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QED PROCESSES IN PERIPHERAL KINEMATICS AT POLARIZED PHOTON–PHOTON AND PHOTON–ELECTRON COLLIDERS

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Бакмаев С. и др. E2-2004-33
КЭД-процессы в периферической кинематике
на фотон-фотонных и фотон-электронных коллайдерах

Для экспериментов, планируемых на фотон-электронных и фотон-фотонных коллайдерах, вычислены сечения для частиц, детектируемых при малых углах рассеяния. Эти процессы описывают образование частиц, движущихся достаточно близко к оси направления пучков. Выражения для процессов с образованием струй, содержащих две и три частицы, включая заряженные лептоны, фотоны и псевдоскалярные мезоны, приведены в явной форме. Полученные результаты могут быть использованы для построения численных программ.

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QED Processes in Peripheral Kinematics
at Polarized Photon–Photon and Photon–Electron Colliders

The calibration QED process cross sections for experiments planned on electron–photon and photon–photon colliders for detecting particles scattered at small angles are calculated. These processes describe the creation of two jets moving sufficiently close to the beam axis directions. The jets that contain two and three particles including charged leptons, photons, and pseudoscalar mesons are considered explicitly. Considering the pair production subprocesses we take into account both bremsstrahlung and double photon mechanisms. The obtained results are suitable for further numerical calculations.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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INTRODUCTION

QED processes of the type $2 \to 3, 4, 6$ at colliders of high energies have attracted both theoretical and experimental attention during the last four decades. Accelerators with high-energy colliding e^+e^-, γe, $\gamma\gamma$ and $\mu^+\mu^-$ beams are now widely used or designed to study fundamental interactions [1]. Some processes of quantum electrodynamics (QED) might play an important role at these colliders, especially those inelastic processes whose cross section does not drop with increasing energy. The planned colliders will be able to work with polarized particles, so these QED processes are required to be described in more detail, including the calculation of cross sections with definite helicities of the initial particles — leptons ($l = e$ or μ) and photons γ. These reactions have the form of a two-jet process with the exchange of a virtual photon γ^* in the t channel (see Fig. 1).

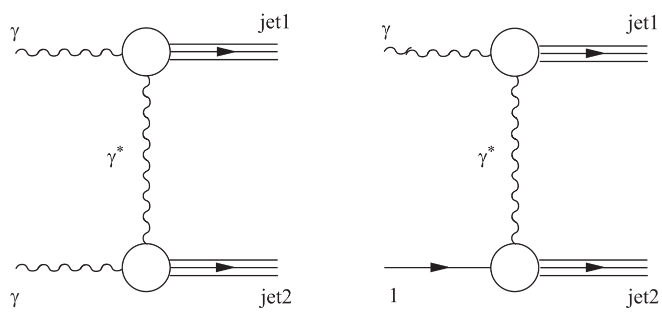

Fig. 1. The processes $\gamma\gamma$, γl ($l = e, \mu$) with the exchange of a virtual photon γ^* in the t channel

Much attention to the calculation of helicity amplitudes of QED processes at high-energy colliders was paid in the literature (see [2] and references therein). Keeping in mind the physical programs at planned $\gamma\gamma$ and lepton-γ colliders, a precise knowledge of a set of calibration and monitoring processes is needed. The calibration processes are the QED processes with sufficiently large cross sections and clear signatures for detection. A rather rich physics can be investigated in peripheral processes such as heavy leptons and mesons (scalar and pseudoscalar) creation, where the relevant QED monitoring processes must be measured. Let us remind the general features of peripheral processes, namely, the important fact of their nondecreasing cross sections in the limit of high total energies $\sqrt{s}$ in the center of mass frame of the initial particles. The possibility of measuring the jets containing two or three particles can be relevant. This is a motivation of our paper. It is organized in the following way. In Sec. 1, the kinematics of peripheral processes is briefly described. In Sec. 2, the impact factors describing the conversion of initial photon to the pair of charged particles (fermions or spinless mesons) are calculated. In Secs. 3, 4, and 5 a similar calculation is made for the initial polarized electron and photon, in particular subprocesses such as the single and the double Compton process, and the processes of pair creation are considered. Since the helicity amplitudes for subprocesses of type $2 \to 3$ have in general a complicated form, we do not put explicit expressions for the corresponding cross sections indicating only the strategy to obtain it.

1. KINEMATICS

Throughout the paper it is implied that the energy fractions of a jet component are positive quantities of the order of unity in magnitude (the sum of energy fractions of each jet is unity) and the values of transversal to the beam direction component of their 3-momenta are much larger compared to their rest masses. Thus, we neglect the mass of jet particles. The corresponding amplitudes include a large amount of the Feynman diagrams (FDs). Fortunately, in the high-energy limit a number of essential FD contributing the "leading" approximation greatly reduces. The method used permits one to estimate the uncertainty caused by "nonleading" contributions which have the following magnitudes of the order:

$$\frac{m^2}{s_1}, \quad \frac{s_1}{s}, \quad \frac{s_2}{s}, \quad \frac{\alpha}{\pi} \ln \frac{s}{m^2}, \tag{1}$$

where s_1 and s_2 are the jet invariant mass squares* compared with the terms of order unity. The last term in Eq. (1) is caused by the absence of radiative corrections in our analysis. The angles θ_i of particle emission to the corresponding projectile direction of motion is assumed to be of the order (see Fig. 2)

$$\frac{m_i}{\sqrt{s}} \ll \theta_i \sim \frac{\sqrt{s_i}}{\sqrt{s}} \ll 1, \tag{2}$$

*These are supposed to be small.

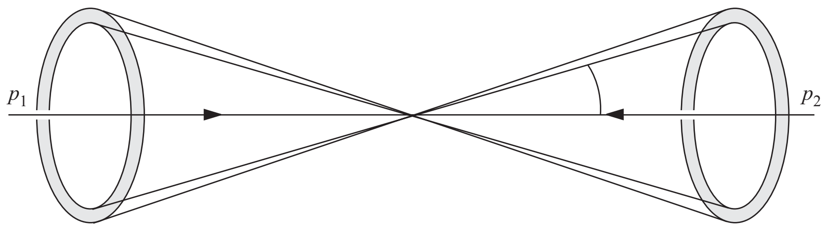

Fig. 2. The scheme of collision of initial beams with detection of two jets moving in the cones within the angles θ_i

where m_i is the typical mass of the jet particle.

