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**SOME UNSETTLED QUESTIONS
IN THE PROBLEM OF NEUTRINO OSCILLATIONS**

I. INTRODUCTION

The suggestion that, by analogy with K^o, \bar{K}^o oscillations, there could be neutrino oscillations (i.e., that there could be neutrino-antineutrino oscillations $\nu \rightarrow \bar{\nu}$) was considered by Pontecorvo [1] in 1957. It was subsequently considered by Maki et al. [2] and Pontecorvo [3] that there could be mixings (and oscillation) of neutrinos of different aromas (i.e., $\nu_e \rightarrow \nu_\mu$ transitions).

The problem of solar neutrinos arose after the first experiment had been performed in order to measure the flux of neutrinos from the Sun by the $^{37}\text{Cl} - ^{37}\text{Ar}$ [4] method. The flux was found to be several times smaller than expected from calculations made in accordance with the standard solar model (SSM) [5]. It was suggested in [6] that the solar neutrino deficit could be explained by Majorana neutrino oscillations. Subsequently, when the result of the experiment at Kamiokande [7] confirmed the existence of the deficit relative to the SSM calculations, one of the attractive approaches to the explanation of the solar neutrino deficit became resonant enhancement of neutrino oscillations in matter [8]. Resonant enhancement of neutrino oscillations in matter was obtained from Wolfenstein's equation for neutrinos in matter [9]. It was noted in Ref. [10] that Wolfenstein's equation for neutrinos in matter is an equation for neutrinos in matter in which they interact with matter not through the weak but through a hypothetical weak interaction that is left-right symmetric. Since only left components of neutrinos participate in the standard weak interactions, the results obtained from Wolfenstein's equation have no direct relation to real neutrinos.

Later experimentalists obtained the first results on the Gran Sasso $^{71}\text{Ga} - ^{71}\text{Ge}$ experiment [11], that within a 3σ limit did not disagree with the SSM calculations. The new data from the SAGE experiment [12] is fairly close to the Gran Sasso results.

After the discovery of neutrino transitions (oscillations) at Super-Kamiokande [13, 14] and at SNO [15], it is necessary to analyze the situation, which arises in the problem of neutrino oscillations.

This work is devoted to discussion of some theoretical and experimental questions, which have been kept unsettled in the problem of neutrino

oscillations. These are: construction of the correct neutrino oscillations theory; quest for the type of neutrino transitions (oscillations); determination of the concrete mechanism of neutrino transitions; real observation of the neutrino oscillations in experiments; solution of some contradictions in experiments (CHOOZ problem, inconsistency SNO, SK, Homestake and GNO, SAGE data).

II. THEORY

1. The Modern Theory of Neutrino Oscillations

In the old theory of neutrino oscillations [6, 16], constructed in the framework of Quantum Mechanics by analogy with the theory of K^0, \bar{K}^0 oscillation, it is supposed that mass eigenstates are ν_1, ν_2, ν_3 neutrino states but not physical neutrino states ν_e, ν_μ, ν_τ , and that the neutrinos ν_e, ν_μ, ν_τ are created as superpositions of ν_1, ν_2, ν_3 states. This meant that the ν_e, ν_μ, ν_τ neutrinos have no definite mass; i.e., their masses may vary depending on the ν_1, ν_2, ν_3 admixture in the ν_e, ν_μ, ν_τ states.

On the example of K^0, \bar{K}^0 mesons (eigenstates of the strong interactions) we can well see that on the time $10^{-23}s$ (a typical time of the strong interactions) the K_1^0, K_2^0 mesons-eigenstates of the weak interactions cannot be created since its typical time is $10^{-6} - 10^{-8}s$. Besides, every particle must be created on its mass shell and it will be left on its mass shell, while passing through vacuum. It is clear that this picture is incorrect.

Originally, it was supposed [6, 16] that these neutrino oscillations are real oscillations; i.e., that real transition of electron neutrino ν_e into muon neutrino ν_μ (or tau neutrino ν_τ) takes place. Then the neutrino $x = \mu, \tau$ is decayed in electron neutrino plus something

$$\nu_x \rightarrow \nu_e + \dots, \quad (1)$$

as a result, we get energy from vacuum, which equals the mass difference (if $m_{\nu_x} > m_{\nu_e}$)

$$\Delta E \sim m_{\nu_x} - m_{\nu_e}. \quad (2)$$

Then, again this electron neutrino transits into muon neutrino, which is

decayed again and we get energy, and etc. **So we got a perpetuum mobile!** Obviously, the law of energy conservation cannot be fulfilled in this process. The only way to restore the law of energy conservation is to demand that this process is virtual one. Then, these oscillations will be the virtual ones and they are described in the framework of the uncertainty relations. The correct theory of neutrino oscillations can be constructed only into the framework of the particle physics theory, where the conception of mass shell is present [17], [18], [20].

