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SPIN OBSERVABLES IN THE $NN \to Y\Theta^+$ REACTION AT THE THRESHOLD AND QUANTUM NUMBERS OF THE Θ^+ PENTAQUARK

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Спиновые наблюдаемые в реакции $NN \to Y\Theta^+$ вблизи порога
и квантовые числа пентакварка Θ^+

Получены общие формулы для параметров спин-спиновой корреляции $C_{i,j}$ и коэффициентов передачи поляризации K_i^j в реакции $NN \to Y\Theta^+$ вблизи порога для произвольного спина пентакварка Θ^+. Показано, что измерение знака $C_{y,y}$ или наблюдение ненулевой передачи поляризации от нуклона к гиперону Y позволяет однозначно определить P-четность пентакварка Θ^+ независимо от значения его спина. Измерение этих наблюдаемых как в pp-, так и в pn-канале реакции позволяет определить изоспин Θ^+.

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Spin Observables in the $NN \to Y\Theta^+$ Reaction at the Threshold
and Quantum Numbers of the Θ^+ Pentaquark

General formulae for the spin–spin correlation parameters $C_{i,j}$ and spin-transfer coefficients K_i^j are derived for the reaction $NN \to Y\Theta^+$ at the threshold for an arbitrary spin of the pentaquark Θ^+. It is shown that measurement of the sign of $C_{y,y}$ or observation of the non-zero polarization transfer from the nucleon to the hyperon Y allows one to determine the P-parity of the Θ^+ unambiguously and independently of the spin of the Θ^+. Measurement of these spin observables in both the pp- and pn-channels of this reaction determines also the isospin of the Θ^+.

The investigation has been performed at the Dzhelepov Laboratory of Nuclear Problems, JINR.

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1. INTRODUCTION

Experimental indications [1–7] of existence of an exotic baryon with the strangeness $S=+1$, called as the $\Theta^+(1540)$, which presumably consists of five constituent quarks, stimulated many theoretical works. An important task now is an experimental determination of the quantum numbers of the Θ^+. Model independent methods for determination of the P-parity of the pentaquark Θ^+ in the reaction $NN\to Y\,\Theta^+$ were suggested in Refs. [8–11]. These methods are based on such general properties of the reaction amplitude as angular momentum and P-parity conservation and on the generalized Pauli principle for nucleons. It was shown that the sign of the spin–spin correlation parameter $C_{y,y}$ determines unambiguously the P-parity of the Θ^+, π_Θ, in the reaction $pp\to\Sigma^+\Theta^+$ [9]. Another strong correlation between $C_{y,y}$ and π_Θ is also valid for the $pn\to\Lambda^0\Theta^+$ reaction [10, 11] if the isospin of the Θ^+ equals zero. Furthermore, measurement of the spin transfer coefficients $K_y^y=K_x^x$ or K_z^z of these reactions also allows one to determine the P-parity unambiguously [10, 11]. Measurement of the polarization transfer from the initial nucleon to the hyperon in the reaction $NN\to Y\Theta^+$ can be perfomed by a single spin experiment with polarized beam or target, because the polarization of the hyperon can be measured via its weak decay. However, the results of Refs. [8–11] are based on the assumption that the spin of the Θ^+ is equal to $1/2$. Up to now the spin of the Θ^+ is not known, as well as the P-parity and isospin, and within some models its value can be $3/2$. In this work we derive formulae for the spin observables of the reaction $NN\to Y\,\Theta^+$ at the threshold for the general case of an arbitrary spin of the Θ^+. Analysis is based on common properties of the reaction amplitude and the standard method of the spin-tensor operators [12]. We also derive a full spin structure of the cross section of this reaction for the case of the spin-$1/2$ particles taking into account all polarizations in the initial an final states.

