingly the prescription of Ref. [37].

The transition amplitude corresponding to the diagram $b)$ in Fig 1. is

$$M_{1\gamma}^{(l)} = \frac{eG_{ud}}{\sqrt{2}} \cdot \epsilon_m^{(l)} \cdot (\bar{u}_e(\mathbf{p}_e)\mathcal{P}^{m\alpha}u_\nu(\mathbf{p}_\nu)) \cdot (\bar{U}_p(\mathbf{p}_p)h_{0\alpha}U_n(\mathbf{p}_n)), \quad (m,l) = (1,2,3), \tag{3.7}$$

where $\boldsymbol{\epsilon}^{(l)}$ is the $\gamma-$ray polarization vector.

After all, upon lumping all the terms of (3.1-3.7) together, we left with the corrected transition amplitude accounting for electromagnetic interactions to the lowest order in electric charge e:

$$M_{tot}^{(l)} = M_0 + M_{1\gamma}^{(l)} + M_2, \tag{3.8}$$

where

$$M_2 = M_R + M_{2\gamma} \tag{3.9}$$

is proportional to e^2, whereas $M_{1\gamma}^{(l)}$ is linear in e.

The decay probability we aim to calculate is expressed through the absolute square

$$|M_{tot}^{(l)}|^2 = |M_0 + M_2 + M_{1\gamma}^{(l)}|^2 \tag{3.10}$$

which reduces in the first α-order to

$$|M_{tot}^{(l)}|^2 \approx |M_0|^2 + M_0^*M_R + M_0M_R^* + |M_{1\gamma}^{(l)}|^2 + M_0^*M_{2\gamma} + M_0M_{2\gamma}^*\,. \tag{3.11}$$

To allow for the polarizations of the particles involved in the process considered, we rewrite, as usually (see, for instance, [1]), the terms incorporated in (3.11) making use of the polarization matrices $\rho_e, \rho_\nu, \rho_n, \rho_p$ of the electron, the antineutrino, the neutron and the proton, respectively. The value of the first term in (3.11) is, of course, well known,

$$|M_{0if}|^2 = \frac{G_{ud}^2}{2} Sp[\rho_p h_0^\alpha \rho_n \bar{h}_0^\beta] \cdot Sp[\rho_e l_{0\alpha} \rho_\nu \bar{l}_{0\beta}], \tag{3.12}$$

and evaluating the quantity

$$[M_0^* M_R + M_0 M_R^*]_{if} \tag{3.13}$$

in (3.11) is straightforward, allowing for the equations (3.1), (3.3). The quantities $h_0^\alpha, l_{0\alpha}$ have been defined in (3.1), (3.2), and, as usually, $\bar{a} \equiv \gamma^0 a^+ \gamma^0$.

The fourth term in (3.11) is

$$|M_{1\gamma}^{(l)}|^2 = \frac{\alpha G_{ud}^2}{2} \epsilon_\mu^{(l)} \epsilon_\kappa^{(l)*} Sp[\rho_p h_{0\alpha} \rho_n \bar{h}_{0\beta}] Sp[\rho_e \mathcal{P}^{\mu\alpha} \rho_\nu \bar{\mathcal{P}}^{\kappa\beta}], \tag{3.14}$$

where $\mathcal{P}^{\mu\alpha}$ is defined by (3.5). With the equations (3.4), (3.5) accounted for, the sum of the last two terms in (3.11) can be presented in the following form

$$(M_S)_{if}^2 \equiv (M_0^* M_{2\gamma} + M_0 M_{2\gamma}^*)_{if} =$$
$$\frac{\alpha G_{ud}^2}{2} \cdot \frac{i}{(2\pi)^4} \cdot \int d^4k F(k) [T^{\mu\lambda\alpha} S_{\mu\lambda\alpha} + (T^{\mu\lambda\alpha} S_{\mu\lambda\alpha})^*], \tag{3.15}$$

$$T^{\mu\lambda\alpha} = Sp[\rho_p (h_\gamma)^{\mu\lambda} \rho_n (h_0)^\alpha], \quad S_{\mu\lambda\alpha} = Sp[\rho_e \mathcal{P}_{\mu\lambda} \rho_\nu (l_0)_\alpha]. \tag{3.16}$$

The distribution (2.7) studied in this work corresponds with the experiment on the $\beta-$decay of unpolarized neutrons, where only the absolute value of the proton momentum $P = |\mathbf{P}|$ is registered, whereas electrons, antineutrinos and $\gamma-$rays are not considered at all, as well as the proton angular distribution. Consequently, the polarization matrices in (3.12)-(3.16) are given as follows

$$\rho_p = M_p \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho_n = M_n \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho_e = (\hat{p}_e + m)/2, \quad \rho_\nu = \hat{p}_\nu, \tag{3.17}$$

and we are to evaluate henceforth the yield of protons with a given value $P = |\mathbf{P}|$, that is, the $\beta-$decay probability integrated over the momenta of electrons, antineutrinos, $\gamma-$rays and over the proton escape direction as well as summarized over the polarizations of all the particles in the final state. It is instructive to recall that in the previous work [38] we have treated, the other way round, the electron momentum distribution (2.2) in the decay of a polarized neutron just regardless of the final proton state.

4. The distribution of protons with momentum absolute value.

The quantity $|M_0|^2$ in (3.11) provides the well-known uncorrected yield of protons at a given value of the momentum $P = |\mathbf{P}|$ [19, 27, 39]

$$dW_0(P) = dP{\cdot}P\frac{G_{ud}^2}{4\pi^3}(1+3g_A^2)\int\limits_{\varepsilon_1}^{\varepsilon_2} d\varepsilon\, \varepsilon\, \omega_{\nu 0}[1 + a_0\mathcal{N}(\varepsilon, P)]\,. \tag{4.1}$$

Here, the limits of the integration over the electron energy are

$$\varepsilon_{1,2} = -\frac{1}{2}[\pm P - \Delta + \frac{m^2}{\pm P - \Delta}]\,, \tag{4.2}$$

P itself varying within

$$P_1 = 0 \le P \le P_2 = \sqrt{\Delta^2 - m^2}\,, \tag{4.3}$$

and the notation is introduced:

$$\mathcal{N}(\varepsilon, P) = (\mathbf{v}_\nu \mathbf{v}) = -\frac{1}{2\omega_{\nu 0}\varepsilon}(p_e^2 - P^2 - \omega_{\nu 0}^2)\,, \tag{4.4}$$

where $p_e = \varepsilon v$ is the electron momentum, $\mathbf{v}_\nu$ and $\mathbf{v}$ are the antineutrino and electron velocities, restrictively.

After the unsophisticated familiar calculations, the contribution owing to (3.14) into the distribution (2.7) is put into the simple, but slightly long and cumbersome form

$$\begin{aligned} dW_{1\gamma}(P) = dP{\cdot}P(1+3g_A^2)\frac{G_{ud}^2\alpha}{8\pi^4}\int\limits_{\varepsilon_1}^{\varepsilon_2} v d\varepsilon \int\limits_0^{k_m}\frac{dk{\cdot}k^2}{\omega}\int\limits_{-1}^{x_2}\frac{dx}{l{\cdot}s} \times \\ \Big\{\varepsilon\omega_\nu[(\omega+\varepsilon)v^2u + \frac{\omega^2}{\varepsilon}(1-xv)] - \\ \frac{a_0}{2}(\omega_\nu^2 - P^2 + l^2)[\frac{k}{l^2}\sqrt{s}(k + xv\varepsilon) + v^2\varepsilon(1 + \frac{\omega\varepsilon(1-xv)}{l^2})u]\Big\}\,. \end{aligned} \tag{4.5}$$

Here we have denoted:

$$\begin{aligned} u = 1 - x^2(1 - \frac{\lambda^2}{\omega^2})\,,\quad k_m = \sqrt{(\Delta-\varepsilon)^2 - \lambda^2}\,,\quad \omega(k) = \sqrt{k^2+\lambda^2}\,, \\ l = \sqrt{k^2 + {p_e}^2 + 2p_e kx}\,,\quad s = (\omega - xvk)^2\,,\quad \omega_\nu = \Delta - \varepsilon - \omega\,, \end{aligned} \tag{4.6}$$

and the upper limit of the integration over x is

$$x_2(\omega, p_e) = \frac{1}{2\omega p_e}[(\omega_\nu + P)^2 - (p_e + \omega)^2] + 1 \le 1\,. \tag{4.7}$$

Herein, feasible further reducing the quantity (4.5), being rather complicated and time-consuming, runs nevertheless, as a matter of fact, along a conventional familiar way, so that there sees no reason to put out here all the calculations at full length; the outcome is incorporated into the final result (4.15) for the distribution (2.7), along with the contributions from all other diagrams in Fig. 1.