In the modern theory on neutrino oscillations [17, 18], constructed in the framework of the particle physics theory, it is supposed that:

1) The physical states of the ν_e, ν_μ, ν_τ neutrinos are eigenstates of the weak interaction and, naturally, the mass matrix of ν_e, ν_μ, ν_τ neutrinos is diagonal. All the available experimental results indicate that the lepton numbers l_e, l_μ, l_τ are well conserved; i.e., the standard weak interactions do not violate the lepton numbers.

2) Then, in order to violate the lepton numbers, it is necessary to introduce an interaction violating these numbers (by analogy with the $K^0 \leftrightarrow \bar{K}^0$ oscillations when the strangeness is violated by the weak interactions by using Cabibbo matrix). It is equivalent to introducing nondiagonal mass terms in the mass matrix of ν_e, ν_μ, ν_τ . By diagonalizing this matrix, we go to the ν_1, ν_2, ν_3 neutrino states. Exactly like it was in the case of K^0 mesons created in strong interactions, when mainly K^0, \bar{K}^0 mesons were produced, in the considered case ν_e, ν_μ, ν_τ , but not ν_1, ν_2, ν_3 , neutrino states are mainly created in the weak interactions (this is so, because the contribution of the lepton numbers violating interactions in this process is too small). And in such case no oscillations take place.

3) Then, when the ν_e, ν_μ, ν_τ neutrinos are passing through vacuum, they will be converted into superpositions of the ν_1, ν_2, ν_3 owing to the presence of the interactions violating the lepton number of neutrinos and will be left on their mass shells. And, then, oscillations of the ν_e, ν_μ, ν_τ neutrinos will take place according to the standard scheme [16-18]. Whether these oscillations are real or virtual, it will be determined by the masses of the physical neutrinos ν_e, ν_μ, ν_τ .

i) If the masses of the ν_e, ν_μ, ν_τ neutrinos are equal, then the real oscillation of the neutrinos will take place.

ii) If the masses of the ν_e, ν_μ, ν_τ are not equal, then the virtual oscillation of the neutrinos will take place. To make these oscillations real, these neutrinos must participate in the quasi-elastic interactions, in order to undergo transition to the mass shell of other appropriate neutrinos by analogy with $\gamma - \rho^0$ transition in the vector meson dominance model.

2. Neutrino Oscillation Types

The mass matrix of ν_e and ν_μ neutrinos has the form

$$\begin{pmatrix} m_{\nu_e} & 0 \\ 0 & m_{\nu_\mu} \end{pmatrix}. \quad (3)$$

Due to the presence of the interaction violating the lepton numbers, a nondiagonal term appears in this matrix and then this mass matrix is transformed into the following nondiagonal matrix (CP is conserved):

$$\begin{pmatrix} m_{\nu_e} & m_{\nu_e\nu_\mu} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu} \end{pmatrix}, \quad (4)$$

then the lagrangian of mass of the neutrinos takes the following form ($\nu \equiv \nu_L$):

$$\begin{aligned} \mathcal{L}_M &= -\frac{1}{2} [m_{\nu_e} \bar{\nu}_e \nu_e + m_{\nu_\mu} \bar{\nu}_\mu \nu_\mu + m_{\nu_e\nu_\mu} (\bar{\nu}_e \nu_\mu + \bar{\nu}_\mu \nu_e)] \equiv \\ &\equiv -\frac{1}{2} (\bar{\nu}_e, \bar{\nu}_\mu) \begin{pmatrix} m_{\nu_e} & m_{\nu_e\nu_\mu} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \end{aligned} \quad (5)$$

which is diagonalized by turning through the angle θ and (see ref. in [16]) and then this lagrangian (5) transforms into the following one:

$$\mathcal{L}_M = -\frac{1}{2} [m_1 \bar{\nu}_1 \nu_1 + m_2 \bar{\nu}_2 \nu_2], \quad (6)$$

where

$$m_{1,2} = \frac{1}{2} \left[(m_{\nu_e} + m_{\nu_\mu}) \pm \left((m_{\nu_e} - m_{\nu_\mu})^2 + 4m_{\nu_e\nu_\mu}^2 \right)^{1/2} \right],$$

and angle θ is determined by the following expression:

$$tg 2\theta = \frac{2m_{\nu_e\nu_\mu}}{(m_{\nu_\mu} - m_{\nu_e})}, \quad (7)$$

$$\begin{aligned}\nu_e &= \cos\theta\nu_1 + \sin\theta\nu_2, \\ \nu_\mu &= -\sin\theta\nu_1 + \cos\theta\nu_2.\end{aligned}\tag{8}$$

