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D. Cirilo Lombardo*

STATIC AND ROTATING BLACK HOLES IN EINSTEIN–BORN–INFELD THEORIES

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*E-mail: diego@thsun1.jinr.ru, diego77jcl@yahoo.com

1 Introduction and results:

The four dimensional solutions with spherical symmetry of the Einstein equations coupled to Born-Infeld fields have been well studied in the literature [1,2,9,10]. In particular, the electromagnetic field of the Born Infeld monopole, in contrast to Maxwell counterpart, contributes to the ADM mass of the system (it is, the four momentum of assymptotic flat manifolds). B. Hoffmann was the first who studied such static solutions in the context of the general relativity with the idea of to obtain a consistent particle-like model [2]. Unfortunately, these static Einstein-Born-Infeld models generate conical singularities at the origin [2,9]. This type of singularities cannot be removed as in global monopoles or other non-localized defects of the spacetime [6,8]. In this work a *new* static spherically symmetric solution with Born-Infeld charge is obtained. The new metric, when the intrinsec mass of the system is zero, is *regular* anywhere in the sense that was given by B. Hoffmann and L. Infeld in 1937[9], it is, without discontinuities and singularities in the electromagnetic fields, the metric tensor and their derivatives in all the manifold. The fundamental feature of this solution is the lack of conical singularities at the origin. A distant observer will associate with this solution an electromagnetic mass that is a twice of the mass of the electromagnetic geon founded by M. Demianski in 1986 [10]. The energy-momentum tensor and the electric field are both regular with zero value at the origin and new parameters appear, given to the new metric surprising behaviours. The used convention is the *spatial* of Landau and Lifshitz (1974), with signatures of the metric, Riemann and Einstein all positives (+++) [4,5].

2 The Born-Infeld theory:

The most significative non-linear theory of electrodynamics is, by excellence, the Born-Infeld theory [1,3]. Among its many special properties is an exact SO(2) electric-magnetic duality invariance. The Lagrangian density describing Born-Infeld theory (in arbitrary spacetime dimensions) is

$$\mathcal{L}_{BI} = \sqrt{g}L_{BI} = \frac{4\pi}{b^2}\left\{\sqrt{g} - \sqrt{|\det(g_{\mu\nu} + bF_{\mu\nu})|}\right\} \tag{1}$$

where b is a fundamental parameter of the theory, with dimensions of field. In open superstring theory, for example, loop calculations lead to this Lagrangian with $b = 2\pi\alpha' (\alpha' \equiv inverse of the string tension)$. In four spacetime dimensions the determinant in (2) may be expanded out to give

$$L_{BI} = \frac{4\pi}{b^2}\left\{1 - \sqrt{1 + \frac{1}{2}b^2 F_{\mu\nu}F^{\mu\nu} - \frac{1}{16}b^4\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2}\right\}$$

which coincides with the usual Maxwell Lagrangian in the weak field limit.

It is useful to define the second rank tensor $P^{\mu\nu}$ by

$$P^{\mu\nu} = -\frac{1}{2}\frac{\partial L_{BI}}{\partial F_{\mu\nu}} = \frac{F^{\mu\nu} - \frac{1}{4}b^2\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)\widetilde{F}^{\mu\nu}}{\sqrt{1 + \frac{1}{2}b^2 F_{\mu\nu}F^{\mu\nu} - \frac{1}{16}b^4\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2}} \tag{2}$$

(so that $P^{\mu\nu} \approx F^{\mu\nu}$ for weak fields) and satisfies the electromagnetic equations of motion

$$\nabla_\mu P^{\mu\nu} = 0 \tag{3}$$

which are highly non linear in $F_{\mu\nu}$. The energy-momentum tensor may be written as

$$T_{\mu\nu} = \frac{1}{4\pi}\left\{\frac{F_\mu{}^\lambda F_{\nu\lambda} + \frac{1}{b^2}\left[\mathbb{R} - 1 - \frac{1}{2}b^2 F_{\mu\nu}F^{\mu\nu}\right] g_{\mu\nu}}{\mathbb{R}}\right\} \tag{4}$$

$$\mathbb{R} \equiv \sqrt{1 + \frac{1}{2}b^2 F_{\mu\nu}F^{\mu\nu} - \frac{1}{16}b^4\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2}$$

Although it is by no means obvious, it may verified that equations (3)-(5) are invariant under electric-magnetic duality $F \longleftrightarrow *G$. We can show that the SO(2) structure of the Born-Infeld theory is more easy to see in quaternionic form [13,14]

$$\frac{1}{R}\left(\sigma_0 + i\sigma_2\overline{\mathbb{P}}\right) L = \mathbb{L}$$

$$\frac{\mathbb{R}}{\left(1 + \overline{\mathbb{P}}^2\right)}\left(\sigma_0 - i\sigma_2\overline{\mathbb{P}}\right)\mathbb{L} = L$$

$$\overline{\mathbb{P}} \equiv \frac{\mathbb{P}}{b}$$

where we have been defined

$$L = F - i\sigma_2\widetilde{F}$$

$$\mathbb{L} = P - i\sigma_2 \widetilde{P}$$

the pseudoescalar of the electromagnetic tensor $F^{\mu\nu}$

$$\mathbb{P} = -\frac{1}{4} F_{\mu\nu} \widetilde{F}^{\mu\nu}$$

and σ_0 , σ_2 the well know Pauli.´s matrix.