In this approach we can consider initial particles (having the 4-momenta p_1, p_2) as massless and use the Sudakov parameterization of 4-momenta of any particle of the problem:

$$q_i = \alpha_i p_2 + \beta_i p_1 + q_{i\perp}, \qquad (3)$$
$$q_{i\perp} p_{1,2} = 0, \quad q_{i\perp}^2 = -\mathbf{q}_i^2 < 0.$$

The Sudakov parameters β_i are the quantities of order of unity for the momenta of the particles belonging to the $\mathrm{jet1}$ and obeying the conservation law $\sum_{\mathrm{jet1}} \beta_i = 1$, whereas the components of the $\mathrm{jet1}$ particle momenta along the 4-momentum p_2 are small positive numbers which can be determined from the on-mass-shell conditions of the $\mathrm{jet1}$ particles $q_i^2 = s\alpha_i\beta_i - \mathbf{q}_i^2 = 0, \alpha_i = \mathbf{q}_i^2/(s\beta_i) \ll 1$.

The same is valid for the 4-momenta of the particles belonging to the $\mathrm{jet2}$, namely, $\alpha_j \sim 1$, $\sum_{\mathrm{jet2}} \alpha_j = 1$, $\beta_j = \mathbf{q}_j^2/(s\alpha_j) \ll 1$.

Among the large amount of FDs, describing the process in the lowest (Born) order of perturbation theory (PT) (tree approximation), only those survive (i. e., give a contribution to the cross section which does not decrease with increasing s) which have a photonic *t*-channel one-particle state.

It is known [3] that the matrix elements of the peripheral processes have a factorized form and the cross section can be written in terms of the so-called impact factors, each of which describes the subprocess of interaction of the internal virtual photon with one of the initial particles to produce a jet moving in the direction close to this projectile momentum. So the problem can be formulated in terms of computation of impact factors. For processes with initial photons with definite state of polarization described in terms of Stokes' parameters we construct the relevant chiral matrices from bilinear combinations of chiral amplitudes. The last step consists in the construction of differential cross sections.

The matrix element, which corresponds to the main («leading») contribution, to the cross section, has the form

$$M = iJ_1^\mu \frac{g_{\mu\nu}}{q^2} J_2^\nu, \tag{4}$$

where J_1^μ and J_2^ν are the currents of the upper (associated with $\mathrm{jet}1$) and lower blocks of the relevant Feynman diagram, respectively, and $g_{\mu\nu}$ is the metric tensor. The current J_1^μ describes the scattering of an incoming particle of momentum p_1 with a virtual photon and subsequent transition to the first jet (similar to J_2^ν). Matrix elements (4) can be written in the form (see the appendices in [3])

$$\begin{gathered} M = 2i\frac{s}{q^2} I_1 I_2, \\ I_1 = \frac{1}{s} J_1^\mu p_{2\mu}, \quad I_2 = \frac{1}{s} J_2^\nu p_{1\nu}. \end{gathered} \tag{5}$$

Really, it follows from the Gribov representation of the metric tensor:

$$g^{\mu\nu} = \frac{2}{s}(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu) + g_\perp^{\mu\nu} \approx \frac{2}{s} p_2^\mu p_1^\nu. \tag{6}$$

Invariant mass squares of jets can also be expressed in terms of the Sudakov parameters of the exchanged photon:

$$\begin{gathered} q = \alpha p_2 + \beta p_1 + q_\perp, \quad (q + p_1)^2 = s_1 = -\mathbf{q}^2 + s\alpha, \\ (-q + p_2)^2 = s_2 = -\mathbf{q}^2 - s\beta, \quad q^2 = s\alpha\beta - \mathbf{q}^2 \approx -\mathbf{q}^2. \end{gathered} \tag{7}$$

Here and below we mean by the symbol «$\approx$» the equation with neglect of the terms which do not contribute to the limit $s \to \infty$.

The singularity of the matrix element (5) at $\mathbf{q} = 0$ is fictitious (excluding the elastic scattering). Really, one can see that it disappears due to the current conservation:

$$q_\mu J_1^\mu \approx (\alpha p_2 + q_\perp)_\mu J_1^\mu = 0, \quad p_{2\mu} J_1^\mu = \frac{s}{s\alpha} \mathbf{q} \mathbf{J}_1, \tag{8}$$

$$q_\nu J_2^\nu \approx (\beta p_1 + q_\perp)_\nu J_2^\nu = 0, \quad p_{1\nu} J_2^\nu = \frac{s}{s\beta} \mathbf{q} \mathbf{J}_2. \tag{9}$$

We arrive at the modified form of the matrix element of peripheral process

$$M(a(p_1,\eta_1) + b(p_2,\eta_2)) \to \mathrm{jet}_{1\lambda_1} + \mathrm{jet}_{2\lambda_2} = i(4\pi\alpha)^{\frac{n_1+n_2}{2}} \frac{2s}{\mathbf{q}^2} m_{1\lambda_1}^{\eta_1} m_{2\lambda_2}^{\eta_2}, \tag{10}$$

where η_i describe the polarization states of the projectile $i = a, b$; λ_i describe the polarization states of constituents of the corresponding jet. The numbers of QED vertices in the upper and lower blocks of FD (see Fig. 1) are denoted by $n_{1,2}$.

We give here two alternative forms for the matrix elements $m_{1,2}$ of the subprocesses $\gamma^*(q) + a(p_1, \eta_1) \to \mathrm{jet}_{1(\lambda_1)}$ and $\gamma^*(q) + b(p_2, \eta_2) \to \mathrm{jet}_{2(\lambda_2)}$:

$$m^{\eta_1}_{1\lambda_1} = \frac{\mathbf{q}\mathbf{J}^{\eta_1}_{1\lambda_1}}{s_1 + \mathbf{q}^2}, \tag{11}$$

$$m^{\eta_1}_{1\lambda_1} = \frac{1}{s} p_{2\mu} J^{\eta_1 \mu}_{1\lambda_1}; \tag{12}$$

and the similar expressions for the lower block. We use the second representation (12). The form (11) can be used as a check of validity of gauge invariance, namely turning the matrix elements to zero in the limit $\mathbf{q} \to 0$.

