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SCHEMES OF NEUTRINO MIXINGS (OSCILLATIONS) AND THEIR MIXING MATRICES

Бештоев Х. М. E2-2004-58
Схемы смешивания нейтрино и их матрицы смешивания

В работе рассматриваются три схемы смешивания (осцилляций) нейтрино вместе с их матрицами смешивания, аналогичные матрицам смешивания Кабиббо–Кобаяши–Маскавы. В этих схемах нейтринные переходы являются виртуальными, если массы нейтрино различаются. Две из них принадлежат к так называемой схеме массовых смешиваний (параметры смешивания выражаются через элементы массовой матрицы), а треть является схемой зарядовых смешиваний (параметры смешивания выражаются через заряды). Для первой схемы получена система из 6 уравнений для определения всех элементов массовой матрицы (масс нейтрино и ширин переходов) с использованием экспериментальных данных. Во второй и третьей углы смешивания нейтрино равны и близки к максимальным углам смешивания $(\pi/4)$. Предполагается, что эксперимент должен дать ответ на вопрос: которая из этих схем реализуется в действительности?

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Schemes of Neutrino Mixings (Oscillations)
and Their Mixing Matrices

Three schemes of neutrino mixings (oscillations) together with their mixing matrices (analogous to Kabibbo–Kobayashi–Maskawa matrices) are considered. In these schemes neutrino transitions are virtual if neutrino masses are different. Two of them belong to the so-called mass mixing schemes (mixing parameters are expressed by elements of mass matrices) and the third scheme belongs to the charge mixing one (mixing parameters are expressed through charges). In the first scheme, six equations for determination of all the elements of the mass matrix (neutrino masses and transition widths) are given using experimental data. In the second and third ones the mixing angles are equal or close to maximal angles $(\pi/4)$. It is obvious that the experiment must give an answer to the following question: Which of these schemes is realized indeed?

The investigation has been performed at the Laboratory of Particle Physics, JINR, and at the Institute of Applied Mathematics and Automation, KBSC of RAS, Nalchik, Russia.

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INTRODUCTION

In the quark sector, mixings between d, s, b quarks are described by Kabibbo–Kobayashi–Maskawa matrices [1]. At present, we know that the lepton numbers are not conserved [2–5] and ν_e, ν_μ, ν_τ neutrinos are also mixed. Then, for lepton sector we can also introduce similar matrices. Unfortunately, we do not know if there are neutrino oscillations or only neutrino mixings without oscillations. Therefore, it is necessary to consider all the realistic schemes of neutrino mixings and oscillations. Usually, only the standard scheme of neutrino oscillations is considered [6]. Since in this scheme the law of energy-momentum conservation is not fulfilled [7], we suppose that this scheme is not a realistic one for description of neutrino oscillations. We proposed three schemes for description of neutrino mixings and oscillations [7]. The first scheme is the development of the standard scheme in the framework of the particle physics. In these schemes neutrino transitions are virtual if neutrino masses are different. You are invited to study these schemes of neutrino mixings and oscillations. In the paper we also obtain mixing matrices for these schemes.

SCHEMES (TYPES) OF NEUTRINO MIXING (OSCILLATION) AND THEIR MIXING MATRICES

In the general case there can be two schemes (types) of neutrino mixings (oscillations): mass mixing schemes and charge mixings scheme (as it takes place in the vector dominance model or vector boson mixings in the standard model of electroweak interactions).

1. Two Schemes of Neutrino Mass Mixings (Oscillations) and Their Mixing Matrices. In the standard approach [6] it is supposed that neutrinos have already been created in superposition states, i.e., mass matrix is a nondiagonal one. If mass matrix is nondiagonal at once, then we must diagonalize this matrix in order to find eigenstates of neutrinos. Then eigenstates are ν_1, ν_2, ν_3 neutrinos, i.e., there must be created ν_1, ν_2, ν_3 neutrinos but not ν_e, ν_μ, ν_τ neutrinos. It is obvious that it cannot be coordinated with experimental data. In the weak interactions only physical neutrinos (ν_e, ν_μ, ν_τ) are created, i.e., mass matrix is a diagonal one, and then at violation of the lepton numbers this matrix is transformed into nondiagonal one [7]. We stress this point for its fundamental importance.