The contribution to the decay probability caused by (3.15) sets out as follows:

$$dW_P(\mathbf{P},\mathbf{p}_e,\mathbf{p}_\nu) = P^2 dP d\mathbf{n}_P d\mathbf{p}_e d\mathbf{p}_\nu \delta(\mathbf{P}+\mathbf{p}_e+\mathbf{p}_\nu)\delta(\Delta-\varepsilon-\omega_{\nu 0}(\mathbf{P},\mathbf{p}_e))\frac{\alpha G^2_{ud}}{4(2\pi)^8\varepsilon\omega_{\nu 0}}\times$$
$$\int d^4k[(iF(k)T^{\mu\lambda\alpha}S_{\mu\lambda\alpha})+(iF(k)T^{\mu\lambda\alpha}S_{\mu\lambda\alpha})^*]\,,\quad \mathbf{n}_P=\frac{\mathbf{P}}{P}\,. \qquad (4.8)$$

The evaluation of all the expression (4.8) comes out to be plain and unsophisticated except for the piece incorporating the integral

$$I = 2M_p i\int d^4kF(k) = \frac{M_p\pi^2}{2}J\,, \qquad (4.9)$$

the careful study of which is relegated to Appendix. In Eq. (4.8), all the emerging integrals of the type

$$\int d^4kF(k)k^n_\alpha k^m_\beta \;\; (n,m=0,1)\,, \qquad (4.10)$$

but one given by (4.9), are evidently real and their straightforward evaluating runs in a general way, with allowance for the relations $\Delta/M_N\to 0$, $(\Delta/M_N)\ln(\Delta/M_N)\to 0$ which hold true so far the nucleon mass is presumed tending to infinity, $M_N\to\infty$. Then, upon integrating (4.8) over $d\mathbf{n}_P d\mathbf{p}_e d\mathbf{p}_\nu$, the contribution caused by (4.8) into the distribution (2.7) results as

$$dW_P(P) = -dP\cdot P\frac{G^2_{ud}\alpha}{8\pi^2}\int\limits_{\varepsilon_1}^{\varepsilon_2} d\varepsilon\,\varepsilon\,\omega_{\nu 0}\times$$
$$\Big\{[\frac{2}{v}(\ln(\frac{\lambda}{m})\cdot\ln(x)-\mathcal{J})+2\ln(\frac{m}{M_p})]\cdot[(3g_A^2+1)+\mathcal{N}(1-g_A^2)]+$$
$$\frac{1}{v}\ln(x)[v^2(3g_A^2+1)+\mathcal{N}(1-g_A^2)]-\frac{1}{2}\Big(\frac{3}{2}+2\ln(\frac{\Lambda}{M_p})\Big)\times \qquad (4.11)$$
$$\Big(5+12g_A+15g_A^2+5\mathcal{N}(1-g_A^2)\Big)+[2+3g_A+3g_A^2+2\mathcal{N}(1-g_A^2)]\Big\}\,,$$

with allowance for all what is expounded in Appendix. Here

$$\mathcal{J} = [\Big(\frac{1}{2}\ln(x)\Big)^2-\mathcal{F}(1/x-1)+\pi^2\frac{v}{\tilde{v}_P(P,\varepsilon)}]\,,\quad x=\frac{1-v}{1+v}\,, \qquad (4.12)$$

$\mathcal{F}(z)$ is the Spens function [40]

$$\mathcal{F}(z) = \int\limits_0^z \frac{dt}{t}\ln(1+t),$$

and the quantity $\tilde{v}_P(P,\varepsilon)$ is defined by Eq. (A.21).

As was discussed above, see Sec. 1., the ad hoc effective cut-off parameter Λ emerges in (4.11) in order to prevent the ultraviolet divergency of the integrals

$$2M_p i\int d^4kF(k)k_\alpha k_\beta = -g^{\alpha\beta}\frac{\pi^2}{4}\Big(\frac{3}{2}+2\ln(\frac{\Lambda}{M_p})-2\delta_{0\alpha}\Big)\,, \tag{4.13}$$

which occur in calculating (4.8).

The contributions to the distribution (2.7) from the diagrams $a), e), f)$, that is, due to the first three terms in (3.11), when combined, result as

$$dW_s(P) = dP{\cdot}P\frac{G^2_{ud}}{4\pi^3}\int\limits_{\varepsilon_1}^{\varepsilon_2} d\varepsilon\,\varepsilon\,\omega_{\nu 0}[(3g_A^2+1)+\mathcal{N}(1-g_A^2)]\times$$
$$\frac{\alpha}{\pi}\Big(-\ln(\frac{\Lambda}{M_p})+\frac{1}{2}\ln(\frac{M_p}{m})-\frac{9}{4}-2\ln(\frac{\lambda}{m})\Big)\,. \tag{4.14}$$

Upon summarizing all the results (4.1), (4.5), (4.11), (4.14), after a good deal of plain, but slightly cumbersome calculations, the distribution (2.7) sets out in the eventual form

$$dW(P) = dW_0(P) + dP{\cdot}P\frac{G^2_{ud}\alpha}{8\pi^4}\int\limits_{\varepsilon_1}^{\varepsilon_2} d\varepsilon\,\varepsilon\,\omega_{\nu 0}\Big\{(1+3g_A^2)\big\{(1+a_0\mathcal{N})\times$$
$$\Big(\frac{2}{v}\mathcal{J}+3\ln(\frac{M_p}{m})-\frac{9}{2}-4\ln(\frac{2\omega_{\nu 0}}{m})(1+\frac{1}{2v}\ln(x))-\frac{2}{v}\mathcal{K}-$$
$$v^2\int\limits_{-1}^{1}dy\frac{1-y^2}{(1-vy)^2}\ln\Big(\frac{v\varepsilon+y\omega_{\nu 0}+\sqrt{\omega_{\nu 0}{}^2+v^2\varepsilon^2+2yv\omega_{\nu 0}\varepsilon}}{2v\varepsilon}\Big)\Big)$$
$$-\frac{1}{v}\ln(x)(v^2+a_0\mathcal{N})+\frac{v}{\omega_{\nu 0}}\{\int\limits_{-1}^{1}dx[v^2((\omega_{\nu 0}-\varepsilon)I_0-I_1)\frac{1-x^2}{b^2}+\frac{1}{b\varepsilon}(\omega_{\nu 0}I_1-I_2)]+ \tag{4.15}$$
$$\int\limits_{\tilde{k}(\varepsilon)}^{k(\varepsilon)}\frac{dk}{k}(\omega_{\nu 0}-k)[v^2(k+\varepsilon)(i_0-i_2)+\frac{k^2}{\varepsilon}(i_0-vi_1)]\}-$$
$$\frac{a_0v}{2\varepsilon\omega_{\nu 0}}[\int\limits_0^{\omega_{\nu 0}}dk(A(1)+B(1)+C(1))+\int\limits_{\tilde{k}(\varepsilon)}^{\omega_{\nu 0}}dk(A(x_2)+B(x_2)+D(x_2))]\big\}+$$
$$\frac{7}{4}+6g_A+\frac{33}{4}g_A^2+\ln(\frac{\Lambda}{M_p})(3\mathcal{N}(1-g_A^2)+9g_A^2+12g_A+3)\Big\}\,.$$

Here, the quantities $\omega_{\nu 0}\,,\mathcal{J},\mathcal{N},a_0,x_2,l^2$ have been defined afore, and the following notations are introduced anew:

$$\mathcal{K} = \frac{1}{2}(\mathcal{F}(x) - \mathcal{F}(1/x) - \ln(1/x)\cdot\ln(\frac{1-v^2}{4}))$$
$$-v + \frac{1}{2}\ln(x) + \mathcal{F}(v) - \mathcal{F}(-v),$$
$$\tilde{k}(\varepsilon) = (\omega_{\nu 0} + P - \varepsilon v)/2\,,\quad k(\varepsilon) = (\omega_{\nu 0} - P + \varepsilon v)/2\,,\quad b = 1 - xv\,,$$
$$i_n(k,\varepsilon) = \int\limits_{-1}^{x_2(k,\varepsilon)} \frac{dx x^n}{rb^2}\,,\quad I_n(\omega_{\nu 0}) = \int\limits_{0}^{\omega_{\nu 0}} \frac{dk k^n}{r}\,,$$
$$A(x) = (\omega_\nu^2 - P^2)\int\limits_{-1}^{x} \frac{dx\cdot d(x)}{l^3}\,,\quad B(x) = \int\limits_{-1}^{x} \frac{dx\cdot d(x)}{l}\,, \tag{4.16}$$
$$C(x) = \int\limits_{-1}^{x} \frac{dx 2\varepsilon v^2(1-x^2)}{lb^2}(xv\varepsilon - \omega_\nu)\,,\quad D(x) = \int\limits_{-1}^{x} \frac{dx}{lkb^2}(\omega_\nu^2 + l^2 - P^2)\varepsilon(1-x^2)v^2\,,$$
$$r(k,x) = \sqrt{v^2\varepsilon^2 + 2vkx\varepsilon + k^2}\,,\quad d(x) = [l^2 + \varepsilon^2(1 - xv^2) + k\varepsilon b - \varepsilon(\varepsilon + k)]/b\;.$$

For briefness's sake, we don't pull out explicitly the vast expressions of the integrals in (4.16), though they all are amenable to straightforward analytical evaluation.

