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NEAR THRESHOLD RADIATIVE 3π PRODUCTION IN e^+e^- ANNIHILATION

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1 Introduction

The new Brookhaven experimental result for the anomalous magnetic moment of the muon [1] induced considerable excitement in the physics community, because it was interpreted as indicating towards a new physics beyond the Standard Model [2]. However such claims, too premature in our opinion, assume that the theoretical prediction for the muon anomaly is well understood at the level of necessary precision. And hadronic uncertainties raise the main concern [3]. Fortunately the leading hadronic contribution is related to the hadronic corrections to the photon vacuum polarization function, which can be accurately calculated provided that the precise experimental data about the low energy hadronic cross sections in e^+e^- annihilation are in our disposition.

In last years new, high statistics, experimental data was collected in the ρ-ω region in Novosibirsk experiments at VEPP-2M collider [4]. In this region the hadronic cross sections are dominated by $e^+e^- \to 2\pi$ and $e^+e^- \to 3\pi$ channels. The former is of uppermost importance for reduction of errors in evaluation of the hadronic vacuum polarization contribution to muon g-2. Considerable progress was reported for this channel by CMD-2 collaboration [5]. The $e^+e^- \to 3\pi$ channel, which gives less important but still significant contribution to the hadronic error, was also investigated in the same experiment in ω-meson region [6]. Such high precision experiments require accurate knowledge of various backgrounds. Among them $e^+e^- \to 3\pi\gamma$ channel provides an important background needed to be well understood. This experimental necessity motivated our investigation of the three pion radiative production presented here. Besides being interesting as the important background source, this process could be interesting by itself, because a detailed experimental study of the final state radiation will allow to get important information about pion-photon dynamics at low energies. But such experimental investigation will require much more statistics than available in VEPP-2M experiments and maybe would be feasible only at ϕ-factories, where the low energy region can be reached by radiative return technique as was recently demonstrated in KLOE experiment [7].

2 Initial state radiation

Let J_μ be the matrix element of the electromagnetic current between the vacuum and the $\pi^+\pi^-\pi^0$ final state. Then the initial state radiation (ISR) contribution to the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ process cross section is given at $O(\alpha)$ by the standard expression [8]

$$\begin{aligned} d\sigma_{ISR}(e^+e^- \to 3\pi\gamma) = \frac{e^6}{4(2\pi)^8(Q^2)^2}\Bigg\{\frac{Q^2}{4E^2}\, J\cdot J^*\left(\frac{p_+}{k\cdot p_+}-\frac{p_-}{k\cdot p_-}\right)^2 - \\ -\frac{Q^2}{2E^2}\,\frac{p_+\cdot J\; p_+\cdot J^* + p_-\cdot J\; p_-\cdot J^*}{k\cdot p_+ k\cdot p_-} - \frac{J\cdot J^*}{2E^2}\left(\frac{k\cdot p_+}{k\cdot p_-}+\frac{k\cdot p_-}{k\cdot p_+}\right) + \\ +\frac{m_e^2}{E^2}\left(\frac{p_+\cdot J}{k\cdot p_-}-\frac{p_-\cdot J}{k\cdot p_+}\right)\left(\frac{p_+\cdot J^*}{k\cdot p_-}-\frac{p_-\cdot J^*}{k\cdot p_+}\right)\Bigg\}\, d\Phi \equiv \frac{e^6}{4(2\pi)^8}\,|A_{ISR}|^2 d\Phi, \end{aligned} \quad (1)$$

where $d\Phi$ stands for the Lorentz invariant phase space

$$d\Phi = \frac{d\vec{k}}{2\omega}\,\frac{d\vec{q}_+}{2E_+}\,\frac{d\vec{q}_-}{2E_-}\,\frac{d\vec{q}_0}{2E_0}\,\delta(p_+ + p_- - k - q_+ - q_- - q_0)$$

and $Q^2 = (q_+ + q_- + q_0)^2 = 4E(E-\omega)$ is the photon virtuality, E being the beam energy and ω – the energy of the γ quantum. Particle 4-momenta assignment can be read from the corresponding diagrams presented on Fig.1

Current matrix element J_μ has a general form

$$J_\mu = \epsilon_{\mu\nu\sigma\tau} q_+^\nu q_-^\sigma q_0^\tau\; F_{3\pi}(q_+, q_-, q_0). \quad (2)$$