Originally it was supposed [6] that these neutrino oscillations are real oscillations, i.e., that there takes place a real transition of electron neutrino ν_e into muon neutrino ν_μ (or tau neutrino ν_τ). Then the neutrino $x = \mu, \tau$ will decay in electron neutrino plus something

$$\nu_x \to \nu_e + ..., \tag{1}$$

as a result, we get energy from vacuum, which equals the mass difference (if $m_{\nu_x} > m_{\nu_e}$)

$$\Delta E \sim m_{\nu_x} - m_{\nu_e}. \tag{2}$$

Then, again this electron neutrino transits into muon neutrino, which decays again and we get energy and etc. **So we have got a perpetuum mobile!** Obviously, the law of energy conservation cannot be fulfilled in this process. The only way to restore the law of energy conservation is to demand that this process is a virtual one. Then, these oscillations will be the virtual ones and they are described in the framework of the uncertainty relations. The correct theory of neutrino oscillations can be constructed only in the framework of the particle physics theory, where the concept of mass shell is present [8], [9].

We can also see that there are two cases of neutrino transitions (oscillations) in the scheme of mass mixings [9].

1.1. Development of the standard scheme of neutrino mixings (oscillations). The standard scheme belongs to the so-called mass mixings scheme, since mixing parameters are expressed through elements of mass matrix. In this case the probability of $\nu_e \rightarrow \nu_\mu$ transition (oscillation) is described by the following expression (for simplification we consider two neutrino ν_e, ν_μ mixings cases):

$$P(\nu_e \rightarrow \nu_\mu, t) = \sin^2 2\theta \sin^2 \left[\pi t \frac{|m^2_{\nu_1} - m^2_{\nu_2}|}{2p_{\nu_e}}\right], \tag{3}$$

where p_{ν_e} is a momentum of ν_e neutrino,

$$\sin^2 2\theta = \frac{4m^2_{\nu_e\nu_\mu}}{(m_{\nu_e} - m_{\nu_\mu})^2 + 4m^2_{\nu_e\nu_\mu}}, \tag{4}$$

and

$$m_{1,2} = \frac{1}{2}\left[(m_{\nu_e} + m_{\nu_\mu}) \pm \left((m_{\nu_e} - m_{\nu_\mu})^2 + 4m^2_{\nu_\mu\nu_e}\right)^{1/2}\right], \tag{5}$$

At this transitions (oscillations) neutrinos remain on their mass shell and transitions (oscillations) must be virtual.

It is interesting to remark that expression (4) can be obtained from the Breit–Wigner distribution [10]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}, \tag{6}$$

using the following substitutions:

$$E = m_{\nu_e}, \quad E_0 = m_{\nu_\mu}, \quad \Gamma/2 = 2m_{\nu_e\nu_\mu}, \tag{7}$$

where $\Gamma/2 \equiv W(...)$ is a width of $\nu_e \to \nu_\mu$ transition, then we can use a standard method [9, 11] for calculating this value. Then, the probability of $\nu_e \to \nu_\mu$ transitions is defined by these neutrino masses and width of their transitions.