Surely, upon integrating (2.7), (4.15) over dP within the limits (4.3), one gets the total decay probability, $W = 1/\tau$. Consequently, the quantity

$$\delta W = \int\limits_{P_1}^{P_2} [dW(P) - dW_0(P)] \tag{4.17}$$

stands for the effect of electromagnetic interactions on the total decay probability. The quantity (4.17) was evaluated earlier in Ref. [38], starting with the same Lagrangian (1.1)-(1.3), but by integrating the general expression (2.1) first of all over $d\mathbf{P}$ and then over $d\mathbf{p}_\nu d\mathbf{p}_\mathbf{e} d\mathbf{k}$, just the other way round as compare to the order of integrating in this work.

Having at our disposal the result (4.15), we are in position to ascertain the effect of electromagnetic interactions on the proton distribution (2.7), which is our outright objective.

5. The calculation of the proton distribution and discussion of the results.

Before to set forth the results of the numerical calculations, the main features of the ultimate formula (4.15) deserve to be spotlighted.

Surely, upon adding the contributions (4.11), (4.14), (4.1), (4.5) from all the diagrams in Fig. 1, the ad hoc artificial photon mass λ has disappeared from the final formula (4.15), amenably to the received handling of the infrared radiation; see, for instance, [1, 2, 29].

With the effective interaction (1.1)-(1.3) underling the inquiry, the ad hoc effective cut-off parameter Λ has emerged in order to prevent the ultraviolet divergencies which would come from integrating over the four-momenta of the virtual photons in the diagrams $d), e), f)$. As was proclaimed in Sec.1, we are not on the point of treating the whole problem how to remove the ultraviolet divergency out of the radiative corrections to the neutron $\beta-$decay. In the course of our upright calculation, we just take for granted the received recipe, first set forth in Refs. [13] and perfectly confirmed in the profound papers [14], which prescribes the Λ value to be equal to the mass of $Z-$ or $W-$boson. So, in the present work, as well as in the previous one [38], we presume $\Lambda = M_Z$ (or $\Lambda = M_W$) in the main calculations whose results are to be confronted with experimental data. For comprehension's sake, we set out also the results gained with $\Lambda = M_N$ which get us to realize several interesting features of the distribution (4.15). While the dependence of (4.15) on the Λ value is slight enough as being due to the terms $\sim \ln(\Lambda/M_p)$, the contribution of these terms into (4.15) is of value, so far the accuracy about 1% or better goes. What is to emphasize here is that the contribution $\sim \ln(\Lambda/M_p)$ in (4.15) would vanish at all, as in the case of the decay of the $\mu-$meson [41], if the relation $g_A = -g_V$ (i.e. $g_A = -1$ in the notations adopted here) held true, in perfect agreement with the general theorem ascertained in Ref. [42]. It might be well to point out that, till now, this stringent constrain has not been adhered to in many a calculation (see, for instance, [31, 32]).

It goes as a matter of course that the genuine quantity a (2.3) prefixed to $\mathbf{v}\mathbf{v}_\mathrm{e}$ in (2.2), rendering the electron-antineutrino correlation, will never occur in the distribution (2.7), (4.15). Moreover, considering the distribution (4.15), one realizes just away that beside the terms depending on the combination a_0 (2.4), there exist ones depending on g_A immediately, but not via a_0. The evaluations carried out make us realize that the contribution of these terms into (4.15) is not negligible as compare to the contribution of the terms which are multiple of a_0 (2.4). Thus, confronting the calculated distribution (2.7), (4.15) with the correspondent experimental data, we are not in position to acquire directly the quantity a itself, or even a_0, but can judge the value of g_A only, so far the accuracy about 1% or better goes.

Starting with the effective interaction (1.1)-(1.3), we have consistently evaluated, in the unified way, upright and straightforward, all the contributions into the distribution (4.15) from all the diagrams in Fig. 1, without treating separately the so called "Coulomb correction" and the "model-independent" and the "model-dependent" parts of the contribution from the diagram d). So, to the first $\alpha-$order, we have acquired the complete effect of electromagnetic interactions on the proton distribution (2.7). The final result (4.15) stands in one-to-one correspondence with the form of the original effective interaction (1.1)-(1.3), being immediately expressed just in terms of the quantities $G_{ud}, g_A, |V_{ud}|, ...$ involved in (1.2).

In the calculations (see, for instance, [30–36]) pursuing the approach launched long ago by Ref. [37], the whole contribution from the diagram d) in Fig. 1, upon rather artificial extracting the "Coulomb correction", is divided into the "model-dependent" and "model-independent" parts. In this, strictly speaking, ambiguous separating, the "model-independent" part of the amplitude $M_{2\gamma}$ (3.4) is chosen to be a multiple of M_0 (3.1). Subsequently, the corrected eventual distribution (4.15) would be a multiple of the uncorrected one (4.1), which is apparently not our case. In that approach, the whole effect of the remained "model-dependent" part is presumed to be absorbed into the quantities $G_{ud}, g_V, g_A, ...$ in (1.2) determining (4.1), so that they have got the new values $G'_{ud}, g'_V, g'_A, ...$ instead of the original ones in (1.2). Thus, the experimental data show up to be described in terms of these "new" quantities. However, any strict quantitative one-to-one correspondence between $G'_{ud}, g'_V, g'_A, ...$ and $G_{ud}, g_V, g_A, ...$ has never been asserted explicitly and definitely in the aforecited investigations [30–37]. But the guide tenet is to inquire outright into the original effective interaction in order to ascertain, as precisely as possible, the genuine values of the quantities in (1.2). In particular, we are in need of the stringent $|V_{ud}|$ value in order to judge the validity of the CKM identity (1.5) [5]. So, the aforesaid calculations [30–37], making use of the very handy, but rather untenable simplifications, cannot be said to be eligible for now. Consequently, we perform our calculation accordingly to the result (4.15) which lumps together all the effect of electromagnetic interactions on the distribution (2.7) up to the first $\alpha-$order.

The main results to discuss, depicted by the solid curves in Figs. 2-4, are obtained with $\Lambda = M_Z = 94\, GeV$; the value g_A=1.2662 is adopted [4,16]. At first, we merely offer in Fig. 2 , for the sake of comprehension, the relative distribution

$$\frac{dW(P)}{dP}\cdot\frac{1}{W}, \tag{5.1}$$

where W stands for the total decay probability which is obtained by integrating (4.15) over dP ,

$$W = \int\limits_{P_1}^{P_2} dP\Big(\frac{dW(P)}{dP}\Big), \tag{5.2}$$

the limits P_1, P_2 given by (4.3). As a matter of fact, what is measured in the experiments [20–22] is just the quantity displayed in Fig. 2. In Fig. 3, the solid curve sets forth the modification (in percent) of the quantity depicted in Fig. 2, namely,

$$\delta\Big(\frac{dW(P)}{dP}\cdot\frac{1}{W}\Big) = \Big(\frac{dW(P)}{dP}\cdot\frac{1}{W} - \frac{dW_0(P)}{dP}\cdot\frac{1}{W_0}\Big)/\Big(\frac{dW_0(P)}{dP}\cdot\frac{1}{W_0}\Big), \tag{5.3}$$

where the quantity $dW_0(P)/dP$ stands for the uncorrected proton distribution (4.1), and

$$W_0 = \int\limits_{P_1}^{P_2} dP\Big(\frac{dW_0(P)}{dP}\Big) \tag{5.4}$$

is the total uncorrected decay probability. The solid curve in Fig. 4 offers the modification (in percent) of the proton distribution itself:

$$\delta\Big(\frac{dW(P)}{dP}\Big) = \Big(\frac{dW(P)}{dP} - \frac{dW_0(P)}{dP}\Big)/\Big(\frac{dW_0(P)}{dP}\Big). \tag{5.5}$$

Certainly, the modification of the total probability evaluated through (4.15), (4.1),

$$\delta W = \int\limits_{P_1}^{P_2} dP\Big(\frac{dW(P)}{dP} - \frac{dW_0(P)}{dP}\Big)/\int\limits_{P_1}^{P_2} dP\Big(\frac{dW_0(P)}{dP}\Big), \tag{5.6}$$

comes out to be strictly equal to the value $\delta W = 8.05\%$ obtained earlier in the Ref. [38], starting with the same effective interaction (1.1)-(1.3), but performing calculations in a different way.

