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CROSSING SYMMETRY VIOLATION
IN THE PROCESS OF LEPTON PAIR PRODUCTION
IN RELATIVISTIC ION COLLISIONS COMPARED
WITH THE CROSSING PROCESS

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1 Introduction

During the past decade the growing interest to the process of lepton pair production in the strong Coulomb fields appeared. This is connected mainly with beginning of the operation of the relativistic heavy ion collider RHIC (Lorentz factor $\gamma = \frac{E}{m} = 108$) and the new collider LHC ($\gamma = 3000$) which will operate in the nearest future. At such energies the lepton pair yield becomes huge (according to [1, 2]), so that a detail analysis of the process

$$A + B \rightarrow A + B + e^+ + e^-$$

(1)

accounting the Coulomb corrections (CC) is required. Such work has been done during last years and a lot of papers are devoted to this subject [3, 4, 5, 6, 7, 8, 11, 12, 13]. Nevertheless, the problem turn out to be more complex than it seems from the first glance. We want only to notice the exciting result obtained in the papers [3, 4, 7]: the Coulomb corrections to the process (1) enter the amplitude of this process in such a way that its cross section is determined solely by the lowest (Born) term. At present we understand that this result is the incorrect application of crossing symmetry property which, as it is known long ago (see, e.g., [14]), is valid only on the Born level. As an obvious example of the crossing symmetry violation we want to cite the process of lepton pair photoproduction on the nuclei and its counterpart, the bremsstrahlung in lepton–nucleus scattering. Amplitudes of both processes are determined by Coulomb phase which is infrared stable in the case of pair photoproduction whereas it is infrared divergent in the case of bremsstrahlung and this difference cannot be adjust by trivial crossing change of variables. Taking into account the importance of the problem and permanent interest to it from the scientific society, we calculated the full amplitude for the process (2) accounting all possible photon exchanges among the colliding relativistic particles. Comparing it with the amplitude of the process (1) we had shown that the crossing symmetry property becomes invalid whereas one takes into account the final state interaction of the lepton pair with the Coulomb field of ions.

2 The Born amplitude of the process $3 \rightarrow 3$

Let us construct the amplitude of the process $3 \rightarrow 3$ represented in Fig. 1 (a, b)

$$A_1(p_1) + A_2(p_2) + C(p_3) \rightarrow A_1(p'_1) + A_2(p'_2) + C(p'_3).$$

(2)

We consider the kinematics when all the energy invariants which determined the process (2) are large, compared with the masses of involved particles and
the transfer momenta

\[ s = (p_1 + p_2)^2, \quad s_1 = (p_1 + p_3)^2, \quad s_2 = (p_3 + p_2)^2, \]
\[ q_1^2 = (p_1 - p_1')^2, \quad q_2^2 = (p_2 - p_2')^2, \quad q_3^2 = (p_3 - p_3')^2, \]
\[ p_1^2 = p_1'^2 = m_1^2, \quad p_2^2 = p_2'^2 = m_2^2, \quad p_3^2 = p_3'^2 = m_3^2, \]
\[ s \gg s_1 \sim s_2 \gg -q_1^2 \sim -q_2^2 \sim -q_3^2. \]

For the Born amplitude of the process (2) one can write

\[ M^{(1)}_{(1)} = -i(4\pi\alpha)^2 Z_1 Z_2 \bar{u}(p_1') \gamma_\mu u(p_1) \bar{u}(p_2') \gamma_\nu u(p_2) \frac{\bar{u}(p_3') O_{\rho\sigma} u(p_3) g^{\mu\alpha} g^{\nu\beta}}{q_1^2 q_2^2}, \]

where \( Z_{1,2} \) are the charge numbers of the colliding nuclei. We use Sudakov parameterization for all four–momenta entering the problem (for details see [12])