A remarkable feature of the peripheral processes is that their differential cross sections do not depend on the total center-of-mass energy $\sqrt{s}$. To see this property, let us first rearrange the phase volume $d\Phi$ of the final two-jet kinematics state to a more convenient form:

$$\begin{gathered} d\Phi = (2\pi)^4 \delta^4 (p_1 + p_2 - \textstyle\sum_i p_i^{(1)} - \sum_j p_j^{(2)}) dF^{(1)} dF^{(2)} = \\ = (2\pi)^4 d^4 q \delta^4_{(1)} \delta^4_{(2)} dF^{(1)} dF^{(2)}, \\ \delta^4_{(1)} = \delta^4 (p_1 + q - \textstyle\sum_i p_i^{(1)}), \quad \delta^4_{(2)} = \delta^4 (p_2 - q - \sum_j p_j^{(2)}), \\ dF_{(1,2)} = \Pi_i \dfrac{d^3 p_i^{(1,2)}}{2\varepsilon_i^{(1,2)} (2\pi)^3} \ . \end{gathered} \tag{13}$$

Using Sudakov's parameterization for the transferred 4-momentum q phase volume

$$d^4 q = \frac{s}{2} d\alpha d\beta d^2 q_\perp = \frac{1}{2s} ds_1 ds_2 d^2 q_\perp \tag{14}$$

with the invariant mass squares of the jets $s_{1,2}$, we put the phase volume in the factorized form

$$d\Phi = \frac{(2\pi)^4}{2s} d^2 q_\perp ds_1 dF^{(1)} \delta^4_{(1)} ds_2 dF^{(2)} \delta^4_{(2)}. \tag{15}$$

Using the modified form of the matrix element and the phase volume for peripheral process cross section in the case of polarized initial particles (photons or electrons), we have

$$d\sigma^{\eta_1 \eta_2} = \frac{\alpha^{n_1+n_2} \pi^2 (4\pi)^{2+n_1+n_2} d^2 q_\perp}{(\mathbf{q}^{\ 2})^2} \Phi_1^{\eta_1}(\mathbf{q}) \Phi_2^{\eta_2}(\mathbf{q}) \tag{16}$$

with the impact factors $\Phi_i^{\eta_i}$ in the form

$$\Phi_i^{\eta_i}(\mathbf{q}) = \int ds_i \sum_{\lambda_j} |m^{\eta_i}_{i\lambda_j}|^2 dF_i \delta^4_{(i)}, \quad i = 1, 2. \tag{17}$$

The matrix elements with the definite chiral states of all particles $m^{\eta_i}_{i(\lambda)}$, where the subscript (λ) denotes the set of chiral parameters of the final state, are calculated and listed below.

In the case of initial polarized photons the description in terms of Stokes' parameters $\xi_{1,2,3}$, $\xi_1^2+\xi_2^2+\xi_3^2 \ll 1$ is commonly used. The matrix element squared in the r. h. s. of (17) must be replaced by [4]

$$T_\gamma = \mathrm{Tr}\,(\mathcal{M}\rho) = \frac{1}{2}\mathrm{Tr}\begin{pmatrix} m^{++} & m^{+-} \\ m^{-+} & m^{--} \end{pmatrix}\begin{pmatrix} 1+\xi_2 & i\xi_1-\xi_3 \\ -i\xi_1-\xi_3 & 1-\xi_2 \end{pmatrix} \tag{18}$$

with the spin $\mathcal{M}$-matrix elements

$$\begin{gathered} m^{++} = \sum_\lambda |m^+_{(\lambda)}|^2, \quad m^{+-} = \sum_\lambda m^+_{(\lambda)}(m^-_{(\lambda)})^*, \\ m^{--} = \sum_\lambda |m^-_{(\lambda)}|^2, \quad m^{-+} = (m^{+-})^*. \end{gathered} \tag{19}$$

We choose $\lambda = +1$ for the initial fermion

$$T_e = \sum_\lambda |m^+_\lambda|^2. \tag{20}$$

The cross sections $d\sigma_{n_1,n_2}$ of the process of type $2 \to n_1 + n_2$ with production of two jets

$$\begin{aligned} a(p_1,\eta_1) + b(p_2,\eta_2) \to a_1(r_1\lambda_1) +_{\dots} +a_{n_1}(r_{n_1},\lambda_{n_1}) + \\ + b_1(q_1,\sigma_1) +_{\dots} +b_{n_2}(q_{n_2},\sigma_{n_2}), \end{aligned} \tag{21}$$

where energy fractions $x_1,\dots x_{n_1}, \sum x_i = 1$ and transversal components of momenta $\mathbf{r}_1,\dots \mathbf{r}_{n_1}$, $\sum \mathbf{r}_i = \mathbf{q}$ of jet a and the similar quantities $y_i, \mathbf{q}_i, \sum y_i = 1$, $\sum \mathbf{q}_i = -\mathbf{q}$ for the other jet b, have the form

$$d\sigma_{22} = \frac{\alpha^4}{2^2\pi^4} T_2^{(1)} T_2^{(2)} \frac{d^2q}{(\mathbf{q}^2)^2} d^2r_1 d^2q_1 \frac{dx_1dy_1}{x_1x_2y_1y_2}, \tag{22}$$

$$d\sigma_{23} = \frac{\alpha^5}{2^4\pi^6} T_2^{(1)} T_3^{(2)} \frac{d^2q}{(\mathbf{q}^2)^2} d^2r_1 d^2q_1 d^2q_2 \frac{dx_1dy_1dy_2}{x_1x_2y_1y_2y_3}, \tag{23}$$

$$d\sigma_{33} = \frac{\alpha^6}{2^6\pi^8} T_3^{(1)} T_3^{(2)} \frac{d^2q}{(\mathbf{q}^2)^2} d^2q_1 d^2q_2 d^2r_1 d^2r_2 \frac{dx_1dx_2dy_1dy_2}{x_1x_2x_3y_1y_2y_3}\ . \tag{24}$$

2. SUBPROCESSES $\gamma^*\gamma \to e^+e^-$, $\pi^+\pi^-$

Let us first consider the contribution to the photon impact factor from the lepton pair production subprocess

$$\gamma(k_1, \eta) + \gamma^*(q) \to e^-(q_-, \lambda) + e^+(q_+, -\lambda). \tag{25}$$

The matrix element of the subprocess has the form (we suppress the factor $4\pi\alpha$)

$$\begin{gathered} m_{1\lambda}^{\eta\mu} = -\bar{u}_\lambda(q_-)\Big[\hat{\varepsilon}^\eta \frac{\hat{q}_- - \hat{k}_1}{\kappa_{1-}}\gamma^\mu + \gamma^\mu \frac{-\hat{q}_+ + \hat{k}_1}{\kappa_{1+}}\hat{\varepsilon}^\eta\Big] v_\lambda(q_+), \\ \bar{u}_\lambda = \bar{u}\,\omega_{-\lambda}, v_\lambda = \omega_{-\lambda} v. \end{gathered} \tag{26}$$