From Exp.(7) one can see that if $m_{\nu_e} = m_{\nu_\mu}$, then the mixing angle is equal to $\pi/4$ independently of the value of $m_{\nu_e\nu_\mu}$:

$$\begin{aligned}\sin^2 2\theta &= \frac{(2m_{\nu_e\nu_\mu})^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + (2m_{\nu_e\nu_\mu})^2}, \\ &\begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix}.\end{aligned}\tag{9}$$

It is interesting to remark that expression (9) can be obtained from the Breit-Wigner distribution [19]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2},\tag{10}$$

by using the following substitutions:

$$E = m_{\nu_e}, \quad E_0 = m_{\nu_\mu}, \quad \Gamma/2 = 2m_{\nu_e\nu_\mu},$$

where $\Gamma/2 \equiv W(\dots)$ is a width of $\nu_e \rightarrow \nu_\mu$ transition, then we can use a standard method [18, 20] for calculating this value.

The expression for time evolution of ν_1, ν_2 neutrinos (see (6), (8)) with masses m_1 and m_2 is

$$\nu_1(t) = e^{-iE_1 t} \nu_1(0), \quad \nu_2(t) = e^{-iE_2 t} \nu_2(0),\tag{11}$$

where

$$E_k^2 = (p^2 + m_k^2), \quad k = 1, 2.$$

If neutrinos are propagating without interactions, then

$$\begin{aligned}\nu_e(t) &= \cos\theta e^{-iE_1 t} \nu_1(0) + \sin\theta e^{-iE_2 t} \nu_2(0), \\ \nu_\mu(t) &= -\sin\theta e^{-iE_1 t} \nu_1(0) + \cos\theta e^{-iE_2 t} \nu_2(0).\end{aligned}\tag{12}$$

Using the expression for ν_1 and ν_2 from (6), and putting it into (10), one can get the following expression:

$$\nu_e(t) = [e^{-iE_1 t} \cos^2\theta + e^{-iE_2 t} \sin^2\theta] \nu_e(0) +$$

$$\begin{aligned}
& + [e^{-iE_1 t} - e^{-iE_2 t}] \sin\theta \cos\theta \nu_\mu(0), \\
\nu_\mu(t) = & [e^{-iE_1 t} \sin^2\theta + e^{-iE_2 t} \cos^2\theta] \nu_\mu(0) + \\
& + [e^{-iE_1 t} - e^{-iE_2 t}] \sin\theta \cos\theta \nu_e(0).
\end{aligned} \tag{13}$$

The probability that neutrino ν_e created at the time $t = 0$ will be transformed into ν_μ at the time t is an absolute value of amplitude $\nu_\mu(0)$ in (13) squared; i. e.,

$$\begin{aligned}
P(\nu_e \rightarrow \nu_\mu) = & |(\nu_\mu(0) \cdot \nu_e(t))|^2 = \\
= & \frac{1}{2} \sin^2 2\theta [1 - \cos((m_2^2 - m_1^2)/2p)t],
\end{aligned} \tag{14}$$

where it is supposed that $p \gg m_1, m_2$; $E_k \simeq p + m_k^2/2p$.

The expression (12) presents the probability of neutrino aroma oscillations. The angle θ (mixing angle) characterizes value of mixing. The probability $P(\nu_e \rightarrow \nu_\mu)$ is a periodical function of distances, where the period is determined by the following expression:

$$L_o = 2\pi \frac{2p}{|m_2^2 - m_1^2|}. \tag{15}$$

And probability $P(\nu_e \rightarrow \nu_e)$ that the neutrino ν_e created at time $t = 0$ is preserved as ν_e neutrino at time t is given by the absolute value of the amplitude of $\nu_e(0)$ in (13) squared. Since the states in (13) are normalized states, then

$$P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) = 1. \tag{16}$$

So, we see that aromatic oscillations caused by nondiagonality of the neutrinos mass matrix violate the law of the $-\ell_e$ and ℓ_μ lepton number conservations. However, in this case, as one can see from Exp. (16), the full lepton numbers $\ell = \ell_e + \ell_\mu$ are conserved.

We can also see that there are two cases of ν_e, ν_μ transitions (oscillations) [18, 20].

1. If we consider the transition of ν_e into ν_μ particle, then

$$\sin^2 2\theta \simeq \frac{4m_{\nu_e, \nu_\mu}^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + 4m_{\nu_e, \nu_\mu}^2}, \tag{17}$$

How can we understand this $\nu_e \rightarrow \nu_\mu$ transition?