Expression for length of these oscillations has the following form:

$$L_o = 2\pi \frac{2p}{|m_2^2 - m_1^2|}. \tag{8}$$

Above, we considered the case of two neutrino transitions (oscillations). In the general case we must consider three neutrino transitions (oscillations). For a complete description of three neutrino oscillations we must have six parameters (we suppose that this mass matrix is symmetric about the diagonal one)

$$\begin{pmatrix} m_{\nu_e} & m_{\nu_e\nu_\mu} & m_{\nu_e\nu_\tau} \\ m_{\nu_e\nu_\mu} & m_{\nu_\mu} & m_{\nu_\mu\nu_\tau} \\ m_{\nu_e\nu_\tau} & m_{\nu_\mu\nu_\tau} & m_{\nu_\tau} \end{pmatrix}; \tag{9}$$

three diagonal terms of this matrix are masses of three physical neutrinos m_{ν_μ}, m_{ν_τ}, m_{ν_e} and three nondiagonal mass terms of this matrix are $m_{\nu_e\nu_\mu}$, $m_{\nu_\mu\nu_\tau}$, $m_{\nu_e\nu_\tau}$-neutrino transition widths. Since in the expression for neutrino transition probabilities, mass differences (in squared form) are used in reality, we need only five parameters (and for further simplification physical neutrino masses are used). Besides, if mass matrix is complex, there appears one parameter, connected with CP violation.

Let us consider back problem, i.e., problem of finding of these (six) parameters from experiments. From experiments on neutrino transitions (oscillations) we can determine the following six values: three values from amplitudes

$$\sin^2 2\theta_{ij} = \frac{4m^2_{\nu_i\nu_j}}{(m_{\nu_i} - m_{\nu_j})^2 + 4m^2_{\nu_i\nu_j}} \tag{10}$$

and three values from oscillation lengths (or differences of squared masses $m^2_{\nu_\alpha} - m^2_{\nu_\beta}$)

$$P'(\nu_i \to \nu_j, t) = \sin^2\left[\pi t \frac{|m^2_{\nu_\alpha} - m^2_{\nu_\beta}|}{2p_{\nu_i}}\right], \tag{11}$$

where

$$i < j, \qquad i, j = e, \mu, \tau; \qquad \alpha, \beta = 1, 2, 3.$$

Using these parameters we can obtain values of six neutrino mass matrix parameters: three values for neutrino mass (or two mass differences) and three nondiagonal mass parameters (widths of neutrino transitions).

These mixing angles can be connected with mixing matrix V as in the case of Kabibbo–Kobayashi–Maskawa matrix in standard manner [1]. We will choose a parameterization of the mixing matrix V in the form proposed by Maiani [12]:

$$V=\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix}\begin{pmatrix} c_\beta & 0 & s_\beta\exp(-i\delta) \\ 0 & 1 & 0 \\ -s_\beta\exp(i\delta) & 0 & c_\beta \end{pmatrix}\begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{12}$$

$$\begin{gathered} c_{e\mu}=\cos\theta, \quad s_{e\mu}=\sin\theta, \quad c_{e\mu}^2+s_{e\mu}^2=1; \\ c_{e\tau}=\cos\beta, \quad s_{e\tau}=\sin\beta, \quad c_{e\tau}^2+s_{e\tau}^2=1; \\ c_{\mu\tau}=\cos\gamma, \quad s_{\mu\tau}=\sin\gamma, \quad c_{\mu\tau}^2+s_{\mu\tau}^2=1; \\ \exp(i\delta)=\cos\delta+i\sin\delta. \end{gathered} \tag{13}$$

In our approximation, the value of δ can be considered to be equal to zero.

Equations for mixing angles expressed through elements of mass matrix have the following form:

$$s_{e\mu}=\sin\theta=\frac{1}{\sqrt{2}}\left[1-\frac{|m_{\nu_\mu}-m_{\nu_e}|}{\sqrt{(m_{\nu_\mu}-m_{\nu_e})^2+(2m_{\nu_e\nu_\mu})^2}}\right], \tag{14}$$

$c_{e\mu}^2=1-s_{e\mu}^2$;

$$s_{e\tau}=\sin\beta=\frac{1}{\sqrt{2}}\left[1-\frac{|m_{\nu_\tau}-m_{\nu_e}|}{\sqrt{(m_{\nu_\tau}-m_{\nu_e})^2+(2m_{\nu_e\nu_\tau})^2}}\right], \tag{15}$$