And for the $F_{3\pi}$ form-factor, which depends only on invariants constructed from the pions 4-momenta, we will take the expression from [9]

$$F_{3\pi} = \frac{\sqrt{3}}{(2\pi)^2 f_\pi^3}\left[\sin\theta\cos\eta\; R_\omega(Q^2) - \cos\theta\sin\eta\; R_\phi(Q^2)\right](1 - 3\alpha_K - \alpha_K H)\,. \quad (3)$$

Here $\alpha_K \approx 0.5$, $f_\pi \approx 93$ MeV is the pion decay constant, $\eta = \theta - \arcsin\frac{1}{\sqrt{3}} \approx 3.4°$ characterizes the departure of the ω-ϕ mixing from the ideal one, and

$$H = R_\rho(Q_0^2) + R_\rho(Q_+^2) + R_\rho(Q_-^2),$$

where

$$Q_0^2 = (q_+ + q_-)^2,\;\; Q_+^2 = (q_0 + q_+)^2,\;\; Q_-^2 = (q_0 + q_-)^2.$$

Dimensionless Breit-Wigner factors have the form

$$R_V(Q^2) = \left[\frac{Q^2}{M_V^2} - 1 + i\frac{\Gamma_V}{M_V}\right]^{-1}, \quad R_\rho(Q^2) = \left[\frac{Q^2}{M_\rho^2} - 1 + i\frac{\sqrt{Q^2}\Gamma_\rho(Q^2)}{M_\rho^2}\right]^{-1},$$

where $V = \omega, \phi$ and for the ρ meson the energy dependent width is used

$$\Gamma_\rho(Q^2) = \Gamma_\rho \frac{M_\rho^2}{Q^2}\left(\frac{Q^2 - 4m\pi^2}{M_\rho^2 - 4m\pi^2}\right)^{3/2}.$$

The last term in (1) is completely irrelevant for VEPP-2M energies if the hard photon is emitted at large angle. So we will neglect it in the following.

3 Final state radiation

To describe final state radiation (FSR), we use effective low energy Wess-Zumino-Witten Lagrangian [10]. The relevant piece of this Lagrangian is reproduced below.

$$\begin{aligned} \mathcal{L} = \frac{f_\pi^2}{4} Sp\Big[D_\mu U (D_\mu U)^+ + \chi U^+ + U\chi^+ \Big] - \\ -\frac{e}{16\pi^2}\epsilon^{\mu\nu\alpha\beta} A_\mu Sp\Big[Q\Big\{ (\partial_\nu U)(\partial_\alpha U^+)(\partial_\beta U)U^+ - (\partial_\nu U^+)(\partial_\alpha U)(\partial_\beta U^+)U \Big\}\Big] - \\ -\frac{ie^2}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu A_\nu)A_\alpha Sp\Big[Q^2(\partial_\beta U)U^+ + Q^2U^+(\partial_\beta U) + \frac{1}{2}QUQU^+(\partial_\beta U)U^+ - \\ -\frac{1}{2}QU^+QU(\partial_\beta U^+)U\Big]. \end{aligned} \tag{4}$$

Here $U = \exp\left(i\frac{\sqrt{2}P}{f_\pi}\right)$, $D_\mu U = \partial_\mu U + ieA_\mu[Q, U]$, $Q = diag\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$ is the quark charge matrix, and terms with $\chi = B\, diag\,(m_u, m_d, m_s)$ introduce explicit chiral symmetry breaking due to nonzero quark masses. The constant B has dimension of mass and is determined through the equation $Bm_q = m_\pi^2$, $m_q = m_u \approx m_d$. The pseudoscalar meson matrix P has its standard form

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

It is straightforward to get from (4) the relevant interaction vertexes shown on Fig.2