2. FORMALISM

Assuming dominance of the S-wave in the relative motion in the final system, the most general expression for the amplitude of the binary reaction $1+2\to 3+4$ at the threshold can be written as [13]

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}=\sum_{\substack{J\,M\\ S\,M_S\,L\,m}}(j_1\mu_1\,j_2\mu_2|SM_S)(j_3\mu_3\,j_4\,\mu_4|J\,M)\times$$
$$\times\,(S\,M_S\,L\,m|J\,M)Y_{Lm}(\hat{\mathbf{k}})a_J^{LS}. \quad (1)$$

Here j_i and μ_i are the spin of the ith particle and its z-projection, J and M are the total angular momentum and its z-projection; S and L are the spin and orbital momentum of the initial system, respectively, and M_S and m are the corresponding z-projections. Information on the reaction dynamics is contained in the complex amplitudes $a_J^{L\,S}$. The sum over J in Eq. (1) is restricted by the conditions $J = j_3 + j_4, j_3 + j_4 - 1, ..., |j_3 - j_4|$. We choose the z-axis along the vector of the initial momentum $\hat{\mathbf{k}}$, therefore $Y_{Lm}(\hat{\mathbf{k}}) = \sqrt{(2\,L+1)/4\pi}\,\delta_{m\,0}$. Due to P-parity conservation, the orbital momentum L in Eq. (1) is restricted by the condition $(-1)^L = \pi$, where $\pi = \pi_1\,\pi_2\,\pi_3\pi_4$ is the product of internal parities of the participating particles, π_i. We consider here mainly transitions without mixing the total isospin T in this reaction*. For the fixed T and π the spin of the initial nucleons S is fixed unambiguously by the generalized Pauli principle: $(-1)^S = \pi(-1)^{T+1}$. Therefore, in order to determine the P-parity π of the system at a given isospin T, it is sufficient to determine the spin of the NN-system in the initial state of this reaction.

Let us consider here a particular case of $j_3 = \frac{1}{2}$ and j_4 being half-integer, $j_4 = \frac{1}{2}, \frac{3}{2}, \frac{3}{2}, \ldots$. For this case there are two total angular momenta $J_p = j_4 + \frac{1}{2}$ and $J_m = j_4 - \frac{1}{2}$. For the spin-singlet initial state $S = 0$ only one orbital momentum is allowed, $L = J$, and therefore there is only one scalar amplitude, $a_J^{L\,S} = a_J^{J\,0}$. For $S = 1$ and $j_4 \geqslant \frac{3}{2}$ there are the following three scalar amplitudes $a_J^{L\,S} \equiv a_J^L$:
(i) $a_{J_p}^{J_p}$, $a_{J_m}^{J_m+1}$ and $a_{J_m}^{J_m-1}$, if $(-1)^{J_p} = \pi$,
or
(ii) $a_{J_m}^{J_m}\,(J_m \neq 0)$, $a_{J_p}^{J_p+1}$ and $a_{J_p}^{J_p-1}$, if $(-1)^{J_p} = -\pi$.
For particular case of $j_4 = j_3 = \frac{1}{2}$, one has $J_m = 0$ and $J_p = 1$. For this case only two triplet amplitudes are allowed for $\pi = +1$, i.e., a_1^0 and a_1^2, whereas the amplitude a_0^0 is forbidden by conservation of the total angular momentum. For $\pi = -1$ one also has only two triplet amplitudes, one of them corresponds to $J = 1$, a_1^1, and another one is allowed for $J = 0$, i.e. a_0^1.

Using Eq. (1) one can find the polarized cross section $d\sigma(\mathbf{p}_1, \mathbf{p}_2)$ as follows:

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = \Phi \sum_{\mu_3\,\mu_4} |T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 = \frac{1}{4\pi} \sum_M \left(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | S M\right)^2 \times$$
$$\times \sum_{J\,M\,L\,L'} \sqrt{(2L+1)(2L'+1)} (S\,M\,L\,0|J\,M) \times$$
$$\times\,(S\,M\,L'\,0|J\,M)\,a_J^{LS}\,(a_J^{L'S})^*, \quad (2)$$

where Φ is a kinematical factor. Using the relations $(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2|00) = \chi^+_{\mu_1} \frac{i\sigma_y}{\sqrt{2}} \chi^{(T)+}_{\mu_2}$

*The isospin mixing is possible, for example, in the reaction $p + n \to \Sigma^0 + \Theta^+$, if the Θ^+ is an isotriplet. In this case the P-parity cannot be determined by using the method in question.