$c_{e\tau}^2=1-s_{e\tau}^2$;

$$s_{\mu\tau}=\sin\gamma=\frac{1}{\sqrt{2}}\left[1-\frac{|m_{\nu_\tau}-m_{\nu_\mu}|}{\sqrt{(m_{\nu_\tau}-m_{\nu_\mu})^2+(2m_{\nu_\mu\nu_\tau})^2}}\right], \tag{16}$$

$c_{\mu\tau}^2=1-s_{\mu\tau}^2$.

*1.2. Analysis of present status of neutrino mixing parameters.*Super-Kamiokande data [3] on atmospheric neutrino transitions for $\nu_\mu \to \nu_\tau$ are

$$\sin^2 2\beta \cong 1, \quad \Delta m_{23}^2 \cong 2.5\cdot 10^{-3}\ \text{eV}^2. \tag{17}$$

The KamLAND detector [5] on the $\bar\nu_e \to \bar\nu_\mu$ transition presented the following data:

$$\sin^2 2\theta \cong 1, \quad \Delta m_{21}^2 \cong 6.9\cdot 10^{-5}\ \text{eV}^2. \tag{18}$$

Using the above and the SNO [4] data we can come to conclusion that for $\nu_e \to \nu_\tau$ transitions we have

$$\sin^2 2\gamma \cong 1, \tag{19}$$

but the value of Δm_{31}^2 remains unknown.

The vicinity of $\sin^2 2\theta$, $\sin^2 2\beta$, $\sin^2 2\gamma$ to unity allows the expansion of these values around unity and then expressions for $\sin^2 2\theta_{ij}$ and mass differences will have the following form:

$$(2m_{ij})^2 \gg (m_j - m_i)^2 \quad j > i; \;\; i, j = e, \mu, \tau$$

$$\sin^2 2\theta_{ij} \cong 1 - \frac{(m_{\nu_i} - m_{\nu_j})^2}{4m^2_{\nu_i,\nu_j}}, \tag{20}$$

$$\Delta m^2_{21} = m^2_2 - m^2_1 = (m_{\nu_\mu} + m_{\nu_e})\sqrt{(m_{\nu_\mu} - m_{\nu_e})^2 + (2m_{\nu_e\nu_\mu})^2}, \tag{21}$$

if $2m_{\nu_e\nu_\mu} \gg |m_{\nu_\mu} - m_{\nu_e}|$ then

$$\Delta m^2_{21} = (m_{\nu_\mu} + m_{\nu_e})2m_{\nu_e\nu_\mu}\left[1 + \frac{(m_{\nu_\mu} - m_{\nu_e})^2}{2(2m_{\nu_e\nu_\mu})^2}\right], \tag{21$'$}$$

and if $2m_{\nu_e\nu_\mu} \ll |m_{\nu_\mu} - m_{\nu_e}|$ then

$$\Delta m^2_{21} = (m^2_{\nu_\mu} - m^2_{\nu_e})\left[1 + \frac{(2m_{\nu_e\nu_\mu})^2}{2(m_{\nu_\mu} - m_{\nu_e})^2}\right]; \tag{21$''$}$$

$$\Delta m^2_{31} = m^2_3 - m^2_1 = (m_{\nu_\tau} + m_{\nu_e})\sqrt{(m_{\nu_\tau} - m_{\nu_e})^2 + (2m_{\nu_e\nu_\tau})^2}, \tag{22}$$

if $2m_{\nu_e\nu_\tau} \gg |m_{\nu_\tau} - m_{\nu_e}|$ then

$$\Delta m^2_{31} = (m_{\nu_\tau} + m_{\nu_e})2m_{\nu_e\nu_\tau}\left[1 + \frac{(m_{\nu_\tau} - m_{\nu_e})^2}{2(2m_{\nu_e\nu_\tau})^2}\right], \tag{22$'$}$$