Using these Feynman rules, one can calculate $\gamma^* \to \pi^+\pi^-\pi^0\gamma$ amplitude originated from the diagrams shown on Fig.3
The result is

$$A_{\mu\nu}(\gamma^*_\mu \to 3\pi\gamma_\nu) = \frac{ie^2}{4\pi^2 f_\pi^3} T_{\mu\nu}, \tag{5}$$

where ($Q = q_+ + q_- + q_0 + k$ is the virtual photon 4-momentum)

$$\begin{aligned} T_{\mu\nu} &= \epsilon_{\mu\nu\alpha\beta} Q^\alpha k^\beta \left(1 - \frac{(q_+ + q_-)^2 - m_\pi^2}{(Q-k)^2 - m_\pi^2}\right) + \epsilon_{\mu\nu\alpha\beta}(Q+k)^\alpha q_0^\beta + \\ &+ \epsilon_{\mu\lambda\alpha\beta} Q^\alpha q_0^\beta \left(\frac{(2q_- + k)_\nu q_+^\lambda}{2q_- \cdot k} + \frac{(2q_+ + k)_\nu q_-^\lambda}{2q_+ \cdot k}\right) - \\ &- \epsilon_{\nu\lambda\alpha\beta} k^\alpha q_0^\beta \left(\frac{(2q_- - Q)_\mu q_+^\lambda}{Q^2 - 2q_- \cdot Q} + \frac{(2q_+ - Q)_\mu q_-^\lambda}{Q^2 - 2q_+ \cdot Q}\right). \end{aligned} \tag{6}$$

The tensor $T_{\mu\nu}$ is gauge invariant

$$Q^\mu T_{\mu\nu} = k^\nu T_{\mu\nu} = 0.$$

Note that our result for $A_{\mu\nu}(\gamma^*_\mu \to 3\pi\gamma_\nu)$ is in agreement with the known result [11, 12] for $\gamma^*\gamma^* \to 3\pi$ amplitude (these two amplitudes are connected by crossing symmetry, of course).

If $J_\mu^{(\gamma)}$ is the amplitude of the transition $\gamma^*_\mu \to \pi^+\pi^-\pi^0\gamma$, then the final state radiation (FSR) contribution to the $e^+e^- \to \gamma^* \to \pi^+\pi^-\pi^0\gamma$ process cross section is given by [8]

$$\begin{aligned} d\sigma_{FSR} &= \frac{e^2}{(2\pi)^8\, 64E^4} \sum_\epsilon \left\{ \frac{Re(p_+ \cdot J^{(\gamma)}\ p_- \cdot J^{(\gamma)*})}{E^2} - J^{(\gamma)} \cdot J^{(\gamma)*} \right\} d\Phi \approx \\ &\approx \frac{1}{(2\pi)^8\, 64E^4} \sum_\epsilon \left[|J_1^{(\gamma)}|^2 + |J_2^{(\gamma)}|^2 \right] d\Phi, \end{aligned}$$

where the sum is over the photon polarization ϵ and z axis was assumed to be along $\vec{p}_-$. But $J_\mu^{(\gamma)} = \epsilon^\nu A_{\mu\nu}(\gamma^* \to 3\pi\gamma)$. So we can perform the polarization sum by using $\sum_\epsilon \epsilon_\mu \epsilon^*_\nu = -g_{\mu\nu}$. Introducing gauge invariant real 4-vectors t_1 and t_2 by $t_{1\mu} = T_{1\mu}$, $t_{2\mu} = T_{2\mu}$, the result can be casted in the form (note that the norm of gauge invariant 4-vector is always negative)

$$d\sigma_{FSR}(e^+e^- \to 3\pi\gamma) = \frac{e^4}{(2\pi)^8\, 64E^4} \frac{1}{(2\pi)^4 f_\pi^6} \left[-t_1 \cdot t_1 - t_2 \cdot t_2\right] d\Phi. \tag{7}$$

But for the photon virtualities of real experimental interest vector meson effects can no longer be neglected. So we replace (7) by

$$d\sigma_{FSR}(e^+e^- \to 3\pi\gamma) = \frac{e^6}{(2\pi)^8\, 64E^4} \frac{1}{(2\pi)^4 f_\pi^6} \left[-t_1 \cdot t_1 - t_2 \cdot t_2\right] K_{BW}\, d\Phi \equiv$$

$$\equiv \frac{e^6}{(2\pi)^8} \; |A_{FSR}|^2 d\Phi, \tag{8}$$

where we have introduced a phenomenological Breit-Wigner factor

$$K_{BW} = 3 \left| \sin\theta \cos\eta R_\omega(4E^2) - \cos\theta \sin\eta R_\phi(4E^2) \right|^2 .$$

This factor is similar to one presented in ISR (see (3)) and tends to unity then $E \to 0$. It gives about order of magnitude increase in σ_{FSR} for energies $2E = 0.65 \div 0.7$ GeV.