In flat space, and for purely electric configurations, the Lagrangian (2) reduces to

$$L_{BI} = \frac{4\pi}{b^2} \left\{ 1 - \sqrt{1 + b^2 \overrightarrow{E}^2} \right\}$$

so there is an upper bound on the electric field strength $\overrightarrow{E}$

$$\left| \overrightarrow{E} \right| \leq \frac{1}{b} \tag{5}$$

3 Statement of the problem:

We propose the following line element for the static Born-Infeld monopole

$$ds^2 = -e^{2\Lambda} dt^2 + e^{2\Phi} dr^2 + e^{2F(r)} d\theta^2 + e^{2G(r)} \sin^2\theta \, d\varphi^2 \tag{6}$$

where the components of the metric tensor are

$$\begin{array}{rlrl} g_{tt} &= -e^{2\Lambda} & g_{tt} &= -e^{-2\Lambda} \\ g_{rr} &= e^{2\Phi} & g_{rr} &= e^{-2\Phi} \\ g_{\theta\theta} &= e^{2F} & g_{\theta\theta} &= e^{-2F} \\ g_{\varphi\varphi} &= \sin^2\theta \, e^{2G} & g^{\varphi\varphi} &= \frac{e^{-2G}}{\sin^2\theta} \end{array} \tag{7}$$

For the obtention of the Einstein-Born-Infeld equations system we use the Cartan´s structure equations method [11], that is most powerful and direct where we work with differential forms and in a orthonormal frame (tetrad). The line element (6) in the 1-forms basis takes the following form

$$ds^2 = -\left(\omega^0\right)^2 + \left(\omega^1\right)^2 + \left(\omega^2\right)^2 + \left(\omega^3\right)^2 \tag{8}$$

were the forms are

$$\begin{array}{lcr} \omega^0 = e^{\Lambda} dt & \Rightarrow & dt = e^{-\Lambda}\omega^0 \\ \omega^1 = e^{\Phi} dr & \Rightarrow & dr = e^{-\Phi}\omega^1 \\ \omega^2 = e^{F(r)} d\theta & \Rightarrow & d\theta = e^{-F(r)}\omega^2 \\ \omega^3 = e^{G(r)} \sin\theta\, d\varphi & \Rightarrow & d\varphi = e^{-G(r)} (\sin\theta)^{-1}\, \omega^3 \end{array} \tag{9}$$

Now, we follow the standard procedure of the structure equations (Appendix):

$$d\omega^\alpha = -\omega^\alpha{}_\beta \wedge \omega^\beta \tag{10}$$

$$\mathcal{R}^\alpha{}_\beta = d\,\omega^\alpha{}_\beta + \omega^\alpha{}_\lambda \wedge \omega^\lambda{}_\beta \tag{11}$$

The components of the Riemann tensor are easily obtained from the well know geometrical relation of Cartan:

$$\mathcal{R}^\alpha{}_\beta = R^\alpha{}_{\beta\rho\sigma}\, \omega^\rho \wedge \omega^\sigma$$

where we obtain explicitly

$$R^0{}_{110} = e^{-2\Phi} \left(\partial_r\partial_r\Lambda - \partial_r\Phi\, \partial_r\Lambda + (\partial_r\Lambda)^2\right) \tag{12}$$

$$R^2{}_{112} = e^{-2\Phi} \left(\partial_r\partial_r F - \partial_r\Phi\, \partial_r F + (\partial_r F)^2\right)$$

$$R^3{}_{113} = e^{-2\Phi} \left(\partial_r\partial_r G - \partial_r\Phi\, \partial_r G + (\partial_r G)^2\right)$$

$$R^3{}_{213} = e^{-(F+\Phi)}\, \partial_r\,(G - F)\, \frac{\cos\theta}{\sin\theta}$$

$$R^3{}_{123} = e^{-(F+\Phi)}\, \partial_r\,(G - F)\, \frac{\cos\theta}{\sin\theta}$$