\[ q_1 = \alpha_1 \tilde{p}_2 + \beta_1 \tilde{p}_1 + q_{1\perp}, \quad q_2 = \alpha_2 \tilde{p}_2 + \beta_2 \tilde{p}_1 + q_{2\perp}, \]
\[ p_1' = \alpha_1' \tilde{p}_2 + \beta_1' \tilde{p}_1 + p_{1\perp}', \quad p_2' = \alpha_2' \tilde{p}_1 + \beta_2' \tilde{p}_1 + p_{2\perp}', \]
\[ p_3' = \alpha_3 \tilde{p}_1 + \beta_3 \tilde{p}_1 + p_{3\perp}, \quad p_3' = \alpha_3' \tilde{p}_1 + \beta_3' \tilde{p}_1 + p_{3\perp}', \]

and the Gribov decomposition of the metric tensor into the longitudinal and transverse parts

\[ g_{\mu\nu} = g_{\perp\mu\nu} + \frac{2}{s} (\tilde{p}_{1\mu} \tilde{p}_{2\nu} + \tilde{p}_{1\nu} \tilde{p}_{2\mu}), \]

with light–like 4–vectors \( \tilde{p}_{1,2} \). For the kinematics of the process we will use the following relations

\[ s = 2\tilde{p}_1 \tilde{p}_2, \quad \beta_1 + \beta_3 = \beta_3', \quad \alpha_2 + \alpha_3 = \alpha_3', \]
\[ g_{\mu\sigma} = \frac{2}{s} \tilde{p}_{1\sigma} \tilde{p}_{2\mu}, \quad g_{\nu\rho} = \frac{2}{s} \tilde{p}_{1\nu} \tilde{p}_{2\rho}, \]
\[ q_1^2 = q_{1\perp}^2 = -q_1^2, \quad q_2^2 = q_{2\perp}^2 = -q_2^2, \]

where \( \mathbf{q}_i \) are two–dimensional vectors in the plane transverse to the z–axes, which we choose along 3–vector \( \tilde{p}_1 = -\tilde{p}_2 \) in the center of mass frame of initial particles \( A_1, A_2 \). Using the gauge invariant condition

\[ q_{1\rho} \bar{u}(p_3') O_{\rho\sigma} u(p_3) \approx (\beta_1 \tilde{p}_1 + q_{1\perp}) \rho \bar{u}(p_3') O_{\rho\sigma} u(p_3) = 0, \]
\[ q_{2\sigma} \bar{u}(p_3') O_{\sigma\rho} u(p_3) \approx (\alpha_2 \tilde{p}_2 + q_{2\perp}) \sigma \bar{u}(p_3') O_{\sigma\rho} u(p_3) = 0, \]

one gets the Born amplitude in the form

\[ M_{(1)}^{(1)}(q_1, q_2) = -4i s N_1 N_2 (4\pi\alpha Z_1)(4\pi\alpha Z_2) B(q_1, q_2), \]
\[ B(q_1, q_2) = \frac{\bar{u}(p'_3)O_{\rho\sigma}u(p_3)q_{1\perp}q_{2\perp}}{s\alpha_2\beta_1 q_1^2 q_2^2}, \]
\[ N_1 = \frac{1}{s} \bar{u}(p'_1)\hat{p}_2 u(p_1), \quad N_2 = \frac{1}{s} \bar{u}(p'_2)\hat{p}_1 u(p_2), \quad s\alpha_2\beta_1 = -q_3^2 - (q_1 - q_2)^2 \sim m^2. \]

The values of \( N_i \) for every polarization state of initial particles (or for spinless particles) are unity and

\[ \bar{u}(p'_3)O_{\rho\sigma}u(p_3)q_{1\perp}q_{2\perp} = \]
\[ \bar{u}(p'_3) \left( \frac{\hat{p}_3 + \hat{q}_1 + m}{(p_3 + q_1)^2 - m^2} \hat{q}_{1\perp} + \frac{\hat{p}_3 + \hat{q}_2 + m}{(p_3 + q_2)^2 - m^2} \hat{q}_{2\perp} \right) u(p_3). \]