4 Monte-Carlo event generator

Although what follows can be considered as a textbook material [13], nevertheless we will give somewhat detailed description of the Monte Carlo algorithm for reasons of convenience.

The important first step is the following transformation of the Lorentz invariant phase space. Let $R_n(p^2; m_1^2, \ldots, m_n^2)$ be n-particle phase space

$$R_n(p^2; m_1^2, \ldots, m_n^2) = \int \prod_{i=1}^{n} \frac{d\vec{q}_i}{2E_i} \, \delta(p - \sum_{i=1}^{n} q_i).$$

Inserting the identity

$$1 = \int dk_1 \; d\mu_1^2 \; \delta(p - q_1 - k_1)\delta(k_1^2 - \mu_1^2)$$

we get

$$R_4(p^2; m_1^2, m_2^2, m_3^2, m_4^2) = \int \frac{d\vec{q}_1}{2E_1} R_3((p - q_1)^2; m_2^2, m_3^2, m_4^2) =$$

$$= \int \frac{d\vec{q}_1}{2E_1} dk_1 d\mu_1^2 R_3(k_1^2; m_2^2, m_3^2, m_4^2)\delta(p - q_1 - k_1)\delta(k_1^2 - \mu_1^2).$$

But (note that $(p - q_1)_0 = E_2 + E_3 + E_4 > 0$)

$$\int \frac{d\vec{q}_1}{2E_1} dk_1 \delta(k_1^2 - \mu_1^2)\delta(p - q_1 - k_1) = \int \frac{d\vec{q}_1}{2E_1} \frac{d\vec{k}_1}{2k_{10}} \delta(p - q_1 - k_1) = R_2(p^2; m_1^2, \mu_1^2).$$

Therefore

$$R_4(p^2; m_1^2, m_2^2, m_3^2, m_4^2) = \int d\mu_1^2 R_3(\mu_1^2; m_2^2, m_3^2, m_4^2) R_2(p^2; m_1^2, \mu_1^2). \tag{9}$$

But [13]

$$R_2(p^2; m_1^2, \mu_1^2) = \int \frac{\lambda^{1/2}(p^2; m_1^2, \mu_1^2)}{8p^2} d\Omega_1^*$$

where λ stands for the triangle function and Ω_1^* describes the orientation of the $\vec{q}_1$ vector in the p-particle rest frame.

It is more convenient to integrate over q-particle energy E^* instead of mass μ, the two being interconnected by the relation $\mu^2 = p^2+q^2-2\sqrt{p^2}E^*$ in the p-particle rest frame.

Using the relation [13]

$$\frac{\lambda^{1/2}(p^2;m^2,\mu^2)}{2\sqrt{p^2}} = \mu\sqrt{\bar{\gamma}^2-1},$$

where $\bar{\gamma}$ is the γ-factor of the "particle" (subsystem) with the invariant mass μ, we get after repeatedly using (9)

$$R_4 = \int \frac{1}{2}\sqrt{\bar{\gamma}_1^2-1}\; dE_1^* d\Omega_1^* \frac{1}{2}\mu_1\sqrt{\bar{\gamma}_2^2-1} dE_2^* d\Omega_2^* \frac{1}{2}|\vec{p}_3^{\,*}| d\Omega_3^*,$$

where $\vec{p}_3^{\,*}$ momentum is in the rest frame of the (3,4) subsystem and E_2^*, Ω_2^*, $\bar{\gamma}_2$ are in the rest frame of the (2,3,4) subsystem.