$$R^3{}_{223} = e^{-2\Phi} \partial_r G\ \partial_r F - e^{-2F}$$

$$R^0{}_{330} = e^{-2\Phi}\, \partial_r\Lambda\ \partial_r G$$

$$R^0{}_{220} = e^{-2\Phi}\, \partial_r\Lambda\ \partial_r F$$

From the components of the Riemann tensor we can construct the Einstein equations

$$G^0{}_0 = -\left(R^{12}{}_{12} + R^{23}{}_{23} + R^{31}{}_{31}\right) \tag{13}$$

$$G^1{}_1 = -\left(R^{02}{}_{02} + R^{03}{}_{03} + R^{23}{}_{23}\right) \tag{14}$$

$$G^0{}_1 = R^{02}{}_{12} + R^{03}{}_{13} \tag{15}$$

$$G^1{}_2 = R^{10}{}_{20} + R^{13}{}_{23} \tag{16}$$

explicitly

$$G^1{}_2 = -e^{-(F+G)}\frac{\cos\theta}{\sin\theta}\partial_r\left(G - F\right) \tag{17}$$

$$G^0{}_0 = e^{-2\Phi}\Psi - e^{-2F} \tag{18}$$

where

$$\Psi = \left[\partial_r\partial_r\left(F+G\right) - \partial_r\Phi\,\partial_r\left(F+G\right) + \left(\partial_r F\right)^2 + \left(\partial_r G\right)^2 + \partial_r F\,\partial_r G\right]$$

$$G^1{}_1 = e^{-2\Phi}\left[\partial_r\Lambda\,\partial_r\left(F+G\right) + \partial_r F\,\partial_r G\right] - e^{-2F} \tag{19}$$

$$G^2{}_2 = e^{-2\Phi}\left[\partial_r\partial_r\left(\Lambda+G\right) - \partial_r\Phi\,\partial_r\left(\Lambda+G\right) + \left(\partial_r\Lambda\right)^2 + \left(\partial_r G\right)^2 + \partial_r\Lambda\,\partial_r G\right] \tag{20}$$

$$G^3{}_3 = e^{-2\Phi}\left[\partial_r\partial_r\left(F+\Lambda\right) - \partial_r\Phi\,\partial_r\left(F+\Lambda\right) + \left(\partial_r\Lambda\right)^2 + \left(\partial_r F\right)^2 + \partial_r F\,\partial_r\Lambda\right] \tag{21}$$

$$G^1{}_3 = G^2{}_3 = G^0{}_3 = G^0{}_2 = G^0{}_1 = 0 \tag{22}$$

4 Energy-momentum tensor of the Born-Infeld monopole

In the tetrad that was selected by us, the energy-momentum tensor takes a diagonal form, being its components the following

$$-T_{00} = T_{11} = \frac{b^2}{4\pi}\left(\frac{\mathbb{R}-1}{\mathbb{R}}\right) \tag{23}$$

$$T_{22} = T_{33} = \frac{b^2}{4\pi}\left(1-\mathbb{R}\right) \tag{24}$$

where we have been defined

$$\mathbb{R} \equiv \sqrt{1-\left(\frac{F_{01}}{b}\right)^2} \tag{25}$$

in this manner, one can see from the Einstein equation (16) the characteristic property of this spacetime

$$G^1{}_2 = -e^{-(F+G)}\frac{\cos\theta}{\sin\theta}\partial_r\left(G-F\right) = 0 \qquad \Rightarrow \qquad G = F \tag{26}$$

5 Equations for the electromagnetic fields of Born-Infeld in the tetrad

The equations that describe the dynamic of the electromagnetic fields of Born-Infeld in a curved spacetime are

$$\nabla_a \mathbb{F}^{ab} = \nabla_a\left[\frac{F^{ab}}{\mathbb{R}} + \frac{P}{b^2\mathbb{R}}\widetilde{F}^{ab}\right] = 0 \qquad (field\, equations) \tag{27}$$

$$\nabla_a\, \widetilde{F}^{ab} = 0 \qquad (\, Bianchi's\ identity) \tag{28}$$

where we was defined

$$P \equiv -\frac{1}{4}F_{\alpha\beta}\widetilde{F}^{\alpha\beta} \tag{29}$$

$$S \equiv -\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} \tag{30}$$

$$\mathbb{R} \equiv \sqrt{1 - \frac{2S}{b^2} - \left(\frac{P}{b^2}\right)^2} \tag{31}$$