If $2m_{\nu_e, \nu_\mu} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of ν_e particle is m_{ν_e} and this mass is distributed by $\sin^2 2\theta$ (or by the Breit-Wigner formula) and the probability of the $\nu_e \rightarrow \nu_\mu$ transition differs from zero and it is defined by masses of ν_e and ν_μ particles and m_{ν_e, ν_μ} , which is computed in the framework of the standard method, as pointed out above.

Another interpretation of the Exp. (17) is: If m_{ν_e, ν_μ} differs from zero then Exp. (17) gives probability of $\nu_e \leftrightarrow \nu_\mu$ transitions. If $m_{\nu_e, \nu_\mu} = 0$ then the $\nu_e \leftrightarrow \nu_\mu$ transitions are forbidden.

So, this is a solution to the problem of the origin of mixing angle in the theory of vacuum oscillations.

In this case the probability of $\nu_e \rightarrow \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \rightarrow \nu_\mu, t) = \sin^2 2\theta \sin^2 \left[\pi t \frac{|m_{\nu_1}^2 - m_{\nu_2}^2|}{2p_{\nu_e}} \right], \quad (18)$$

where p_{ν_e} is a momentum of ν_e neutrino.

2. If we consider the virtual transition of ν_e into ν_μ neutrino at $m_{\nu_e} = m_{\nu_\mu}$ (i.e., without changing the mass shell), then

$$tg2\theta = \infty, \quad (19)$$

$\theta = \pi/4$, and

$$\sin^2 2\theta = 1. \quad (20)$$

In this case the probability of the $\nu_e \rightarrow \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \rightarrow \nu_\mu, t) = \sin^2 \left[\pi t \frac{4m_{\nu_e, \nu_\mu}^2}{2p_a} \right]. \quad (21)$$

In order to make these virtual oscillations real, their participation in quasi-elastic interactions is necessary for the transitions to their own mass shells [20].

It is clear that the $\nu_e \rightarrow \nu_\mu$ transition is a dynamical process.

3. The third type of transitions (oscillations) can be realized by mixings of the fields (neutrinos) by analogy with the vector dominance model ($\gamma - \rho^0$ and $Z^0 - \gamma$ mixings) in a way as it takes place in the particle physics. Since the weak couple constants $g_{\nu_e}, g_{\nu_\mu}, g_{\nu_\tau}$ of ν_e, ν_μ, ν_τ neutrinos are nearly equal in reality, i.e., $g_{\nu_e} \simeq g_{\nu_\mu} \simeq g_{\nu_\tau}$ the angle mixings are nearly maximal:

$$\sin\theta_{\nu_e\nu_\mu} \simeq \frac{g_{\nu_e}}{\sqrt{g_{\nu_e}^2 + g_{\nu_\mu}^2}} = \frac{1}{\sqrt{2}} \simeq \sin\theta_{\nu_e\nu_\tau} \simeq \sin\theta_{\nu_\mu\nu_\tau}. \quad (22)$$

Therefore, if the masses of these neutrinos are equal (which is hardly probable), then transitions between neutrinos will be real and if the masses of these neutrinos are not equal then transitions between neutrinos will be virtual by analogy with $\gamma - \rho^0$ transitions. In this approach we can hardly see neutrino oscillations, though it is possible, in principle.

Probably a more realistic type of neutrino oscillations is the third one since oscillations can arise by a dynamics. Exactly the third type of mixings is realized through dynamical charges $g_{\nu_e}, g_{\nu_\mu}, g_{\nu_\tau}$ of neutrinos but not through neutrino masses. Nevertheless, we must get answer to the question: which of the above three types of neutrino oscillations is realized in the Nature?

3. Mechanisms of Neutrino Oscillations

3.1. Impossibility of Resonance Enhancement of Neutrino Oscillations in Matter

By three different approaches: by using mass Lagrangian [10, 21, 22], by using the Dirac equation [21, 22], and by using the operator formalism [23], the author of this work has discussed the problem of the mass generation in the standard weak interactions and has come to a conclusion that the standard weak interaction cannot generate masses of fermions since the right-handed components of fermions do not participate in these interactions. It is also shown [24] that the equation for Green function of the weak-interacting fermions (neutrinos) in the matter coincides with the equation for Green function of fermions in vacuum and the law of

conservation of the energy and the momentum of neutrino in matter will be fulfilled [23] only if the energy W of polarization of matter by the neutrino or the corresponding term in Wolfenstein equation, is zero (it means that neutrinos cannot generate permanent polarization of matter). These results lead to the conclusion: resonance enhancement of neutrino oscillations in matter does not exist.