Now it is straightforward to rewrite the differential cross-section in the following form

$$d\sigma(e^+e^- \to 3\pi\gamma) = \frac{\alpha^3}{2\pi^2}|A|^2 f d\Phi^*, \tag{10}$$

where $|A|^2 = |A_{ISR}|^2 + |A_{FSR}|^2$ (we do not take into account interference between initial and final state radiations. This interference integrates to zero if we do not distinguish between negative and positive π-mesons),

$$f = \mu_1(\omega_{max}-\omega_{min})(E_{0\,max}^* - E_{0\,min}^*)\sqrt{(E_-^{*2}-m_\pi^2)(\bar{\gamma}_1^2-1)(\bar{\gamma}_2^2-1)}, \tag{11}$$

and

$$d\Phi^* = \frac{d\omega}{(\omega_{max}-\omega_{min})}\frac{d\varphi}{2\pi}\frac{d\cos\theta}{2}\frac{dE_0^*}{(E_{0\,max}^*-E_{0\,min}^*)}\frac{d\varphi_0^*}{2\pi}\frac{d\cos\theta_0^*}{2}\frac{d\varphi_-^*}{2\pi}\frac{d\cos\theta_-^*}{2}. \tag{12}$$

The upper and lower limits for energies are

$$\omega_{max} = \frac{s-9m_\pi^2}{2\sqrt{s}},\quad E_{0\,max}^* = \frac{\mu_1^2-3m_\pi^2}{2\mu_1},\quad E_{0\,min}^* = m_\pi.$$

The minimal photon energy ω_{min} is an external experimental cut. At last, $|A_{ISR}|^2$ and $|A_{FSR}|^2$ can be read from the corresponding expressions (1) and (8) respectively.

According to (10), we can generate $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ events in the cms frame by the following algorithm:

• generate the photon energy ω as a random number uniformly distributed from ω_{min} to ω_{max}. Calculate for the $S_1 = (\pi^+\pi^-\pi^0)$ subsystem the energy $\bar{E}_1 = 2E - \omega$, invariant mass $\bar{\mu}_1 = \sqrt{4E(E-\omega)}$ and Lorentz factor $\bar{\gamma}_1 = \bar{E}_1/\bar{\mu}_1$.

• generate a random number $\bar{\varphi}_1$ uniformly distributed in the interval $[0, 2\pi]$ and take it as the azimuthal angle of the S_1 subsystem velocity vector in the cms frame. Generate another uniform random number in the interval $[-\cos\theta_{min}, \cos\theta_{min}]$ and take it as a $\cos\bar{\theta}_1$, $\bar{\theta}_1$ being the polar angle of the S_1 subsystem velocity vector in the cms frame. This defines the unit vector $\vec{n}_1 = (\sin\bar{\theta}_1\cos\bar{\varphi}_1, \sin\bar{\theta}_1\sin\bar{\varphi}_1, \cos\bar{\theta}_1)$ along S_1 subsystem velocity. θ_{min} is the minimal photon radiation angle – an external experimental cut.

• construct the photon momentum in the cms frame $\vec{k} = -\omega\vec{n}_1$.

• generate the π^0-meson energy E_0^* in the S_1 rest frame as a random number uniformly distributed from $E_{0\,min}^*$ to $E_{0\,max}^*$. Calculate for the $S_2 = (\pi^+, \pi^-)$ subsystem the energy $\bar{E}_2 = \bar{\mu}_1 - E_0^*$, invariant mass $\bar{\mu}_2 = \sqrt{\bar{\mu}_1^2 + m_\pi^2 - 2\bar{\mu}_1 E_0^*}$ and Lorentz factor $\bar{\gamma}_2 = \bar{E}_2/\bar{\mu}_2$.

• generate a random number $\bar{\varphi}_2$ uniformly distributed in the interval $[0, 2\pi]$ and take it as the azimuthal angle of the S_2 subsystem velocity vector in the S_1 rest frame. Generate another uniform random number in the interval $[-1, 1]$ and take it as a $\cos\bar{\theta}_2$, $\bar{\theta}_2$ being the polar angle of the S_2 subsystem velocity vector in the S_1 rest frame. This defines the unit vector along S_2 subsystem velocity in the S_1 rest frame $\vec{n}_2 = (\sin\bar{\theta}_2\cos\bar{\varphi}_2, \sin\bar{\theta}_2\sin\bar{\varphi}_2, \cos\bar{\theta}_2)$.

• construct $\vec{q}_0^{\,*} = -\sqrt{E_0^{*2} - m_\pi^2}\,\vec{n}_2$ – the π^0-meson momentum in the S_1 rest frame.