and $(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|1\lambda) = \chi^+_{\mu_1}\sigma_\lambda\frac{i\sigma_y}{\sqrt{2}}\chi^{(T)+}_{\mu_2}$, where $\sigma_i\,(i = y, \lambda)$ is the Pauli matrix and χ_μ is the 2-spinor, one can find

$$\left(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|00\right)^2 = \frac{1}{4}(1 - \mathbf{p}_1\cdot\mathbf{p}_2), \tag{3}$$

$$\left(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|1M\right)^2 = \begin{cases}\frac{1}{4}(1 + \mathbf{p}_1\cdot\mathbf{p}_2 - 2p_{1z}p_{2z}), & M = 0,\\ \frac{1}{4}[1 \pm (p_{1z} + p_{2z}) + p_{1z}p_{2z}], & M = \pm 1.\end{cases} \tag{4}$$

In Eqs. (2), (3), and (4) $\mathbf{p}_i$ is the polarization vector of the ith particle with the spin $j_i = \frac{1}{2}$ being in the pure spin state χ_{μ_i}. The unpolarized cross section is given as

$$d\sigma_0 = \Phi\,\frac{1}{4}\sum_{\mu_1\,\mu_2\,\mu_3\,\mu_4}|T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 = \frac{1}{16\,\pi}\Phi\sum_{J,L}(2J+1)|a^{L\,S}_J|^2. \tag{5}$$

2.1. The Spin-Singlet Initial State. Using Eqs. (2), (3), and (5) one can find for the spin-singlet polarized cross section the following formula:

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = d\sigma_0(1 - \mathbf{p}_1\cdot\mathbf{p}_2). \tag{6}$$

In notations of Ref. [14], non-zero spin–spin correlation parameters for this case are the following: $C_{x,x} = C_{y,y} = C_{z,z} = -1$.

In order to find spin-transfer coefficients, one should consider the following cross section:

$$d\sigma(\mathbf{p}_1, \mathbf{p}_3) = \Phi\sum_{\mu_2,\mu_4}|T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2. \tag{7}$$

The polarization vector $\mathbf{p}_1$ of the 1st particle in the right-hand side of Eq. (7) can be found only in the following sum:

$$\sum_{\mu_2}(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|00)^2 = \frac{1}{2}\sum_{\mu_2}(\chi^+_{\mu_1}i\sigma_y\chi^{(T)+}_{\mu_2})(\chi^T_{\mu_2}(-i\sigma_y)\chi_{\mu_1}) =$$
$$= \frac{1}{4}\,\mathrm{Sp}\,(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_1) = \frac{1}{2}. \tag{8}$$

Since the vector $\mathbf{p}_1$ is absent actually in the right-hand side of Eq. (8), one should conclude that all the polarization transfer coefficients are zero for the spin-singlet initial state: $K^j_i = 0\,(i,j = x,y,z)$. The obtained results for $C_{i,j}$ and K^j_i are valid for any values of the spins j_3 and j_4, both of them being integer or half-integer.

2.2. The Spin-Triplet Initial State. For $S = 1$ and $M = 0$, Eq. (2) can be written as

$$d\sigma^{M=0}(\mathbf{p}_1, \mathbf{p}_2) = \frac{\Phi}{16\,\pi}(1 + \mathbf{p}_1 \cdot \mathbf{p}_2 - 2p_{1z}p_{2z}) \times \\ \times \sum_J |\sqrt{J}\, a_J^{J-1} - \sqrt{J+1}\, a_J^{J+1}|^2. \quad (9)$$

We arrive at this formula from Eq. (2) using Eq. (4) and the following formulae for the Clebsh–Gordan coefficients: $(1\,0\,J\,0|J\,0) = 0, (1\,0\,J-1\,0|J\,0) = \sqrt{J/(2J-1)}$, $(1\,0\,J+1\,0|J\,0) = \sqrt{(J+1)/(2J+1)}$. In order to simplify the notations, we omit in Eq. (9) and below the superscript $S = 1$ in $a_J^{L\,S}$. The sum over the projections $M = +1$ and $M = -1$ into the right-hand side of Eq. (2) gives

$$d\sigma^{M=\pm 1}(\mathbf{p}_1, \mathbf{p}_2) = \frac{\Phi}{16\,\pi}(1 + p_{1z}p_{2z}) \times \\ \times \begin{cases} \sum_J |\sqrt{J}\, a_J^{J+1} + \sqrt{J+1}\, a_J^{J-1}|^2, & \text{if } (-1)^{J+1} = \pi, \\ \sum_J (2J+1)\, |a_J^J|^2, & \text{if } (-1)^J = \pi. \end{cases} \quad (10)$$