We imply all the particles to be massless. A definite chiral state initial photon polarization vector has the form [5]

$$\hat{\varepsilon}_1^\lambda = N_1[\hat{q}_-\hat{q}_+\hat{k}_1\omega_{-\lambda} - \hat{k}_1\hat{q}_-\hat{q}_+\omega_\lambda], \tag{27}$$

where

$$N_1^2 = \frac{2}{s_1\kappa_+\kappa_-}, \quad s_1 = 2q_+q_-, \quad \kappa_{1\pm} = 2k_1q_\pm. \tag{28}$$

Chiral amplitudes $m_\lambda^\eta = (1/s)m_{1\lambda}^{\eta\mu}p_{2\mu}$ have the form

$$\begin{aligned} m_{1+}^+ &= -\frac{N_1}{s}\bar{u}\hat{q}_+\hat{q}\hat{p}_2\omega_+ v, \quad m_{1-}^+ = -\frac{N_1}{s}\bar{u}\hat{p}_2\hat{q}\hat{q}_-\omega_- v, \\ m_{1-}^- &= -\frac{N_1}{s}\bar{u}\hat{q}_+\hat{q}\hat{p}_2\omega_- v, \quad m_{1+}^- = -\frac{N_1}{s}\bar{u}\hat{p}_2\hat{q}\hat{q}_-\omega_+ v. \end{aligned} \tag{29}$$

The elements of the spin $\mathcal{M}$-matrix in the case of lepton pair production are

$$\begin{aligned} m_{e^+e^-}^{++} = m_{e^+e^-}^{--} &= \frac{2\mathbf{q}^2}{\mathbf{q}_+^2\mathbf{q}_-^2}x_+x_-(x_+^2 + x_-^2), \\ m_{e^+e^-}^{+-} = (m_{e^+e^-}^{-+})^* &= -\frac{4\mathbf{q}^2}{\mathbf{q}_+^2\mathbf{q}_-^2}(x_+x_-)^2 \mathrm{e}^{2i\theta}; \end{aligned} \tag{30}$$

x_+ and x_- are the energy fractions carried out by pair components; $x_+ + x_- = 1$ and θ is the angle between two Euclidean vectors; $\mathbf{q} = \mathbf{q}_- + \mathbf{q}_+$ and $\mathbf{Q} = x_+\mathbf{q}_- - x_-\mathbf{q}_+$.

In the case of charged pion pair production

$$\gamma(p_1, e_1^\eta) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-), \tag{31}$$

we have

$$m^\eta = \frac{1}{s}\varepsilon_{1\nu}^\eta p_2^\mu m_\mu^\nu = \frac{x_+}{p_1q_-}\varepsilon_1^\eta q_- + \frac{x_-}{p_1q_+}\varepsilon_1^\eta q_+ - \frac{2}{s}(\varepsilon_1^\eta p_2)\ . \tag{32}$$

Using the photon polarization vector written as

$$\varepsilon_{1\mu}^{\eta} = N_1[(q_+p_1)q_{-\mu} - (q_-p_1)q_{+\mu} + i\eta\varepsilon_{\mu\alpha\beta\gamma}q_-^{\alpha}q_+^{\beta}p_1^{\gamma}], \tag{33}$$

we obtain the chiral amplitude of the pion pair production process (we define $(p_1p_2q_-q_+) = \epsilon_{\alpha\beta\gamma\delta}p_1^{\alpha}p_2^{\beta}q_-^{\gamma}q_+^{\delta} = (s/2)[\mathbf{q}_-\mathbf{q}_+]_z$)

$$m^{\eta} = -N_1(\mathbf{Q}\mathbf{q} + i\eta[\mathbf{Q},\mathbf{q}]_z) = -N_1|\mathbf{q}|\,|\mathbf{Q}|\mathrm{e}^{i\eta\theta}, \quad \theta = \widehat{\mathbf{q}\mathbf{Q}}, \tag{34}$$

where we imply the z-axis direction along the photon 3-vector and use the relation $[\mathbf{q}_-,\mathbf{q}_+]_z = [\mathbf{Q},\mathbf{q}]_z$. For the pion chiral matrix we have

$$\begin{aligned} m_{\pi^+\pi^-}^{++} = m_{\pi^+\pi^-}^{--} &= \frac{2\mathbf{q}^2}{\mathbf{q}_+^2\mathbf{q}_-^2}(x_+x_-)^2, \\ m_{\pi^+\pi^-}^{+-} = (m_{\pi^+\pi^-}^{-+})^* &= \frac{2\mathbf{q}^2}{\mathbf{q}_+^2\mathbf{q}_-^2}(x_+x_-)^2\mathrm{e}^{2i\theta}. \end{aligned} \tag{35}$$

For the two-pair production process

$$\begin{aligned} &\gamma_1(p_1,\boldsymbol{\xi}_1) + \gamma_2(p_2,\boldsymbol{\xi}_2) \to a(q_-) + \bar{a}(q_+) + b(p_-) + \bar{b}(p_+), \\ &q_{\pm} = \alpha_{\pm}p_2 + x_{\pm}p_1 + q_{\pm\perp}; \quad p_{\pm} = y_{\pm}p_2 + \beta_{\pm}p_1 + p_{\pm\perp}, \end{aligned} \tag{36}$$

the differential cross section (assuming that the pair $a\bar{a}$ moves along the photon-1 direction and the pair $b\bar{b}$ moves along the photon-2 direction) has the form (22) with

$$T^{(1)} = \frac{\mathbf{q}^2}{\mathbf{q}_+^2\mathbf{q}_-^2}(x_+x_-)^2[1 - \xi_3\cos(2\theta) + \xi_1\sin(2\theta)], \quad \text{for} \quad \pi^+,\pi^-, \tag{37}$$

$$T^{(1)} = \frac{\mathbf{q}^2}{\mathbf{q}_+^2\mathbf{q}_-^2}(x_+x_-)\{x_+^2 + x_-^2 + 2x_+x_-[\xi_3\cos(2\theta) + \xi_1\sin(2\theta)]\}, \quad \text{for} \quad e^+,e^- \tag{38}$$

and the similar expression for $T^{(2)*}$. We remind that the obtained formulae are valid at large, compared to masses of particles, transverse components of jet particles

$$\mathbf{q}_-^2 \sim \mathbf{q}_+^2 \sim \mathbf{p}_+^2 \sim \mathbf{p}_-^2 \gg m^2, \quad \mathbf{q}_+ = \mathbf{q} - \mathbf{q}_-; \quad \mathbf{p}_+ = -\mathbf{q} - \mathbf{p}_-, \tag{39}$$

and finite energy fractions $x_{\pm} \sim y_{\pm} \sim 1$, which correspond to the emission angles of jet particles $\theta_i = |\mathbf{q}_i|/(x_i\varepsilon) \gg m/\varepsilon$ that are considerably larger than the mass-to-energy ratio.