• generate φ_-^* and $\cos\theta_-^*$ in the manner analogous to what was described above for $\bar{\varphi}_2$ and $\cos\bar{\theta}_2$, construct the unit vector along the π^- meson velocity in the S_2 rest frame $\vec{n}_3 = (\sin\theta_-^*\cos\varphi_-^*, \sin\theta_-^*\sin\varphi_-^*, \cos\theta_-^*)$.

• construct the π^--meson 4-momentum in the S_2 rest frame $E_-^* = \bar{\mu}_2/2$, $\vec{q}_-^{\,*} = \sqrt{E_-^{*2} - m_\pi^2}\,\vec{n}_3$.

• construct the π^+-meson 4-momentum in the S_2 rest frame $E_+^* = \bar{\mu}_2/2$, $\vec{q}_+^{\,*} = -\vec{q}_-^{\,*}$.

• transform π^0-meson 4-momentum from the S_1 rest frame back to the cms frame.

• transform π^- and π^+ mesons 4-momenta firstly from the S_2 rest frame to the S_1 rest frame and then back to the cms frame.

• for generated 4-momenta of the final state particles, calculate $z = |A|^2 f$.

• generate a random number z_R uniformly distributed in the interval from 0 to z_{max}, where z_{max} is some number majoring $|A|^2 f$ for all final state 4-momenta allowed by 4-momentum conservation.

• if $z \geq z_R$, accept the event, that is the generated 4-momenta of the π^+, π^- and π^0 mesons and the photon. Otherwise repeat the whole procedure.

5 Soft and collinear photon corrections

We assume that the photon in the $e^+e^- \to 3\pi\gamma$ reaction is hard enough $\omega > \omega_{min}$ and radiated at large angle $\theta > \theta_{min}$ so that it could be detected by experimental equipment (the detector). But in any process with accelerated charged particles soft photons are emitted without being detected because the detector has finite energy resolution. Even moderately hard photons can escape detection in some circumstances. How important are such effects? Naively every photon emitted brings extra factor α in the amplitude and so a small correction is expected. But this argument (as well as the perturbation theory) breaks down for soft photons. When electron (positron) emits soft enough photon, it nearly remains on the mass shell so bringing a very large propagator in the amplitude. Formal application of the perturbation theory gives an infinite answer for the correction due to soft photon emission because of this pole singularity. It is well known [14] how to deal with this infrared divergence. In real experiments very low energy photons are never formed because of finite size of the laboratory. So we have a natural low energy cut-off. A remarkable fact, however, is that the observable cross sections do not depend on the actual form of the cut-off because singularities due to real and virtual soft photons cancel each other [15]. The net effect is that the soft photon corrections, summed to all orders of the perturbation theory, factor out as some calculable, so called Yennie-Frautschi-Suura exponent [14].

Collinear radiation of (not necessarily soft) photons by highly relativistic initial electrons (positrons) is another source of big corrections which also should be treated non-perturbatively. Unlike soft photons, however, the matrix element for a radiation of an arbitrary number of collinear photons is not known. Nevertheless there is a nice method (the so called Structure Functions method) [16] which enables to sum leading collinear (and soft) logarithms. The corrected cross-section, when radiation of unnoticed

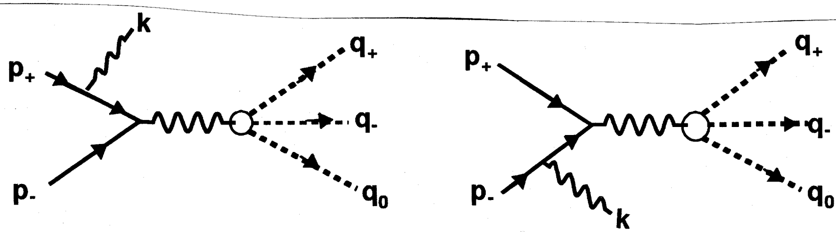
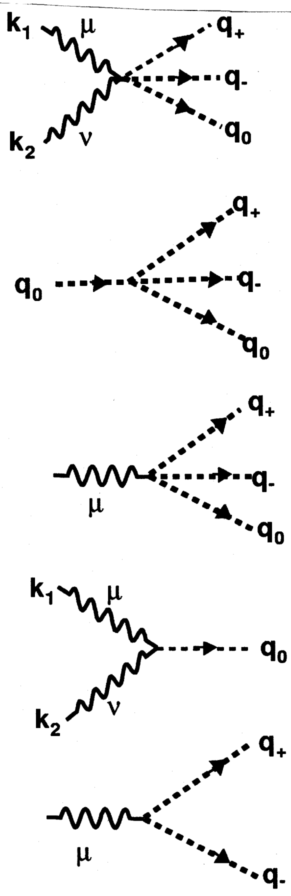