Here we used the following relations: $(1 1\,J\,-1|J\,0) = \frac{1}{\sqrt{2}}$, $(1 1\,J\,-1|J-1\,0) = \sqrt{J+1}/\sqrt{2(2J+1)}$, $(1 1\,J\,-1|J+1\,0) = \sqrt{J}/\sqrt{2(2J+1)}$. Using Eqs. (9), (10) and (5), one can present the polarized cross section (2) in the following standard form [14]:

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = d\sigma_0\,(1 + C_{x,x}\, p_{1x}\, p_{2x} + C_{y,y}\, p_{1y}p_{2y} + C_{z,z}\, p_{1z}\, p_{2z}), \quad (11)$$

where the spin–spin correlation parameters are given as

$$C_{x,x} = C_{y,y} = \frac{\sum_J |\sqrt{J}\, a_J^{J-1} - \sqrt{J+1}\, a_J^{J+1}|^2}{\sum_{J\,L}(2J+1)\,|a_J^L|^2}, \quad (12)$$

$$C_{z,z} = 1 - 2\,C_{y,y}. \quad (13)$$

As seen from Eq. (12), the spin–spin correlation parameters are non-negative for transversal polarization. One can see from Eq. (12) that the diagonal term a_J^J does not contribute into the numerator of $C_{x,x} = C_{y,y}$. The obtained results for $C_{i,j}$ are valid for any values of the spins j_3 and j_4, both of them being integer or half-integer.

Using Eq. (12), one can find, for example, for the particular case of $j_3 = \frac{1}{2}$ and $j_4 = \frac{3}{2}$

$$C_{x,x} = C_{y,y} = \frac{|\sqrt{2}\, a_2^1 - \sqrt{3}\, a_2^3|^2}{3|a_1^1|^2 + 5|a_2^1|^2 + 5|a_2^3|^2}, \quad (14)$$

if $\pi = -1$ ($S = 1$, $T = 1$), and

$$C_{x,x} = C_{y,y} = \frac{|a_1^0 - \sqrt{2}\, a_1^2|^2}{3|a_1^0|^2 + 3|a_1^2|^2 + 5|a_2^2|^2}, \qquad (15)$$

if $\pi = +1$ ($S = 1$, $T = 0$).

Considering the sum $\sum_{\mu_2} (\frac{1}{2}\mu_1 \frac{1}{2}\mu_2|1M)(\frac{1}{2}\mu_1 \frac{1}{2}\mu_2|1M')$, one can find that this sum explicitly contains the polarization vector $\mathbf{p}_1$. Therefore, in contrast to the case of $S = 0$, the spin-triplet initial state $S = 1$ allows a non-zero polarization transfer in this reaction. In order to get the spin-transfer coefficients we use below a general method developed in Ref. [12].

3. THE GENERAL METHOD

According to Ref. [12], the amplitude in Eq. (1) can be written as

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \chi^+_{j_3\,\mu_3} \chi^+_{j_4\,\mu_4} \hat{F} \chi_{j_1\,\mu_1}\, \chi_{j_2\,\mu_2}, \qquad (16)$$

where $\hat{F}$ is an operator acting on the spin states of the initial and final particles. This operator can be written as

$$\hat{F} = \sum_{m_1\, m_2\, m_3\, m_4} T^{m_3\, m_4}_{m_1\, m_2}\, \chi^+_{j_1\, m_1}(1) \chi^+_{j_2\, m_2}(2) \chi_{j_3\, m_3}(3)\, \chi_{j_4\, m_4}(4), \qquad (17)$$

where $\chi_{j_k\, m_k}(k)$ is the spin function of the kth particle with the spin j_k and z-projection m_k and $T^{m_3\, m_4}_{m_1\, m_2}$ is defined by Eq. (1). The operator $\hat{F}$ is normalized to the unpolarized cross section as

$$d\sigma_0 = \frac{\Phi}{(2j_1 + 1)\,(2j_2 + 1)}\, \mathrm{Sp}\, F F^+. \qquad (18)$$