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*In paper [6] formula (38) contains a misprint in the sign of $\xi_3^{(1,2)}$.

3. SUBPROCESSES $\gamma^*\gamma \to e^+e^-\gamma, \pi^+\pi^-\gamma$

Here and below for subprocesses of type $2 \to 3$ we restrict ourselves to calculating the chiral amplitudes and checking their gauge invariance properties.

The subprocess

$$\gamma(k,\lambda) + \gamma^*(q) \to e^+(q_+, -\lambda_-) + e^-(q_-, \lambda_-) + \gamma(k_1, \lambda_1) \tag{40}$$

is described by 6 FDs. A standard calculation of chiral amplitudes $m^{\lambda}_{\lambda_1\lambda_-}$ leads to

$$\begin{aligned} m^{+}_{++} &= -\frac{s_1 N N_1}{s}\bar{u}(q_-)\hat{q}_+\hat{q}\hat{p}_2\omega_+ v(q_+) = (m^{-}_{--})^*, \\ m^{+}_{+-} &= -\frac{s_1 N N_1}{s}\bar{u}(q_-)\hat{p}_2\hat{q}\hat{q}_-\omega_- v(q_+) = (m^{-}_{-+})^*, \\ m^{+}_{-+} &= \frac{N N_1}{s}\bar{u}(q_-)A^{+}_{-+}\omega_+ v(q_+) = (m^{-}_{+-})^*, \\ m^{+}_{--} &= \frac{N N_1}{s}\bar{u}(q_-)A^{+}_{--}\omega_- v(q_+) = (m^{-}_{++})^* \end{aligned} \tag{41}$$

with $A^{+}_{--}(k,k_1) = A^{+}_{-+}(-k_1,-k)$,

$$\begin{gathered} N^2 = \frac{2}{s_1\kappa_-\kappa_+}, \quad N_1^2 = \frac{2}{s_1\kappa_{1+}\kappa_{1-}}, \quad s_1 = 2q_+q_-, \\ \kappa_\pm = 2kq_\pm, \quad \kappa_{1\pm} = 2k_1q_\pm \end{gathered} \tag{42}$$

and a rather cumbersome expression for A^{+}_{-+}

$$\begin{aligned} A^{+}_{-+} = \frac{s_1}{(q_+-q)^2}\hat{k}\hat{q}_+\hat{k}_1(-\hat{q}_+ + \hat{q})\hat{p}_2 - \hat{q}_+(\hat{q}_- - \hat{k})\hat{p}_2(\hat{q}_+ + \hat{k}_1)\hat{q}_- - \\ - \frac{s_1}{(q_- - q)^2}\hat{p}_2(\hat{q}_- - \hat{q})\hat{k}\hat{q}_-\hat{k}_1. \end{aligned} \tag{43}$$

Substituting

$$\hat{p}_2 \approx \frac{1}{\alpha}(\hat{q} - \hat{q}_\perp) = \frac{s}{s\alpha}[\hat{q}_+ + \hat{k_1} + (\hat{q}_- - \hat{k}) - \hat{q}_\perp]\,,$$

in the second term of the r. h. s. of (43) we have

$$\begin{aligned} A^{+}_{-+} = -ss_1\kappa_{1+}\left[\frac{x_+}{(q_+-q)^2} + \frac{1}{s\alpha}\right]\hat{k} - ss_1\kappa_-\left[\frac{x_-}{(q_--q)^2} + \frac{1}{s\alpha}\right]\hat{k}_1 + \\ + \frac{s_1}{(q_+-q)^2}\hat{k}\hat{q}_+\hat{k}_1\hat{q}_\perp\hat{p}_2 + \frac{s_1}{(q_--q)^2}\hat{p}_2\hat{q}_\perp\hat{k}\hat{q}_-\hat{k}_1 + \\ + \frac{s}{s\alpha}\hat{q}_+(\hat{q}_- - \hat{k})\hat{q}_\perp(\hat{q}_+ + \hat{k}_1)\hat{q}_- \end{aligned} \tag{44}$$

with

$$(q_\pm - q)^2 = -\mathbf{q}^2 + 2\mathbf{q}\mathbf{q}_\pm - s\alpha x_\pm, \quad s\alpha = \frac{\mathbf{k}_1^2}{x_1} + \frac{\mathbf{q}_-^2}{x_-} + \frac{\mathbf{q}_+^2}{x_+},$$
$$x_1 + x_- + x_+ = 1, \quad \kappa_\pm = \frac{\mathbf{q}_\pm^2}{x_\pm}, \quad \kappa_{1\pm} = \frac{1}{x_1 x_\pm}(x_1\mathbf{q}_\pm - x_\pm \mathbf{k}_1)^2. \tag{45}$$

A gauge property (the chiral amplitudes must vanish as $\mathbf{q} \to 0$) can be seen explicitly.

A further procedure of constructing the chiral matrix is straightforward and can be performed in terms of simple traces. We will not touch it here.

Consider the subprocess

$$\gamma(k,\lambda) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-) + \gamma(k_1,\lambda_1)\ . \tag{46}$$

There are 12 FDs describing a rather cumbersome expression for the matrix element. It can be considerably simplified when using the modified expressions for the photon polarization vectors in the form [8]

$$\varepsilon_\mu^\lambda(k) = \frac{N}{2}\mathrm{Sp}\,\gamma_\mu \hat{q}_- \hat{q}_+ \hat{k}\omega_\lambda,$$
$$\varepsilon_\mu^{\lambda_1}(k_1) = \frac{N_1}{2}\mathrm{Sp}\,\gamma_\mu \hat{q}_- \hat{q}_+ \hat{k}_1\omega_\lambda \tag{47}$$

with the same expressions for N, N_1 as in the case of the $\gamma\gamma^* \to e^+e^-\gamma$ subprocess. Polarization vectors chosen in such a form satisfy the Lorentz condition $\varepsilon(k)k = 0$, $\varepsilon(k_1)k_1 = 0$ and also the gauge condition $\varepsilon(k)q_- = \varepsilon(k_1)q_- = 0$.