Figure 1. Initial state radiation diagrams and particle 4-momenta assignment.



$$\frac{ie^2}{4\pi^2 f_\pi^3}\varepsilon_{\mu\nu\alpha\beta}\left[\frac{4}{3} k_1^\alpha k_2^\beta - (k_1 - k_2)^\alpha (q_+ + q_-)^\beta\right]$$

$$\frac{i}{f_\pi^2}\left[(q_+ + q_-)^2 - m_\pi^2 - \frac{1}{3}(q_0^2 - m_\pi^2)\right]$$

$$\frac{ie}{4\pi^2 f_\pi^3}\varepsilon_{\mu\nu\alpha\beta}\, q_+^\nu q_-^\alpha q_0^\beta$$

$$-\frac{ie^2}{4\pi^2 f_\pi}\varepsilon_{\mu\nu\alpha\beta}\, k_1^\alpha k_2^\beta$$

$$ie(q_+ - q_-)_\mu$$

Figure 2. Interaction vertexes relevant for final state radiation.

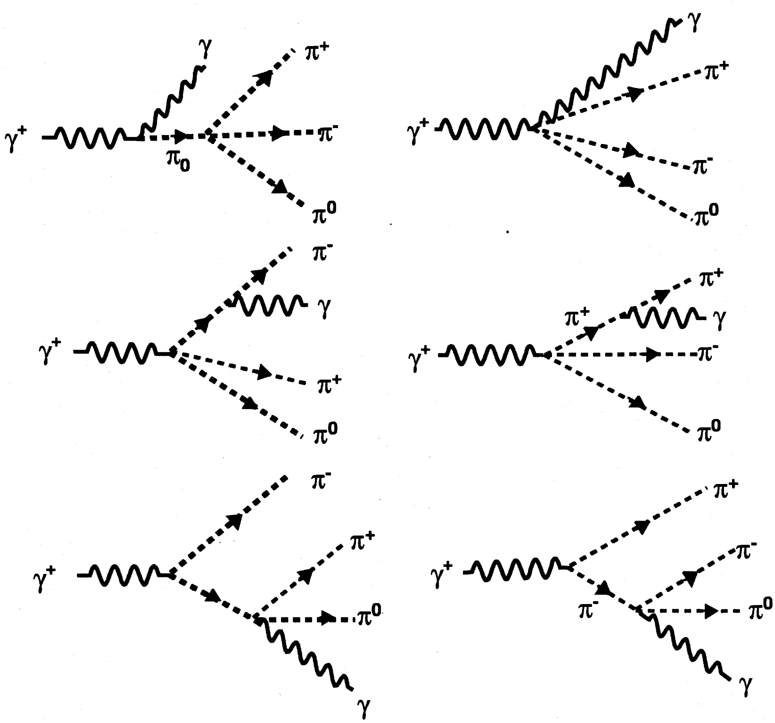


Figure 3. $\gamma^* \to \pi^+\pi^-\pi^0\gamma$ transition diagrams.

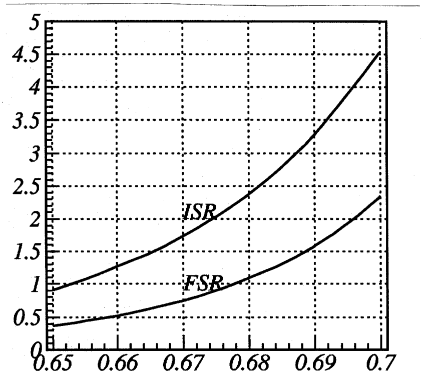


Figure 4. ISR and FSR contributions to the $e^+e^- \to 3\pi\gamma$ cross section.