All the results obtained make us realize that the whole effect of electromagnetic interactions amounts to several percent. Nowadays, this is of value to ascertain with a high accuracy the genuine form of the effective interaction (1.1)-(1.3), especially to gain the strict g_A value, in processing the experimental data of [20–22]. Apparently, this conclusion is seen to be appreciably different as compare to one asserted in Refs. [30, 35, 36], where the radiative corrections were inferred to be rather beyond interest in studying the proton momentum distribution (2.7). Bearing in mind what has been expounded afore, we realize the primary origin of this mismatch and see no reason to be confounded.

The difference between the results obtained with utilizing $\Lambda = M_Z = 94MeV$ (presented in Figs. 2-4) and ones with $\Lambda = M_W = 80MeV$ comes out to be less than 0.1%, alike what was inferred in the previous work [38]. So, for now, there sees no practical difference between the calculations with $\Lambda = M_Z$ and with $\Lambda = M_W$. The short-dashed curves in the pictures render the results obtained with assuming $\Lambda = M_N$ instead of $\Lambda = M_{Z,W}$ the solid curves were obtained with. As seen, the deviations are visible. Especially, they are noticeable for the quantity (5.5) depicted in Fig. 4 and for the quantity (5.6) whose value at $\Lambda = M_N$ comes out to be $\delta W = 5.04\%$ as gained before in Ref. [38].

Certainly, the various terms in the distribution (4.15) are of different species and significance. To the best of our believe, it is pretty much relevant to visualize the effect of the very term $\pi^2 v/\tilde{v}_P$ in (4.11), (4.12), (4.15), which is scrutinized in Appendix, on the whole results depicted in Figs. 2-4. The results of the calculations without allowance for this term $\pi^2 v/\tilde{v}_P$ are displayed in Figs. 3,4 by the long-dashed lines. In appearance, their shape shows up to be utterly unlike the shape of the solid curves standing for the main results obtained amenably to the complete formula (4.15).

Let us now pay attention to some curious peculiarity which shows up on the very curves at $P = \tilde{P} = \Delta - m = 0.7833MeV$. Surely, this tiny cusp isn't visible in the shape of the curve in Fig. 2. When P tends to this value $\tilde{P}$, the lower limit of the integration over $d\varepsilon$ in (4.15) evidently tends to m, $\varepsilon_1 \rightarrow m$. Consequently, there appears $v \rightarrow 0$ at the lower limit, whereas the quantity $\tilde{v}_P$ (A.21) remains though very small, but still finite, $\tilde{v}_P \rightarrow (\Delta - m)/M_p$, so far we don't presume at this point the infiniteness of the proton mass (see Appendix). To all intents and purposes, what rules the behaviour of the distribution (4.15) in the vicinity of $\tilde{P}$ is just the part of (4.15) containing the very quantity $1/\tilde{v}_P$, namely,

$$\frac{d\tilde{W}(P)}{dP} = \tilde{w}(P) = \frac{G_{ud}^2 \alpha}{8\pi^4} P(1+3g_A^2) \int\limits_{\varepsilon_1}^{\varepsilon_2} d\varepsilon\, \varepsilon\, \omega_{\nu 0}(1 + a_0 \mathcal{N}(\varepsilon, P)) \frac{2\pi^2}{\tilde{v}_P}\,, \tag{5.7}$$

so far the remainder of (4.15) is seen to be very flatten at $P{\sim}\tilde{P}$ (the long-dashed lines in Figs. 3,4). First thing, what is to emphasize here is that, even on replacing $\tilde{v}_P = v$, the function (5.7) does not diverge at $P{\to}\tilde{P}$, $\varepsilon_1{\to}m$, when $v{=}(\sqrt{\varepsilon^2 - m^2}/\varepsilon){\to}0$ at the lower limit, $\varepsilon = \varepsilon_1$. Moreover, the difference between the results obtained with $\tilde{v}_P$ and with replacing $\tilde{v}_P \doteq v$ comes out to be as good as negligible, amounting to $\sim$0.3% to the whole radiative correction itself. For that matter, it is instructive to recall that the electron momentum distribution, treated in Ref. [38], includes the quantity (A.18) which would apparently diverge with replacing $\tilde{v}{=}v$, when $\varepsilon{\to}m$, that is, at the end of the electron spectrum.

Though the values of the function (5.7) itself with v and $\tilde{v}_P$ do coincide practically, the derivatives of (5.7) for these two cases show up to be quite different. Indeed, the term ruling the behaviour of the derivative of (5.7) at $P = \tilde{P} + \delta$, $\delta{\to}\pm0$ stems from differentiating the function (5.7) with respect to the lower limit, namely,

$$\frac{d\tilde{w}(P)}{dP}\Big|_{P=\tilde{P}+\delta} \approx \frac{G_{ud}^2 \alpha}{8\pi^4} \tilde{P}(1+3g_A^2)(1 + a_0\mathcal{N})m(\Delta - m)2\pi^2 \frac{\delta}{m} \cdot \frac{1}{\tilde{v}_{\tilde{P}}}\,. \tag{5.8}$$

As a matter of fact, the quantity (5.8) renders at $P{\sim}\tilde{P}$ the whole derivative of (4.15), so far all the quantity (4.15), but the term (5.7), is as good as constant in the vicinity of $\tilde{P}$, as one recognizes considering the long-dashed curves in Figs. 3, 4. With $\tilde{v}_{\tilde{P}}{\to}(\Delta - m)/M_p$ being very small but finite, the quantity (5.8), remaining continuity, changes its sign at $\delta = 0$. So, the function (5.7) itself, and thereby the very quantity (4.15), having got the very sharp maximum at $P = \tilde{P}$, do not suffer a fracture. Yet, when $\tilde{v}_P$ was replaced by v, the behaviour of the derivative (5.8) and, consequently, of the function (5.7) itself would become quite different. In fact, as $v = |\delta|/m$ at $P = \tilde{P} + \delta$, the derivative (5.8) would suffer discontinuity at $P = \tilde{P}$, with its sign changing when $\delta{\to}\pm0$. Consequently, in the case when $\tilde{v}_P$ was replaced by v, the very function (5.7) and, subsequently, (4.15) itself would have got a fracture at $P = \tilde{P}$.

Thus, the properties of the distribution (4.15), especially its dependence on P, have been thoroughly expounded.

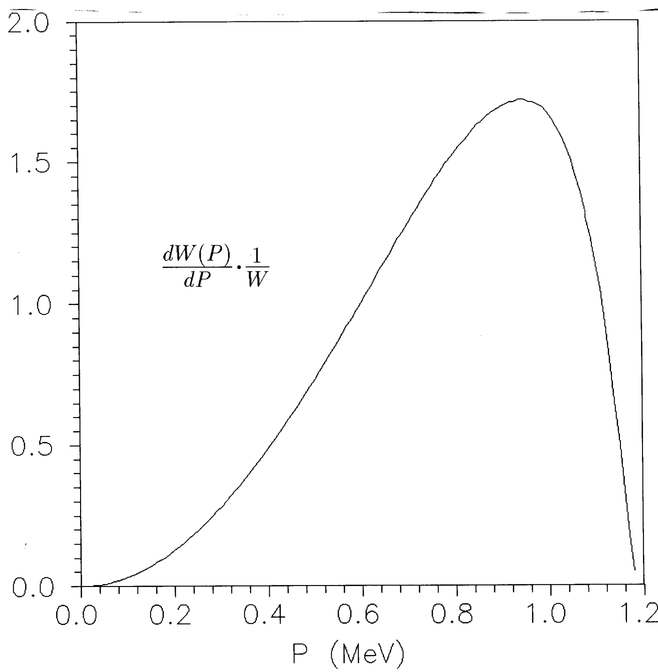

Fig. 2. The relative distribution (5.1) of protons with absolute value of the momentum $P = |\mathbf{P}|$ obtained accordingly (4.15) with $\Lambda = M_Z = 94 GeV$.