The above equations can be expressed geometrically of the following manner

$$dF = 0 \tag{32}$$

$$d^*\mathbb{F} = 0 \tag{33}$$

in explicit form

$$dF = d\left(F_{01}\omega^0 \wedge \omega^1\right) = \partial_\theta\left(e^{\Lambda+\Phi}F_{01}\right) d\theta \wedge dt \wedge dr = 0 \Rightarrow F_{01} = A\left(r\right) \tag{34}$$

$$\begin{aligned} d^*\mathbb{F} &= d\left(-\mathbb{F}_{01}\omega^3 \wedge \omega^2\right) = \partial_r\left(-e^{F+G}\mathbb{F}_{01}\sin\theta\right) dr \wedge d\varphi \wedge d\theta = 0 \\ &\Rightarrow e^{2G}\mathbb{F}_{01} = f\left(\theta\right) \end{aligned} \tag{35}$$

we can see from equation (25) and (27) that

$$\mathbb{F}_{01} = \frac{F_{01}}{\sqrt{1 - \left(\overline{F}_{01}\right)^2}}$$

where we obtains the following form for the electric field of the self-gravitating B-I monopole

$$F_{01} = \frac{b}{\sqrt{\left(\frac{b}{f(\theta)}e^{2G}\right)^2 + 1}} \tag{36}$$

we can to associate (see reference [1])

$$f\left(\theta\right) = br_0^2\left(\theta\right) \equiv Q \qquad \Rightarrow \qquad F_{01} = \frac{b}{\sqrt{\left(\frac{e^G}{r_0}\right)^4 + 1}} \tag{37}$$

Where r_0 is a constant, that in principle we put dependent of θ, with units of longitude. This election of a constant θ-dependent is because F_{01} , well know problem in many solutions of electromagnetic configurations in General Relativity, not necesarilly will be have the same symmetry of the spacetime.

6 System of equations for the self-gravitating BI-monopole

From the Einstein equations we have

$$F = G \tag{38}$$

$$G^0{}_0 = e^{-2\Phi}\left[2\,\partial_r\partial_r G - \partial_r\Phi\, 2\,\partial_r G + 3\left(\partial_r G\right)^2\right] - e^{-2G}$$

$$G^1{}_1 = e^{-2\Phi}\left[2\,\partial_r\Lambda\,\partial_r G + \left(\partial_r G\right)^2\right] - e^{-2G} \tag{39}$$

$$G^2{}_2 = G^3{}_3 = e^{-2\Phi}\,\Xi \tag{40}$$

$$\Xi \equiv \left[\partial_r\partial_r\left(\Lambda + G\right) - \partial_r\Phi\,\partial_r\left(\Lambda + G\right) + \left(\partial_r\Lambda\right)^2 + \left(\partial_r G\right)^2 + \partial_r\Lambda\,\partial_r G\right]$$

and the components of the energy-momentum tensor takes its explicit form reemplacing the F_{01}that we was found in equations (37)

$$-T_{00} = T_{11} = \frac{b^2}{4\pi}\left(1 - \sqrt{\left(\frac{r_0}{e^G}\right)^4 + 1}\right) \tag{41}$$

$$T_{22} = T_{33} = \frac{b^2}{4\pi}\left(1 - \frac{1}{\sqrt{\left(\frac{r_0}{e^G}\right)^4 + 1}}\right) \tag{42}$$

and from

$$G^a{}_b = 8\pi T^a{}_b$$

we obtains the Einstein´s tensor components for our problem.

6.1 Reduction and solutions of the system of Einstein-Born-Infeld equations

Of the above expressions, we can see that $G^0{}_0 = G^1{}_1$,then

$$\partial_r\partial_r G + \left(\partial_r G\right)^2 - \partial_r G\partial_r\left(\Phi + \Lambda\right) = 0 \tag{43}$$

This equation (43) we can to reduce it of following manner. First we make

$$\partial_r G \equiv \xi \tag{44}$$

with this change of variables, in the equation (43) we have first derivatives only

$$\partial_r \xi + \xi^2 - \xi \partial_r (\Phi + \Lambda) = 0 \tag{45}$$

dividing the above expression (45) by ξ and making the substitution

$$\chi \equiv \ln \xi \tag{46}$$

we have been obtained the following inhomogeneous equation

$$\partial_r \chi + e^{\chi} = \partial_r (\Phi + \Lambda) \tag{47}$$

the homogeneus part of the last equation is easy to integrate

$$\chi_h = -\ln r \tag{48}$$

Now, of the usual manner, we makes in (47) the following substitution

$$\chi = \chi_h + \chi_p = -\ln r + \ln \mu = -\ln r + \ln (1 + \eta) \tag{49}$$

then

$$\begin{aligned} \partial_r \ln (1+\eta) + \frac{\eta}{r} &= \partial_r (\Phi + \Lambda) \Rightarrow \\ \partial_r [\ln (1+\eta) + \mathcal{F}(r) - (\Phi + \Lambda)] &= 0 \\ \ln (1+\eta) + \mathcal{F}(r) - (\Phi + \Lambda) &= cte = 0 \end{aligned} \tag{50}$$

where $\frac{d\mathcal{F}(r)}{dr} \equiv \frac{\eta(r)}{r}$. The constant must be put equal to zero for to obtains the correct limit. Finally the form of the exponent G is

$$G = \ln r + \mathcal{F}(r) \tag{51}$$

The next step is to put Φ in function of Λ and G in the expression (39). After of tedious computations and integrations, we obtains

$$e^{2\Lambda} = 1 + a_0 \, e^{-G} + e^{2G} \frac{2b^2}{3} - 2b^2 e^{-G} \int^{Y(r)} \sqrt{Y^4 + (r_0)^4} dY \tag{52}$$