### 3 The Coulomb corrections to the process 3 → 3

Let us consider the set of six Feynman diagrams (FD) with one virtual photon connected the \( p_3 \) line with the particle \( A_1 \) and the two ones connected \( p_3 \) line with the particle \( A_2 \) (see Fig. 2). The loop momentum integration in the relevant matrix element can be performed accounting that

\[ d^4k = (2\pi i)^2 \frac{1}{2s} \frac{d(s\alpha_k)}{2\pi i} \frac{d(s\beta_k)}{2\pi i} d^2k_{\perp}, \quad k = \alpha_k \vec{p}_2 + \beta_k \vec{p}_1 + k_{\perp}. \]

It can be shown that only 4 FD amplitudes works (Fig. 2 (a–d)). Really, when one write explicitly denominators in Fig. 2 (e, f) through longitudinal Sudakov variables, i.e.,

\[ (p_3 + k)^2 - m^2 + i0 \approx s\alpha_k\beta_3 + i0, \]
\[ (p_3 - q_2 + k)^2 - m^2 + i0 \approx s\alpha_k\beta_3 + i0, \]
\[ (p_2 - k)^2 - m^2 + i0 \approx -s\beta_k + i0, \]
\[ (p'_2 + k)^2 - m^2 + i0 \approx s\beta_k + i0, \]

one can see that both poles in \( \alpha_k \) complex plane are situated in the same half-plane, so their contribution to the amplitude is zero (suppressed by factor \( |q_3^2/s| \sim |s_1/s| \)). This result is in agreement with one obtained in [8].

It is convenient to introduce 8 FD (including four ones depicted in Fig. 2 (a–d) and additional four FD with interchanged photons absorbed by nucleus \( A_2 \) line). To avoid the double counting we multiply the relevant matrix
element by statistical factor $1/2!$. This trick permits one to perform the integration over $\alpha_k, \beta_k$ with the result

$$
\int_{-\infty}^{\infty} \frac{d(s\alpha_k)}{2\pi i} \left( \frac{\beta_3}{s\alpha_k\beta_3 + i0} + \frac{\beta_3}{-s\alpha_k\beta_3 + i0} \right) =
$$

$$
\int_{-\infty}^{\infty} \frac{d(s\beta_k)}{2\pi i} \left( \frac{1}{s\beta_k + i0} + \frac{1}{-s\beta_k + i0} \right) = 1. \quad (13)
$$

Now let us show how the cancellations of contribution arising from FD with absorption of $n + 1$ number of exchanged photons between particle $C$ and nucleus $A_1$, sandwiched between two exchanges between particle $C$ and nucleus $A_2$ (Fig. 3) take place. The algebraic symmetrization procedure described above (13), with using the relations

$$
l_i = \alpha_{li} \bar{\beta}_2 + \beta_{li} \bar{\beta}_1 + l_{i\perp}, \quad \alpha_{li} < \alpha_3 < \alpha_k \leq 1, \quad \beta_k < \beta_3 < \beta_{li} \leq 1, \quad (14)
$$

leads to the product of factors

$$
\prod_{i=1}^{n} \left( \frac{\beta_3}{s\alpha_{li}\beta_3 + i0} + \frac{\beta_3}{-s\alpha_{li}\beta_3 + i0} \right) \prod_{j=1}^{n} \left( \frac{1}{s\beta_{l_{ij}} + i0} + \frac{1}{-s\beta_{l_{ij}} + i0} \right), \quad n \geq 1.
$$

(15)

In the terms of notation used in [9, 10] our assumptions (14) read

$$
E = p_{1+} \gg p_{3+} \gg p_{2+} = \frac{m_2^2}{E}, \quad \frac{m_1^2}{E} = p_{1-} \ll p_{3-} \ll p_{2-} = E. \quad (16)
$$

It is easy to see, that in this case, no dependence on $p_{3\pm}$ sign appears for the eikonal amplitudes corresponding to the situation in Fig. 3.