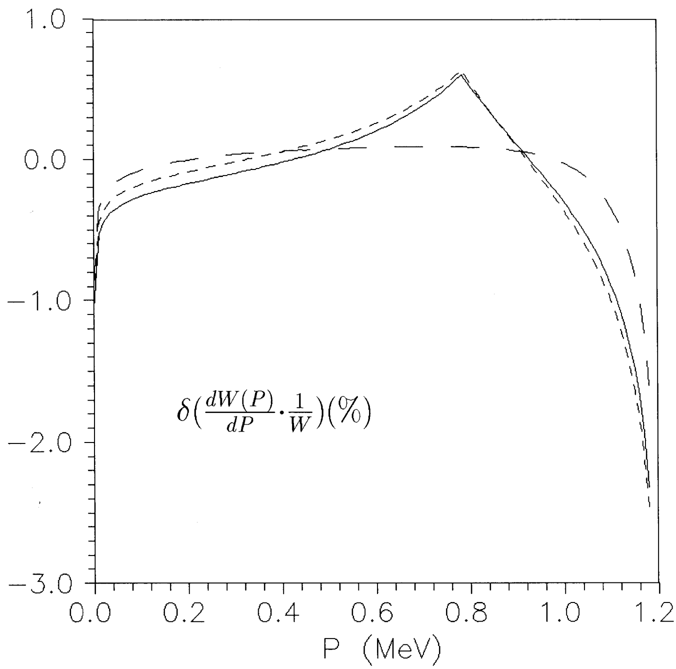

Fig. 3. The modification (5.3) (in per cent) of the relative distribution (5.1) of protons (5.2) owing to electromagnetic interactions. The solid and short-dashed curves represent the results obtained accordingly to (4.15) with $\Lambda = M_Z = 94GeV$ and with $\Lambda = M_N = 939MeV$, respectively. The long-dashed curve stands for the result obtained with $\Lambda = M_Z$, the term with $1/\tilde{v}_P(P,\varepsilon)$ extracted from (4.11), (4.12), (4.15).

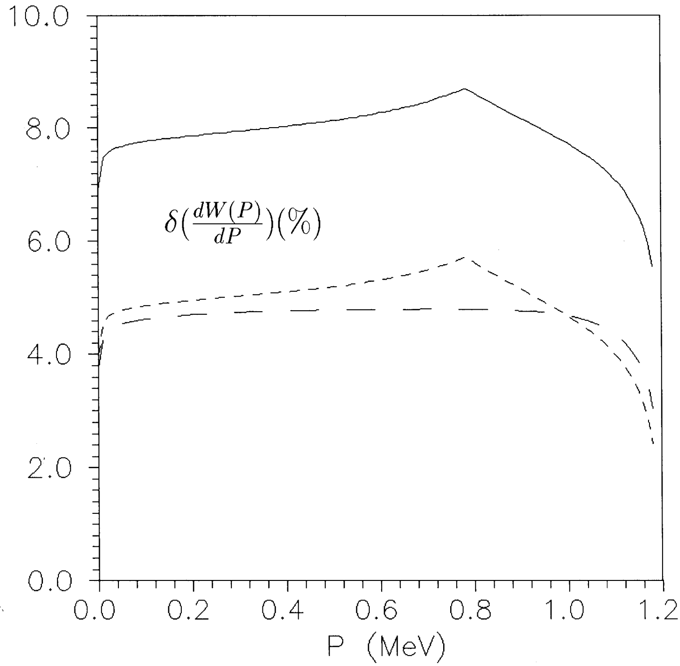

Fig. 4. The modification (5.5) (in per cent) of the proton distribution owing to electromagnetic interactions. The solid, short-dashed and long-dashed curves have the same meaning as in the Fig. 3.

The results obtained bring to light that if we are in need to ascertain the g_A value with an accuracy of $\sim$1% or better from the high-precision measurements of the proton spectra, the effect of electromagnetic interactions is bound to be properly accounted for. As to the value of the quantity a itself in (2.2), or even a_0 in (2.5), thy can't be immediately gained in studying the distribution (2.7), (4.15), so far a precision of $\sim$1% or better goes. Surely, it is to recall that the corrections to the distribution (4.1) owing to the proton mass finiteness [26] must be properly taken into consideration as well, which, in turn, entails the corrections to (4.1) caused by the terms with g_{WM}, g_{IP} in (1.2).

6. Concluding remarks.

Despite the study of neutron $\beta-$decay has been lasting long since, properly speaking, for all the time of neutron physics itself existing, it was restricted until a little while ago by the high-precision reliable inquiry into the neutron lifetime τ [6] and the electron momentum distribution (2.2) [16] only, as a matter of fact. Indeed, registering antineutrinos is apparently unattainable, past questions, and all the attempts to deal with the proton distributions (2.6), (2.7) [18, 20] have turned out to be rather untenable by now, all the more that a precision of $\sim$1% or better goes we are in need to-day. As one might reword, only the lifetime τ and the coefficient A in the distribution (2.2) have been procured safely from experimental data processing, until now.

Nowadays, the situation is thought to alter towards the study of the proton distribution in the final state of neutron $\beta-$decay. The various new experiments [23, 24] as well as the substantial improvements of some previous ones [21, 22] have recently been launched in order to obtain, with an accuracy better than 1%, both the proton distribution itself solely and the electron and proton distributions simultaneously; the appropriate measurements are believed to come to fruition before long. Consequently, obtained such high-precision experimental data, the observed proton distributions are to be evaluated, in turn, with the same high accuracy outright in terms of the received effective Lagrangian (1.1)-(1.3), underlying the inquiry. It is what this work has dealt with accordingly the tenet proclaimed from the very first.

For that matter, it is to expound here that the corrections entailed by the finiteness of the nucleon mass were thoroughly elaborated in Ref. [26] just to the distribution (2.2), but not in the least to the distributions (2.6) and (2.7). So, such corrections are to be re-evaluated just to the very distributions (2.6) and (2.7) in order to acquire the high-precision values of the quantities $G_{ud}, |V_{ud}|, g_A, ...$ in (1.2) by processing the correspondent experimental data [21–24].

What is to realize here is that we shall never deal with the coefficients B or a in (2.2) immediately, but only with the quantities $G_{ud}, |V_{ud}|, g_V, g_A, ...$ in (1.2) themselves, so far antineutrinos and $\gamma-$rays are left beyond registering in studying such conceivable proton or proton-electron distributions. It stands to reason that the triple proton-electron-$\gamma-$ray distributions, if observed, could enable us to reconstruct the antineutrino kinematics and, subsequently, to try the B and a values with an enough precision, as was expounded in Ref. [25].

Surely, to repose full confidence in the values of $G_{ud}, |V_{ud}|, g_A, g_V, ...$ in (1.2), the results acquired from processing the data of the various aforecited, see Sec. 2, experiments must be properly compared and investigated simultaneously; the prescription of such recapitulation was offered in Ref. [43].

The methods of calculation elaborated and applied in studying the distribution (2.7), (4.15) are certain to be relevant and eligible in treating other phenomena, where the proton distribution goes, and we are on the point of pursuing this way in our subsequent work.

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Appendix.

Here, we pick out from the general expression (4.8) the piece including the real part of the quantity (4.9), especially interesting and worth-while for an inquiry,

$$dW_M(\mathbf{P},\mathbf{p}_e,\mathbf{p}_\nu) = P^2 dP d\mathbf{n}_P d\mathbf{p}_e d\mathbf{p}_\nu \delta(\mathbf{P}+\mathbf{p}_e+\mathbf{p}_\nu)\delta(\Delta-\varepsilon-\omega_{\nu 0})\frac{G_{ud}^2\alpha\varepsilon}{\pi^3(2\pi)^5}(\Re\mathbf{e}I)\times$$
$$\Big(1+3g_A^2+\mathbf{v}\mathbf{v}_\nu(1-g_A^2)+2g_A\mathbf{v}\boldsymbol{\xi}(1-g_A)+2g_A\mathbf{v}_\nu\boldsymbol{\xi}(1+g_A)\Big)\,, \quad (A.1)$$

which stems out of the non-relativistic part of the proton propagator (3.6), with $Q_0+M_p\to 2M_p$ assumed therein.