The simplest method to prove the absence of the resonance enhancement of neutrino oscillations in matter is:

If we put an electrical (e) (or strong (g)) charged particle a in vacuum, there arises polarization of vacuum. Since the field around particle a is spherically symmetrical, the polarization must also be spherically symmetrical. Then the particle will be left at rest and the law of energy and momentum conservation is fulfilled.

If we put a weakly (g_W) interacting particle b (a neutrino) in vacuum, then since the field around the particle has a left-right asymmetry (weak interactions are left interactions with respect to the spin direction [25, 29]), polarization of vacuum must be nonsymmetrical; i.e., on the left side there arises maximal polarization and on the right there is zero polarization. Since polarization of the vacuum is asymmetrical, there arises asymmetrical interaction of the particle (the neutrino) with vacuum and the particle cannot be at rest and will be accelerated. Then the law of energy momentum conservation will be violated. The only way to fulfill the law of energy and momentum conservation is to demand that polarization of vacuum be absent in the weak interactions. The same situation will take place in matter. It is necessary to remark that for above-considered proof it is sufficient to know that the field around of the weakly interacting particle is asymmetrical (and there is not necessary to know the precise form of this field). It is necessary also to remark that the Super-Kamiokande data on day-night asymmetry [13] is

$$A = (D - N) / \left(\frac{1}{2}(D + N)\right) = -0.021 \pm 0.020(stat) + 0.013(-0.012)(syst). \quad (23)$$

and it does not leave hope on possibility of the resonance enhancement of neutrino oscillations in matter.

It means that the forward scattering amplitude of the weak interactions have a specific behavior.

In the Sun neutrino experiments, it is impossible to distinguish muon and tau neutrinos since energies of the Sun neutrinos are less than the threshold energy of muon and tau lepton creations. However we can distinguish these neutrinos on the Earth long baseline accelerator neutrino experiments with neutrino energies higher than the threshold energy of muon and tau lepton creations. Probably such experiments put an end to the problem of resonance enhancement neutrino oscillations in matter (see the strong proof absence of this effect given above).

It is interesting to remark that in the gravitational interaction the polarization does not exist either [25].

An Amplitude of Forward Scattering at the Weak Interactions

Connection between refraction coefficient n and amplitude of forward scattering $f_k(k, 0)$ in matter is given by the following expression:

$$n - 1 = \frac{2\pi}{k^2} \sum_i \text{Re} f_i(k, 0), \quad (24)$$

where i is index of summation and k, p , respectively, are momentum and transfer momentum.

Since at the weak interactions the polarization is absent, then

$$n = 1, \quad (25)$$

and

$$\text{Re} f_i(k, 0) = 0, \quad (26)$$

i.e.,

$$\begin{aligned} \text{Re} f_i(k, p) &\rightarrow 0, \quad (27) \\ p &\rightarrow 0. \end{aligned}$$

The amplitude of forward scattering goes to zero when transfer momentum goes to zero in contrast to the strong and electromagnetic interactions, where it differs from zero.

3.2. Majorana Neutrino Oscillations

At present it is supposed [26] that the neutrino oscillations can be connected with Majorana neutrino oscillations. It will be shown that we cannot put Majorana neutrinos in the standard Dirac theory. It means that in experiments the Majorana neutrino oscillations cannot be observed.

Majorana fermion in Dirac representation has the following form [6, 16, 27]:

$$\begin{aligned}\chi^M &= \frac{1}{2}[\Psi(x) + \eta_C \Psi^C(x)], \\ \Psi^C(x) &\rightarrow \eta_C C \bar{\Psi}^T(x),\end{aligned}\tag{28}$$

where η_C is a phase, C is a charge conjunction, T is a transposition.

From Exp. (28) we see that Majorana fermion χ^M has two spin projections $\pm\frac{1}{2}$ and then the Majorana spinor can be rewritten in the following form:

$$\chi^M(x) = \begin{pmatrix} \chi_{+\frac{1}{2}}(x) \\ \chi_{-\frac{1}{2}}(x) \end{pmatrix}.\tag{29}$$

The mass Lagrangian of Majorana neutrinos in the case of two neutrinos χ_e, χ_μ ($-\frac{1}{2}$ components of Majorana neutrinos, and $\bar{\chi}...$ is the same Majorana fermion with the opposite spin projection) in the common case has the following form:

$$\mathcal{L}'_M = -\frac{1}{2}(\bar{\chi}_e, \bar{\chi}_\mu) \begin{pmatrix} m_{\chi_e} & m_{\chi_e \chi_\mu} \\ m_{\chi_\mu \chi_e} & m_{\chi_\mu} \end{pmatrix} \begin{pmatrix} \chi_e \\ \chi_\mu \end{pmatrix}.\tag{30}$$

Diagonalizing this mass matrix by standard methods, one obtains the following expression:

$$\mathcal{L}'_M = -\frac{1}{2}(\bar{\nu}_1, \bar{\nu}_2) \begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix},\tag{31}$$

where

$$\nu_1 = \cos\theta\chi_e - \sin\theta\chi_\mu,$$

$$\nu_2 = \sin\theta\chi_e + \cos\theta\chi_\mu.$$

These neutrino oscillations are described by expressions (11)-(22) with the following substitution of $\nu_{e\mu} \rightarrow \chi_{e\mu}^M$.