and if $2m_{\nu_e\nu_\tau} \ll |m_{\nu_\tau} - m_{\nu_e}|$ then

$$\Delta m^2_{31} = (m^2_{\nu_\tau} - m^2_{\nu_e})\left[1 + \frac{(2m_{\nu_e\nu_\tau})^2}{2(m_{\nu_\tau} - m_{\nu_e})^2}\right]; \tag{22$''$}$$

$$\Delta m^2_{31} = m^2_3 - m^2_2 = (m_{\nu_\tau} + m_{\nu_\mu})\sqrt{(m_{\nu_\tau} - m_{\nu_\mu})^2 + (2m_{\nu_\mu\nu_\tau})^2}, \tag{23}$$

if $2m_{\nu_\mu\nu_\tau} \gg |m_{\nu_\tau} - m_{\nu_\mu}|$ then

$$\Delta m^2_{31} = (m_{\nu_\tau} + m_{\nu_\mu})2m_{\nu_\mu\nu_\tau}\left[1 + \frac{(m_{\nu_\tau} - m_{\nu_\mu})^2}{2(2m_{\nu_\mu\nu_\tau})^2}\right], \tag{23$'$}$$

and if $2m_{\nu_\mu\nu_\tau} \ll |m_{\nu_\tau} - m_{\nu_\mu}|$ then

$$\Delta m^2_{31} = (m^2_{\nu_\tau} - m^2_{\nu_\mu})\left[1 + \frac{(2m_{\nu_\mu\nu_\tau})^2}{2(m_{\nu_\tau} - m_{\nu_mu})^2}\right]. \tag{23$''$}$$

In the general case from the neutrino oscillation experiments, we can obtain six values — $\sin^2 2\theta$, $\sin^2 2\beta$, $\sin^2 2\gamma$, Δm_{21}, Δm_{32}, Δm_{31}, which can be used for determination of six parameters m_{ν_e}, $m_{\nu_\mu}, m_{\nu_\tau}$, $m_{\nu_e \nu_\mu}$, $m_{\nu_\mu \nu_\tau}$, $m_{\nu_e \nu_\tau}$, using 3 equations (20) for $\sin^2 2\ldots$ and 3 equations (21)–(23). Unfortunately, these equation are transcendental ones and they can be solved only numerically.

In the simplest case

$$\sin^2 2\theta \cong \sin^2 2\beta \cong \sin^2 2\gamma \cong 1, \tag{24}$$

$$m_{\nu_e} \cong m_{\nu_\mu} \cong m_{\nu_\tau} = m_\nu \tag{25}$$

and we get

$$\begin{gathered} \Delta m^2_{21} \cong 4 m_{\nu_e \nu_\mu} \cdot m_\nu \cong 2.5 \cdot 10^{-3} \text{ eV}^2, \\ \Delta m^2_{31} \cong 4 m_{\nu_e \nu_\tau} \cdot m_\nu \cong \text{remains unknown for the present}, \\ \Delta m^2_{32} \cong 4 m_{\nu_e \nu_\mu} \cdot m_\nu \cong 6.9 \cdot 10^{-5} \text{ eV}^2, \end{gathered} \tag{26}$$

and there is no possibility to obtain values of masses of physical neutrinos.

*1.3. The case of neutrino mixings without mass shell changing.*Above we considered the case when virtual neutrino transitions take place with change of neutrino masses. Another case is also possible, when ν_e neutrino transits into ν_μ neutrino without changing mass, i. e., $m^*_{\nu_\mu} = m_{\nu_e}$ then

$$\mathrm{tg}\, 2\theta = \infty, \tag{27}$$

$\theta = \pi/4$, and

$$\sin^2 2\theta = 1. \tag{28}$$

In this case the probability of the $\nu_e \to \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \to \nu_\mu, t) = \sin^2 \left[\pi t \frac{4 m^2_{\nu_e \nu_\mu}}{2 p_a} \right]. \tag{29}$$

Expression for length of oscillations in this case has the following form:

$$L_o = 2\pi \frac{2p}{(2 m_{\nu_e \nu_\mu})^2}.$$

In order to make these virtual oscillations real, their participation in quasi-elastic interactions is necessary for their transitions to their own mass shells [9].