Accordingly [1, 28, 29], the integral J in (4.9) reduces as follows

$$J=\int_{-1}^{1}\frac{dt}{p_t^2}\ln\Big(\frac{p_t^2}{\lambda^2}\Big)\,, \quad (A.2)$$

where $2p_t=(1+t)p_e+P(t-1)$. In (A.2) and hereafter in Appendix, $P=(E_P,\mathbf{P})$, $p_e=(\varepsilon,\mathbf{p}_e)$ denote the proton and electron four-momenta, respectively. Surely, in further calculating J (A.2), we are to retain only the terms whose contributions into I (4.9) do not vanish when the nucleon mass tends to infinity.

With the roots

$$t_{1,2}=-\frac{m^2-M_p^2\pm\sqrt{(Pp_e)^2-4M_p^2m^2}}{m^2+M_p^2+2(Pp_e)} \quad (A.3)$$

of the equation

$$p_t^2=0\,, \quad (A.4)$$

both positive, $t_1,t_2<1$ and $t_2\geq t_1$, the consequent singularities of the integrand in (A.2) reside within the range of integration. As usually, see, for instance, [1, 28, 29], these singularities are to be treated by adding an infinitesimal negative imaginary part to p_t^2 in (A.2); then the singularities of the integrand in (A.2) are at t_2+i0 and t_1-i0. We also choose the branch cuts for the logarithms of the type $\ln(p_t^2)$, occurring in (A.2) and hereafter, so that they do not cross the real axis; the branch must be chosen so that $\Im\mathbf{m}\,[\ln(p_t^2)]=0$ for $t=1$ or $t=-1$. That is what we are to keep in mind hereinafter, in course of the subsequent calculations.

Then, by introducing the new variable

$$p_z^2=\frac{1}{4}(p_e(z+1)+P(1-z))^2=\frac{1}{4}(m^2+M_p^2+2p_eP)(1-t_1z)(1-t_2z)\,, \quad (A.5)$$

the integral (A.2) proves to be transformed as follows

$$J = \int\limits_{-\infty}^{\infty} \frac{dz}{p_z^2} \ln \frac{p_z^2}{\lambda^2} - \int\limits_{-1}^{1} \frac{dz}{p_z^2} \ln \frac{p_z^2}{\lambda^2} + \int\limits_{-1}^{1} \frac{dt}{p_t^2} \ln(t^2) \,. \tag{A.6}$$

Let us now focus on the first integral in (A.6) which reduces to

$$J_1 = \frac{1}{A} \int\limits_{-\infty}^{\infty} \frac{dz}{(1-t_1 z)(1-t_2 z)} [\ln \frac{A}{\lambda^2} + \ln(1-t_1 z) + \ln(1-t_2 z)] \,, \tag{A.7}$$

where $A = (m^2 + M_p^2 + 2p_e P)/4$. The infinitesimal imaginary shifts of the singularities, $t_2 + i0$, $t_1 - i0$, give rise to the imaginary part of the integral (A.7). Yet, so far the desirable distribution (A.1) does depend just on $\Re \mathbf{e} I$, the emerged imaginary part of (A.7) will not contribute to (A.1) at all. Integrating the term containing $\ln(A/\lambda^2)$ in (A.7) results in the pure imaginary value

$$\frac{1}{A} \ln \frac{A}{\lambda^2} \cdot \frac{2\pi i}{t_2 - t_1} \,. \tag{A.8}$$

With allowance for the discussion set out before Eq. (A.5), the rest of the integral (A.7) transforms to

$$\frac{2\pi i}{A} \cdot \frac{1}{t_2 - t_1} \cdot \ln[(t_2 - t_1)^2 (-\frac{1}{t_2 t_1} - i0 \cdot (t_2 - t_1))] = \frac{2\pi i}{A} \cdot \frac{1}{t_2 - t_1} [-i\pi + \ln \frac{|t_2 - t_1|^2}{|t_2 t_1|}] \,. \tag{A.9}$$

So, the real part of the integral (A.7) we are in need of is merely

$$\Re \mathbf{e} J_1 = \frac{2\pi^2}{A} \cdot \frac{1}{t_2 - t_1} \,. \tag{A.10}$$

If anything, it is to note that when we assumed for the positions of the singularities $t_2 - i0$, $t_1 + i0$ instead $t_2 + i0$, $t_1 - i0$, the imaginary part of (A.7) (which are, though, beyond our need here) would change its sign, whereas the actually desirable real part (A.10) would not modify at all.

In the second integral in (A.6), the integrand has got no singularities within the range of integration. The straightforward evaluation of this integral results in

$$\frac{2}{v M_p \varepsilon} \Big(\ln \frac{m}{\lambda} \cdot \ln(x) + \frac{1}{4} (\ln(x))^2 - \mathcal{F}(1/x - 1) \Big) \,, \quad x = \frac{1-v}{1+v} \,, \tag{A.11}$$

where $\mathcal{F}(z)$ is the Spens function [40] (4.12).

Recalling the quantity I is a multiple of $2M_p$, the contributions from (A.10), (A.11) into the decay probability (A.1) do not vanish when $M_p \to \infty$, unlike the contribution into (A.1) from the third integral in (A.6) which proves to be proportional to

$$\sim \frac{\pi^2}{v\varepsilon}\int\limits_0^1 dt\cdot \ln(t)\,[\frac{1}{t-1-\delta_2}-\frac{1}{t-1-\delta_1}]\,, \qquad (A.12)$$

where $\delta_{1,2}=-2\varepsilon(1\pm v)/M_p$. Indeed, after variable change $t=(1+\delta_{1,2})(1+z_{1,2})$, the integrals in (A.12) transform as

$$\int\limits_{-1}^{-\delta_i/(1+\delta_i)} \frac{dz}{z}\ln[(1+\delta_i)(1+z)]\,, \qquad (A.13)$$

and then the Eq. (A.12) reduces to

$$\frac{2\pi^2}{M_p}[2\ln\frac{2(1+v)\varepsilon}{M_p}-(1+\frac{1}{v})\ln(x)]\,, \qquad (A.14)$$

which apparently tends to zero when $M_p\to\infty$, even in the very case $v\to 0$. So, the last integral in (A.6) does not contribute to (A.1) and, consequently, to (4.9). Thus, after all, we left with

$$\Re\mathrm{e}I=\frac{M\pi^2}{2}\Re\mathrm{e}J=\frac{\pi^2}{v\varepsilon}[\ln\frac{m}{\lambda}\cdot\ln(x)+(\frac{1}{2}\ln(x))^2-\mathcal{F}(1/x-1)+\frac{M_p\pi^2}{A}\cdot\frac{v\varepsilon}{t_2-t_1}]\,, \qquad (A.15)$$

which specifies the value of (A.1).

By integrating (A.1) over $d\mathbf{P}d\mathbf{p}_\nu$ one gets the contribution to the electron momentum distribution with respect to $\boldsymbol{\xi}$, studied in the previous work [38]. The contribution of (A.1) into the antineutrino momentum distribution would be acquired by integrating (A.1) over $d\mathbf{P}d\mathbf{p}_e$. Surely, upon integrating (A.1) over $d\mathbf{P}d\mathbf{p}_e d\mathbf{p}_\nu$ and averaging over the neutron polarizations we obtain the contribution to the total decay probability. In order to acquire the distribution of protons with a given $|\mathbf{P}|$ value we are to integrate (A.1) over $d\mathbf{n}_P d\mathbf{p}_e d\mathbf{p}_\nu$. In integrating the first three terms in (A.15), there occur no hitches and complexities. Let us focus on integrating the last term in (A.15). What is of value to realize here is that there would emerge the divergency at $v\to 0$ if we merely put in this term $M_p=\infty$, without any special cautions. Indeed, on integrating this term over $d\mathbf{n}_P d\mathbf{p}_\nu$ we get the value

$$d\mathbf{p}_e d|\mathbf{P}|\frac{|\mathbf{P}|\,2\pi^5\omega_{\nu 0}}{v\varepsilon}\Big(v^2\varepsilon^2-m\frac{\omega_{\nu 0}^2}{M_p}+\frac{m\mathbf{P}^2}{M_p^2}(m+M_p)\Big)^{-1/2}\,, \qquad (A.16)$$

which apparently would diverge when we put $M_p\to\infty$, $v\to 0$ simultaneously. Thus, this is one and only point in all the course of our calculation where the presumption $M_p\to\infty$ shows up to be self-contradicting, though in the very special case $v \to 0$. Given an ε value, the value of $|\mathbf{P}|$ varies within the limits

$$\omega_{\nu 0} - p_e \leq |\mathbf{P}| \leq \omega_{\nu 0} + p_e \,. \tag{A.17}$$