3.1. Polarization Transfer Coefficients. The spin-transfer coefficient is given by the following formula [12]:

$$K^{\kappa}_{\lambda} = \frac{\mathrm{Sp}\, F \sigma_\lambda(1) F^+ \sigma_\kappa(3)}{\mathrm{Sp}\, F F^+}, \qquad (19)$$

where $\lambda, \kappa = 0, \pm 1$. For $j_1 = j_3 = \frac{1}{2}$ we found from Eqs. (1), (16) and (19) the spin-transfer coefficient in the following general form:

$$\mathrm{Sp}\, FF^{+}\, K_{\lambda}^{\kappa} = \delta_{\lambda,-\kappa}\, \frac{3}{2\pi} \sum_{S\,S'\,J\,J'\,L\,L'\,J_0} \sqrt{(2L+1)\,(2L'+1)} \times$$
$$\times \sqrt{(2S+1)\,(2S'+1)}(2J+1)\,(2J'+1)\,(-1)^{j_2+j_4+S'+J'+L} \times$$
$$\times\,(1\,-\lambda\,1\,\lambda|J_0\,0)(L'\,0\,L\,0\,|J_0 0)\times$$
$$\times \left\{ \begin{array}{ccc} \frac{1}{2} & j_2 & S \\ S' & 1 & \frac{1}{2} \end{array} \right\} \left\{ \begin{array}{ccc} \frac{1}{2} & j_4 & J' \\ J & 1 & \frac{1}{2} \end{array} \right\} \left\{ \begin{array}{ccc} J & S & L \\ J' & S' & L' \\ 1 & 1 & J_0 \end{array} \right\} a_J^{L\,S} (a_{J'}^{L'\,S'})^{*}. \tag{20}$$

Here we used the standard notations for the 6j- and 9j-symbols [15]. From Eq. (20) one can find the following relations:

$$K_{+1}^{-1} = K_{-1}^{+1} = -K_x^x = -K_y^y, \tag{21}$$

and $K_i^j = 0$ at $i \neq j$, where $i, j = x, y, z$. From Eq. (20) we also find that there is no polarization transfer ($K_i^j = 0,\ i, j = x, y, z$) for $S = S' = 0$ in accordance with the above discussion. These coefficients are also equal to zero for $J = J' = 0$. For the spin-triplet transitions $S = S' = 1$, we find from Eq. (20) that $K_x^x = K_y^y \neq 0$ and $K_0^0 = K_z^z \neq 0$. Eq. (20) is valid for arbitrary spins j_2 and j_4.

As an example, let us consider the reaction with the minimal spins $j_i = \frac{1}{2},\ i = 1, ..., 4$. For the total isospin $T = 0$ and parity $\pi = +1$ one has $S = 1$. For this case Eq. (20) gives (using the notation $a_J^{L\,1} = a_J^L$)

$$K_x^x = K_y^y = \frac{|\sqrt{2}\, a_1^0 + a_1^2|^2 - 3\,\mathrm{Re}\,(\sqrt{2}\, a_1^0 + a_1^2) a_1^{2\,*}}{3\,(|a_1^0|^2 + |a_1^2|^2)}, \tag{22}$$

$$K_z^z = \frac{|\sqrt{2}\, a_1^0 + a_1^2|^2}{3\,(|a_1^0|^2 + |a_1^2|^2)}. \tag{23}$$

The formulae (22) and (23) coincide with those obtained previously in Ref. [10] by a different method. For the case of $T = 1$ and $\pi = -1$ one has $S = 1$. In this case Eq. (20) gives

$$K_x^x = K_y^y = \frac{\sqrt{6}\,\mathrm{Re}\, a_0^1\, a_1^{1\,*}}{|a_0^1|^2 + 3|a_1^1|^2}, \tag{24}$$

$$K_z^z = \frac{3|a_1^1|^2}{|a_0^1|^2 + 3|a_1^1|^2}, \tag{25}$$

which coincide (except for notations) with those obtained recently in Ref. [11] in the $\boldsymbol{\sigma}$-representation for the amplitude. For higher spins of the 4th particle

$j_4 \geqslant \frac{3}{2}$, Eq. (20) also gives non-zero coefficients K_x^x and K_z^z, but the formulae are more cumbersome and thus we do not present them here.