The matrix, analogous to Kabibbo–Kobayashi–Maskawa in this case, is a trivial one and it has the following form:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} c_\beta & 0 & s_\beta \exp(-i\delta) \\ 0 & 1 & 0 \\ -s_\beta \exp(i\delta) & 0 & c_\beta \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{30}$$

$$
\begin{aligned}
c_{e\mu} = \cos\theta = \frac{1}{\sqrt{2}}, &\quad s_{e\mu} = \sin\theta = \frac{1}{\sqrt{2}};\\
c_{e\tau} = \cos\beta = \frac{1}{\sqrt{2}}, &\quad s_{e\tau} = \sin\beta = \frac{1}{\sqrt{2}};\\
c_{\mu\tau} = \cos\gamma = \frac{1}{\sqrt{2}}, &\quad s_{\mu\tau} = \sin\gamma = \frac{1}{\sqrt{2}};\\
\exp(i\delta) &= 1.
\end{aligned} \tag{31}
$$

In our approximation the value of δ can be considered to be equal to zero.

In case of

$$\sin^2 2\theta = \sin^2 2\beta = \sin^2 2\gamma = 1, \tag{32}$$

we have

$$
\begin{aligned}
\Delta m^2_{21} &= (2m_{\nu_e\nu_\mu})^2,\\
\Delta m^2_{31} &= (2m_{\nu_e\nu_\tau})^2,\\
\Delta m^2_{32} &= (2m_{\nu_e\nu_\mu})^2,
\end{aligned} \tag{33}
$$

and we can obtain values of nondiagonal mass terms (widths of neutrino transitions) but there is no possibility of obtaining values of masses of physical neutrinos.

It is necessary to remark that in physics all the processes are realized through dynamics. Unfortunately, in this mass mixings scheme the dynamics is absent. Probably, this is an indication of the fact that these schemes are incomplete ones, i.e., these schemes demand a physical substantiation (see section 1.2).

Obviously, these schemes will work only if neutrino oscillations take place in reality (it is clear that there also can be neutrino mixings in absence of neutrino oscillations).

2. The Scheme of Neutrino Charge Mixings (Oscillations). The third scheme (type) of mixing or transition of neutrinos can be realized by mixings of the neutrino fields by analogy with the vector dominance model ($\gamma - \rho^o$ and $Z^o - \gamma$ mixings) the way it takes place in the particle physics. Then, in the case of two neutrinos, we have

$$
\begin{aligned}
\nu_1 &= \cos\,\theta\nu_e - \sin\,\theta\nu_\mu,\\
\nu_2 &= \sin\,\theta\nu_e + \cos\,\theta\nu_\mu.
\end{aligned} \tag{34}
$$

In the case of three neutrinos we can also choose parameterization of the mixing matrix V in the form proposed by Maiani [12]:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} c_\beta & 0 & s_\beta \\ 0 & 1 & 0 \\ -s_\beta & 0 & c_\beta \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{35}$$

$$
\begin{aligned}
c_{e\mu} &= \cos\theta \quad s_{e\mu} = \sin\theta, \quad c_{e\mu}^2 + s_{e\mu}^2 = 1; \\
c_{e\tau} &= \cos\beta, \quad s_{e\tau} = \sin\beta, \quad c_{e\tau}^2 + s_{e\tau}^2 = 1; \\
c_{\mu\tau} &= \cos\gamma, \quad s_{\mu\tau} = \sin\gamma, \quad c_{\mu\tau}^2 + s_{\mu\tau}^2 = 1;
\end{aligned} \tag{36}
$$