Then, upon integrating (A.1) over $d\mathbf{P}d\mathbf{p}_\nu$ with accounting for the limits (A.17), the contribution due to (A.15) into the correspondent electron momentum distribution (2.2) proves to be

$$\frac{G_{ud}^2 \alpha \omega_{\nu 0}^2 d\mathbf{p}_e}{8 v \pi^5}[(1+3g_A^2) + (\mathbf{v}\boldsymbol{\xi})2g_A(1-g_A)] \times$$
$$\Big(\ln\frac{m}{\lambda}\ln(x) + (\frac{1}{2}\ln(x))^2 - \mathcal{F}(1/x - 1) + \pi^2\frac{v}{\tilde{v}}\Big) \,, \tag{A.18}$$

where

$$\tilde{v}(\varepsilon) = \frac{1}{2}\left(\sqrt{(v + \frac{m\omega_{\nu 0}^2}{M_p \varepsilon})^2 + 2v\frac{\omega_{\nu 0}}{\varepsilon}(\frac{m}{M_p})^2} + \sqrt{(v - \frac{m\omega_{\nu 0}^2}{M_p \varepsilon})^2 - 2v\frac{\omega_{\nu 0}}{\varepsilon}(\frac{m}{M_p})^2}\right) \,, \tag{A.19}$$

with no divergency occurring herein. The replacement $\tilde{v} \to \sqrt{v^2 + \omega_{\nu 0}^2/M_p^2}$ is seen to be rather a good approximation [38].

Next, given a $|\mathbf{P}|$ value, the value of ε varies within the limits (4.2), the $|\mathbf{P}|$ value itself varying within the limits (4.3). So far neutron is unpolarized, there are no contributions from the terms including $\boldsymbol{\xi}$, so that the desirable contribution from (A.1) into the proton distribution (2.7), (4.15) results as follows

$$d\mathbf{P}\cdot|\mathbf{P}|\frac{G_{ud}^2 \alpha}{4\pi^4}\int_{\varepsilon_1}^{\varepsilon_2} d\varepsilon \frac{\varepsilon \omega_{\nu 0}}{v}[\ln\frac{m}{\lambda}\cdot\ln(x) + (\frac{1}{2}\ln(x))^2 - \mathcal{F}(1/v - 1) + \pi^2\frac{v}{\tilde{v}_P}] \times$$
$$\Big((1+3g_A^2) + (1-g_A^2)\mathcal{N}(P,\varepsilon)\Big) \,, \tag{A.20}$$

where

$$\tilde{v}_P(|\mathbf{P}|,\varepsilon) = \sqrt{v^2 - \frac{m}{M_p \varepsilon^2}[\omega_{\nu 0}^2 - \mathbf{P}^2(\frac{m}{M_p} + 1)]} \,. \tag{A.21}$$

Let us recall that the terms in (A.18) and (A.20) containing $1/\tilde{v}, 1/\tilde{v}_P$ which originate from (A.10) are usually associated with the so called "Coulomb correction" (see, for instance, [31–33, 44, 45]). What is of value to emphasize here is that these terms have been wrought up in our treatment, simultaneously with all other electromagnetic corrections, in upright consistent evaluating the contribution from the diagram *d*), Fig. 1, dictated, in turn, directly by the original effective interaction (1.1)-(1.3).

Here, it is relevant to point out that the method of Refs. [1, 28, 29] we pursue here is appropriate to describe consistently, including the "Coulomb interaction", the electromagnetic effect only in the very case where electron and proton are in continuum, but never form a bound state, neither real or even virtual. In summarizing the contributions from the processes to an arbitrary-high α-order in frame-work of this method, the multiple electron-proton re-scattering with vanishing relative momenta, leading to the formation of a (Pe^-) bound-state, isn't allowed for. Thus, this approach provides true handling the infrared $\gamma-$radiation accompanying $\beta-$decay [38], but does not describe the feasible production of the (Pe^-) bound-state, that is, an H-atom. As one may reword, a certain species of the Bethe-Solpeter equation ought to be drawn into consideration in order to reproduce such bound-state formation in calculating the neutron $\beta-$decay, but this is not our task here, all the more that the probability of the H-production is known to be negligibly small [46].

References

[1] V.B. Berestezky, E.M. Lifshitz and L.P. Pitajevsky, *Relativistic Quantum Field Theory, part I*, Nauka, Moscow (1971).

E.M. Lifshitz and L.P. Pitajevsky, *Relativistic Quantum Field Theory, part II*, Nauka, Moscow (1971).

[2] E.D. Commins and P.H. Bucksbaum, *Weak Interactions of Leptons and Quarks*, Cam. Univ. Press, Cambridge England (1983).

E.D. Commins, *Weak Interactions*, McGraw-Hill Book Company, New York (1973).

P.H. Frampton and W.K. Tung, Phys. Rev. D **3** 1114 (1971).

[3] C. Itzykson and J.-B. Zuber, *Quantum Field Theory*, McGgaw-Hill book company, New York (1981).

[4] Rev. Part. Prop., Phys. Rev. D **50** (Part I) 1177 (1994).

[5] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

M. Kobajashi and T. Maskawa, Prog. Theor. Phys. **49**, 625 (1973).

[6] A. Pichlmeier, P. Geltenbort, V. Nesvizhevsky et al., in *Proc. of ISINN-6*, E3-98-202, JINR, Dubna, Russia, 15-18 May, 1998, p. 220.

S. Arzumanov, L. Bondarenko, S. Chernyavski et al., in *Proc. of ISINN-5*, E3-97-213, JINR, Dubna, Russia, 14-17 May, 1997, p. 53.

K. Schreckenbach and W. Mampe, J. Phys. **G12**, 1 (1992).

W. Mampe, P. Aregon, J. C. Bates et al., NIM **A284**, 111, (1989).
W. Mampe, L. Bondarenko, V.I.Morosov et al., JETP Lett. **57**, 82 (1993).

J. Byrne, P. G. Dawber, C. G. Habeck et al., Europhys. Lett. **33**, 187 (1996).

[7] A. P. Serebrov and N. V. Romanenko, JETP Lett. **55**, 503 (1992).

[8] B. R. Hostein and S. B. Treiman, Phys. Rev. D **16**, 2369 (1977).

M. A. Beg, R. V. Budny, R. Mohapatra et al., Phys. Rev. Lett. **38**, 1252 (1977).

[9] R. N. Mohapatra and D. P. Sidhu, Phys. Rev. Lett. **38**, 667 (1977).

R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975).

[10] *Time Reversal Invariance and Parity Violation in Neutron Reactions*, Dubna, Russia, 4-7 May 1993, ed. by C.R. Gould, J.B. Bowman, Y.P. Popov, World Scientific, Singapure (1994).

L. Beck, K. Schrekenbach, T. Soldner et al., in *Proc. of ISINN-5*, E3-97-213, JINR, Dubna, Russia, 14-17 May, 1997, p. 199.

[11] T. Kinoshita and A. Sirlin, Phys. Rev. **113** 1652 (1959).

[12] S.M. Berman and A. Sirlin, Ann. of Phys. **20** 20 (1962).

[13] T.D. Lee, Phys. Rev. **128** 899 (1962).

R.A. Shaffer, Phys. Rev. **128** 1452 (1962).

G. Dorman, Nuov. Chim. **32** 1226 (1964).

D. Bailin, Phys. Rev. **135** B166 (1964).

[14] A. Sirlin, Nucl. Phys. B **71** 29 (1974); Nucl. Phys. B **100** 291 (1975); Nucl. Phys. B **196** 83 (1982); Rev. Mod. Phys. **50** 573 (1978); Phys. Rev. D **22** 971 (1980).

[15] S. Weinberg, Phys. Rev. Lett. **19** 1264 (1967); Rev. Mod. Phys. **46** 255 (1974).

A. Salam, *Elementary Particle Theory: Relative Groups and Analyticity (Nobel Symposium No. 9)* ed. by N. Svartholm (Stockholm 1968 Almquist and Wiksells) p. 367.

[16] J. Reich, H. Abele, M. A. Hoffman et al., in *Proc. of ISINN-6*, E3-98-209, JINR, Dubna, Russia, 15-18 May, 1998, p. 226.

H. Abele, S. Bäßler, D. Dübbers et al., Phys. Lett. B **407**, 212, (1997).

P. Liaud, K. Schreckenbach, R. Kossakowski et al., Nucl. Phys. A **612**, 53 (1997).

K. Schreckenbach, P. Liaud, R. Kossakowski et al., Phys. Lett. B **349**, 427 (1995).

[17] B. G. Yerosolimsky, Uspehy Fiz. Nauk **116**, 145 (1975).