3.2. Spin-Spin Correlation Coefficients. For the spin-spin correlation coefficient, defined as [14]

$$C_{\lambda,\,\kappa} = \frac{\mathrm{Sp}\, F\sigma_\lambda(1)\,\sigma_\kappa(2)\, F^+}{\mathrm{Sp}\, F\, F^+}, \tag{26}$$

we found for the case of $j_1 = j_2 = \frac{1}{2}$

$$\begin{aligned}\mathrm{Sp}\, FF^+\, C_{\lambda,\kappa} = \delta_{\lambda,-\kappa}\,\frac{3}{2\pi}\sum_{S\,S'\,J}(-1)^{S+J}(2J+1)\times\sqrt{(2S+1)(2S'+1)}\times\\ \times\sum_{L\,L\,J_0}(-1)^{L'}(2J_0+1)\sqrt{2L'+1}\times(1\lambda\,1-\lambda\,|J_0\,0)\,(J_0\,0\,L'\,0\,|L\,0)\times\\ \times\begin{Bmatrix} S' & S & J_0\\ L & L' & J\end{Bmatrix}\begin{Bmatrix} S' & \frac{1}{2} & \frac{1}{2}\\ S & \frac{1}{2} & \frac{1}{2}\\ J_0 & 1 & 1\end{Bmatrix} a_J^{L\,S}(a_{J'}^{L'\,S'})^*.\end{aligned} \tag{27}$$

We found from Eq. (27) the following relations: $C_{+1,-1} = C_{-1,+1} = -C_{x,x} = -C_{y,y} \neq 0$, $C_0^0 = C_z^z \neq 0$, whereas $C_{i,j} = 0$ at $i \neq j$ $(i,j = x,y,z)$. This formula is valid for arbitrary values of the spins j_3 and j_4 both of them being integer or half-integer.

One can see from Eq. (26) that the sum $\Sigma = C_{x,x} + C_{y,y} + C_{z,z}$ is equal to $\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)$. Therefore, Σ is fixed by the spin S: $\Sigma = -3$ for $S = 0$ and $\Sigma = +1$ for $S = 1$ in accordance with the above results given in Eqs. (12), (13) and (6). From Eq. (27) one can find that $C_{x,x} = C_{y,y} = C_{z,z} = -1$ for $S = S' = 0$. For $S = S' = 1$ we did not find here a transformation from a rather cumbersome formula (27) to a more compact form of Eqs. (12) and (13). However, one can check by straightforward calculations that these formulae lead to the same result.

4. FULL SPIN STRUCTURE FOR THE REACTION $\frac{1}{2}+\frac{1}{2}\to\frac{1}{2}+\frac{1}{2}$

For completeness, in this section we give the full spin structure of the binary reaction at $j_1 = j_2 = j_3 = j_4 = \frac{1}{2}$ discussed in part in Ref. [10]. For the case $T = 0$ and $\pi = -1$, one has $S = 0$, and the amplitude (1) can be written as

$$M_{\mu_1\,\mu_2}^{\mu_3\,\mu_4} = \sum_{\alpha=x,y,z}\left(\chi_{\mu_3}^+\sigma_\alpha\frac{i\sigma_y}{\sqrt{2}}\,\chi_{\mu_4}^{(T)+}\right)\left(\chi_{\mu_1}^{(T)}\frac{-i\sigma_y}{\sqrt{2}}\,\chi_{\mu_2}\right)\hat{k}_\alpha\sqrt{\frac{3}{4\,\pi}}\,a_1^{1\,0}. \tag{28}$$

When deriving Eq. (28) from Eq. (1) we used for the Clebsh–Gordan coefficients the formulae given above after Eq. (2). The unpolarized cross section corresponding to the amplitude (28) takes the following form:

$$d\sigma_0 = \frac{1}{4}\Phi \sum_{\mu_1\,\mu_2\,\mu_3\,\mu_4} |M_{\mu_1\,\mu_2}^{\mu_3\,\mu_4}|^2 = \frac{3}{16\pi}\Phi\,|a_1^{1\,0}|^2, \tag{29}$$

that is in agreement with Eq. (5). In order to calculate the polarized cross section we use the density matrix for the spin-$1/2$ particle being in the pure spin state χ_{μ_i} in the following form:

$$\chi_{\mu_i}\,\chi_{\mu_i}^{+} = \frac{1}{2}(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_i). \tag{30}$$

Using Eqs. (30) and (28) one can write the cross section with polarized both initial and final particles as

$$\begin{aligned} d\sigma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_3, \mathbf{p}_4) &= \Phi\,|M_{\mu_1\,\mu_2}^{\mu_3\,\mu_4}|^2 = \\ &= \frac{1}{4}d\sigma_0\,(1 - \mathbf{p}_1\cdot\mathbf{p}_2)\,[1 + \mathbf{p}_3\cdot\mathbf{p}_4 - 2(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}})]. \end{aligned} \tag{31}$$