The standard theory of weak interactions is constructed on the base of local gauge invariance of Dirac fermions. In this case Dirac fermions have the following lepton numbers l_l , which are conserved,

$$l_l, l = e, \mu, \tau, \quad (32)$$

and Dirac antiparticles have lepton numbers with the opposite sign

$$\bar{l} = -l_l. \quad (33)$$

Gauge transformation of Majorana fermions can be written in the form:

$$\begin{aligned} \chi'_{+\frac{1}{2}}(x) &= \exp(-i\beta)\chi_{+\frac{1}{2}}(x), \\ \chi'_{-\frac{1}{2}}(x) &= \exp(+i\beta)\chi_{-\frac{1}{2}}(x). \end{aligned} \quad (34)$$

Then lepton numbers of Majorana fermions are

$$l^M = \sum_i l_i^M(+1/2) = - \sum_i l_i^M(-1/2),$$

i. e., antiparticle of Majorana fermion is the same fermion with the opposite spin projection.

Now we come to discussion of the problem of the place of Majorana fermion in the standard theory of weak interactions [28].

To construct the standard theory of weak interactions [29], Dirac fermions are used. The absence of contradiction of this theory with the experimental data confirms that all fermions are Dirac particles.

Now, if we want to put the Majorana fermions into the standard model, we must take into account that, in the common case, the gauge charges of the Dirac and Majorana fermions are different (especially it is well seen in the example of Dirac fermion having an electrical charge since it cannot have a Majorana charge (it is worth to remind that in the weak currents the fermions are included in the couples form)). In this case we cannot just include Majorana fermions in the standard theory of weak

interactions by gauge invariance manner. Then, in the standard theory the Majorana fermions cannot appear.

If we include Majorana neutrinos into the standard model, then in experiments we must see the following reactions:

$$\chi_l + A(Z) \rightarrow l^- + A(Z + 1),$$

with probability 1/2 and

$$\chi_l + A(Z) \rightarrow l^+ + A(Z - 1),$$

with the same probability (where $x = e, \mu, \tau$), since Majorana neutrinos are superpositions of Dirac neutrinos and antineutrinos. Obviously, all the available experimental data [30] does not confirm this predictions, therefore we cannot consider this mechanism as realistic one for neutrino oscillations.

3.3. Transitions (Oscillations) of Aromatic Neutrinos

In the work [2] Maki et al. supposed that there could exist transitions between aromatic neutrinos ν_e, ν_μ . Afterwards ν_τ was found and then ν_e, ν_μ, ν_τ transitions could be possible. The author of this work has developed this direction (see [21, 25]). It is necessary to remark that only this scheme of oscillations is realistic for neutrino oscillations (see also this work). The expressions, which described neutrino oscillations in this case are given above in expressions (9)-(22).

III. EXPERIMENTS

1. Experimental Observation of the Neutrino Oscillations

At present, it is supposed that the neutrino oscillations were observed [13-15]. In these experiments only transitions between the Sun or atmospheric neutrinos have been observed. Since we suppose that the neutrino oscillations take place, therefore we must observe (the Sun) neutrino oscillations in reality. Since the length of neutrino oscillations is sufficiently

great, we cannot observe higher modes in terrestrial experiment. But we have another possibility to observe the Sun neutrino oscillations using the fact that the Earth orbit is elliptic one with:

$$\text{Earth's perihelion } R_P = 147.117 \cdot 10^6 km ,$$

$$\text{Earth's aphelion } R_A = 152.083 \cdot 10^6 km,$$

and their difference ΔR is $\Delta R = 4.866 \cdot 10^6 km$. Since the Sun neutrinos include all energies up to $15 MeV$, we must divide this energy spectrum into energy regions (also distances must be divided into regions) and observe these neutrino fluxes as function of energy and the Earth's distances from the Sun. On these conditions we must observe the neutrino oscillations if the length of neutrino oscillation R_{osc} is bigger than the region Δ , where these (high energy) neutrinos are generated on the Sun, i.e.,

$$\Delta \sim 0.05 R_{sun} \sim 10^4 km, \quad (35)$$

$$R_{osc} > \Delta.$$

It is obvious that in these experiments it is impossible to register $\nu_\mu \leftrightarrow \nu_\tau$ oscillations since their length is small enough, as we know from Super-Kamiokande experiments. The Super-Kamiokande and SNO detectors wholly fit for such observations.