The charged current in the standard model of weak interactions for two lepton families has the following form:

$$
j^\alpha = \left(\begin{array}{c} \bar{e}\bar{\mu} \end{array} \right)_L \gamma^\alpha V \left(\begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right)_L,
$$

$$
V = \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right), \tag{37}
$$

and then the interaction Lagrangian is

$$
\mathcal{L} = \frac{g}{\sqrt{2}} j^\alpha W_\alpha^+ + \text{h.c.} \tag{38}
$$

and

$$
\begin{array}{c} \nu_e = \cos\theta\nu_1 + \sin\theta\nu_2 \\ \nu_\mu = -\sin\theta\nu_1 + \cos\theta\nu_2. \end{array} \tag{39}
$$

Then, taking into account that the charges of ν_1, ν_2 neutrinos are g_1, g_2 we have

$$
g\cos\theta = g_1, \quad g\sin\theta = g_2, \tag{40}
$$

i.e.

$$
\cos\theta = \frac{g_1}{g}, \quad \sin\theta = \frac{g_2}{g}. \tag{41}
$$

Since $\sin^2\theta + \cos^2\theta = 1$, then

$$
g = \sqrt{g_1^2 + g_2^2}
$$

and

$$
\cos\theta = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad \sin\theta = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}. \tag{42}
$$

Since we suppose that $g_1 \cong g_2 \cong \frac{g}{\sqrt{2}}$, then

$$
\cos\theta \cong \sin\theta \cong \frac{1}{\sqrt{2}}. \tag{43}
$$

It is not difficult to come to consideration of the case of three neutrino types ν_e, ν_μ, ν_τ. Since the weak couple constants $g_{\nu_e}, g_{\nu_\mu}, g_{\nu_\tau}$ of ν_e, ν_μ, ν_τ neutrinos are approximately equal in reality, i.e., $g_{\nu_e} \simeq g_{\nu_\mu} \simeq g_{\nu_\tau}$ then the angle mixings are nearly maximal:

$$\begin{aligned}\cos\theta = \cos\theta_{\nu_e\nu_\mu} \cong \sin\theta_{\nu_e\nu_\mu} \cong \frac{1}{\sqrt{2}},\\ \cos\beta = \cos\theta_{\nu_e\nu_\tau} \cong \sin\theta_{\nu_e\nu_\tau} \cong \frac{1}{\sqrt{2}},\\ \cos\gamma = \cos\theta_{\nu_\mu\nu_\tau} \cong \sin\theta_{\nu_\mu\nu_\tau} \cong \frac{1}{\sqrt{2}}.\end{aligned} \tag{44}$$

As it is stressed above in the case of mass mixings scheme, we have no dynamical substantiation in contrast to the case of charge mixings scheme, but these schemes may be jointed if neutrino masses have the following form:

$$m_{\nu_i} = g_i v, \quad i = e, \mu, \tau, \tag{45}$$

where v is constant, as in the Higgs mechanism [13]. And then the problem of dynamical substantiation in this scheme is solved.

CONCLUSION

Unfortunately, we do not know if there are neutrino oscillations or only neutrino mixings without oscillations. Therefore, it is necessary to consider all the realistic schemes of neutrino mixings and oscillations. In this work three schemes of neutrino mixings (oscillations) together with their mixing matrices (analogous to Kabibbo–Kobayashi–Maskawa matrices) were considered. In these schemes neutrino transitions are virtual if neutrino masses are different. Two of them belong to the so-called mass mixing schemes (mixing parameters are expressed by elements of mass matrices) and the third scheme belongs to the charge mixing one (mixing parameters are expressed through charges). For the first scheme, the equations for determination of all the elements of mass matrix (neutrino masses and transition widths) by using experimental data were given. In the second and third ones the mixing angles are equal or close to the maximal angles ($\pi/4$). It is obvious that the experiment must get an answer to the following question: Which of these schemes is realized indeed?

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