The matrix element has (we lost the Bose symmetry at this stage) the form

$$m_{\lambda_1}^\lambda = \frac{1}{s}p_2^\rho \varepsilon^\mu(k)\varepsilon_1^{*\sigma}(k_1)O_{\rho\mu\sigma} = \frac{4x_-}{(q_- - q)^2}\Big[\frac{(\varepsilon_1 q_+)(\varepsilon q)}{\kappa_{1+}} - \frac{(\varepsilon_1 q)(\varepsilon q_+)}{\kappa_+}\Big] +$$
$$+ \frac{4(\varepsilon p_2)(\varepsilon_1 q_+)}{s\kappa_{1+}} - \frac{4(\varepsilon_1 p_2)(\varepsilon q_+)}{s\kappa_+} + 2(\varepsilon\varepsilon_1)\Big[\frac{x_+}{(q_+ - q)^2} - \frac{x_-}{(q_- - q)^2}\Big], \tag{48}$$

where we imply $\varepsilon = \varepsilon^\lambda$, $\varepsilon_1 = \varepsilon_1^{\lambda_1}$ and $x_\pm = 2p_2 q_\pm / s$, $x_1 = 2p_2 k_1/s$, where $x_+ + x_- + x_1 = 1$.

For $\lambda_1 = \lambda$ we have

$$m_\lambda^\lambda = s_1 N N_1[A_1 + i\lambda B_1], \quad A_1 = -\mathbf{Q}\mathbf{q}, \quad B_1 = [\mathbf{Q}\mathbf{q}]_z. \tag{49}$$

For the case of opposite chiralities we have

$$
\begin{aligned}
&m^{\lambda}_{-\lambda} = s_1 N N_1 [A + i\lambda B],\\
&A = -\mathbf{Q}\mathbf{q} + \frac{1}{2x_1 x_- x_+}\times\\
&\quad\times [\mathbf{Q}^2\mathbf{k}_1^2 - \mathbf{q}_-^2(x_1\mathbf{q}_+ - x_+\mathbf{k}_1)^2 - \mathbf{q}_+^2(x_1\mathbf{q}_- - x_-\mathbf{k}_1)^2]\times\\
&\quad\times\Big(\frac{x_+}{(q_+-q)^2} - \frac{x_-}{(q_- - q)^2}\Big),\\
&B = \Big(\frac{x_+}{(q_+-q)^2} + \frac{x_-}{(q_- - q)^2}\Big)\times\\
&\quad\times (s\alpha[\mathbf{q}_-\mathbf{q}_+]_z - s\alpha_-[\mathbf{q}\mathbf{q}_+]_z + s\alpha_+[\mathbf{q}\mathbf{q}_-]_z)+\\
&\quad + 2[\mathbf{q}_-\mathbf{q}_+]_z - [\mathbf{Q}\mathbf{q}]_z,\\
&\quad s\alpha_\pm = \frac{\mathbf{q}_\pm^2}{x_\pm},\quad s\alpha = \frac{\mathbf{k}_1^2}{x_1} + s\alpha_+ + s\alpha_-.
\end{aligned}
\tag{50}
$$

We can see that the Bose symmetry is restored.

4. SUBPROCESSES $e\gamma^* \to e\gamma; e+\gamma+\gamma$

Consider first the Compton subprocess*

$$\gamma^*(q) + e(p,\lambda_1) \to \gamma(k,\lambda) + e(p',\lambda_1). \tag{51}$$

For the chiral matrix elements we have (we choose $\lambda_1 = +1$)

$$
\begin{aligned}
m^+_\lambda &= \frac{N}{s}\bar{u}(p')[-\hat{p}\omega_\lambda(\hat{p}' + \hat{k})\hat{p}_2 - \hat{p}_2(\hat{p} - \hat{k})\hat{p}'\omega_{-\lambda}]\omega_+ u(p),\\
m^+_+ &= -\frac{N}{s}\bar{u}(p')\hat{p}\hat{q}\hat{p}_2\omega_+ u(p),\quad m^+_- = -\frac{N}{s}\bar{u}(p')\hat{p}_2\hat{q}\hat{p}'\omega_+ u(p).
\end{aligned}
\tag{52}
$$

The sum of modulo square of the matrix elements is

$$T_e = \sum_\lambda |m^+_\lambda|^2 = 2\frac{\mathbf{q}^2}{\kappa\kappa'}[1 + (1-x)^2] \tag{53}$$

with

$$\kappa = 2kp = \frac{\mathbf{k}^2}{x},\quad \kappa' = 2kp' = \frac{1}{x(1-x)}(\mathbf{p}'x - \mathbf{k}(1-x))^2, \tag{54}$$

---

*The case of real initial photons was considered in paper [7].

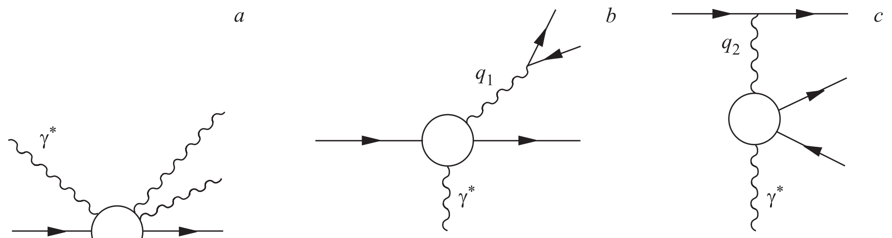

Fig. 3. Feynman diagrams describing the subprocess $\gamma^* e^- \to \gamma\gamma e^-$ (a) and pair production $\gamma^* e \to e a \bar{a}$ subprocess by the bremsstrahlung (b) and double photon (c) mechanisms

and $x = 2kp_2/2p_1p_2, 1-x$ are the energy fractions of photon and electron in the final state.