The polarization vectors of the final particles $\mathbf{p}_3$ and $\mathbf{p}_4$ are determined by the reaction amplitude (28) and can be found using the standard methods [12, 14]. After performing this step and substituting the obtained vectors $\mathbf{p}_3$ and $\mathbf{p}_4$ into Eq. (31), one can find the polarized cross section $d\sigma(\mathbf{p}_1, \mathbf{p}_2)$ given by Eq. (6). However, the calculation of $\mathbf{p}_3$ and $\mathbf{p}_4$ is not necessarily and Eq. (31) is sufficient to find all the spin observables for the reaction described by the amplitude (28). In particular, one can see from Eq. (31) that there is no polarization transfer in this reaction ($K_i^j = 0$, $i, j = x, y, z$), but there are spin–spin correlations in both the initial and final states.

For $T = 0$ and $\pi = +1$ we have $S = 1$, and the amplitude in Eq. (1) can be written as

$$M_{\mu_1\,\mu_2}^{\mu_3\,\mu_4} = \sum_{\alpha=x,y,z} \left(\chi_{\mu_3}^{+}\sigma_\alpha \frac{i\sigma_y}{\sqrt{2}}\,\chi_{\mu_4}^{(T)+}\right)\left(\chi_{\mu_1}^{(T)}\frac{-i\sigma_y}{\sqrt{2}}\,\Pi_\alpha\chi_{\mu_2}\right), \tag{32}$$

where Π_α is the following spin operator:

$$\Pi_\alpha = G\sigma_\alpha + F\,\hat{k}_\alpha(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}) \tag{33}$$

with

$$G = \frac{1}{\sqrt{4\pi}}\,(a_1^0 + \frac{1}{\sqrt{2}}\,a_1^2) \tag{34}$$

and

$$F = -\frac{3}{\sqrt{8\pi}}\, a_1^2. \tag{35}$$

The cross section with polarized initial and final particles is the following:

$$\begin{aligned} d\sigma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_3, \mathbf{p}_4) &= \Phi\, |M^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 = \\ &= \sum_{\alpha\,\beta = x,y,z} \frac{1}{8}\,\mathrm{Sp}\,\{\sigma_\alpha\,(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_4)\,\sigma_\beta(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_3)\}\times \\ &\qquad \times \frac{1}{8}\,\mathrm{Sp}\,\{\Pi^+_\alpha\,(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_2)\,\Pi_\beta(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_1)\}. \end{aligned} \tag{36}$$