2. The Earth Neutrinos

At present, there exist big detectors for observation of the Sun and atmospheric neutrinos. The problem study of the Earth neutrino sources represents enormous interest, therefore, using the same detectors, it is possible to research the Earth neutrino sources. More detailed consideration of this question will be published later. It is important that there is no necessity to reconstruct the detectors. It is only necessary to collect data from the Earth as it is fulfilled for the Sun neutrinos but from opposite to the Sun direction [31]. In this way we can obtain the Earth neutrino sources map using the detectors located in different places of the Earth surface.

3. The Problem with the GNO (GALLEX), SAGE Data

As stressed above, transitions between neutrinos with different flavor have been already observed and the neutrino mixing angles are nearly maximal. The Super-Kamiokande [13], SNO [15], Homestake [32] data (D) normalized on SSM calculations (see [33])) are in good agreement.

Homestake 1970-1994, $E_{thre} = 0.814$:

$$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-, \quad \frac{D^{exp}}{D^{BP2000}} = 0.34 \pm 0.03$$

Super-Kamiokande 1996-2001, $E_{thre} = 4.75\text{MeV}$:

$$\nu_e + e^- \rightarrow \nu_e + e^-, \quad \frac{D^{exp}}{D^{BP2000}} = 0.465 \pm 0.015;$$

SNO $E_{thre} = 6.9\text{MeV}$

$$\nu_e + d \rightarrow p + p + e^-, \quad \frac{D^{exp}}{D^{BP2000}} = 0.35 \pm 0.02,$$

$E_{thre} = 2.2\text{MeV}$

$$\nu_e + d \rightarrow p + n + e^-, \quad \frac{D^{exp}}{D^{BP2000}} = 1.01 \pm 0.13,$$

$E_{thre} = 5.2\text{MeV}$

$$\nu + e^- \rightarrow \nu + e^-, \quad \frac{D^{exp}}{D^{BP2000}} = 0.47 \pm 0.05,$$

But normalized on SSM [33] GNO (GALLEX) [34], SAGE [12, 35] data are higher than above data on values 0.16 – 0.20

GNO (GALLEX) 1998-2000, $E_{thre} = 0.233\text{MeV}$

$$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-, \quad \frac{D^{exp}}{D^{BP2000}} = 0.51 \pm 0.08;$$

SAGE 1990-2001, $E_{thre} = 0.233\text{MeV}$

$$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-, \quad \frac{D^{exp}}{D^{BP2000}} = 0.54 \pm 0.05;$$

Since the length of neutrino transitions (oscillations) is proportional to their energy (see Exp.(13)), therefore, the lower the neutrino energies, the smaller the lengths of transitions (oscillations), hence the norm of transitions (oscillations) at the low energies must be the same as at the high ones. In order to resolve this problem, it is probably necessary to examine calibration of the last experiments. It is more likely that the Standard Sun model [33] requires a revision in these energy regions. It is very important to organize an experiment with neutral currents in this energy region, in order to register the full (the Sun) neutrino fluxes. It is clear that there is no sense in drawing the allowed regions pictures [13] until this problem is solved.

4. The CHOOZ Problem

In the reactor experiment on $\bar{\nu}_e$ neutrinos in France, there was observed a very small angle mixing for $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ transitions [36]. If this result is correct then only the $\nu_e \rightarrow \nu_\mu$ neutrino transitions there should be observed in SNO [15] and the $\nu_e \rightarrow \nu_\tau$ transitions must be suppressed, i.e., relation between ν_e and ν_μ neutrino fluxes must be equal

$$\Phi_{\nu_e} \simeq \Phi_{\nu_\tau}. \quad (36)$$

However, in the SNO experiments on neutral currents, we can see approximate equality in the numbers of the three types of neutrinos

$$\Phi_{\nu_e} \simeq \frac{1}{2}(\Phi_{\nu_\tau} + \Phi_{\nu_\mu}). \quad (37)$$

It is clear that the angle of $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ transition cannot be small. Since the distance from the reactor is small, the detector can register only a small mixing angle (in order to see a correct mixing angle, the detector must be at a distance which is an order of the oscillations length).

The big angle mixing obtained in KamLAND [37] also does not confirm the CHOOZ data.