Consider now the double Compton subprocess (see Fig. 3, a)

$$e(p,\eta) + \gamma^*(q) \to e(p',\eta) + \gamma(k_1,\lambda_1) + \gamma(k_2,\lambda_2). \tag{55}$$

The chiral matrix elements $m^\eta_{\lambda_1\lambda_2}$ are

$$\begin{aligned} m^+_{++} &= (m^-_{--})^* = -\frac{s_1 N_1 N_2}{s}\bar{u}(p')\hat{p}\hat{q}\hat{p}_2\omega_+ u(p); \\ m^+_{--} &= (m^-_{++})^* = -\frac{s_1 N_1 N_2}{s}\bar{u}(p')\hat{p}_2\hat{q}\hat{p}'\omega_+ u(p); \\ m^+_{+-} &= (m^-_{-+})^* = \frac{N_1 N_2}{s}\bar{u}(p')A^+_{+-}\omega_+ u(p); \\ m^+_{-+} &= (m^-_{+-})^* = \frac{N_1 N_2}{s}\bar{u}(p')A^+_{-+}\omega_+ u(p) \end{aligned} \tag{56}$$

with $A^+_{-+}(k_1,k_2) = A^+_{+-}(k_2,k_1)$ and

$$A^+_{+-}(k_1,k_2) = \frac{s_1}{(p'-q)^2}\hat{p}_2(\hat{p}'-\hat{q})\hat{k}_1\hat{p}'\hat{k}_2 + \hat{p}(\hat{p}'+\hat{k}_1)\hat{p}_2(\hat{p}-\hat{k}_2)\hat{p}' + \\ + \frac{s_1}{(p+q)^2}\hat{k}_1\hat{p}\hat{k}_2(\hat{p}+\hat{q})\hat{p}_2 \tag{57}$$

with

$$s_1 = 2pp', \quad N_i^2 = \frac{2}{s_1\kappa_i\kappa_i'}, \quad \kappa_i = 2pk_i, \quad \kappa_i' = 2p'k_i. \tag{58}$$

To see the gauge invariance property of two last amplitudes we perform a substitution $p_2 = (q - q_\perp)/\alpha_q$ in the second term of the r. h. s. of Eq. (57) and arrive at the form

$$A^{+}_{+-}(k_1,k_2) = ss_1\kappa_1'\left(\frac{x'}{(p'-q)^2}+\frac{1}{s\alpha_q}\right)\hat{k}_2 + ss_1\kappa_2\left(\frac{1}{(p+q)^2}-\frac{1}{s\alpha_q}\right)\hat{k}_1 + \\ + \frac{s_1}{(p+q)^2}\hat{k}_1\hat{p}\hat{k}_2\hat{q}_\perp\hat{p}_2 - \frac{s_1}{(p'-q)^2}\hat{p}_2\hat{q}_\perp\hat{k}_1\hat{p}'\hat{k}_2 - \\ - \hat{p}(\hat{p}'+\hat{k}_1)\hat{q}_\perp(\hat{p}-\hat{k}_2)\hat{p}'\frac{s}{s\alpha_q}. \quad (59)$$

We can verify that this expression turns to zero at $\mathbf{q}=0$. Really, we can use

$$(p'-q)^2 = -\mathbf{q}^2 + 2\mathbf{p}^{'}\mathbf{q} - sx'\alpha_q, \quad (p+q)^2 = -\mathbf{q}^2 + s\alpha_q, \\ \alpha_q = \alpha' + \alpha_1 + \alpha_2, \quad x' + x_1 + x_2 = 1, \quad s\alpha' = \frac{(s\mathbf{p}^{'})^2}{x'}, \quad s\alpha_i = \frac{\mathbf{k}_i^2}{x_i}, \quad (60) \\ \kappa_i = s\alpha_i, \quad \kappa_i' = \frac{1}{x'x_i}(\mathbf{k}_i x' - \mathbf{p}^{'}x_i)^2.$$

A further strategy is similar to the one mentioned above (45).

5. SUBPROCESSES $e\gamma^* \to e\pi^+\pi^-, e\mu^+\mu^-$

The matrix element of the pion pair production subprocess

$$e(p,\eta) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-) + e(p',\eta) \quad (61)$$

can be written in the form

$$m^\eta = \bar{u}(p')[\hat{B}+\hat{D}]\omega_\eta u(p), \quad (62)$$

where bremsstrahlung mechanism contribution is (see Fig. 3, *b*)

$$\hat{B} = \frac{1}{q_1^2}\Big[B\hat{q}_1 + \frac{1}{s(p+q)^2}\hat{q}_1\hat{q}\hat{p}_2 - \frac{1}{s(p'-q)^2}\hat{p}_2\hat{q}\hat{q}_1\Big], \\ q_1 = q_+ + q_-, \; q_2 = p^{'} - p_1; \quad (63) \\ \hat{D} = \frac{1}{q_2^2}\Big[D(2\hat{q}_- + \hat{q}_2) - 2\frac{x_-}{(q-q_-)^2}\hat{q}_\perp + \frac{2(\mathbf{q}^2 - 2\mathbf{q}\,\mathbf{q}_-)}{s(q-q_-)^2}\hat{p}_2\Big].$$

For the squares of module of the chiral amplitudes, which enter in (23) and (24), we have

$$T_3^{(\pi)} = |m^+|^2 = \mathrm{Sp}\,(\hat{\mathrm{p}}'(\hat{\mathrm{B}}+\hat{\mathrm{D}})\hat{\mathrm{p}}(\tilde{\mathrm{B}}+\tilde{\mathrm{D}})\omega_+) \quad (64)$$

with B and D specified below (69).

For the subprocess of the muon pair production we have

$$e(p,\eta)+\gamma^*(q)\to\mu^+(q_+)+\mu^-(q_-)+e(p',\eta). \tag{65}$$

The bremsstrahlung and two-photon mechanisms must be taken into account (see Fig. 3, *b, c*):

$$m_\lambda^+=\frac{1}{q_1^2}\bar u(p')B_\mu\omega_+u(p)\bar u(q_-)\gamma^\mu\omega_\lambda v(q_+)+ \\ +\frac{1}{q_2^2}\bar u(p')\gamma_\nu\omega_+u(p)\bar u(q_-)D_\nu\omega_\lambda v(q_+) \tag{66}$$

with double photon mechanism contribution (not considered in paper [2])

$$D_\nu=D\gamma_\nu+\frac{1}{s(q-q_+)^2}\gamma_\nu\hat q\hat p_2-\frac{1}{s(q-q_-)^2}\hat p_2\hat q\gamma_\nu \tag{67}$$

and bremsstrahlung mechanism contribution

$$B_\mu=B\gamma_\mu-\frac{1}{s(p'-q)^2}\hat p_2\hat q\gamma_\mu+\frac{1}{s(p+q)^2}\gamma_\mu\hat q\hat p_2 \tag{68}$$

with

$$B=\frac{x'}{(p'-q)^2}+\frac{1}{(p+q)^2},\quad D=\frac{x_-}{(q_--q)^2}-\frac{x_+}{(q-q_+)^2}\ , \\ x_\pm=\frac{2p_2q_\pm}{s},\quad x'=\frac{2p_2p'}{s},\quad x_++x_-+x'=1. \tag{69}$$