Calculating the traces in Eq. (36), one can find finally

$$\begin{aligned} d\sigma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_3, \mathbf{p}_4) = \frac{1}{16}\Phi\Big\{ &|G|^2(1 + \mathbf{p}_1\cdot\mathbf{p}_2)\times \\ &\times(3 + \mathbf{p}_3\cdot\mathbf{p}_4) + \big[(|F|^2 + 2\,\mathrm{Re}\,FG^*)(1 + \mathbf{p}_1\cdot\mathbf{p}_2) - \\ &- 2|F|^2\,(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\hat{\mathbf{k}})\big]\,[1 - 2(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}}) + \mathbf{p}_3\cdot\mathbf{p}_4] - \\ &- 2|G|^2\big[(\mathbf{p}_1\cdot\mathbf{p}_2)(1 + \mathbf{p}_3\cdot\mathbf{p}_4) - (\mathbf{p}_1\cdot\mathbf{p}_3)(\mathbf{p}_2\cdot\mathbf{p}_4) - \\ &- (\mathbf{p}_2\cdot\mathbf{p}_3)(\mathbf{p}_1\cdot\mathbf{p}_4)\big] - \\ &- 2\,\mathrm{Re}\,FG^*(\mathbf{p}_2\cdot\hat{\mathbf{k}})\big[(\mathbf{p}_1\cdot\hat{\mathbf{k}})(1 + \mathbf{p}_3\cdot\mathbf{p}_4) - (\mathbf{p}_3\cdot\hat{\mathbf{k}})\times \\ &\times(\mathbf{p}_4\cdot\mathbf{p}_1) - (\mathbf{p}_4\cdot\hat{\mathbf{k}})(\mathbf{p}_1\cdot\mathbf{p}_3)\big] - \\ &- 2\,\mathrm{Re}\,FG^*(\mathbf{p}_1\cdot\hat{\mathbf{k}})\big[(\mathbf{p}_2\cdot\hat{\mathbf{k}})(1 + \mathbf{p}_3\cdot\mathbf{p}_4) - \\ &- (\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\mathbf{p}_2) - (\mathbf{p}_4\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\mathbf{p}_3)\big] - \\ &- 2\,\mathrm{Im}\,FG^*\,\big([\mathbf{p}_1\times\hat{\mathbf{k}}](\mathbf{p}_2\cdot\hat{\mathbf{k}}) + (\mathbf{p}_1\cdot\hat{\mathbf{k}})\,[\mathbf{p}_2\times\hat{\mathbf{k}}]\big)\cdot(\mathbf{p}_3 + \mathbf{p}_4) + \\ &+ 2\,\mathrm{Im}\,FG^*\,(\mathbf{p}_3\cdot\hat{\mathbf{k}})([\hat{\mathbf{k}}\times(\mathbf{p}_1 + \mathbf{p}_2)]\cdot\mathbf{p}_4) + \\ &+ 2\,\mathrm{Im}\,FG^*\,(\mathbf{p}_4\cdot\hat{\mathbf{k}})([\hat{\mathbf{k}}\times(\mathbf{p}_1 + \mathbf{p}_2)]\cdot\mathbf{p}_3) + \\ &+ 2(|G|^2 + \mathrm{Re}\,FG^*)(\mathbf{p}_1 + \mathbf{p}_2)\cdot(\mathbf{p}_3 + \mathbf{p}_4) - \\ &- 2\,\mathrm{Re}\,FG^*(\mathbf{p}_1\cdot\hat{\mathbf{k}} + \mathbf{p}_2\cdot\hat{\mathbf{k}})\times(\mathbf{p}_3\cdot\hat{\mathbf{k}} + \mathbf{p}_4\cdot\hat{\mathbf{k}})\Big\}. \end{aligned} \tag{37}$$

The unpolarized cross section for this case is

$$d\sigma_0 = \frac{1}{4}\Phi\,\left\{|G + F|^2 + 2|G|^2\right\} = \frac{\Phi}{16\pi}3(|a_1^0|^2 + |a_1^2|^2). \tag{38}$$

Using Eq. (38), one can find from Eq. (36) all the spin observables for this reaction. For example, one can see that the spin–spin correlation coefficients $C_{i,j}$ and spin transfer coefficients K_i^j obtained from Eq. (36) coincide with those given by Eqs. (12) and (13) and Eqs. (22) and (23), respectively.

5. CONCLUSION

The obtained formulae (6), (12), (13), (20), and (27) allow us to conclude that for $S = 1$ (i) the spin–spin correlation coefficient $C_{y,y}$ is always non-negative, and (ii) spin transfer coefficients K_y^y and K_z^z are non-zero in the reaction in question $1+2 \to 3+4$ at the threshold independently of the spin j_4 of the 4th particle. On the contrary, for $S = 0$, the spin-spin correlation coefficients $C_{x,x} = C_{y,y} = C_{z,z}$ are equal to -1 and all the spin transfer coefficients are equal to zero. This conclusion is a generalization of the previous results [10, 11] found for the case of $j_4 = \frac{1}{2}$. The obtained result allows one to determine unambiguously the P-parity of the Θ^+ by measurement of either $C_{y,y}$ or K_x^x (or K_z^z) in the reaction $pp \to \Sigma^+ \Theta^+$. The total isospin of this channel is fixed, $T = 1$, therefore the spin S of the initial nucleons is directly related to the P-parity π_Θ of the Θ^+: $(-1)^S = \pi_\Theta$. In the reaction $pn \to \Lambda^0\Theta^+$ one has either $(-1)^S = -\pi_\Theta$, if the isospin of the Θ^+ is even ($I_\Theta = 0,\, 2$), or $(-1)^S = \pi_\Theta$, if $I_\Theta = 1$. Therefore, both the P-parity and the isospin of the Θ^+ can be determined unambiguously by combined measurement of $C_{y,y}$ or K_y^y (or K_z^z) in these two reactions.

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