The poles of the electron Green functions (Fig. 3) are situated at the same half plane of $k_-$ what one allows to safely neglect the contribution of such diagrams.

Performing the integration over longitudinal Sudakov variables $\alpha_{li}, \beta_{li}$ in blocks 1, 2 of FD in Fig. 3 one can see, that dependence on longitudinal Sudakov variables $\alpha_k, \beta_k$ relevant to the lower loop is completely the same as in the previous case (see Fig. 2 (e, f)), therefore contribution of such type of FD to the total amplitude is zero.

The physical reason of this suppression is the same as in the case of bremsstrahlung suppression for fast charged particle moving through the media known as the Landau–Pomeranchuk–Migdal effect. Really, this effect can be explained starting from the fact of power suppression of radiation between
two scattering centers in the case when the distance between these centers is less than the coherence length.

Further integration over transverse momentum is straightforward

\[ \int \frac{d^2k}{\pi} \frac{1}{(k + \lambda)((q_2 - k)^2 + \lambda^2)} = \frac{2}{q_2^2} \ln \frac{q_2^2}{\lambda^2}. \tag{17} \]

For the amplitude \( M_{(2)}^{(1)} \) (see Fig. 4) and the similar amplitude \( M_{(1)}^{(2)} \) we obtained

\[ M_{(2)}^{(1)} + M_{(1)}^{(2)} = M_{(1)}^{(1)}(q_1, q_2) \frac{1}{2!} \left[ 2iZ_1 \alpha \ln \frac{q_1^2}{\lambda^2} + 2iZ_2 \alpha \ln \frac{q_2^2}{\lambda^2} \right]. \tag{18} \]

The amplitude for arbitrary amount of interchanged photons (see Fig. 5) is constructed in the similar way

\[ M_{(\infty)}^{(1)}(q_1, q_2) = M_{(1)}^{(1)}(q_1, q_2)e^{i(\varphi_1(q_1) + \varphi_2(q_2))} \tag{19} \]

with the Coulomb phases

\[ \varphi_1(q_1) = Z_1 \alpha \ln \frac{q_1^2}{\lambda^2}, \quad \varphi_2(q_2) = Z_2 \alpha \ln \frac{q_2^2}{\lambda^2}. \]

Consider now the case with one additional exchanged photon between two nuclei \( A_1, A_2 \). The relevant matrix element \( M_{(1B)} \) reads

\[ . \ M_{(1B)} = i\alpha Z_1 Z_2 \int \frac{d^2k}{\pi(k^2 + \lambda^2)} M_{(1)}^{(1)}(q_1 + k, q_2 + k). \tag{20} \]

Two-dimensional integral in (20) is infrared divergent. To regularize it we introduce the photon mass parameter \( \lambda \).

In the same approach we get for the matrix element with the \( n \) exchanged (between nuclei) photons (see Fig. 6 (a))

\[ M_{(nB)} = \frac{(i\alpha Z_1 Z_2)^n}{n!} \prod_{i=1}^{n} \int \frac{d^2k_i}{\pi(k_i^2 + \lambda^2)} M_{(1)}^{(1)}(q_1 + \sum_{i=1}^{n} k_i, q_2 + \sum_{i=1}^{n} k_i). \tag{21} \]

It is convenient to write down this expression in the impact parameter representation. For this aim we use the following identity

\[ \int d^2k_{n+1} \delta^{(2)}(k_{n+1} - q_1 - \sum_{i=1}^{n} k_i) = \frac{1}{(2\pi)^2} \int d^2k_{n+1} d^2 \rho e^{i(k_{n+1} - q_1 - \sum k_i) \cdot \rho} = 1. \tag{22} \]
Thus the matrix element with arbitrary number of exchanged photons can be cast

\[ M(3 \to 3) = \sum_{n=1}^{\infty} M_{(nB)} = \frac{1}{4} \int \frac{d^2 \rho}{\pi} e^{-i \mathbf{q}_1 \cdot \mathbf{\rho}} e^{i \alpha Z_1 Z_2} \psi(\rho) \mathcal{M}^{(1)}_{(1)}(\rho, q_1, q_2) \]  