To perform the conversion in the Lorentz indices μ,ν in (66), one can use the projection operators. In the case of equal chiralities $\eta=\lambda=+1$ we choose the projection operator as

$$P_+=\frac{\bar u(p)\hat q_+\omega_+u(q_-)}{\bar u(p)\hat q_+\omega_+u(q_-)}\ . \tag{70}$$

Inserting it and using the relation $\omega_+u(p)\bar u(p)=\omega_+\hat p$, we obtain

$$m_+^+=\frac{-2}{\bar u(p)\hat q_+\omega_+u(q_-)}\bar u(p')\Big[\left(\frac{D}{q_2^2}+\frac{B}{q_1^2}\right)\hat q_-\hat q_+\hat p+ \\ +\frac{\hat q_-\hat q_+\hat p\hat q_\perp\hat p_2}{s}\left(\frac{1}{q_2^2(q-q_+)^2}-\frac{1}{q_1^2(p+q)^2}\right)+ \\ +\frac{\hat p_2\hat q_\perp\hat q_-\hat q_+\hat p}{s}\left(\frac{1}{q_2^2(q_--q)^2}-\frac{1}{q_1^2(p'-q)^2}\right)\Big]\omega_+v(q_+)= \\ =\frac{-2}{\bar u(p)\hat q_+\omega_+u(q)_-}\bar u(p')A_+^+\omega_+v(q_+). \tag{71}$$

In the case of opposite chiralities $\eta = -\lambda = +1$ we use the projection operator

$$P_- = \frac{\bar{u}(p)\omega_- u(q_-)}{\bar{u}(p)\omega_- u(q_-)}.$$

The similar calculations lead to the following result:

$$\begin{aligned} m_-^+ = \frac{2}{\bar{u}(p)\omega_- u(q_-)}\bar{u}(p')\Big[\left(\frac{D}{q_2^2} + \frac{B}{q_1^2}\right) 2(pq_-) + 2\frac{\hat{p}\hat{q}_-\hat{q}_\perp\hat{p}_2}{s}\left(\frac{1}{q_2^2(q-q_+)^2} + \right. \\ \left. + \frac{1}{q_1^2(p_1-q_-)^2}\right) - \frac{\hat{p}\hat{q}_\perp\hat{p}_2\hat{q}_-}{s}\left(\frac{1}{q_2^2(q-q_-)^2} + \frac{1}{q_1^2(p+q)^2}\right) - \\ - \frac{\hat{q}_-\hat{p}_2\hat{q}_\perp\hat{p}}{s}\left(\frac{1}{q_2^2(q-q_-)^2} + \frac{1}{q_1^2(p+q)^2}\right)\Big]\omega_- v(q_+) = \\ = \frac{2}{\bar{u}(p)\omega_- u(q_-)}\bar{u}(p')A_-^+\omega_- v(q_+). \quad (72) \end{aligned}$$

The property of A_+^+, A_-^+ tending to zero as $|\mathbf{q}| \to 0$ is explicitly seen from (71) and (72).

For the sum of squares of chiral amplitudes, entering Eqs. (23) and (24), one has

$$\begin{aligned} T_3^{(\mu)} = \sum |m_\lambda^+|^2 = \\ = \frac{1}{(pq_+)(q_-q_+)}\mathrm{Sp}\,(\hat{p}'A_+^+\hat{q}_+\tilde{A}_+^+\omega_+) + \frac{2}{pq_-}\mathrm{Sp}\,(\hat{p}'A_-^+\hat{q}_+\tilde{A}_-^+\omega_+). \quad (73) \end{aligned}$$

A further strategy is straightforward.

CONCLUSION

In our paper [6], we wrote down the explicit expressions for the spin matrix elements $\mathcal{M}_{ij}$ for subprocesses of the type $2 \to 2$, which are reviewed here. For the subprocesses of the type $2 \to 3$, we formulated the algorithm of the calculation of spin matrix elements. We considered all possibilities of pair creation in the mentioned subprocesses as they were not completely considered in the recent work [2]. The gauge condition $\mathcal{M}_{ij}(q) \to 0$ for $|\mathbf{q}| \to 0$ is explicitly fulfilled in all the cases. The subprocesses with the pions in the final state were also considered in the paper for the first time.

The magnitude of the cross sections (22)–(24) is of the order $\alpha^n/\mu^2 \gg \alpha^n/s, n = 4, 5, 6$, where $\mu^2 = \max(s_1, s_2)$ is large enough to be measured, and does not depend on s. The strategy of calculation of cross section, using the helicity amplitudes of subprocesses $2 \to 3$, is described above and can be implemented to numerical programs which take into account details of experiments.

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REFERENCES

1. Zeroth-Order Design Report for the Next Linear Collider, Report No. SLAC-474 1996; JLC Design Study, KEK Report 97-1. 1997;
   Heuer R. D. et al. Technical Design Report of 500 GeV Electron Positron Collider with Integrated X-ray Facility. DESY 2001-011. 2001; Physics Potential and Development of $\mu^+\mu^-$ Colliders / Ed. D. Cline. AIP Conf. Proc. 1997. V. 441.
2. *Carimalo C., Schiller A., Serbo V. G.* // Eur. Phys. J. C. 2002. V. 23. P. 633.
3. *Baier V. N. et al.* // Phys. Rep. C. 1982. V. 78, No. 3. P. 293.
4. *Berestetsky V., Lifshits E., Pitaevski L.* Quantum Electrodynamics. M.: Nauka, 1989.
5. *Berends F. A. et al.* // Nucl. Phys. B. 1982. V. 206. P. 53.
6. *Bartoš E. et al.* // Nucl. Phys. B. 2004. V. 676. P. 481.
7. *Bartoš E. et al.* // Ibid. P. 390.
8. *Borodulin V. I., Rogalev R. N., Slabospitsky S. R.* CORE: Compendium of relations: Version 2.1. Preprint IFVE-95-90, IHEP-95-90. 1995. 108 p.; hep-ph/9507456;
   Sikach S. Institute of Physics, BAS, Minsk. Private communication.

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