B. G. Yerosolimsky, L. N. Bodarenko, Yu. A. Mostovoj et al., Yad. Fiz. **8**, 176 (1968).

B. G. Yerosolimsky, L. N. Bondarenko, Yu. A. Mostovoj et al., Yad. Fiz. **12**, 323 (1970).

[18] I. A. Kuznetsov, A. P. Serebrov, I. V. Stepanenko et al., Phys. Rev. Lett. **75**, 794 (1995).

I. A. Kuznetsov, A. P. Serebrov, I. V. Stepanenko et al., JETP Lett. **60**, 311 (1994).

A. P. Serebrov, I. A. Kuznetsov, I. V. Stepanenko et al., JETP **113**, 1963 (1998).

[19] P. Riehs, Acta Phys. Austriaca **27**, 205 (1968).

[20] Ch. Stratowa, P. Dobrozemsky, P Weinzierl, Phys. Rev. D **18**, 3970 (1978).

[21] J. Byrne, P. G. Dawber, S.R. Lee, NIM **A439**, 454 (1994).

[22] J. Byrne, P.G. Dawber, C. Habeck at al., ILL Experimental Report, Experiment $\mathcal{N}$ 3-07-85, Instrument PL1. Date of report: Feb. the 10th, 1998.

J. Byrne, P.G. Dawber, M.G.D. van der Griten at al., ILL Experimental Report, Experiment $\mathcal{N}$ 3-07-97, Instrument PL1. Date of report: Aug. the 15th, 1999.

P.G. Dawber, J. Byrne, M.G.D. van der Griten at al., NIM **A**, to be published.

[23] A. Young, in *Proceedings of The Second UCN Workshop*, Pushkin, Russia, June 14-17, 1999, Published by the Petersburg Nuclear Physics Institute, 1999, p.485.

S. Hödle, ibid, p.477.

W.S.Wilburn, J.S. Kapustinsky, J.D. Bowman et al., in *1999 Division of Nucl. Phys. Fall Meeting*, October 20-23, 1999, Pacific Grove, CA, Bul. of American Phys. Soc., **44**, [BC.09] (1999).

M.S. Dewey, F.E. Wietfeldt, B.G. Yerozolimsky et al., in *1999 Division of Nucl. Phys. Fall Meeting*, October 20-23, 1999, Pacific Grove, CA, Bul. of American Phys. Soc., **44**, [BC.11], [BC.12] (1999).

A.R. Yong, S. Hödle, C.-Y. Lin at al., in *1999 Division of Nucl. Phys. Fall Meeting*, October 20-23, 1999, Pacific Grove, CA, Bul. of American Phys. Soc., **44**, [BC.08] (1999).

H.P. Mumm, M.C. Browne, R.G.H. Robertson at al., in *1998 Division of Nucl. Phys. Fall Meeting*, October 28-31, 1998, Santa Fe, NM, Bul. of American Phys. Soc., **43**, [B2.05], [B2.06] (1998).

[24] J. Byrne, P.G. Dawber, C. Habeck at al., ILL Experimental Report, Experiment $\mathcal{N}$ 3-07-61, Instrument PL1. Date of report: Aug. the 22th, 1996.

L. Beck, K. Schreckenbach, T. Soldner at al., ILL Experimental Report, Experiment $\mathcal{N}$ 3-07-103, Instrument PL1. Date of report: Aug. the 10th, 1999.

H. Abele, M.A. Hoffmann, S. Bäßler at al., ILL Experimental Report, Experiment $\mathcal{N}$ 3-07-73, Instrument PL1. Date of report: Aug. the 12th, 1997.

[25] G. G. Bunatian, ZhETF **116**, 1505 (1999).

G. G. Bunatian, JETP Lett. **69**, 728 (1999).

[26] S.M. Bilin'ky, R.M. Ryndin, Ya.A. Smorodinsky and Ho Tso-Hsin, ZhETF **37**, 1758 (1959).

[27] O. Nachtman, Zeit. für Phys. **215**, 505 (1968).

[28] A. I. Achiezer and V. B. Berestezky, *Quantum Electrodynamics*, FM, Moscow, 1959.

[29] D.R. Yennie, S.C. Frautschi and H. Suura, Ann. of Phys. **13**, 379 (1961).

N. Meister and D.R. Yennie, Phys. Rev. **130**, 1210 (1963).

[30] R. Christian and H. Kühnelt, Acta Phys. Austriaca **49**, 229 (1978).

[31] K. Toth, K. Szegö and A. Margaritis, Phys. Rev. D **33**, 3306 (1986).

[32] F. Glück and K. Toth, Phys. Rev. D **41**, 2160 (1990).

[33] F. Glück and K. Toth, Phys. Rev. D **46**, 2090 (1992).

[34] F. Glück, I. Joò, J. Last, Nucl. Phys. A **593**, 125 (1995).

[35] F. Glück, Phys. Lett. B **376**, 25 (1996).

[36] F. Glück, Phys. Lett. B **436**, 25 (1998).

[37] A. Sirlin, Phys. Rev. **164**, 1767 (1967).

[38] G. G. Bunatian, Yad. Fiz. **62**, 697 (1999).

[39] O. Kofoed-Hansen, Phys. Rev. **74**, 1785 (1948).

[40] R. Mathuse, Phil. Mag. (Ser. 7) **40**, 351 (1949).

[41] S.M. Berman, Phys. Rev. **112**, 267 (1958).

[42] R.P. Feynman and M. Gell-Mann, Phys. Rev. **109** 193 (1958).
Ya.A. Smorodinsky and Ho Tso-Hsin; ZhETF **38**, 1007 (1960).

[43] Yu. A. Mostovoy, Phys. of Atomic Nucl. **59**, 968 (1996).
Yu. A. Mostovoy, Preprint of Russian Research Centrum "Kurchtov Institute" , IAE-6040/2, Moscow, 1997.

[44] F. Glück, Phys. Rev. D **47**, 2840 (1993).

[45] A. Garcia, Phys. Rev. D **25**, 1348 (1982); D **35**, 232 (1987).

[46] L.L. Nemenov, Yad. Fiz. **15**, 1047 (1971); **16**, 1258 (1972).

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О спектре протонов в β-распаде свободного нейтрона

Мы рассматриваем расчеты, предназначенные для того, чтобы извлечь с высокой точностью, ~1% или лучше, общие характеристики слабого взаимодействия из экспериментов по β-распаду свободного нейтрона; особое значение придается явлениям, связанным с отдачей протонов. Выясняется роль, которую играют электромагнитные взаимодействия в процессе β-распада, причем особое внимание привлечено к влиянию γ-излучения на импульсное распределение частиц в конечном состоянии. В свете экспериментов, проводимых и планируемых в настоящее время, исследуется влияние электромагнитного взаимодействия на спектр протонов отдачи. Результаты расчетов, которые надлежит сравнивать с экспериментальными данными, выражены непосредственно через величины, определяющие эффективный лагранжиан, лежащий в основе исследований. Оказывается, что поправки к энергетическому распределению протонов, обусловленные электромагнитными взаимодействиями, достигают величины нескольких процентов. В настоящее время это является существенным для получения с высокой точностью характеристик слабого взаимодействия из обработки экспериментальных данных по распределению протонов в β-распаде свободного нейтрона.

Работа выполнена в Лаборатории нейтронной физики им. И.М.Франка ОИЯИ.

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Bunatian G.G. E4-2000-19
On the Proton Spectrum in Free Neutron β-decay

We consider the calculations which are appropriate to acquire with a high precision, of ~1% or better, the general characteristics of weak interactions from the experiments on the free neutron β-decay; the principle emphasis is placed on the phenomena associated with the recoil of protons. The part played by electromagnetic interactions in β-decay is visualized, with special attention drawn to the influence of the γ-radiation on the momentum distribution of the particles in the final state. The effect of electromagnetic interactions on the proton recoil spectrum is studied, in the light of the experiments which are carried out and planned for now. The results of the calculations, which are to be confronted with the experimental data, are presented upright in terms of the effective Lagrangian underlying the inquiry. Owing to electromagnetic interactions, the corrections to the energy distribution of protons prove to amount to the value of a few per cent. Nowadays, this is substantial to obtain with a high accuracy the characteristics of weak interactions by processing the data of the experiments on the proton distribution in the free neutron β-decay.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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