(23)

with

\[ \psi(\rho) = \int \frac{d^2 k}{\pi} \frac{e^{-i k \cdot \rho}}{k^2 + \lambda^2} = 2 K_0(\rho \lambda) \approx -2 \ln \left( \frac{C \rho \lambda}{2} \right), \]  

(24)

where \( C \approx 1.781 \) and

\[ \mathcal{M}^{(1)}_{(1)}(\rho, q_1, q_2) = \int \frac{d^2 k}{\pi} e^{-i k \cdot \rho} M^{(1)}_{(1)}(k, k + q_2 - q_1). \]  

(25)

This result confirms the general ansatz given above (see (19)) that the dependence on "photon mass" \( \lambda \) can be represented as a phase factor. As can be seen the whole amplitude (23) cannot be cast solely as a Born amplitude multiplied by the phase factor. The corresponding contributions to the total cross section (except the Born term) will be enhanced only by the first power of logarithm in energy.

Finally, taking into account all photon exchanges between particle \( C \) and nuclei \( A_1, A_2 \) we obtain the general answer by the simple replacement in the expression (25)

\[ M^{(1)}_{(1)} \to M^{(\infty)}_{(1)} = M^{(1)}_{(1)}(k, k + q_2 - q_1) e^{i \varphi_1(k) + i \varphi_2(k + q_2 - q_1)} \]  

(26)

with \( \varphi_1, \varphi_2 \) given in (19).

4 The Coulomb corrections to the process of lepton pair production

As was mentioned above, our goal is to investigate the crossing symmetry property between the amplitudes of the process (2) and the relevant process in Fig. 1 (c, d)

\[ A_1(p_1) + A_2(p_2) \to A_1(p'_1) + A_2(p'_2) + C(p_3) + \bar{C}(p_4), \]  

(27)

with the following kinematics

\[ s = (p_1 + p_2)^2, \quad s_p = (q_+ + q_-)^2, \quad q_1^2 = (p_1 - p'_1)^2, \quad q_2^2 = (p_2 - p'_2)^2, \]  

(28)

\[ p_1^2 = p'_1^2 = m_1^2, \quad q_+^2 = q_-^2 = m_2^2, \quad q_+^2 = q_-^2 = m^2, \quad s \gg -q_1^2 \sim -q_2^2 \sim s_{12}. \]
Using the Sudakov technique the Born amplitude for the process (27) can be represented in the form

\[ M_p = -i s 2^6 \pi^2 Z_1 Z_2 N_1 N_2 B_p(q_1, q_2) \]

with

\[ B_p(q_1, q_2) = \frac{e^\alpha e^\beta \tilde{u}(q_-) T_{\alpha \beta} v(q_+)|q_1||q_2|}{\tilde{s} q_1^2 q_2^2}, \]

\[ \tilde{s} = s \alpha_2 \beta_1 = (q_+ + q_-)^2 + (q_1 + q_2)^2, \]

\[ T_{\alpha \beta} = \gamma_\beta \frac{\hat{q}_1 - \hat{q}_+ + m}{(q_1 - q_+)^2 - m^2 \gamma_\alpha} + \gamma_\alpha \frac{\hat{q}_2 - \hat{q}_+ + m}{(q_2 - q_+)^2 - m^2 \gamma_\beta}. \]

(for details see [12]).