VI. CONCLUSION

It is noted that the theory of neutrino oscillations can be constructed only in the framework of the particle physics theory, where a mass shell conception is present and then transitions (oscillations) between neutrinos with equal masses are real and between neutrinos with different masses are virtual.

There can be three types of neutrino transitions (oscillations). It is necessary to solve the question: which type of neutrino transitions (oscillations) is realized in nature?

At present it is considered that Dirac and Majorana neutrino oscillations can be realized. It is shown that we cannot put Majorana neutrinos in the standard weak interactions theory without violation of the gauge invariance. If we use the Majorana neutrinos then we come to contradiction with the existing experimental data. Then it is obvious that there can be only realized transitions (oscillations) between Dirac neutrinos with different flavors.

It is also shown that the mechanism of resonance enhancement of neutrino oscillations in matter cannot be realized without violation of the law of energy-momentum conservation.

Though it is supposed that we see neutrino oscillations in experiments, indeed there only transitions between neutrinos are registered. In order to register neutrino oscillations, it is necessary to see second or even higher neutrino oscillation modes in experiments. For this purpose we can use the elliptic character of the Earth orbit.

The analysis shows that the SNO experimental results do not confirm the smallness of $\nu_e \rightarrow \nu_\tau$ transition angle mixings, which was obtained in the CHOOZ experiment. It is also noted that there is contradiction between the SNO, Super-Kamiokande, Homestake and SAGE, GNO (GALLEX) data.

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Некоторые нерешенные вопросы в проблеме осцилляции нейтрино

Отмечено, что теорию осцилляции нейтрино можно построить только в рамках физики частиц, где используется понятие массовой поверхности, и тогда переходы (осцилляции) между нейтрино с равными массами являются реальными, а переходы между нейтрино с разными массами являются виртуальными. Показано, что могут существовать три типа нейтринных переходов. В экспериментах необходимо решить вопрос, какой из этих типов нейтринных переходов реализуется в природе.

В настоящее время предполагается, что могут существовать осцилляции дираковских и майорановских нейтрино. Показано, что мы не можем подставить майорановское нейтрино в стандартную теорию без нарушения калибровочной инвариантности. Кроме того, если мы предполагаем, что нейтрино являются майорановскими частицами, то это приводит к экспериментально не подтвержденным результатам. Тогда, очевидно, могут реализоваться только переходы между дираковскими нейтрино различных ароматов. Также показано, что механизм резонансного усиления осцилляции нейтрино в веществе не может реализоваться без нарушения закона сохранения энергии-импульса. Хотя считается, что мы видим осцилляции нейтрино, но на самом деле зарегистрированы только переходы между нейтрино. Чтобы зарегистрировать осцилляции нейтрино, необходимо в эксперименте увидеть вторые и более высокие моды осцилляции. Для этой цели можно использовать эллиптический характер земной орбиты при наблюдении солнечных нейтрино. Анализ показывает, что экспериментальные результаты, полученные в SNO, не подтверждают малость угла смешивания $\nu_e \rightarrow \nu_\tau$ -переходов, полученных в эксперименте CHOOZ. Также отмечается, что имеет место противоречие между данными SNO, Super-Kamiokande, Homestake и SAGE, GNO (GALLEX).

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Перевод автора

Some Unsettled Questions in the Problem of Neutrino Oscillations

It is noted that the theory of neutrino oscillations can be constructed only in the framework of the theory particle physics where a mass shell conception is presented and then transitions (oscillations) between neutrinos with equal masses are real and between neutrinos with different masses are virtual. There can be three types of neutrino transitions. In the experiments it is necessary to decide the question: which type of neutrino transitions is realized in nature?

At present it is supposed that Dirac and Majorana neutrino oscillations can be realized. It is shown that we cannot put Majorana neutrinos in the standard weak interactions theory without violation of the gauge invariance. If we use the Majorana neutrinos then we come to contradiction with the existing experimental data. Then it is obvious that there can be only realized transitions between Dirac neutrinos with different flavors. It is also shown that the mechanism of resonance enhancement of neutrino oscillations in matter cannot be realized without violation of the law of energy-momentum conservation. Though it is supposed that in experiments we see neutrino oscillations, indeed only transitions between neutrinos are registered. To register neutrino oscillations, it is necessary to see second or even higher neutrino oscillation modes in experiments. For this purpose we can use the elliptic character of the Earth orbit at registrations of sun neutrinos. The analysis shows that the SNO experimental results do not confirm smallness of $\nu_e \rightarrow \nu_\tau$ transition angle mixing, which was obtained in the CHOOZ experiment. Also is noted that there is contradiction between the SNO, Super-Kamiokande, Homestake and SAGE, and GNO (GALLEX) data.

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