Where, we have been defined

$$Y(r) = e^G$$

and $a(0)$ is an integration constant.

Hitherto, we know that $\mathcal{F}$ is an arbitrary function of the radial coordinate r , but for to be sure of it, we must to introduce the fuction Λ given for above equation, in the Einstein equation (40) and to verified that $G_{22} = G_{33}$. Successfully, this equality is verified and the functions Λ, Φ and G remains matematically determinate. In this manner the line element of our problem (6) takes the following form

$$ds^2 = -e^{2\Lambda}dt^2 + e^{2\mathcal{F}(r)}\left[e^{-2\Lambda}\left(1 + r\ \partial_r\mathcal{F}(r)\right)^2 dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] \tag{53}$$

6.2 Analysis of the function $\mathcal{F}(r)$ from the physical point of view.

The function $\mathcal{F}(r)$ must to have the behaviour in the form that the electric field of the configuration obey the following requirements for gives to us a regular solution in the sense that was given by B. Hoffmann and L. Infeld [9]

$$F_{01}|_{r=r_o} < b \tag{54}$$

$$F_{01}|_{r=0} = 0 \tag{55}$$

$$F_{01}|_{r\to\infty} = 0 \qquad \textit{assymptotically Coulomb} \tag{56}$$

the simplest function $\mathcal{F}(r)$ that obey the above conditions, is of the type

$$e^{\mathcal{F}(r)} = \left[1 - \left(\frac{r_0}{a\,|r|}\right)^n\right]^m \tag{57}$$

where a is an arbitrary constant, and the exponents n and m will obey the following relation

$$mn > 1 \qquad (m, n \in \mathbb{N}) \tag{58}$$

that put in sure a consistent regularization condition not only for the electric (magnetic) field but for the energy-momentum tensor[1] (41) and (42) and the line element (53).

The analysis of the Riemann tensor indicate us that it is regular every where and its components goes faster than $\frac{1}{r^3}$ when $r \to \infty$. With all this

[1]Will be shown later, that an strongest regularity condition on the energy-momentum tensor at $r = r_0$ at the origin is restricted to $F_{01}|_{r=r_o} < b$

considerations, the metric solution to the problem is

$$ds^2 = -e^{2\Lambda}dt^2 + \left[1 - \left(\frac{r_0}{a\,|r|}\right)^n\right]^{2m} \left\{ e^{-2\Lambda}dr^2 \left[\frac{1 - \left(\frac{r_0}{a|r|}\right)^n (mn-1)}{\left[1 - \left(\frac{r_0}{a|r|}\right)^n\right]}\right]^2 + \right.$$

$$\left. + r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right\} \tag{59}$$

and the electric field takes the form

$$F_{01} = \frac{b}{\sqrt{1 + \left[1 - \left(\frac{r_0}{a|r|}\right)^n\right]^{4m} \left(\frac{r}{r_0}\right)^4}} \tag{60}$$

Note that if in the condition (54) we take $F_{01}|_{r=r_o} = b$ (limiting value for the electric field in BI theory) the energy momentum diverges automatically at $r = r_0$. Strictely, the regularity conditions for the energy-momentum tensor (without divergences in the "border" r_0 of the spherical configuration source of the non-linear electromagnetic fields) are

$$T_{ab}|_{r=r_o} = finite \qquad \Rightarrow \qquad -1 < a < 0 \;\; and \;\; 0 < a < 1$$

and

$$T_{ab}|_{r=0} \to 0 \qquad \Rightarrow \qquad \mathbb{R} \to 1$$

6.3 Interesting cases for particular values of n and m

Because

$$\exp \mathcal{F}(r) = \left[1 - \left(\frac{r_0}{a\,|r|}\right)^n\right]^m$$

is easy to see that for $m = 0$

$$e^G = r$$

and we obtains the spherically symmetric line element of Hoffmann [2]and the electric field $F_{01}$and the energy-momentum tensor T_{ab}take the form of the well know EBI solution for the electromagnetic geon of Demiánski [10].