Generalization for the case of arbitrary number of exchanged photons between colliding nuclei is straightforward. The same approach as was used above leads to the following form of the generalized amplitude

\[ M(2 \to 4) = \frac{1}{4} \int \frac{d^2 \rho}{\pi} e^{-i q_1 \rho - i a Z_1 Z_2 \psi(\rho)} \Phi_B(\rho, q_2), \]

with

\[ \Phi_B(\rho, q_2) = \int \frac{d^2 k}{\pi} e^{ik \rho} M_p(k, q_2 - k). \]

Comparing the expression (30) with the amplitude for the process 3 \to 3 (23) one can see that the crossing symmetry property between the considered processes takes place in the case when one neglects the multiple exchanges of particle C with nuclei. Moreover, this statement is correct even when one takes into account the screening effects between nuclei A_1 and A_2 in both processes, which manifest itself by insertion of light–by–light scattering blocks into Feynman amplitudes. As was shown in [13] accounting of this effect can be provided by the universal factor

\[ \exp \left\{ -\frac{\alpha^2 Z_1 Z_2}{2} L A(\rho) \right\}, \quad L = \ln(\gamma_1 \gamma_2), \]

with the complex quantity A(\rho) connected with the Fourier transformation of light–by–light scattering amplitude.

Nevertheless, crossing symmetry is broken in all orders of perturbation theory if one tries to compare the full amplitude for the process 3 \to 3 (expression (23) with the replacement (26)) and the relevant amplitude for the process 2 \to 4 accounting the multiple interaction of produced particles [12].

Thus the crossing symmetry property takes place only for the colliding nuclei with charge numbers fulfilled the approximation Z_{1,2} \alpha \ll 1.
Fig. 1: Feynman diagrams for Born amplitudes of the process $A_1 + A_2 + C \rightarrow A_1 + A_2 + C$ (a, b) and the process $A_1 + A_2 \rightarrow A_1 + A_2 + C + \bar{C}$ (c, d).

Fig. 2: Feynman diagrams for the process $A_1 + A_2 + C \rightarrow A_1 + A_2 + C$ with three photon exchange.

Fig. 3: Feynman diagram for the process $A_1 + A_2 + C \rightarrow A_1 + A_2 + C$ with $n + 1$ exchanged photons ($n \geq 1$) between particles $A_1$ and $C$. 
Fig. 4: Feynman diagram for the amplitude $M_{(2)}^{(1)}$.

\[ \frac{1}{2!} \left( \begin{array}{c} \text{diagram} \\ \text{diagram} \end{array} \right) + \]

Fig. 5: Feynman diagram for the amplitude $M_{(\infty)}^{(\infty)}(q_1, q_2)$.

Fig. 6: Feynman diagrams for the $n$ photon exchange between nuclei $A_1$ and $A_2$ compared with the Born diagram for the processes $3 \rightarrow 3$ (a) and $2 \rightarrow 4$ (b) (blob in (b) correspond to diagrams in Fig. 1 (c, d)).
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References


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Бартош Э., Геворкян С. Р., Куреев Э. А. Е2-2002-211
Нарушение кроссинг-симметрии, связывающее процесс образования лептонных пар в столкновениях тяжелых ионов с соответствующим кроссинг-процессом

Используя технику Судакова, мы суммируем ряд поправок для процесса 3→3 и получаем компактное аналитическое выражение для амплитуды этого процесса, которое учитывает все возможные кулоновские взаимодействия между сталкивающимися частицами. Сравнивая это выражение с амплитудой образования лептонных пар в столкновениях тяжелых ионов, то есть в процессе 2→4, мы показываем, что кроссинг-симметрия между этими процессами существует, только если пренебречь взаимодействием пары с ионами (то есть в приближении Z1,2α<<1).

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Crossing Symmetry Violation in the Process of Lepton Pair Production in Relativistic Ion Collisions Compared with the Crossing Process

Using the Sudakov technique we sum the perturbation series for the process 3→3 and obtain the compact analytical expression for the amplitude of this process, which takes into account all possible Coulomb interactions between colliding particles. Comparing it with the amplitude of the lepton pair production in heavy ion collisions, i.e., in the process 2→4, we show that crossing symmetry between these processes holds only if one neglects the interaction of the produced pair with ions (i.e., in the approximation Z1,2α<<1).

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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