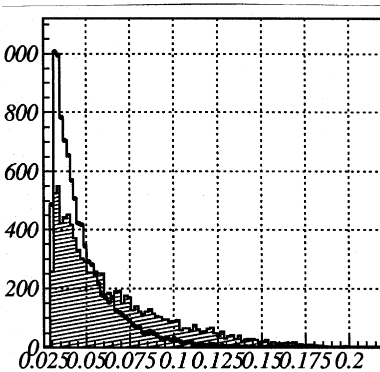


Figure 5. The photon energy distributions for ISR and FSR (hatched histogram).

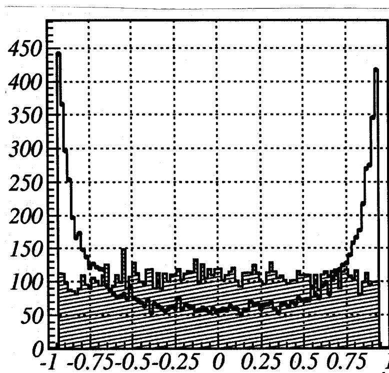


Figure 6. The photon angular distributions for ISR and FSR (hatched histogram).

photons with total energy less than $\Delta E \ll E$ is allowed, looks like [16]

$$\tilde{\sigma}(s) = \int\limits_0^{\Delta E} \frac{d\omega}{\omega}\, \sigma(4E(E-\omega))\, \beta \left(\frac{\omega}{E}\right)^{\beta} \left[1 + \frac{3}{4}\beta + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right)\right], \quad (13)$$

where $\beta = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$ and we have omitted some higher order terms.

In our case the hard photon is well separated (because of $\omega > \omega_{min}$, $\theta > \theta_{min}$ cuts) from the soft and collinear regions of the phase space. So equation (13) is applicable and it indicates that the soft and collinear corrections to the cross-section of the process $e^+e^- \to 3\pi\gamma$ do not exceed 20% when $\Delta E \sim \omega_{min} = 30$ MeV, $\theta_{min} = 20°$ and $E = 0.7$ GeV. Such corrections are irrelevant for the present VEPP-2M statistics but may become important in future high statistics experiments.

6 Numerical results and conclusions

On Fig.4 numerical results are shown for $\sigma(e^+e^- \to 3\pi\gamma)$ with $\omega_{min} = 30$ MeV, $\theta_{\rm min} = 20°$. As expected, the cross section is small, only few picobarns, for energies $0.65 \div 0.7$ GeV.

This figure shows also that FSR contributes significantly at such low energies. So if future ϕ-factory experiments produce high enough statistics in this energy region, the study of FSR will become realistic. FSR and ISR give different angular and energy distributions for the photon as illustrated by Fig.5 and Fig.6 This fact can be used for the FSR separation from the somewhat more intensive ISR.

Acknowledgments

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Радиационное рождение 3π вблизи порога в e^+e^--аннигиляции

Рассмотрен радиационный процесс образования конечного состояния $\pi^+\pi^-\pi^0\gamma$ при электрон-позитронной аннигиляции вблизи порога. Рассматривается излучение жесткого фотона как в начальном, так и в конечном состояниях без их интерференции. Амплитуда, отвечающая излучению в конечном состоянии, получена в соответствии с эффективным лагранжианом Весса–Зумино–Виттена, используемым для пион-фотонных взаимодействий при низких энергиях. В реальных экспериментах энергии никогда не бывают столь малы, чтобы эффектом ρ- и ω-мезонов можно было пренебречь. Поэтому для рассмотрения влияния векторных мезонов в амплитуду излучения в конечном состоянии вводится феноменологический фактор Брейта–Вигнера. При использовании амплитуды рождения 3π был разработан специальный монте-карловский генератор событий, который можно применять в экспериментальных исследованиях.

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Near Threshold Radiative 3π Production in e^+e^- Annihilation

We consider the $\pi^+\pi^-\pi^0\gamma$ final state in electron-positron annihilation at the center-of-mass system energies not far from the threshold. Both initial and final state radiations of the hard photon are considered but without interference between them. The amplitude for the final state radiation is obtained by using the effective Wess–Zumino–Witten Lagrangian for pion-photon interactions valid for low energies. In real experiments the energies are never so small that ρ and ω mesons would have negligible effect. So, a phenomenological Breit–Wigner factor is introduced in the final state radiation amplitude to consider for the vector mesons influence. Using the radiation of 3π production amplitudes, a Monte Carlo event generator was developed which could be useful in experimental studies.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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