For the case: $a = 1$, $n = 4$ and $m = \frac{1}{4}$ we have

$$F_{01} = \frac{b}{\sqrt{1 + \left[1 - \left(\frac{r_0}{|r|}\right)^4\right] \left(\frac{r}{r_0}\right)^4}} = \frac{Q}{r^2}$$

where as is usually taked $br_0^2 \equiv Q$. How we see, with these values of the exponents and parameters, we obtains as solution the maxwellian linear field.

Note that the values of the exponents m and n that we was taked above, are particular *limiting cases* in which the solution loses the regularity.

6.4 Analysys of the metric

We have the metric (53)

$$ds^2 = -e^{2\Lambda}dt^2 + e^{2\mathcal{F}(r)}\left[e^{-2\Lambda}\left(1 + r\ \partial_r \mathcal{F}(r)\right)^2 dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

if we make the substitution

$$Y \equiv r\, e^{\mathcal{F}(r)}$$

and differentiating it

$$dY \equiv e^{\mathcal{F}(r)}\left(1 + r\ \partial_r \mathcal{F}(r)\right) dr$$

the line element (6) takes the form

$$ds^2 = -e^{2\Lambda}dt^2 + e^{-2\Lambda}dY^2 + Y^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

we can see that the metric (in particular the g_{tt} coefficient), in the new coordinate $Y(r)$, takes the similar form like a Demianski solution for the Born-Infeld monopole spacetime :

$$e^{2\Lambda} = 1 - \frac{2M}{Y} - \frac{2b^2 r_o^4}{3\left(\sqrt{Y^4 + r_o^4} + Y^2\right)} - \frac{4}{3} b^2 r_o^2\, {}_2F_1\left[1/4, 1/2, 5/4; -\left(\frac{Y}{r_0}\right)^4\right]$$

here M is an integration constant, which can be interpreted as an intrinsic mass, and ${}_2F_1$ is the Gauss hypergeometric function[12]. We have pass

$$g_{rr} \to g_{YY}, \qquad g_{tt}(r) \to g_{tt}(Y)$$

Specifically, for the form of the $\mathcal{F}(r)$ given by (57), Y is

$$Y \equiv \left[1 - \left(\frac{r_o}{a\,|r|}\right)^n\right]^m r$$

Now, with the metric coefficients fixed to a assymptotically minkowskian form, one can study the assymptotic behaviour of our solution. A regular, assymptotically flat solution with the electric field and energy-momentum tensor both regular, in the sense of B. Hoffmann and L. Infeld is when the exponent numbers of $Y(r)$ take the following particular values:

$$n = 3 \quad and \qquad m = 1$$

In this case, and for $r >> r_0$, we have the following assymptotic behaviour

$$e^{2\Lambda} = 1 - \frac{2M}{r} - \frac{8b^2 r_o^4 K\left(1/2\right)}{3r_o r} - 2\frac{b^2 r_o^4}{r^2} + ...$$

A distant observer will associate with this solution a total mass

$$M_{eff} = M + \frac{4b^2 r_o^4 K\left(1/2\right)}{3r_o}$$

and total charge

$$Q^2 = 2b^2 r_o^2$$

Note that when the intrinsec mass M is zero the line element is regular everywhere, the Riemann tensor is also regular everywhere and hence the space-time is singularity free. The electromagnetic mass

$$M_{el} = \frac{4b^2 r_o^4 K\left(1/2\right)}{3r_o}$$

and the charge Q are the *twice* that the electromagnetic charge and mass of the Demianski solution [10] for the static electromagnetic geon.

Is not very difficult to check that (for $m = 1$ and $n = 3$) the maximum of the electric field (see figures) is not in $r = 0$, but in the *physical border* of the spherycal configuration source of the electromagnetic fields. It means that $Y\left(r\right)$ maps correctely the internal structure of the source in the same form that the quasiglobal coordinate of the reference [7] for the global monopole in general relativity. The lack of the conical singularities at the origin is because the very well description of the manifold in the neighborod of $r = 0$ given by $Y\left(r\right)$. The function $Y\left(r\right)$ for the values of the m and n parameters given above, have two differents behaviours near of $r = 0$ depending of the a sign

$$for\ a < 0 \qquad when\ r \to \infty,\ Y\left(r\right) \to \infty \quad and\ when\ r \to 0,\ Y\left(r\right) \to -\infty$$

$$for\ a > 0 \qquad when\ r \to \infty,\ Y\left(r\right) \to \infty \quad and\ when\ r \to 0,\ Y\left(r\right) \to \infty$$

The metric (see figures) and the energy-momentum tensor remains *both* regulars at the origin (it is: $g_{tt} \rightarrow -1, T_{\mu\nu} \rightarrow 0$ for $r \rightarrow 0$).

Because the metric is regular ($g_{tt} = -1$, at $r = 0$ and at $r = \infty$), its derivative must change sign. In the usual gravitational theory of general relativity the derivative of g_{tt} is proportional to the gravitational force which would act on a test particle in the Newtonian approximation. In Einstein-Born-Infeld theory with this new static solution, it is interesting to note that although this force is attractive for distances of the order $r_0 << r$, it is actually a repulsion for very small r.

7 Conclusions

In this report a *new* exact solution of the Einstein-Born-Infeld equations for a static spherically symmetric monopole is presented. The general behaviour of the geometry, is strongly modified according to the value that takes r_0(Born-Infeld radius[1,3]) and three new parameters: a, m and n.

The fundamental feature of this solution is the lack of conical singularities at the origin when: $-1 < a < 0$ or $0 < a < 1$ and $mn > 1$. In particular, for $m = 1$ and $n = 3$, with the parameter a in the range given above and the intrinsic mass of the system M is zero, the strong regularity conditions given by B. Hoffmann and L. Infeld in [9], holds in all the spacetime. For the set of values for the parameters given above, the solution is assymptotically flat, free of singularities in the electric field, metric, energy-momentum tensor and their derivatives (with derivative values zero for $r \rightarrow 0$); and the electromagnetic mass (ADM) of the system is a twice that the electromagnetic mass of other well known [2,10,14] solutions for the Einstein-Born-Infeld monopole.

This solution have a surprising similitude with the metric for the global monopole in general relativity given in [7] in the sense that the physic of the problem have a correct description only by means of a new radial function $Y(r)$.

Because the metric is regular ($g_{tt} = -1$, at $r = 0$ and at $r = \infty$), its derivative (that is proportional to the the force in Newtonian approximation) must change sign. In Einstein-Born-Infeld theory with this new static solution, it is interesting to note that although this force is attractive for distances of the order $r_0 << r$, it is actually a repulsive for very small r.

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8.1 Appendix: connections and curvature forms from the geometrical Cartan's formulation

·Making the exterior derivatives of ω^{α} we computing the connection 1-forms $\omega^{\alpha}{}_{\beta}$:

$$\begin{aligned} \omega^{0}{}_{1} = \omega^{1}{}_{0} &= e^{-\Phi}\ \partial_r\Lambda\ \omega^{0} \\ \omega^{2}{}_{1} = -\omega^{1}{}_{2} &= e^{-\Phi}\ \partial_r F(r)\ \omega^{2} \\ \omega^{3}{}_{1} = -\omega^{1}{}_{3} &= e^{-\Phi}\ \partial_r G(r)\ \omega^{3} \\ \omega^{3}{}_{2} = -\omega^{2}{}_{3} &= \frac{\cos\theta}{sen\theta} e^{-F(r)}\omega^{3} \end{aligned} \tag{61}$$

·Making the exterior derivatives of $\omega^{\alpha}{}_{\beta}$ we computing the curvature 2-forms $\mathcal{R}^{\alpha}{}_{\beta}$:

$$\mathcal{R}^{0}{}_{1} = e^{-2\Phi}\left(\partial_r\partial_r\Lambda - \partial_r\Phi\,\partial_r\Lambda + (\partial_r\Lambda)^2\right)\omega^1\wedge\omega^0$$

$$\mathcal{R}^{2}{}_{1} = e^{-2\Phi}\left(\partial_r\partial_r F - \partial_r\Phi\,\partial_r F + (\partial_r F)^2\right)\omega^1\wedge\omega^2$$

$$\mathcal{R}^{3}{}_{2} = e^{-(F+\Phi)}\,\partial_r\left(G-F\right)\frac{\cos\theta}{sen\theta}\omega^1\wedge\omega^3 + \left(e^{-2\Phi}\partial_r G\ \partial_r F - e^{-2F}\right)\omega^2\wedge\omega^3$$

$$\begin{aligned} \mathcal{R}^{3}{}_{1} &= e^{-2\Phi}\left(\partial_r\partial_r G - \partial_r\Phi\,\partial_r G + (\partial_r G)^2\right)\omega^1\wedge\omega^3 + \\ &\quad + e^{-(F+\Phi)}\,\partial_r\left(G-F\right)\frac{\cos\theta}{sen\theta}\omega^2\wedge\omega^3 \end{aligned}$$

$$\begin{aligned} \mathcal{R}^{0}{}_{2} &= -e^{-2\Phi}\,\partial_r\Lambda\ \partial_r F\ \omega^0\wedge\omega^2 \\ \mathcal{R}^{0}{}_{3} &= -e^{-2\Phi}\,\partial_r\Lambda\ \partial_r G\ \omega^0\wedge\omega^3 \end{aligned} \tag{62}$$

Figure 1 : electric field F_{10} of the EBI - monopole in function of r, for M = 0, r_0 = 1, m = 1, n = 3 and a = -0.9

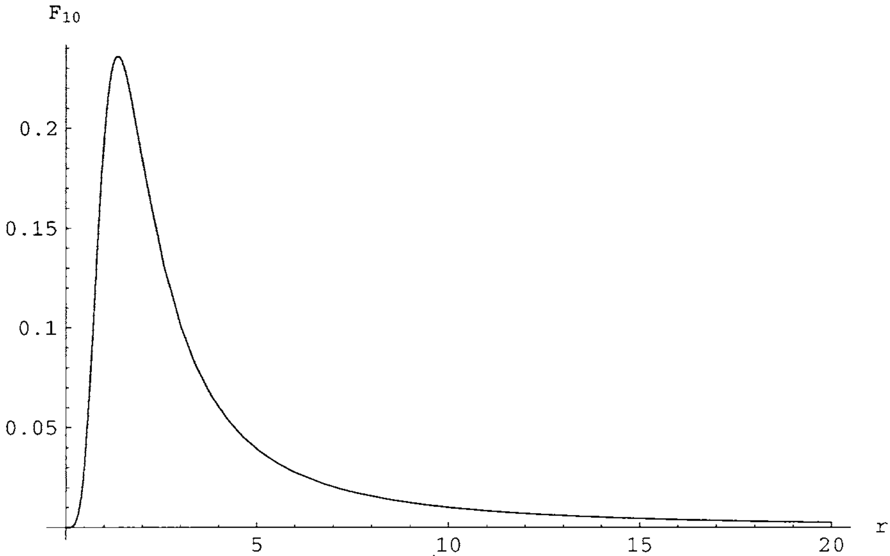
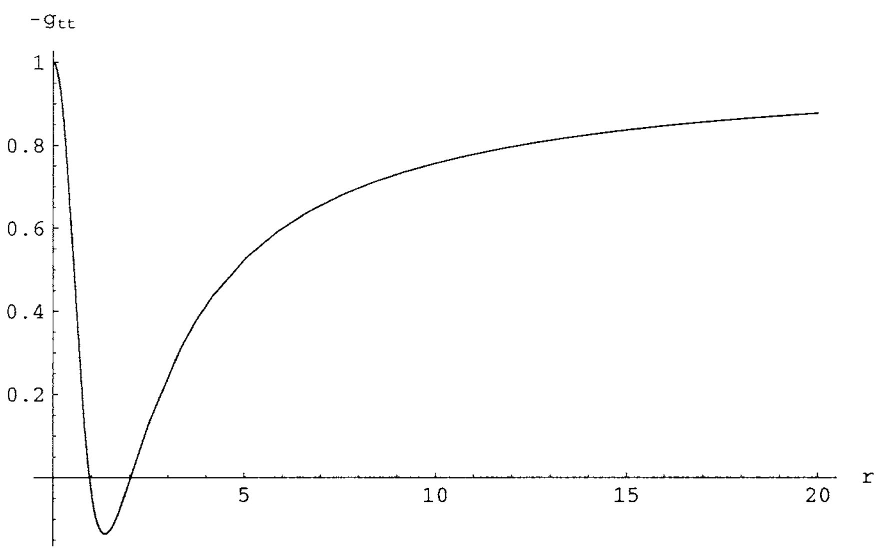


Figure 2 : coefficient $-g_{tt}$ of the EBI - monopole in function of r, for M = 0, r_0 = 1, m = 1, n = 3 and a = -0.9



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Сирилло Ломбардо Д. E2-2003-221

Статические и вращающиеся черные дыры в теории Эйнштейна–Борна–Инфельда

Получено новое асимптотически-плоское решение для сферически-симметричного пространства-времени. При внутренней массе системы, равной нулю, возникающее пространство-время везде регулярно в смысле, указанном Хоффманном и Инфельдом в 1937 году. Это означает, что метрика, электромагнитное поле и их производные не имеют сингулярностей и разрывностей на многообразии. В частности, отсутствует коническая сингулярность в начале координат, в отличие от хорошо известного монопольного решения, данного Хоффманном в 1935 году.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

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Cirilo Lombardo D. E2-2003-221

Static and Rotating Black Holes in Einstein–Born–Infeld Theories

In this work a new asymptotically flat solution of the coupled Einstein–Born–Infeld equations for a static spherically symmetric space-time is obtained. When the intrinsic mass of the system is zero the resulting space-time is regular anywhere in the sense given by B. Hoffmann and L. Infeld in 1937. This means that the metric, the electromagnetic field and their derivatives have not singularities and discontinuities in all the manifold. In particular, there are no conical singularities at the origin, in contrast to well-known monopole solution studied by B. Hoffmann in 1935.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@pds.jinr.ru
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